Interplay between particle transport, zonal flows and zonal density in Dissipative Trapped-Electron Mode turbulence

M. Leconte and Lei Qi

Korea Institute of Fusion Energy (KFE), Korea

Acknowledgts: Raghvendra Singh, P.H. Diamond, I. Dodin, Hongxuan Zhu, T. Kobayashi

TH/P7-23: 28th IAEA FEC 2020 (e-conference), May 10-15, 2021

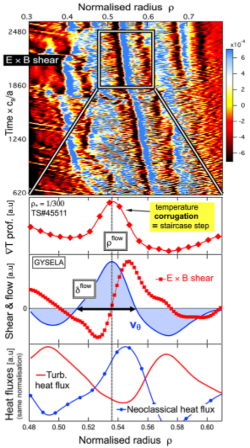
KFE 한국핵융합에너지연구원 KOREA INSTITUTE OF FUSION ENERGY

Outline

1. Introduction
2. Motivation
   - Turbulent flux and transport crossphase
   - Basic Mechanism of crossphase effects on the particle flux
3. Our Results: Analytical theory
   - Wave kinetic approach including crossphase
   - Schematic derivation of the model
   - DTEM wave-kinetic model
4. Our Results: Simulations of the Hasegawa-Wakatani model with BOUT++ (or DTEM for $f_t \to 1$)
   - zonal flows & zonal density corrugations
   - Transport crossphase radial modulation
5. Summary and conclusions

General Intro

- Transitions to enhanced confinement regimes $\rightarrow$ key for future devices (ITER)
- Recently, $E \times B$ **staircase** observed in gyrokinetic simulations of ITG (GYSELA).
- Associated with **profile corrugations** in T_i and other quantities.
- Are profile corrugations universal, independent of the type of turbulence?
- What is the role of the **transport crossphase**?



ExB staircase [**Dif-Pradalier '15]**

General Intro (ct'd)

- H-mode regime well studied both experimentally and theoretically $\rightarrow$ L-H transition explained via 'flow-shear paradigm' **[Biglari Diamond Terry '90], [Moyer '95], [Diamond Itoh^2 Hahm '05]**: flow shear reduces transport by **suppressing** turbulence intensity.
- Crossphase may play a role in the L-H transition [**Kobayashi '17**].
- Zonal density effect on turbulent transport was observed in gyrokinetic simulations of Trapped-electron-Mode (TEM) turbulence **[Lang '08]** and in fluid simulations of resistive drift-wave turbulence **[An '17]**.
- Here we identify a new mechanism for **self-organization of crossphase and zonal density corrugations**.
- In this work, we focus on the **particle transport** channel

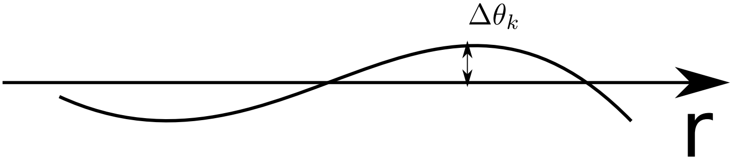
Turbulent flux and crossphase

- the turbulent particle flux can be written:

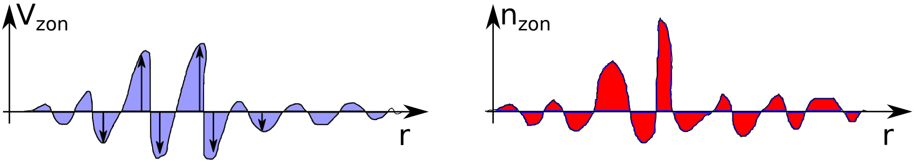
$$\Gamma_{turb} = \sqrt{\langle n^2 \rangle}\sqrt{\langle \phi^2 \rangle}\sin\theta_{n\phi}$$

- with $\theta_{n\phi}$: transport crossphase
- the particle flux can be suppressed by:
  - suppressing the turbulence amplitudes $\sqrt{\langle n^2 \rangle}, \sqrt{\langle \phi^2 \rangle} \searrow$ (eddy-shearing paradigm)
  - and/or
  - suppressing the crossphase $\theta_{n\phi} \searrow$

Basic Mechanism:
$E \times B$ nonlinearity $\rightarrow$ transport crossphase modulation $\rightarrow$ drives zonal density corrugations



modulation of **transport crossphase** θ_k between density and potential fluctuations



Cartoon of zonal flows and zonal density

Model

- fluid model for dissipative trapped-electron mode turbulence (with $\frac{L_n}{L_{Te}} \rightarrow 0$) including **zonal modes**: [1]

$$
\begin{aligned}
\frac{\partial n}{\partial t} &+ v_E \cdot \nabla n + v_* \frac{\partial \phi}{\partial y} = -\nu(\tilde{n} - \tilde{\phi}) \\
\frac{\partial}{\partial t}\left[(1 - f_t)\tilde{\phi} - \nabla_\perp^2 \phi\right] &- v_E \cdot \nabla \nabla_\perp^2 \phi + (1 - f_t) v_* \frac{\partial \phi}{\partial y} \\
&= f_t \nu (\tilde{n} - \tilde{\phi})
\end{aligned}
$$

n	:	effective density
ϕ	:	electric potential
$\nu = \nu_{ei}/f_t$	:	de-trapping rate ($\sim$ e-i collision frequency)
normalizations	:	space (ρ_s), time: (L_n/c_s)
For $f_t \rightarrow 1$	:	reduces to the Hasegawa-Wakatani model with $\frac{\nu_{ei}}{f_t}$

[1] based on **[Baver '02]**

Model (ct'd)

- This model has two nonlinearities:

polarization nonlinearity

$$v_E \cdot \nabla \nabla_\perp^2 \phi = \sum_{k=k'+k''} (k_\perp'^2 - k_\perp''^2)(\hat{\mathbf{z}} \times k') \cdot k'' \phi_{k'} \phi_{k''}$$

ExB convective nonlinearity

$$v_E \cdot \nabla n = \frac{1}{2} \sum_{k=k'+k''} (\hat{\mathbf{z}} \times k') \cdot k'' (n_{k'} \phi_{k''} - \phi_{k'} n_{k''})$$

Linear analysis

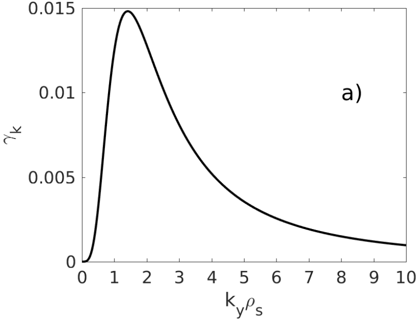


Figure: growth-rate v.s. k_y

dispersion relation

$$\omega \left[1 + \frac{k_\perp^2}{1 + i\omega/\nu} \right] - \omega_* = 0$$

DTEM frequency $\omega \ll \nu$

$$\omega_k = \omega_*/(1 + k_\perp^2)$$

growth-rate

$$\gamma_k \simeq \frac{1}{\nu} \frac{k_\perp^2 \omega_*^2}{(1 + k_\perp^2)^3}$$

with $\omega_* = k_y v_*$

Zonal flows & zonal density

- zonal flows $V_{zon}(x,t) = \langle v_E \rangle$
  & zonal density $n_{zon}(x,t) = \langle n \rangle$
  are nonlinearly driven by DTEM turbulence:

$$
\begin{aligned}
\frac{\partial V_{zon}}{\partial t} &= -\frac{\partial}{\partial x} \sum_k k_r k_\theta |\phi_k|^2 \\
\frac{\partial n_{zon}}{\partial t} &= -\frac{\partial}{\partial x} \sum_k k_\theta \frac{|n_k|}{|\phi_k|} |\phi_k|^2 \sin\theta_k
\end{aligned}
$$

V_{zon}	:	zonal flows
n_{zon}	:	zonal density
$\lvert\phi_k\rvert$, $\lvert n_k\rvert$	:	amplitudes
θ_k	:	crossphase

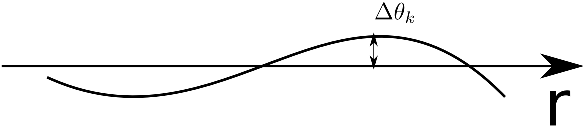
- - zonal flows driven by the polarization nonlinearity
  - zonal density driven by $E \times B$ nonlinearity

Wave kinetic approach

- The turbulent particle flux at wavenumber k can be written:

$$\Gamma_k \propto \; W_k \sin\theta_k$$

- with $W_k = (1+k^2)|\phi_k|^2/\omega_k$ the wave action density (Wigner function) for DW.
- Define $W_k = \langle W_k \rangle + \Delta W_k(r,t)$ and $\theta_k = \theta_k^0 + \Delta\theta_k(r,t)$
- $\Delta\theta_k(r,t)$: **crossphase modulation**



Wave kinetic approach (ct'd)

Wave Kinetic equation (WKE) **[Malkov, Diamond, Rosenbluth '01]**

$$\frac{\partial W_k}{\partial t} + \frac{\partial \omega}{\partial k_r}\frac{\partial W_k}{\partial r} - \frac{\partial \omega}{\partial r}\frac{\partial W_k}{\partial k_r} = \hat{\gamma}_{NL} W_k$$

- with the non-linear growth-rate operator satisfying:

$$\hat{\gamma}_{NL} W_k = 2\gamma_k W_k - \Delta\omega W_k^2$$

- We extend the wave kinetic equation to include the dependence of growth-rate on crossphase

$$\gamma_k \simeq (\theta_k^0 + \Delta\theta_k)\omega_k, \quad \text{(valid for } |\theta_k^0|, \Delta\theta_k \ll 1),$$

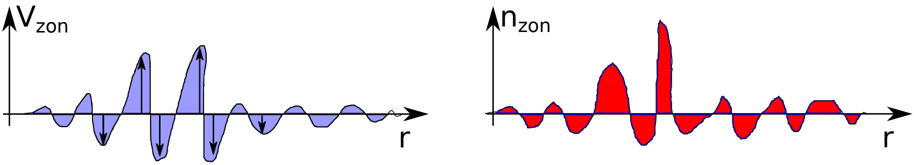
- crossphase modulation $\rightarrow$ growth-rate modulation

Wave Kinetic Equation

- and we obtain the Extended Wave-Kinetic Equation (EWKE):

$$\frac{\partial W_k}{\partial t} + v_{gr}\frac{\partial N_k}{\partial k_r} - k_\theta \nabla_r U \frac{\partial W_k}{\partial k_r} = 2\gamma_k^0 W_k - 2c_k W_k \nabla_r N$$

- 1st term on RHS: linear growth $\gamma_k^0 = \theta_k^0 \omega_k$
- 2nd term: nonlinear stabilization due to **radial modulation of the crossphase $\Delta\theta_k \propto -\nabla_r N$ [Leconte & Kobayashi '21]**
- What causes this radial modulation?
  $\hookrightarrow$ zonal density corrugations (as shown in the following)



Transport crossphase modulation

- consider the weakly-nonlinear density response **[Zhou, Zhu & Dodin '19]**:

Weakly-nonlinear Density Response

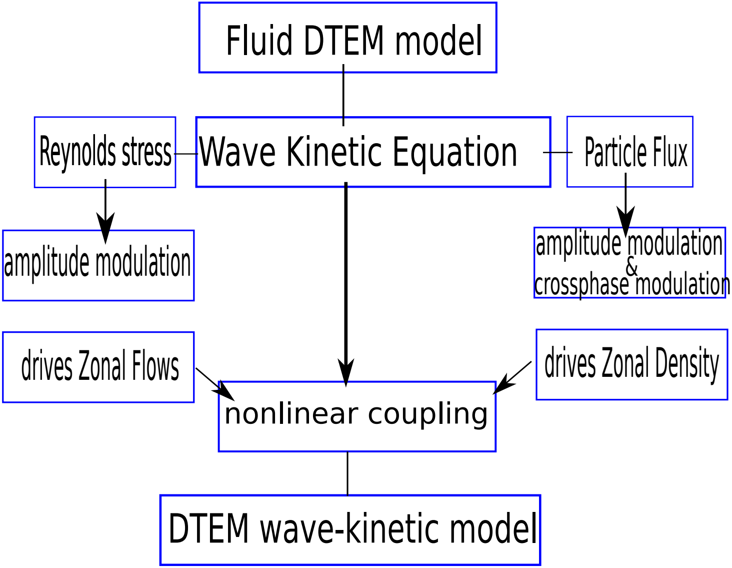
$$\frac{n_k}{\phi_k} \simeq 1 - i\theta_k$$

- $\theta_k \to \theta_k^0$ in the linear regime
- with $\theta_k^0 = \nu^{-1}[k_\perp^2\omega_*/(1+k_\perp^2)$ the linear crossphase
- Writing the nonlinear crossphase: $\theta_k = \theta_k^0 + \Delta\theta_k$, we have:

crossphase modulation similar to **[Holland & Diamond '01]** (ETG)

$$\Delta\theta_k = -\frac{\partial\theta_k^0}{\partial\omega_*} k_\theta \nabla_r N$$

Schematic derivation of the model



DTEM wave-kinetic model

- Wave-kinetic analysis leads to the DTEM wave-kinetic model :

$$\frac{\partial W_k}{\partial t} + \frac{\partial \omega_k}{\partial k_r}\nabla_r W_k - k_\theta \nabla_r U \frac{\partial W_k}{\partial k_r} = 2\gamma W_k - 2c_1 W_k \nabla_r N$$

$$\frac{\partial U}{\partial t} = -c_2 \nabla_r \int W_k dk_r - \mu U$$

$$\frac{\partial N}{\partial t} = -c_3 \nabla_r \int W_k dk_r + c_4 \nabla_r \Big[\int W_k dk_r \nabla_r N \Big] + D_0 \nabla_r^2 N$$

W_k : turbulent wave action density
U : zonal flows
N : zonal density

- 2 ‘predators’: zonal flows and zonal density corrugations
- numerical implementation of wave-kinetic model left for future work

Results from direct fluid turbulence simulations

Hereafter, we present results that support the theory, using **direct numerical simulations** of the fluid **modified Hasegawa-Wakatani model** using BOUT++ framework **[Dudson '09]**

- fluid model for collisional drift-waves (modified Hasegawa-Wakatani) including **zonal modes**
- : limit of DTEM for $f_t \to 1$ (strongly unstable DTEM)

$$
\begin{aligned}
\frac{\partial n}{\partial t} + v_E \cdot \nabla n + \kappa \frac{\partial \phi}{\partial y} &= \nu(\tilde{\phi} - \tilde{n}) \\
\frac{\partial}{\partial t} \nabla_\perp^2 \phi + v_E \cdot \nabla \nabla_\perp^2 \phi &= \nu(\tilde{\phi} - \tilde{n})
\end{aligned}
$$

Results: zonal flows & zonal density corrugations

- 2D BOUT++ Simulations on a grid of size $[L_x.L_y] = 256 \times 256$,
  -constant equilibrium density gradient $\kappa = 0.5$ ($L_n = 2\rho_s$),
  - de-trapping rate $\nu = 1$.

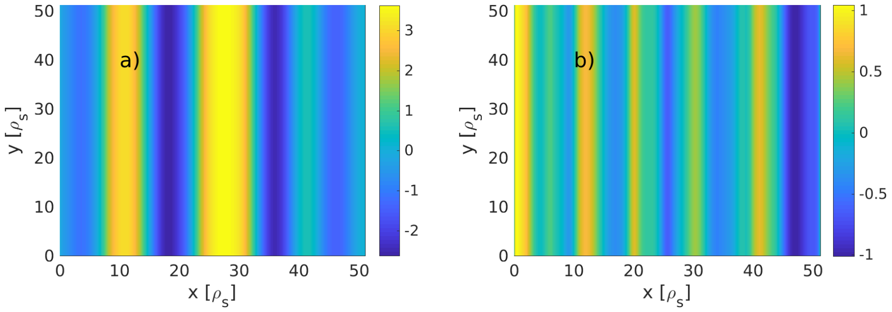
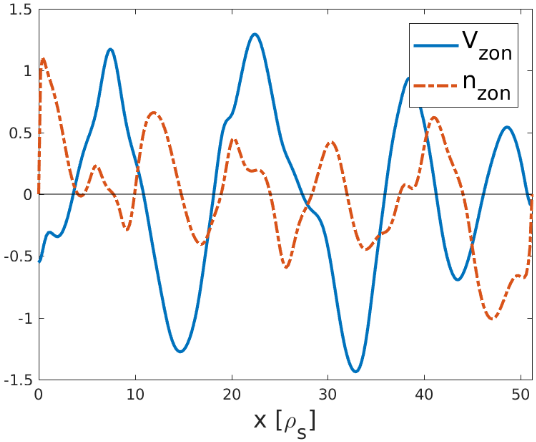


Figure: Snapshot of the simulations in saturated state: ϕ, **n**

- Contours of electric potential $\phi \rightarrow$ **zonal flows**
- Contours of density $n \rightarrow$ **zonal density corrugations**

Results: Evidence of staircase



- zonal density corrugations n_{zon} have different radial scale compared to zonal flows V_{zon} ! $\rightarrow$ suppress different scales?

Results: Transport crossphase radial modulation

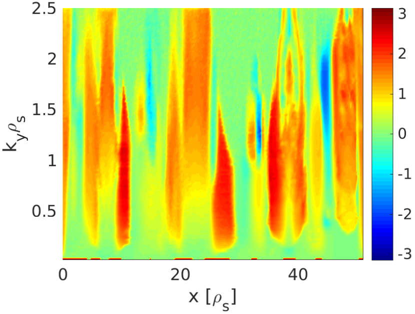


Figure: Time-averaged **nonlinear crossphase** $\theta_k(x)$ v.s. radius and $k_y\rho_s$

- turbulence spectrum divided into the $|\phi_k|^2$, $|n_k|^2$ power spectrum, and cross-spectrum
- cross-spectrum $\rightarrow n_k^*\phi_k$ is complex-valued

crossphase spectrum (crossphase)

$$\theta_k = \arg n_k^*\phi_k$$

- Time-averaged **nonlinear crossphase** $\theta_k(x)$ shows **radial modulations**
- This result supports the theory proposed in this talk

How efficient is zonal density at suppressing transport?

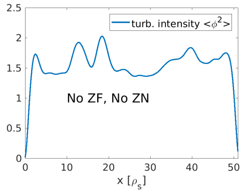


Figure: Turbulence intensity profile

No ZF, No ZN

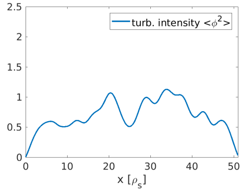


Figure: Turbulence intensity profile

ZN only

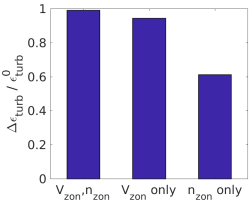


Figure: Zonal efficacy for 3 cases

zonal efficacy

$$\chi = \frac{|I - I_0|}{I_0}$$

$\hookrightarrow$ Zonal density has a significant effect on turbulence suppression

Summary and conclusions

- In the framework of a fluid DTEM model, we showed that zonal density corrugations can be nonlinearly driven by the DTEM turbulence
- The wave kinetic analysis predicts that DTEM turbulence can nonlineary drive zonal density corrugations that can suppress the transport crossphase by radially modulating it, thus reinforcing the zonal density corrugations.
- An extended wave-kinetic equation (EWKE) is derived. Nonlinear coupling to zonal flows and zonal density yields a novel DTEM wave kinetic model.
- Direct numerical simulations of the related Hasegawa-Wakatani model confirm that crossphase modulation can drive zonal density, and zonal density stabilizes turbulence.
- Open Questions
  - Can zonal density corrugations be observed experimentally?
  - Are radial modulations of crossphase observed in nonlinear gyrokinetic simulations? (ongoing work)
  - work in progress: extension to collisionless modes (CTEM)

THANK YOU

This work was supported by R&D Program through National Fusion Research Institute (NFRI) funded by the Ministry of Science, ICT and Future Planning of the Republic of Korea (NFRI-EN 2041-6).