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# An Improved Equation-Free Method for Gyrokinetic Profile Evolution of Tokamak Plasmas


B. Sturdevant[1*], S. E. Parker[2], C. S. Chang[1], and R. Hager[1]

[1]Princeton Plasma Physics Laboratory, [2]University of Colorado at Boulder

*email : bsturdev@pppl.gov



## ABSTRACT

- A multiscale method has been developed for accelerating kinetic plasma simulations based on equation-free projective integration [1,2].
- Scheme is implemented in XGCa [3] and demonstrated for gyrokinetic neoclassical ion heat transport problem.
- Profile evolution is accurately reproduced while achieving a 4x speed-up compared to brute force time stepping.


## BACKGROUND

- Cross-field transport of particles, energy, and momentum due to turbulence and collisional neoclassical orbital dynamics determines confinement level in magnetic confinement fusion devices.
- Transport processes are multiscale, presenting enormous challenges for direct numerical simulations based on first-principles.
- Previous methods have generally focused on core transport and make use of 1D transport equations.
- Equation-free projective integration can provide a flexible framework for extending beyond 1D core transport.

## NUMERICAL APPROACH

**Application to total-*f* gyrokinetic model**

A set of low-order 2D fluid moments is selected as a coarse state variable

$$M_{p,q}(\mathbf{X}) = \int v_\parallel^p \mu^q f(\mathbf{X}, v_\parallel, \mu) B d\mu dv_\parallel$$

A four-step process is followed to advance from $t$ to $t + \Delta t$:

1. Simulate $f$ forward in time to $t + \Delta t_e$ allowing transients to settle
2. Simulate $f$ an additional $\Delta t_l$ while collecting snapshot data of low-order fluid moments
3. Extrapolate low-order fluid moments to $t + \Delta t$ based on snapshot data
4. Apply a "lifting" operator to re-initialize kinetic simulations for the next cycle

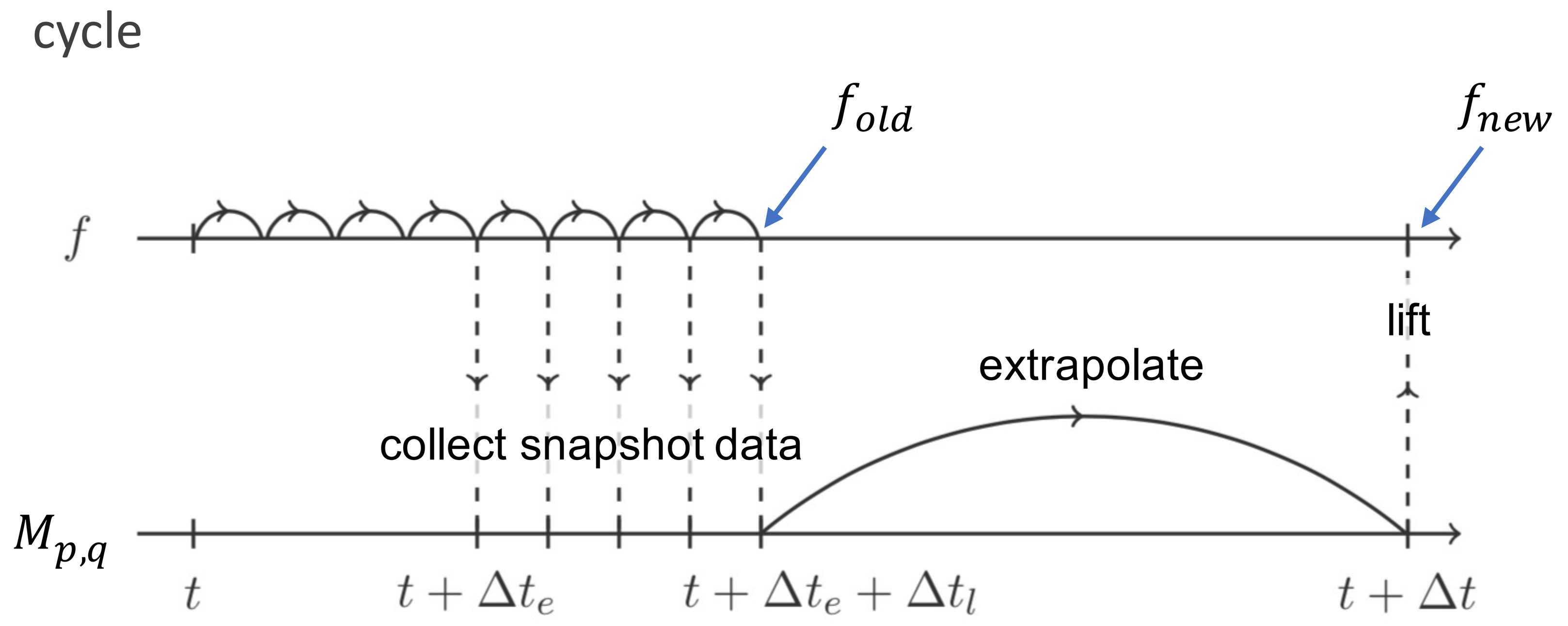
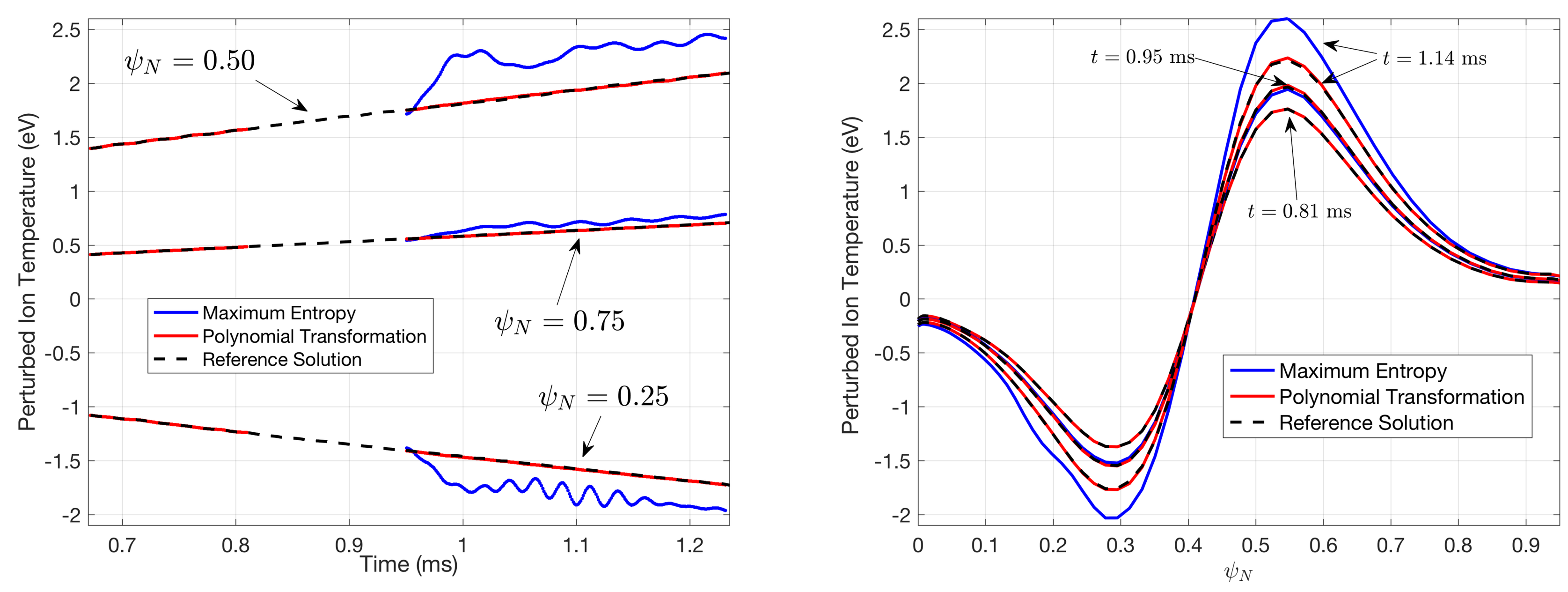



*Illustration of numerical approach*

**Improved Lifting Method**

In lifting, we construct $f_{new}$ whose moments match $M_{p,q}$ at $t + \Delta t$. We attempt to reuse kinetic information from the previous time steps by applying an appropriate scaling to $f_{old}$ at $t + \Delta t_e + \Delta t_l$. This is shown to eliminate spurious transients that were present in previously considered maximum entropy based lifting methods.

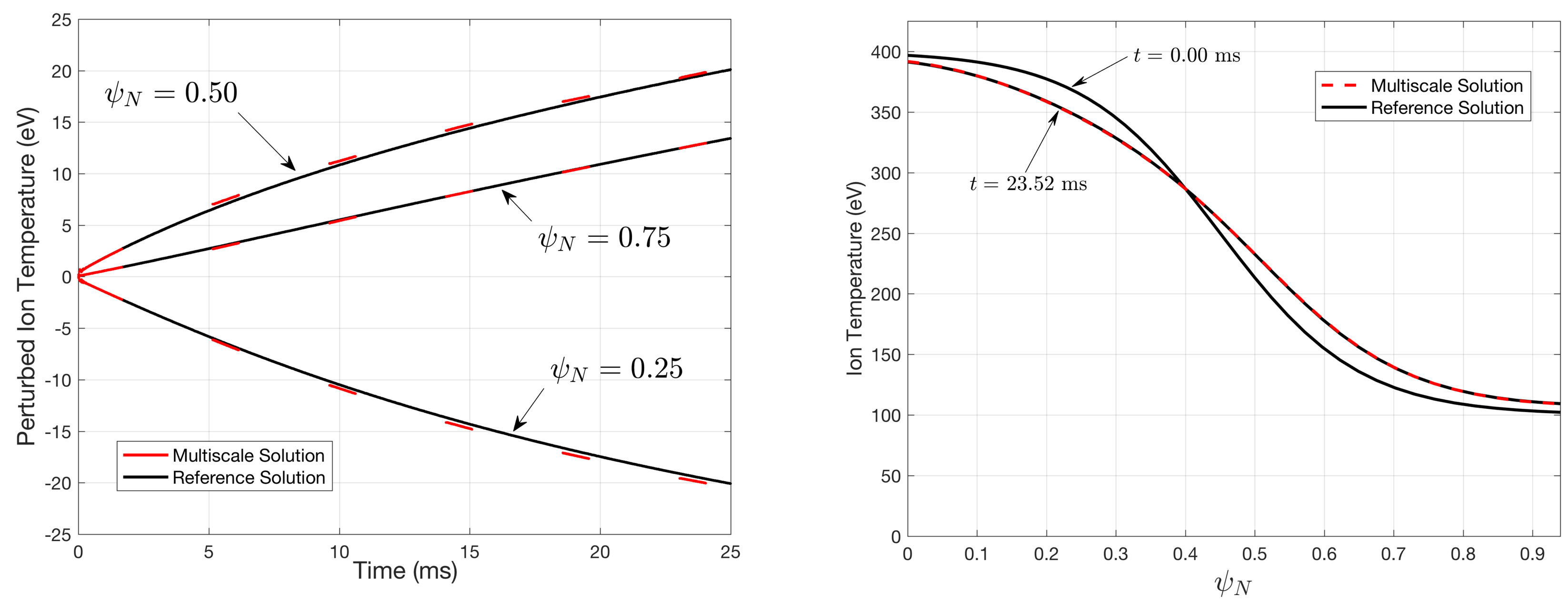



*Evolution of ion temperature after one projective time step is taken. Spurious transient behavior is found for maximum entropy lifting but is eliminated with the improved method. Reprinted from [2] with permission from AIP Publishing.*

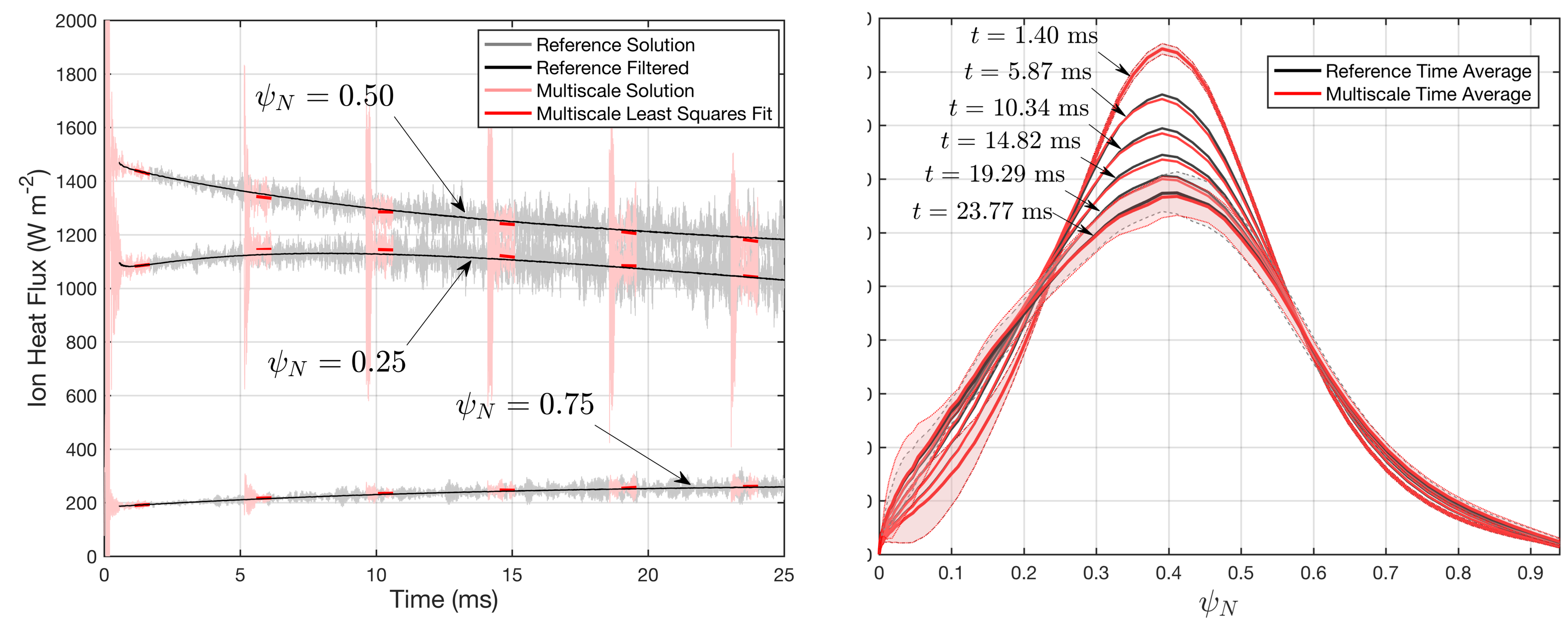
## SIMULATION RESULTS

**Long timescale simulations of neoclassical ion heat transport**

The equation-free method was demonstrated to accurately predict long timescale ion temperature profile relaxation and evolution of the ion heat flux over a 25 ms time interval. A speed-up factor of 4.4x is achieved using the equation-free approach compared to brute force time stepping.




*Time history of ion temperature profile (left) and initial and final temperature profiles (right). Simulation results using the equation-free method are compared to a reference solution using direct numerical simulation. Reprinted from [2] with permission from AIP Publishing.*




*Time history of ion heat flux (left) and snapshots of ion heat flux profiles (right). Simulation results using the equation-free method are compared to a reference solution using direct numerical simulation. Reprinted from [2] with permission from AIP Publishing.*

## CONCLUSION

- Equation-free projective integration has been applied to accelerate kinetic plasma simulations in XGCa
- It has been demonstrated to accurately predict profile evolution in a neoclassical ion heat transport problem
- Key to this work was the development of an improved lifting operator to smoothly re-initialize simulations after each projective integration cycle.
- Work on applying this method to ITG turbulence problems with heat sources is underway.

## ACKNOWLEDGEMENTS / REFERENCES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