Influence of radial electric field on stochastic diffusion in Wendelstein-type stellarators

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in the presence of a radial electric field**

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4. Summary

1. Motivation

Delayed collisionless losses of fast ions

• Confinement of fast ions in optimized Wendelstein-line stellarators is improved due to high β: diamagnetic drift tends to cancel ∇B drift so that average radial magnetic drift vanishes, minimizing prompt losses

• **However**, high β does not prevent delayed losses of 3.5 MeV alpha particles, as shown numerically in [W. Lotz, P. Merkel, J. Nuhrenberg, E. Strumberger, *Plasma Phys. Contr. Fusion* **34** (1992) 1037].

• Suggested loss mechanism: stochastic diffusion due to repeated particle trapping and de-trapping in local magnetic wells: [C.D. Beidler, Ya.I. Kolesnichenko, V.S. Marchenko, I.N. Sidorenko, H. Wobig, *Phys. Plasmas* **8** (2001) 2731].

• Trapping/de-trapping probability and nonadiabatic jumps of $J_{||}$ consistently described in theory of stochastic diffusion developed in [A.V. Tykhyy, *Ukr. J. Phys.* **63**(6) (2018) 495].

Does radial electric field affect stochastic diffusion?

- Radial electric field (E_r) is always present in stellarators.
- E_r creates additional ***E*** x ***B*** drift. $E_r < 0$ **adds** to diamagnetic drift, $E_r > 0$ **reduces** it.
- E_r affects orbits of particles trapped in local magnetic wells by modifying the contours of the longitudinal adiabatic invariant of bounce motion, $J_{||}$:
  $E_r < 0$ improves their confinement, $E_r > 0$ degrades it [1, 2].
  [1] Ya.I. Kolesnichenko, V.V. Lutsenko, A.V. Tykhyy, A. Weller, A. Werner, H. Wobig, J. Geiger, *Phys. Plasmas* **13** (2006) 072504
  [2] J.M. Faustin, W.A. Cooper, J.P. Graves, D. Pfeerle, J. Geiger, *Nucl. Fusion* **56** (2016) 092006
- Effect of E_r on stochastic diffusion has not yet been studied, motivating the present work.

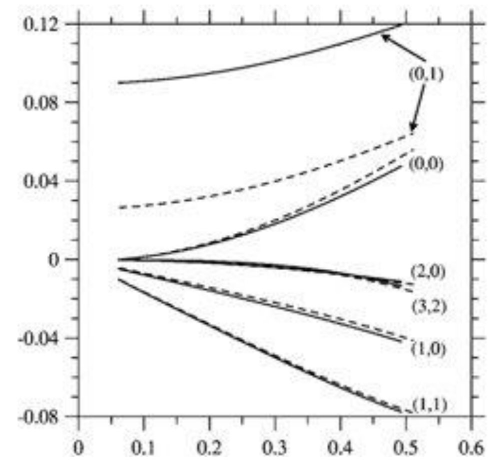
2. Theory

Magnetic field in Wendelstein-line stellarators

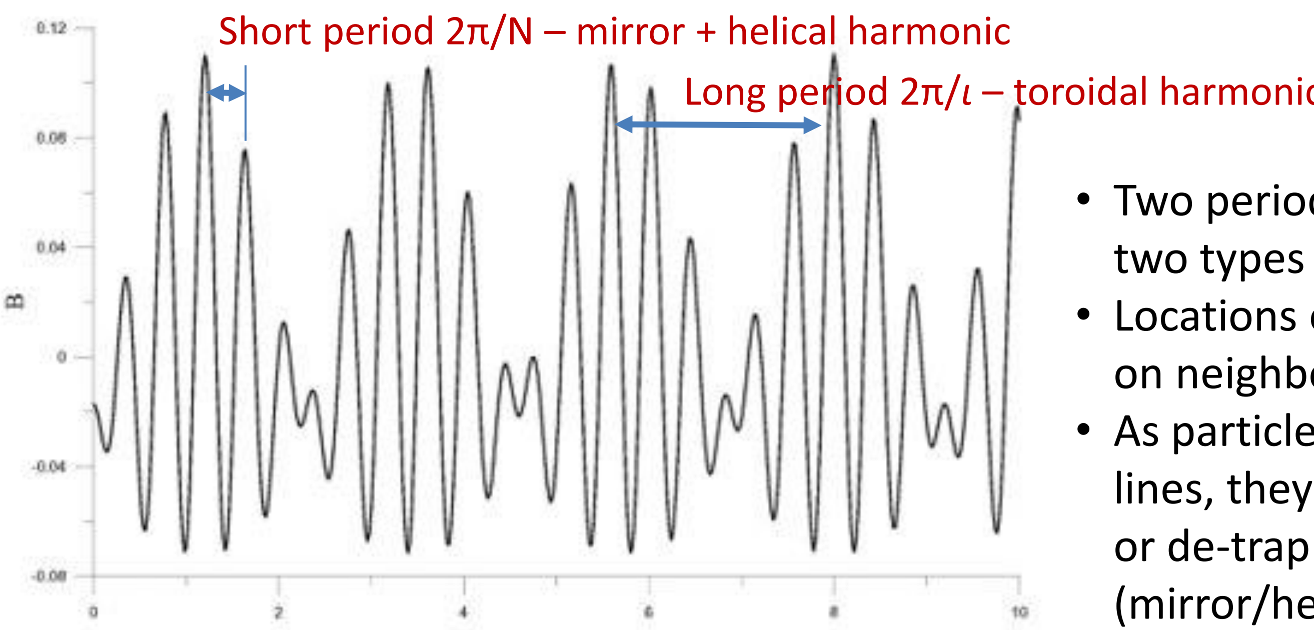
- Magnetic field strength $|B|$ in Wendelstein-line stellarators has multiple harmonics, with mirror, helical, toroidal and diamagnetic being the largest by magnitude
- Model magnetic field includes only these four harmonics:

$$B/\bar{B} = 1 + \epsilon_0 - \epsilon_t \cos\vartheta + \epsilon_m \cos N\varphi - \epsilon_h \cos(N\varphi - \vartheta)$$

$$B/\bar{B} = 1 + \epsilon_0 - \epsilon_t \cos\vartheta + \epsilon_{hm} \cos N(\varphi - \varphi_t)$$



- Number of magnetic field periods $N >> 1$, rotational transform $\iota \sim 1$
- Resulting $|B|$ variation along a field line has two periods:



- Two periods of $|B|$ variation create two types of trapped particles
- Locations of $|B|$ wells not aligned on neighboring field lines due to ι
- As particles drift between field lines, they may become trapped in or de-trap from short-period (mirror/helical) $|B|$ wells

Trapped particles in stellarators

To characterize the type of particle orbits, we introduce the trapping parameter κ and particle pitch parameter α:

$$\kappa^2 = \frac{\alpha - \epsilon_E - \epsilon_0 + \epsilon_t \cos\vartheta + \epsilon_{hm}}{2\epsilon_{hm}}$$

$$\frac{m v_{\parallel}^2}{2} = 2\mu\bar{B}\epsilon_{hm}\left[\kappa^2 - \cos^2\frac{N(\varphi - \varphi_t)}{2}\right]$$

$$\alpha = \frac{W}{\mu\bar{B}} - 1 \qquad \epsilon_E = -\frac{e}{\mu\bar{B}}\int^r E_r(r')dr'$$

$\kappa^2 < 1$ - particle trapped in mirror/helical well, "locally trapped"

$\kappa^2 > 1$ - "locally passing" but may be trapped in toroidal well

κ changes as particle drifts between field lines

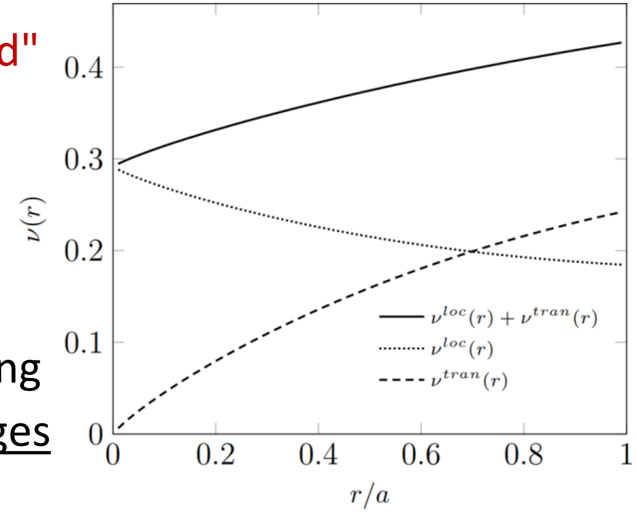
- Localized particles always have $\kappa^2 < 1$
- Passing particles always have $\kappa^2 > 1$
- Transitioning particles switch locally trapped ↔ locally passing

α for localized, transitioning, and passing particles lies in ranges (in narrow orbit approximation):

$$\epsilon_0 - \epsilon_m + \epsilon_h - \epsilon_t < (\alpha^{loc} - \epsilon_E) < \epsilon_0 + \epsilon_m - \epsilon_h - \epsilon_t$$

$$\epsilon_0 + \epsilon_m - \epsilon_h - \epsilon_t < (\alpha^{tran} - \epsilon_E) < \epsilon_0 + \epsilon_m + \epsilon_h + \epsilon_t$$

$$\epsilon_0 + \epsilon_m + \epsilon_h + \epsilon_t < (\alpha^{pass} - \epsilon_E)$$



Radial dependence of fraction of trapped alpha particles born in a Helias reactor. Fraction of born trapped particles is **30-40%.**

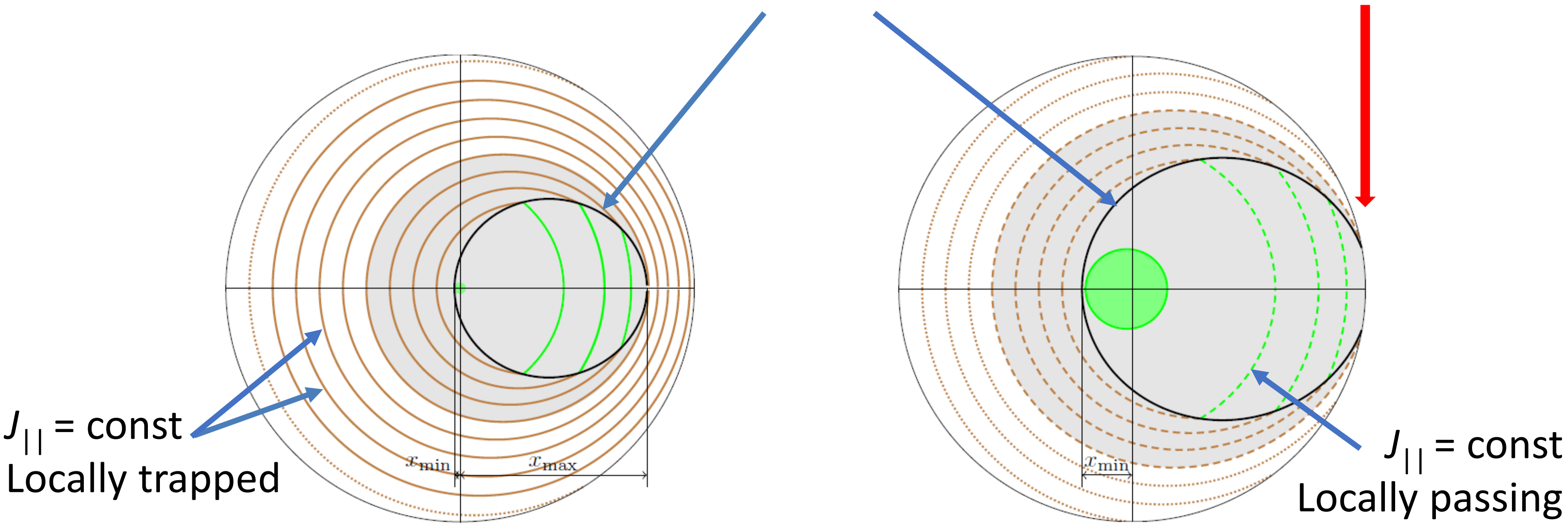
- Stochastic diffusion affects transitioning particles
- Transitioning particles constitute a considerable amount of fast ion population
- Fraction of transitioning particles depends on radial profiles of plasma parameters

Stochastic diffusion of transitioning particles

- Drift causes transitions between locally trapped and locally passing states
- Bounce period becomes very large close to transition point $v_{||} = 0$, **therefore**
- Longitudinal adiabatic invariant $J_{||}$ of bounce motion not conserved during transition
- Random jumps of $J_{||}$ lead to diffusion of transition point along the **separatrix, $\kappa^2 = 1$,** where particles transition between locally trapped and locally passing states

[C.D. Beidler, Ya.I. Kolesnichenko, V.S. Marchenko, I.N. Sidorenko, H. Wobig *Phys. Plasmas* **8** (2001) 2731]

- Transitioning particles can be lost **IF the separatrix crosses plasma boundary:**



Bounce-averaged drift orbits (contours of $J_{||}$) for locally trapped (brown) and locally passing (green) 3.5 MeV alpha particles in the intermediate Helias reactor (option "A") in the poloidal plane in flux coordinates. Left: $\alpha = 0.12$, right: $\alpha = 0.15$

Stochastic diffusion coefficient

$\delta J_{+\to t}$, the $J_{||}$ jump, is given by separatrix transition theory for a shifted non-linear pendulum:

$$2\pi\delta J_{+\to t} = \frac{\Theta_t}{\Theta_+}(b'_+\Theta_t - b'_t\Theta_+)\xi + a\Theta_t \ln \frac{\Gamma(\xi)\Gamma(\xi + \frac{\Theta_+}{\Theta_t})\Gamma(\frac{\Theta_t}{\Theta_+}(1-\xi))}{(2\pi)^{\frac{3}{2}}\left|\xi - 1 + \frac{\Theta_+}{\Theta_t}\right|}$$

where Θ_i are phase volume change velocities for locally trapped (t) and locally passing (±) states, a and b_i are coefficients derived from an expansion of $J_{||}$ near the separatrix, and $0 < \xi < 1$ is a phase-dependent transition parameter, assumed uniformly distributed.

Transition probability P does not depend on the phase parameter:

$$P = \frac{\Theta_t}{\Theta_+} = \frac{2}{\pi\iota r}\frac{\bar{\rho}_l R}{a^2}\frac{\epsilon_h\epsilon_m\left(\epsilon'_0 + \epsilon'_E - \epsilon'_t\cos\vartheta\right) - \epsilon_t\left(\epsilon^2_{hm}\right)'/2}{\sqrt{\epsilon_{hm}}\left(\epsilon_h\epsilon_m + \epsilon_t\epsilon_{hm}\right)}$$

The diffusion coefficient is calculated as

$$D = \left(\frac{dr}{dJ_{||}}\right)^2 \left\langle\left(\delta J_{+\to t} - \langle\delta J_{+\to t}\rangle\right)^2\right\rangle \frac{2P}{\tau}$$

where τ is the mean time between transitions.

Tykhyy, *Ukr. J. Phys.* **63**(6) (2018) 495

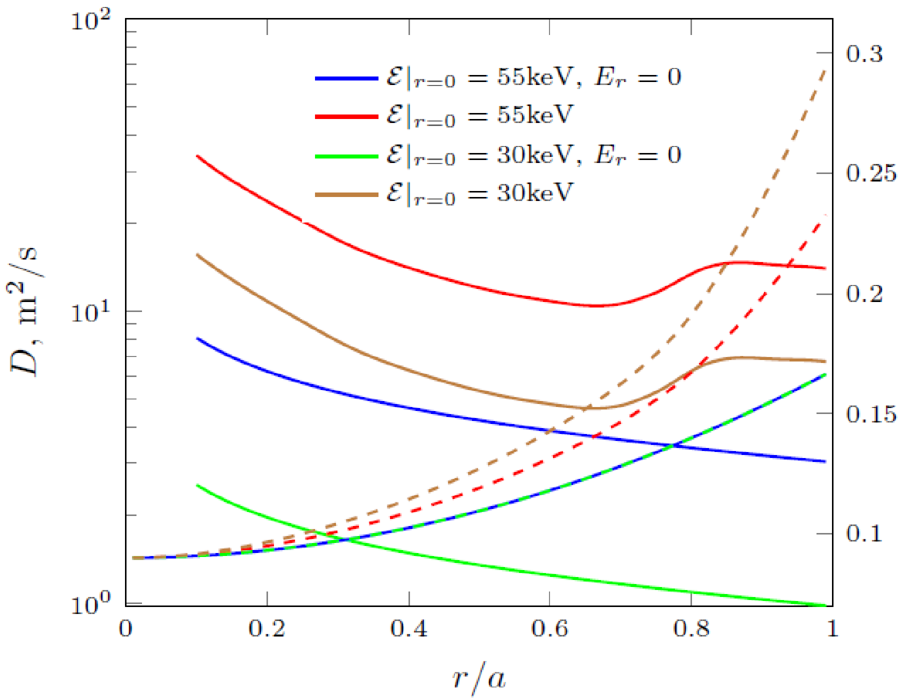
3. Examples: NBI ions in W7-X, fusion alphas in Helias

Example devices and E_r

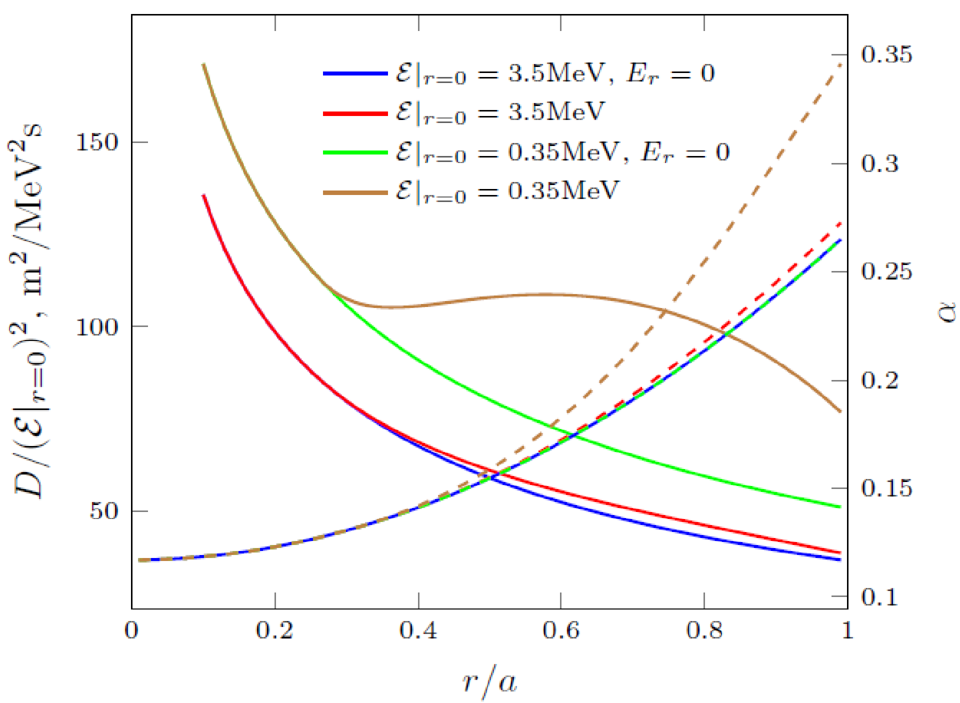
- <u>"intermediate" Helias, option A</u>
  R = 14m, A = 10.5, $n_e = 2\cdot10^{20}$ m^{-3}, $T_e \approx T_i$ = 10 keV
  max E_r ~ -25 kV/m
  potential difference center to edge ~ 10 kV
  Warmer et al. *Plasma Phys. Control. Fusion* **58**(7) 074006 (2016)

- <u>Wendelstein 7-X</u>
  R = 5.5m, A = 10, B = 3 T, $n_e = 7\cdot10^{19}$ m^{-3}, $T_e \approx T_i$ = 3 keV
  max E_r ~ -10 kV/m
  potential difference center to edge ~ 1 kV
  $E_r > 0$ (electron root) also observed
  Pablant et al. *Phys. Plasmas* **25** 022508 (2018)

Effect of E_r on stochastic diffusion coefficient



NBI ions in W7-X

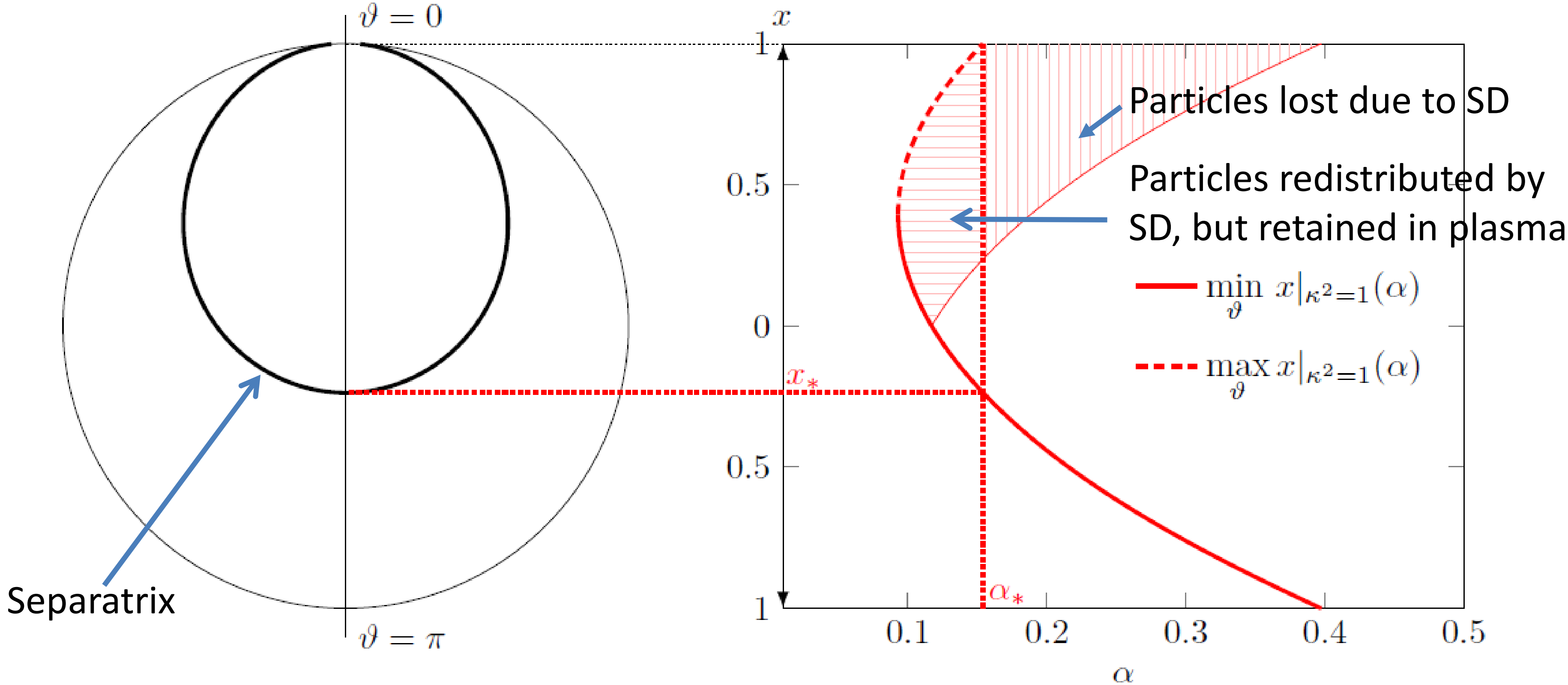


Fusion alphas in Helias

- SD coefficient **strongly depends on particle energy (~$\mathcal{E}^2$)**
- $E_r < 0$ **increases** diffusion coefficient up to 2x
- Is $E_r < 0$ bad for confinement of transitioning particles?

Separatrix diagrams

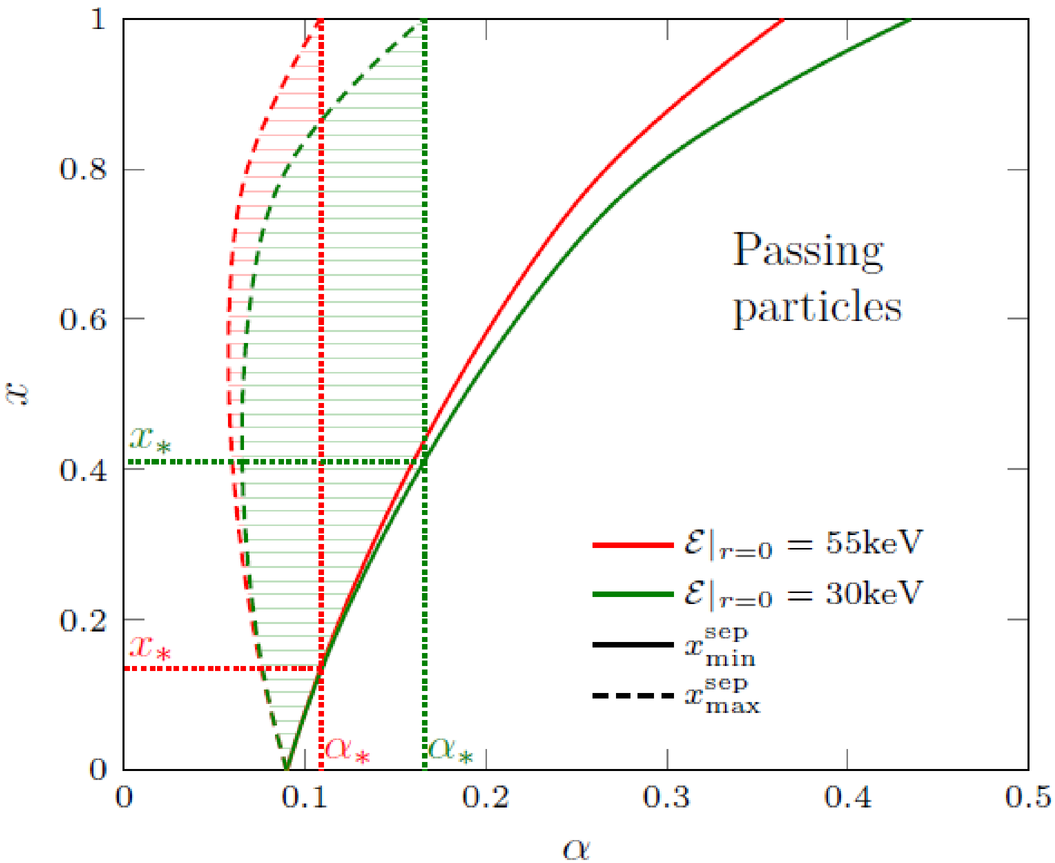
Separatrix shape depends on particle pitch angle. Diagram showing maximum and minimum radius of the separatrix depending on pitch angle helps visualize SD losses.



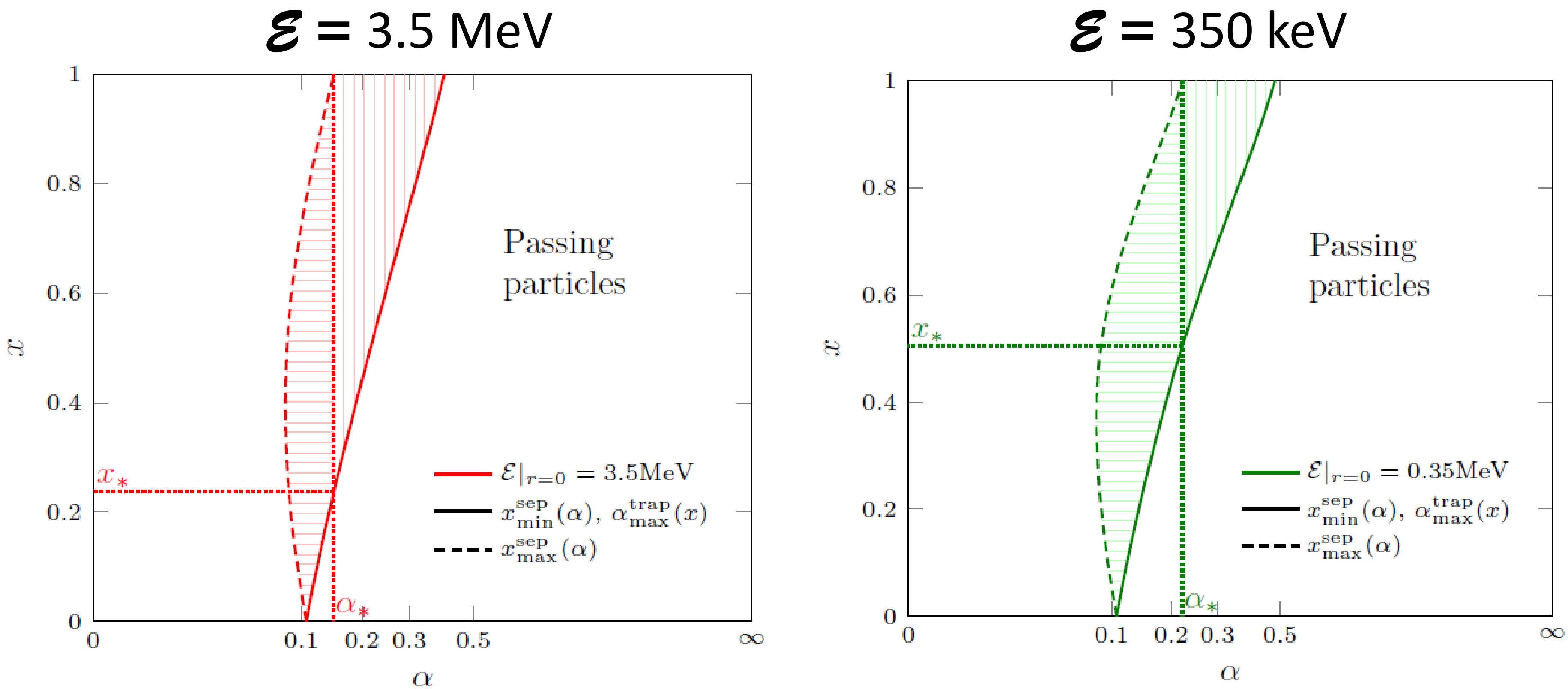
Contours of $\kappa^2 = 1$ (the separatrix) for 3.5 MeV particles in the Helias reactor. Left: poloidal cross section, $\kappa^2(x, \vartheta) = 1$ for $\alpha = \alpha_*$. Right: the minimum and maximum values of $(r/a)|_{\kappa = 1}$ for different α. Horizontal hatch: 3.5 MeV particles that are redistributed by SD, but not lost. Vertical hatch: particles that are lost due to SD.

NBI ions in Wendelstein 7-X

- **For $E_r = 0$, all NBI ions can be lost to SD** but diffusion time τ_D ~ 25ms is close to or larger than slowing down time
- Relatively low plasma β makes it possible for most transitioning NBI ions with $\mathcal{E}$ = 55 keV to be lost even in presence of $E_r < 0$
- Larger effect of E_r at $\mathcal{E}$ = 30 keV significantly improves confinement of transitioning ions



Alphas in the Helias reactor



- E_r is too weak to affect 3.5 MeV particles; about 50% transitioning particles are lost
- Characteristic diffusion time τ_D ~ 2.5ms << slowing down time (85ms in "Option A")
- Enough transitioning particles are retained and slowed down
- Slowed-down transitioning particles have better confinement and much larger τ_D ~ 150ms (180ms without the effect of E_r on diffusion coefficient)

Summary and conclusions

- Transitioning fast ions are subject to stochastic diffusion (SD) in Wendelstein-line stellarators. The fraction of transitioning ions is considerable.
- SD leads to loss / redistribution of fast ions when the separatrix ($\kappa^2 = 1$) intersects / stays within the plasma boundary.
- Because the separatrix location depends on particle pitch, a part of 3.5 MeV alpha particles in a Helias reactor is lost because of SD, another part is confined.
- Radial electric field (E_r) affects both SD coefficient and separatrix location. $E_r < 0$ increases SD coefficient, but shifts the separatrix in such a way that fast ion confinement improves. $E_r > 0$ degrades fast ion confinement.
- 3.5 MeV alpha particles in HELIAS reactor are weakly sensitive to the electric field, but partly thermalized alphas are affected. Due to this, SD may contribute to ash removal when $E_r > 0$.
- Confinement of 55 keV NBI ions in W7-X high-mirror configuration is improved significantly due to $E_r < 0$.
- In general, E_r can be used for both loss mitigation and energy deposition profile optimization.

Thank you