

Turbulence Model Reduction by Deep Learning

Robin A. Heinonen^{1,2}Patrick H. Diamond^{2,a}¹University of Rome, Tor Vergata, ²University of California, San Diego^apdiamond@ucsd.edu

ABSTRACT

- Use machine learning to infer model for transport and mean zonal dynamics in drift-wave turbulence from simulations
- Key results: particle flux contains term driven by zonal flow curvature, which can set up staircase pattern in the profile. Zonal flow formation described by negative viscosity + stabilizing nonlinearity and hyperviscosity
- Methods may be useful for understanding transport in more complicated systems (e.g. gyrokinetics)

BACKGROUND

- (Radial) turbulent transport is described by turbulent fluxes $\Gamma = \langle \tilde{v}_x \tilde{n} \rangle$, $\Pi = \langle \tilde{v}_x \tilde{v}_y \rangle$, etc. Particle flux can lead to confinement losses, Reynolds stress sets up zonal flow (important for L-H transition)
- Traditional modeling approaches, like quasilinear theory, tend to rely on successive and often dubious approximations
- Idea of this work: use supervised machine learning (neural network) to find mean-field closure which expresses fluxes as functions of zonally-averaged variables
- As proof of concept, apply to Hasegawa-Wakatani system

$$\partial_t n + \{\phi, n\} = \alpha(\tilde{\phi} - \tilde{n})$$

$$\partial_t \nabla^2 \phi + \{\phi, \nabla^2 \phi\} = \alpha(\tilde{\phi} - \tilde{n})$$

- Goal is to then obtain closed system

$$\partial_t \langle n \rangle + \partial_x \Gamma = 0$$

$$\partial_t \partial_x^2 \langle \phi \rangle - \partial_x^2 \Pi = 0$$

with expressions for fluxes and partnered with equation for intensity

METHODS

FEATURE SELECTION

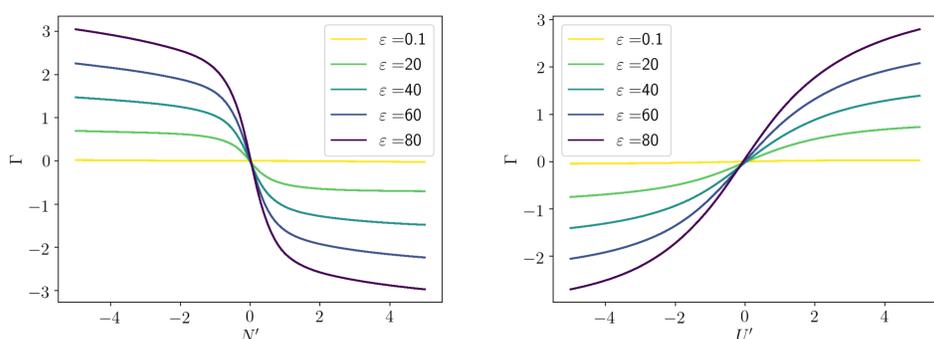
Seek a *local* model: fluxes at each radius, time specified by local mean fields and gradients. Continuous symmetries \rightarrow fluxes should depend only on $N' = \partial_x \langle n \rangle$, $U = -\partial_x^2 \langle \phi \rangle$ and higher-order derivatives. Need turbulence intensity proxy as well: choose turb. PE $\varepsilon = \langle (\tilde{n} - \nabla^2 \tilde{\phi})^2 \rangle$. Input variables/features are then chosen to be $N', U, U', U'', \varepsilon$

SIMULATIONS

Perform 32 simulations of 2D HW on 512 \times 512 grid using BOUT++. Employ variety of initial conditions for mean profile and zonal flow to span parameter space. Simulations fix adiabaticity at $\alpha = 2$

DEEP LEARNING

Use a deep neural network (multilayer perceptron) to learn mapping from input variables to Γ, Π , which are computed at points in radius and time. Serves as a proven form of nonparametric (model-free) regression. Reflection symmetries of HW are enforced by data augmentation



Dependence of particle flux on N' at fixed U' (left) and on U' at fixed N' (right)

OUTCOME

PARTICLE FLUX

For small N', U' , neural net learns model of the form $\Gamma = \varepsilon(-D_n N' + D_u U')$. Second, nondiffusive term depends on flow curvature, modulates profile in presence of zonal flow. Can be derived from quasilinear theory: originates from flow-induced shift in drift-wave frequency

REYNOLDS STRESS

Neural network learns model roughly of the form (for small U)

$$\Pi = \varepsilon f(N', U')(-\chi_1 U + \chi_3 U^3 - \chi_4 U'')$$

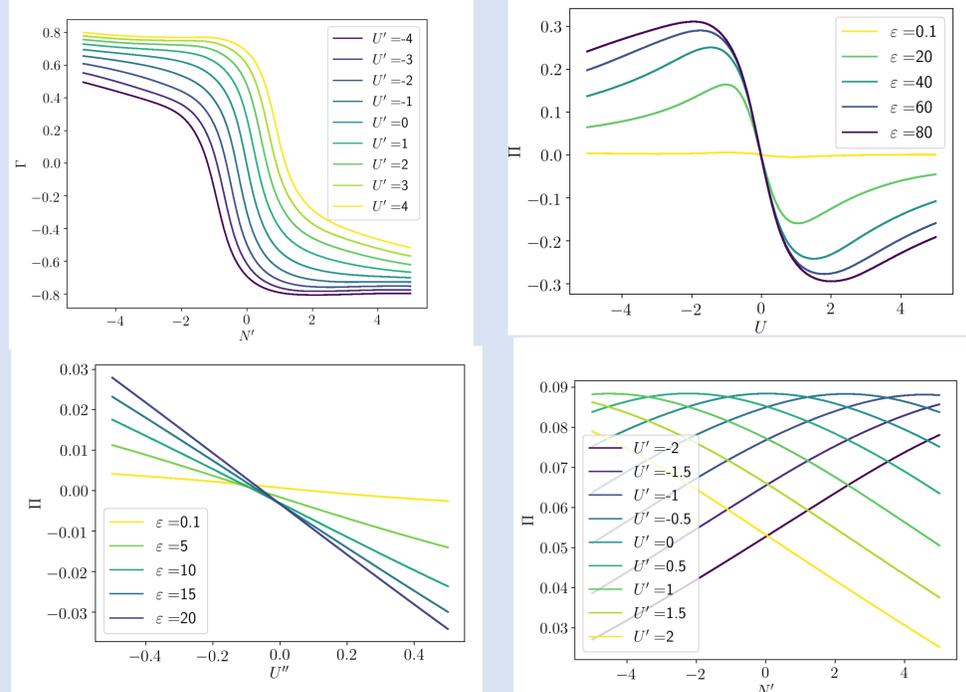
Zonal flows thus destabilized by negative viscosity χ_1 , restabilized by nonlinearity χ_3 , hyperviscosity χ_4 , and f , which $\rightarrow 0$ for large N', U' . Consistent with previous analytic modeling.

1D REDUCED MODEL

Couple with intensity equation

$$\partial_t \varepsilon + 2\varepsilon(N' + U')(\Gamma - \partial_x \Pi) = -\gamma_0 \varepsilon - \gamma_{NL} \varepsilon^2$$

to obtain three-field, 1D reduced model. Can solve numerically



Clockwise from top left: dependence of particle flux on U' and N' ; dependence of Reynolds stress on U (at fixed N', U', U''); dependence of Reynolds stress on U' and N' (at fixed U, ε); dependence of Reynolds stress on U'' (at fixed U, U', U'')

CONCLUSION

- Have used machine learning as novel approach to understand turbulent transport and structure formation in a plasma system
- Identified nondiffusive particle flux as novel route to profile layering. Model for zonal flow generation is of Cahn-Hilliard type (negative viscosity + stabilizing terms)
- Failed to find model for turbulence spreading. May require nonlocal model
- This method could be used to study mean transport in gyrokinetic system or other systems with multiple interacting channels