Integrating tokamak-edge MHD-fluctuation modeling with transport

by

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Goal: Validate NIMROD MHD QH-mode simulations with local perturbation measurements

- Focus on EHO in QH DIII-D discharge 163518 at 2350 ms
- Hypothesis: saturated 3D fluctuations drive particle and thermal transport to maintain steady state pedestal profiles [Snyder NF 2007]
- How well can MHD modeling characterize the low-n perturbations observed during QH-mode?

[K. Burrell PoP 2016]
Extended-MHD codes start from reconstructed state

- **Initial plasma and magnetic configuration:** reconstructed from measurements constrained by force balance

- **Assume:** 2D evolution of this state is on transport time scale
  - Transport requires effects outside the scope of MHD: e.g. neutral-beam, high-k turbulence, neoclassical effects, IOL, neutral interaction

- **Model:** NIMROD code [Sovinec JCP 04] evolves 3D, nonlinear perturbations around 2D steady state
  - Perturbations may modify the axisymmetric (n=0) state
  - Consistent with reconstruction when n=0 modification is small
Best-fit reconstruction is stable → increase instability drive by reducing pedestal width

- EFIT based on “best fit” to experimental data is stable when ExB flow is included

- To destabilize: density and temperature gradients increased in the pedestal region
  - Similar to varyped

- Nonlinear relaxation expected to relax plasma profiles back towards measured state
Simulations saturate to states with $n=1$ dominant, but span from laminar to turbulent.

1.4 times $\nabla p_{\text{exp}}$
Low-amplitude Laminar

2.0 times $\nabla p_{\text{exp}}$
Large-Amplitude Turbulent

Does one underlying drive better match experiment?
Density and temperature relax towards measured state

- Weakly driven case exhibits weak relaxation (profiles too steep)
- Strongly driven case exhibits strong relaxation
  - Density transport greater than temperature consistent with [King et al., Phys Plasma 2016]
- Strongly driven case more consistent with measured profiles
Rotation profiles evolve from initial profiles $\rightarrow$ impacts EHO frequencies

- Initial profile is ExB and neoclassical poloidal flows based on reconstruction
  \[ \mathbf{v} = \Omega_{E\times B} \hat{\mathbf{R}} \cdot \hat{\mathbf{\Phi}} + k_{pol}(\psi) \mathbf{B} \]
  \[ \Omega_{E\times B} = \frac{\mathbf{E} \cdot \nabla \psi}{|\nabla \psi|^2} \]

- With time dynamics, MHD-fluctuation-induced flows are generated and basic FSA quantities are compared
  \[ k_{pol}(\psi) = \left\langle \frac{v_{pol}}{B_{pol}} \right\rangle \]
  \[ \Omega = \left\langle \frac{\mathbf{V}_\Phi}{R} \right\rangle \]

- Large deviation in strongly driven case impacts mode frequency
Strongly-driven turbulent simulation shows large density perturbations

Density at ~440μsec

BES local to outboard mid-plane where flux compression smooths dynamics

2.0 × ∇p_{exp}
Amplitude of BES measurements is bracketed by synthetic diagnostic analysis of simulations

- Both simulation and experiment transition from single to double peak structure when moving radially outward
- Frequencies are not consistent given flow profile modification from MHD fluctuations
Validation promising but imperfect; complicated by

• Changes to magnetic surface location when increasing instability drive
  – Need to keep \( \{p,n,T\}(R,Z) \) fixed for comparison to local measurements at fixed \( R,Z \)
  – Not straight-forward with current profile redistribution which modifies \( \psi(R,Z) \) as profiles often specified as \( \{p,n,T\} (\psi) \)

• Rotation profile modification confounds local diagnostic comparisons which rely on frequency analysis

• Next: consider other sources of momentum transport from effects outside the MHD model
Focus on 2D ion-orbit-loss and neutral momentum transport in DIII-D shot 164988

This shot has minimal 3D magnetic Perturbation → tests 2D model equations
Ion-orbit-loss model from analytic work by Shaing

FSA particle loss rate

\[ \langle \frac{\partial n}{\partial t} \rangle_{orb} = -\frac{2.25}{\sqrt{\pi}} \frac{n \nu_i}{\sqrt{2|S|}} \exp \left[ -\left( \nu_*^{1/4} + \frac{\Omega \sqrt{|S|}}{R B_t v_{Ti}} \frac{|\psi - \psi_s|}{\sqrt{2\epsilon}} \right)^2 \right] \]

Losses are limited by collisions and distance from LCFS

\[ S = 1 + \frac{R B_t^2}{\Omega c_0} \frac{e \Phi''}{m} \]

S is ‘orbit-squeezing’ factor that decreases IOL outside minimum of \( E_r \) well

\[ \Gamma_{orb} = -\hat{r} (\Delta r) (\partial n/\partial t)_{orb} \]

IOL particle flux acts like a current

\[ \langle B_p \cdot \nabla \cdot \Pi_{orb} \rangle = e \Gamma_{orb} \times B \cdot B_p \]

IOL leads to viscous forces & co-current torque

\[ \langle B_t \cdot \nabla \cdot \Pi_{orb} \rangle = e \Gamma_{orb} \times B \cdot B_t \]

Shaing et al., PFB 2 June (1990); Shaing PFB 2 Jan (1992); Shaing PFB 2 Oct (1992)
Model equations predict large IOL torques at edge
Next: investigate torque from full plasma-neutral model

\[ \frac{\partial n_i}{\partial t} + \nabla \cdot (n_i \mathbf{v}) = \Gamma^{\text{ion}} - \Gamma^{\text{rec}} \]

\[ \rho = \sum_{\alpha \neq n} \rho_\alpha \quad \rho \mathbf{v} = \sum_{\alpha \neq n} \rho_\alpha \mathbf{v}_\alpha \quad n_I = \sum_{\alpha \neq n, e} n_\alpha \]

\[ \frac{\partial n_n}{\partial t} + \nabla \cdot (n_n \mathbf{v}_n) = \Gamma^{\text{rec}} - \Gamma^{\text{ion}} \]

\[ \rho \frac{\partial \mathbf{v}}{\partial t} + \rho \mathbf{v} \cdot \nabla \mathbf{v} + \nabla p + \nabla \cdot \Pi = \mathbf{J} \times \mathbf{B} + \left( \Gamma^{\text{ion}} + \Gamma^{\text{cx}} \right) m_i (\mathbf{v}_n - \mathbf{v}) + \mathbf{R}^{\text{cx}}_{in} - \mathbf{R}^{\text{cx}}_{ni} \]

\[ \rho_n \frac{\partial \mathbf{v}_n}{\partial t} + \rho_n \mathbf{v}_n \cdot \nabla \mathbf{v}_n + \nabla p_n + \nabla \cdot \Pi_n = - \left( \Gamma^{\text{rec}} + \Gamma^{\text{cx}} \right) m_i (\mathbf{v}_n - \mathbf{v}) - \mathbf{R}^{\text{cx}}_{in} + \mathbf{R}^{\text{cx}}_{ni} \]

\[ n_I k_B \frac{\partial T_I}{\partial t} + n_I k_B \mathbf{v} \cdot \nabla T_I + (\Gamma - 1) n_I k_B T_I \nabla \cdot \mathbf{v} = -k_B T_I \left( \Gamma^{\text{ion}} - \Gamma^{\text{rec}} \right) \]

\[ + (\Gamma - 1) \left[ \nabla \cdot \mathbf{q} + \Pi : \nabla \mathbf{v} + Q^{\text{ion}} - Q^{\text{rec}} + Q^{\text{cx}} + Q^{\text{cx}}_{in} - Q^{\text{cx}}_{ni} \right] \]

\[ + \left( \Gamma^{\text{ion}} + \Gamma^{\text{cx}} \right) m_i \left( \mathbf{v} - \mathbf{v}_n \right)^2 - \mathbf{R}^{\text{cx}}_{in} \cdot (\mathbf{v} - \mathbf{v}_n) \]

\[ n_n k_B \frac{\partial T_n}{\partial t} + n_n k_B \mathbf{v}_n \cdot \nabla T_n + (\Gamma - 1) n_n k_B T_n \nabla \cdot \mathbf{v}_n = -k_B T_n \left( \Gamma^{\text{ion}} - \Gamma^{\text{rec}} \right) \]

\[ + (\Gamma - 1) \left[ \nabla \cdot \mathbf{q}_n + \Pi_n : \nabla \mathbf{v}_n - Q^{\text{ion}} + Q^{\text{rec}} - Q^{\text{cx}} + Q^{\text{cx}}_{in} + Q^{\text{cx}}_{ni} \right] \]

\[ + \left( \Gamma^{\text{rec}} + \Gamma^{\text{cx}} \right) \frac{m_i}{2} \left( \mathbf{v} - \mathbf{v}_n \right)^2 + \mathbf{R}^{\text{cx}}_{ni} \cdot (\mathbf{v} - \mathbf{v}_n) \]

[Meier & Shumlak POP 19 072508 (2012)]
1D plasma neutral case demonstrates basic physics

- Ballistic expansion force is balanced by charge exchange and advection
- Particle source from wall is ionized inside pedestal
2D neutral model with ion, electrons and neutral fluids produces in/out board asymmetry in heat flux

- Asymmetry is not present without dynamic electron fluid
Neutral torque is balances opposes IOL torque

Torques outside MHD are comparable and should be accounted for in MHD simulations
Conclusions

• **Validation QH-mode simulations scan the underlying drive**
  – Simulations bracket the fluctuation amplitude of BES observations
  – Fluctuation-induced momentum transport modifies rotation
  – Frequency comparison to BES measurements do not agree

• **Sources of momentum transport outside the MHD model are large**
  – Ion-orbit loss produces a co-current parallel torque in the edge
  – Calculation of the neutral CX force shows it balances the IOL torque
  – The torque from the Maxwell and Reynolds stresses of the over-driven QH-mode fluctuations is at most comparable to the IOL/neutral torques

• **Incorporation of IOL/neutral torques is needed during nonlinear QH-mode simulation**