Contribution ID: **1362** Type: **Regular Poster**

Turbulence simulations and Braginskii-style transport coefficients based on high precision gyrokinetic Landau collision operator

Friday 14 May 2021 12:10 (20 minutes)

Trustworthy gyrokinetic (GK) or two-fluid (2F) edge turbulence simulations require an accurate representation of collisions in *gyro-centre coordinates* or by appropriate transport coefficients, respectively. GK collision operators have so far been limited either to *models* for the (trustworthy) Landau-Fokker-Planck operator which have been transformed to the GK gyro-centre coordinates (1), or to the full Landau operator but in a purely drift-kinetic setting (2). 2F transport coefficients have been obtained in several complex analytical calculations and approximations (3) for infinite ion-electron mass ratio, but with rather intransparent ordering schemes and some missing coefficients.
This state is in both cases unsatisfactory. Moreover, differing levels of collision representation in GK (4) and 2F (5) turbulence simulations have precluded stringent comparisons between both frameworks in their overlap region of validity.
Motivated by the successful use of half-sided hermite or similar orthogonal polynomials P_i as velocity space basis functions (4,6), a numerical code for the matrix elements for any species pair a, b,
$\langle f_{a,i,l}|C^{ab}|f_{b,j,l}\rangle, \qquad f_{a,i,l}(\mathbf{v}) = L_l(v_{\parallel\,a}/v_a)P_i(v_a)\exp(-m_a v_a^2/(2T))$
of the linearized but otherwise complete Landau operator C^{ab} was developed. L_l are the Legendre polynomials for the pitch angle distribution, $f_{a,i,l}$ is the basis function for species a and radial and angular indices i, l. This approach turned out to be amazingly successful. It is not even necessary to explicitly demand conservation laws or the H-theorem, as all the matrix elements are computed up to nearly machine precision (14-15 digits) and automatically fulfill those requirements, which has been confirmed by extensive tests, even for unrealistically large particle mass ratios (up to 10^{10}). This in turn simplifies and streamlines the code significantly, which is exactly what allows the high precision. The code is also quite efficient - it takes only seconds on a laptop to calculate the Landau matrix elements for hundreds of polynomials.
As illustration of the convergence the electron/ion Spitzer transport values for infinite mass ratio for increasing number n of polynomials, calculated from an inversion of the collision matrix for given electric field or temperature gradients, are here compared with literature values:

n	λ_{11}	λ_{12}	λ_{22}	α_i
4	1.9729113706547	1.3507439973966	3.5912394745740	3.5751695538083
10	1.9758136156134	1.3887606183611	4.1791659178893	3.9502079279755
20	1.9758224556276	1.3887475542516	4.1791860160905	3.9502080688609
40	1.9758225395211	1.3887474651190	4.1791861130853	3.9502080688623
60	1.9758225395566	1.3887474650753	4.1791861131411	3.9502080688623
(7)	1.9757	1.3889	4.1789	
(8)	1.975	1.389	4.174	3.91

The customary coefficients of (9) are related by $\sigma_\parallel = \lambda_{11} n e^2 \tau_{ei}/m_e$, $R_{T,\parallel} = -\lambda_{12}/\lambda_{11} n \partial_\parallel T_e$, $\kappa^e_\parallel = (\lambda_{22} - \lambda_{12}^2/\lambda_{11})$, $\kappa^i_\parallel = \alpha_i n T_i \tau_{ii}/m_i$. For realistic ion/electron mass ratio certain coefficients are significantly altered, e.g. for Deuterium $\alpha_i = 3.25485015$, or the ion magnetic pumping coefficient $\eta_0^i = 0.913634368$ instead of the Braginskii 0.96 (the slight uncertainty is due to the uncertain mass ratio). Carrying out the same calculations for the Sugama model operator (1) results in more serious deviations, $(\alpha_i, \eta_0^i) = (2.58, 0.760)$.
Here is an exemplary eigenstate with $T_i \neq T_e$ of the collision operator for $m_i/m_e = 100$, which exhibits the interaction of thermal ions and low velocity electrons by a flattening of the low energy electron distribution:

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$f_e(u_e)$, $f_i(u_i)$ are spherically symmetric perturbations of the electron/ion distribution functions, $u_a := \sqrt{2T_a/m_a}$.
For GK simulations the above matrix elements must be transformed to gyro-centre coordinates just as the model operators in [ref1], i.e., the gyro-transformed matrix elements
$\langle f_{a,i_a,l_a}|C^{ab}|f_{b,i_b,l_b}\rangle$, $f_{a,i_a,l_a}(\mathbf{v}_a) = \exp(-i\mathbf{k}\cdot\rho_a) f_{gc,a,i_a,l_a}(v_\perp, v_\parallel)$, $r\rho_a = \frac{m_a}{q_a B}(\mathbf{B}\times\mathbf{v}_a)$,

must be obtained, where f_{gc,a,i_a,l_a} is the gyro-centre distribution basis function for species a, indices i_a, l_a. This computation is carried out with exponential convergence by expanding the angular dependence f_{a,i_a,l_a} in spherical harmonics and the v dependence in the P_i. The wavenumber dependence is treated efficiently by representing it with a Chebyshev interpolant. On a parallel machine, all required GK matrix elements for typical ion-scale turbulence simulations result in seconds, again with 14-15 digits precision.
With the described code it is easy to give much more accurate perpendicular Braginskii-style transport coefficients than currently known (3,9). E.g., for the perpendicular electron heat conduction one gets
$m_e\omega_{ce}^2\tau_{ei}\kappa_{\perp,e}/(nT_e) \approx 4.6642135623731056$,
up to 14 digits equal to $13/4+\sqrt{2}$, an identity which seems not to be known in literature.
In the contribution a comparison survey of GK turbulence simulations for the Sugama and Landau collision operators as well as 2F turbulence simulations using the newly determined high accuracy transport coefficients will be shown. Another area of comparison are high accuracy GAM and zonal flow damping rates.
The inaccuracies of the Landau operator itself are larger than the numerical ones of the presented computations. The accuracy is however beneficial, as it is cheap and takes care of the fundamental properties of the operator (self-adjointness, conservation laws, invariances and the H-theorem). In addition the method can serve as a blueprint to implement more accurate operators, such as a combination of the Balescu-Lenard operator with the Boltzmann operator at large collision angles, which would only render the initial calculation of the matrix elements more costly but not the gyrotransformation.
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Session Classification: P7 Posters 7

Track Classification: Magnetic Fusion Theory and Modelling