Turbulence simulations and Braginskii-style transport coefficients based on high precision gyrokinetic Landau collision operator

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Introduction

- Trustworthy gyrokinetic or two-fluid edge turbulence simulations require accurate representations of collisions
- Gyrokinetic simulations (largely) limited to model operators (e.g. Sugama operator) or drift-kinetic Landau operator
- Fluid simulations limited to analytical Braginskii approximation (Landau based, low order moments, infinite electron-ion mass ratio)
- Meaningful comparisons in fluid limit impossible

➔Calculating (linearized) gyrokinetic complete Landau operator matrix elements nearly exactly can solve both problems

Landau collision operator

- nonlinear: $c^{ab}(f_a, f_b) = \frac{c_{ab}}{m_a}\nabla_a \cdot \int d^3v_b K(\boldsymbol{u}) \cdot \left(\frac{f_b \nabla_a f_a}{m_a} - \frac{f_a \nabla_b f_b}{m_b}\right)$ $\quad \boldsymbol{u} = \boldsymbol{v}_a - \boldsymbol{v}_b$, $\quad K(\boldsymbol{u}) := \frac{\mathbb{1} - \hat{\boldsymbol{u}}\hat{\boldsymbol{u}}}{u}$
- linearized: $C^{ab,T}\delta f_a + C^{ab,F}\delta f_b := c^{ab}(\delta f_a, f_{b0}) + c^{ab}(f_{a0}, \delta f_b)$ $\quad \delta f_s := f_s - f_{s0}$
- matrix elements: $\langle f_{a,i,l}|C^{ab}|f_{b,j,l}\rangle$ $\quad f_{s,i,l}(\boldsymbol{v}) = L_l(v_\parallel/v_s)P_i(v_s)\exp(-m_s v_s^2/(2T))$

  L_l Legendre polynomials $\quad P_i$ half-sided Hermite or similar polynomials

  scalar products: $\langle f_s|g_s\rangle := \int \frac{f_s(v_s)g_s(v_s)}{f_{s0}(v_s)}d^3v_s$ $\quad$ operator notation as in quantum mechanics

- regular (“unmagnetized”) matrix elements extremely efficient, hundreds of them in seconds on a laptop
- machine precision accuracy (14-15 digits) even for unrealistically high mass ratios ($\sim 10^{10}$)

➔automatic conservation of all physical conservation laws, H-theorem

➔can be used to compute standard unmagnetized transport coefficients with unprecedented precision

Unmagnetized transport coefficients

- Spitzer transport coefficients illustrating convergence for increasing number n of radial polynomials (Z=1, infinite mass ratio):

n	λ_{11}	λ_{12}	λ_{22}	α_i
4	1.97291137065 47	1.35074399739 66	3.59123947457 40	3.57516955380 83
10	1.97581361561 34	1.38876061836 11	4.17916591788 93	3.95020792797 55
20	1.97582245562 76	1.38874755425 16	4.17918601609 05	3.95020806886 09
40	1.97582253952 11	1.38874746511 90	4.17918611308 53	3.95020806886 23
60	1.97582253955 66	1.38874746507 53	4.17918611314 11	3.95020806886 23
[Belli 2012]	1.9757	1.3889	4.1789	
[Hinton 83]	1.975	1.389	4.174	3.91

with definitions [Hinton 83]

$$j_{\|,e} = \frac{enT\tau_{ei}}{m_e}(\lambda_{11}A_1 + \lambda_{12}A_2), \quad q_{\|,e} = -\frac{nT^2\tau_{ei}}{m_e}(\lambda_{11}A_1 + \lambda_{12}A_2), \quad q_{\|,i} = -\frac{nT^2\tau_{ii}}{m_i}\alpha_i A_3$$

$$A_1 = \partial_\| \ln p_e + \frac{e}{T_e}\partial_\| \phi, \quad A_2 = \partial_\| \ln T_e, \quad A_3 = \partial_\| \ln T_i$$

Braginskii style transport coefficients are related by [NRL 2019]

$$\sigma_\| = \lambda_{11}ne^2\tau_{ei}/m_e,\ R_{T,\|} = -\lambda_{12}/\lambda_{11}n\partial_\| T_e,\ \kappa_\|^e = (\lambda_{22} - \lambda_{12}^2/\lambda_{11}),\ \kappa_\|^i = \alpha_i nT_i\tau_{ii}/m_i$$

- further Braginskii coefficients compared to best known values (Z=1, infinite mass ratio, all given digits significant):

	[NRL 2019]	accurate value
frictional heat flux coefficient	0.71	0.70287054493606
parallel electron thermal conductivity	3.2	3.203076425585
ion viscosity $\eta_0^i/(nT\tau_{ii})$	0.96	0.96529161180214
electron viscosity $\eta_0^e/(nT\tau_{ei})$	0.73	0.733488956541

- customary transport values not given in truly **independent** set of variables (e.g. parallel velocity depends on density gradient)

➔in general: transport matrix in independent set of variables

Gyrotransformation

- gyrokinetic matrix elements: $\langle f_{gc,a,i_a,l_a}|C^{ab}|f_{gc,b,i_b,l_b}\rangle$

  (i.e. gyrocenter coordinates) $\quad f_{gc,a,i_a,l_a}(\boldsymbol{v}_a) := \exp(-i\boldsymbol{k}\cdot\boldsymbol{\rho}_a)f_{a,i_a,l_a}(v_\perp, v_\|)$ $\quad \boldsymbol{\rho}_a = \frac{m_a}{q_a B}(\boldsymbol{B}\times\boldsymbol{v}_a)$
- gyrotransformation by expansion in regular matrix elements
- k expansion in Chebyshev polynomials
- seconds on parallel machine for reasonable $k\rho_i \lesssim 100$ to machine precision (10-14 digits)
- physical restrictions maintained due to accuracy
- can be used to calculate highly accurate perpendicular transport coefficients

Magnetized transport coefficients

- perpendicular transport coefficients not corresponding to simple numbers in [NRL 2019] (all given digits are significant):

	[NRL 2019]	accurate value
perp. electron heat flux coefficient $m_e\omega_{ce}^2\tau_{ei}\kappa_{\perp,e}/(nT_e)$	4.7	4.6642135623731056
perpendicular electron viscosity $\eta_1^e/(nT\tau_{ei})$	0.51	0.71213203435597
perp./parallel electron viscosity $\eta_2^e/(nT\tau_{ei})$	2.0	2.84853

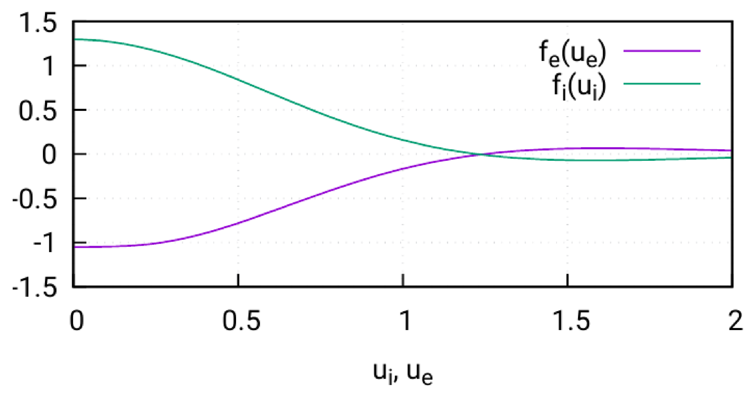
- one value is so accurate that one can guess a (new) analytical expression: $m_e\omega_{ce}^2\tau_{ei}\kappa_{\perp,e}/(nT_e) = 13/4 + \sqrt{2}$
- the deviation by about $\sqrt{2}$ for the other two values hints at a mix-up between τ_{ei} and τ_{ii} in the original publication
- all other values agree with [NRL 2019]

Realistic ion/electron mass ratio

- several degrees of freedom to expand kinetic equation to obtain fluid transport coefficients, one possibility:

  replace stationary infinite mass ratio eigenstates corresponding to T_i, T_e, n_i, n_e, u by slowly changing eigenstates, e.g., one such eigenstate corresponds to $\delta T_i = -\delta T_e,\ n_i = n_e = n_0,\ u = 0,$

  heat exchanging distribution:

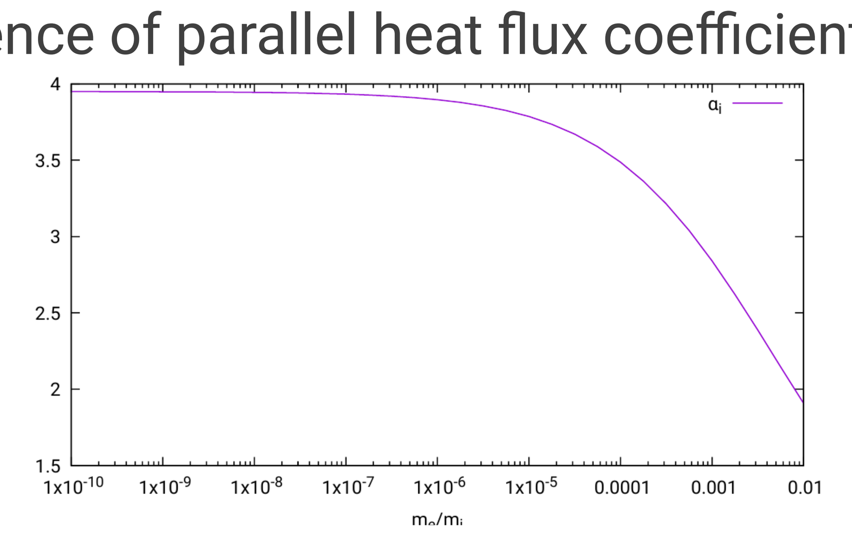


$u_a := \sqrt{2T_a/m_a}$ $\quad m_i/m_e = 100$

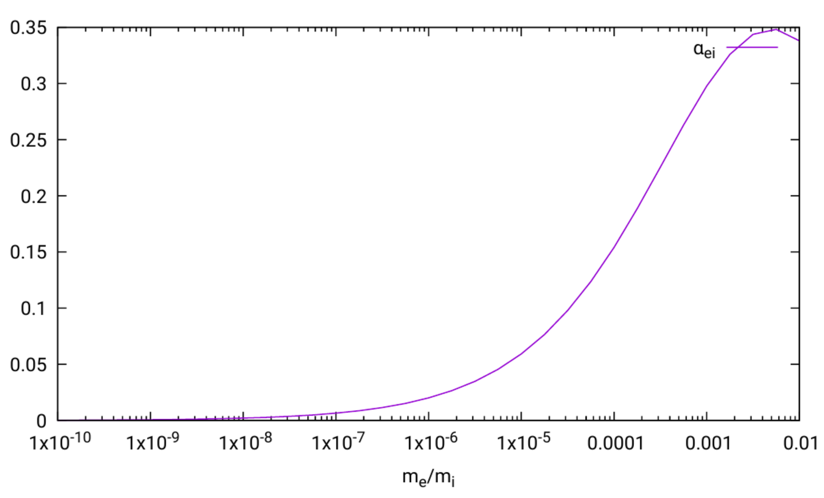
$f_e(u_e),\ f_i(u_i)$ spherically symmetric perturbations of electron, ion distribution function

➔significant changes from infinite mass ratio values

- e.g. for Deuterium: $\alpha_i = 3.25485015$ $\quad \eta_0^i = 0.913634368$
- dependence of parallel heat flux coefficient on mass ratio:



- mass ratio dependent electron-ion cross heat flux significant:



$q_{i,e} = \alpha_{ei}nT_i\tau_{ii}/m_i\partial_\| T_e$

e.g. this term is completely absent in Braginskii.

- Sugama model operator in comparison: $(\alpha_i, \eta_0^i) = (2.58, 0.760)$

Conclusions:

- Inaccuracies of Landau operator larger than numerical ones
- Accuracy takes care of fundamental properties (self-adjointness, conservation laws, invariances, H-theorem)
- model operators/infinite mass ratio both partially far off
- Spin-off: Extremely accurate fluid transport coefficients
- Blueprint for more advanced operators (Balescu, Boltzmann)

[Hinton 83] F.L. Hinton, Handbook of Plasma Physics **1**, 147 (1983)
[Belli 2012] E.A. Belli, J. Candy, Plasma Phys. Control. Fusion **54**, 015015 (2012)
[NRL 2019] A.S. Richardson, NRL plasma formulary, (2019)

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