$\boldsymbol{E} \times \boldsymbol{B}$ staircase-like pattern formation in gyrokinetic simulations: a comparison with experiment and models

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Outlines

I. Introduction of $\boldsymbol{E} \times \boldsymbol{B}$ staircase

II. Gyrokinetic simulation of $\boldsymbol{E} \times \boldsymbol{B}$ staircase in KSTAR plasmas

III. Gyrokinetic simulation of $\boldsymbol{E} \times \boldsymbol{B}$ staircase in comparison with models

IV. Summary and discussions

I. Introduction of $E \times B$ staircase-1

Staircase like structures are commonly observed in the nature: Potential vorticity staircase (*D. G. Dritschel and M. E. McIntyre, J. Atmos. Sci. 2008***)**

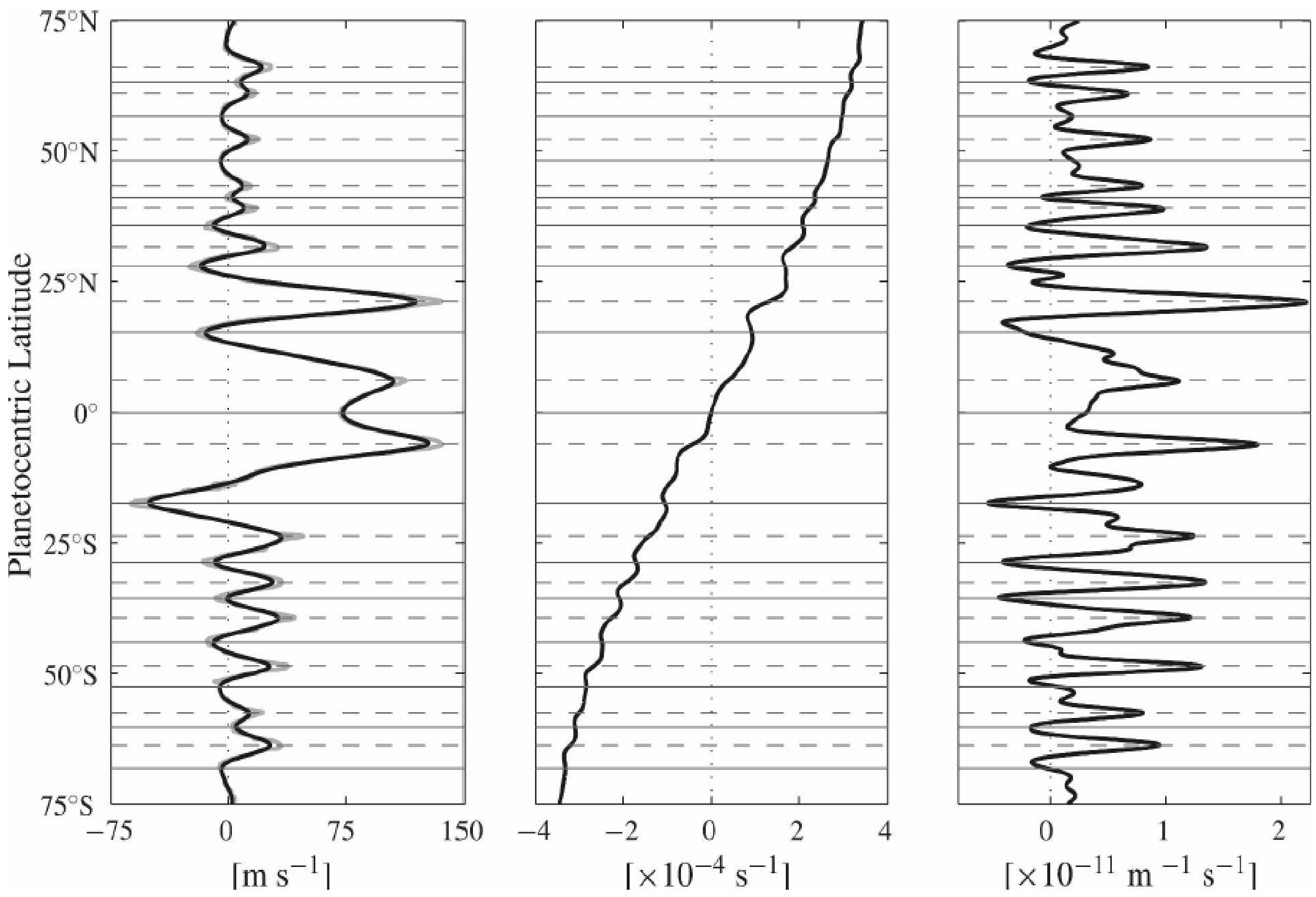


Fig. 5 of F. J. Beron-Vera et al., *J. Atmos. Sci.* **65** (10):3316-3326 (2008)

Potential vorticity staircase observed in *Jupiter's weather layer* based on *Cassini* data. Zonal wind (left), potential vorticity (PV) (middle) and PV gradient (right). Interesting *cross-correlation* between zonal wind and PV gradient: eastward jets are associated with large meridional PV gradients, most westward jets are associated with small meridional PV gradients.

I. Introduction of $E \times B$ staircase-2

Staircase like structures are commonly observed in the nature: thermohaline staircases

Thermohaline staircase indicated by scattering strength (db) from EK80 echhogram

CTD potential temperature profiles

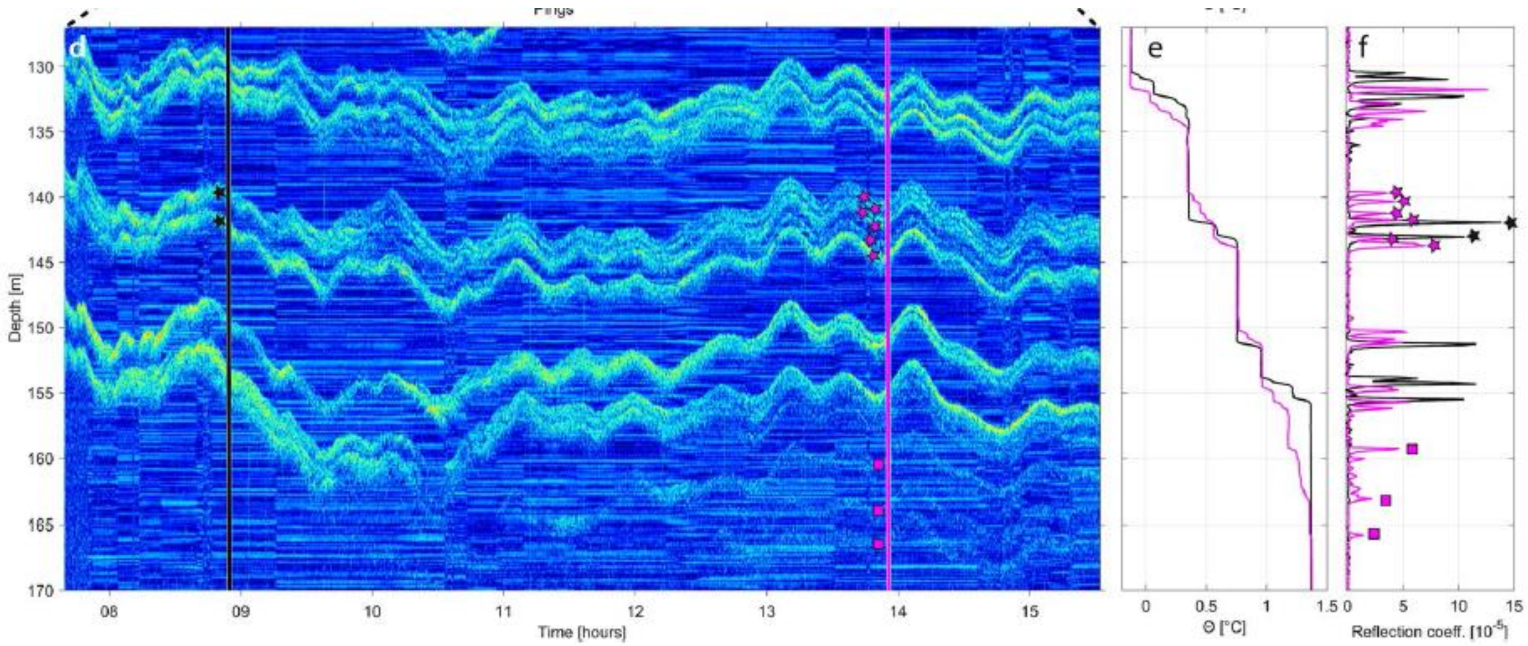


Figure 5 of C. Stranne, et al., *Scientific Reports* **7**, 15192 (2017)

Observations of **thermohaline staircases** in *Arctic Ocean*. Staircase-like structures can be observed in the conductivity, temperature and depth (CTD) potential temperature profile.

I. Introduction of $E \times B$ staircase-3

In magnetic fusion plasmas: $\boldsymbol{E} \times \boldsymbol{B}$ staircases

***What we study here*:**
The global pattern of mean $\boldsymbol{E} \times \boldsymbol{B}$ flow with zero toroidal and poloidal mode number $n = m = 0$ (i.e., zonal flow)

Well known effects:
Reduce the turbulent transport through shearing micro-turbulence and absorbing the energy. (*Z. Lin et al., Science 1998; P. H. Diamond et al., PPCF 2005; L. Qi et al., NF 2017* ⋯ ⋯)

Importance of E × B staircases :
The quasi-stationary and long-lived patterns of $\boldsymbol{E} \times \boldsymbol{B}$ flow staircase have significant effects in the regulation of mesoscopic, non-diffusive and non-local transport avalanches and form transport barriers. (*P. H. Diamond and T. S. Hahm, PoP 1995; B. A. Carreras et al., PoP 1996; G. Dif-Pradalier et al., PRE 2010; T. S. Hahm and P. H. Diamond, JKPS 2018* ⋯ ⋯)

I. Introduction of $E \times B$ staircase-4

Avalanches can be commonly observed in gyrokinetic simulations and tokamak experiments.

Avalanching is intrinsic to the systems exhibiting self-similarity and a consequence of self-organized criticality (**SOC**).

Characteristics of avalanches:

- ✓ Spectral power law scaling $S(f) \sim 1/f$
- ✓ Intermittent burst
- ✓ Mesoscopic scale (tens of ρ_i), non-diffusive and non-local transport events

Self-organized criticality (SOC):
Slowly driven, interaction-dominated systems with threshold, exhibiting self-similarity without tuning, i. e., **power law scaling $1/f$**.

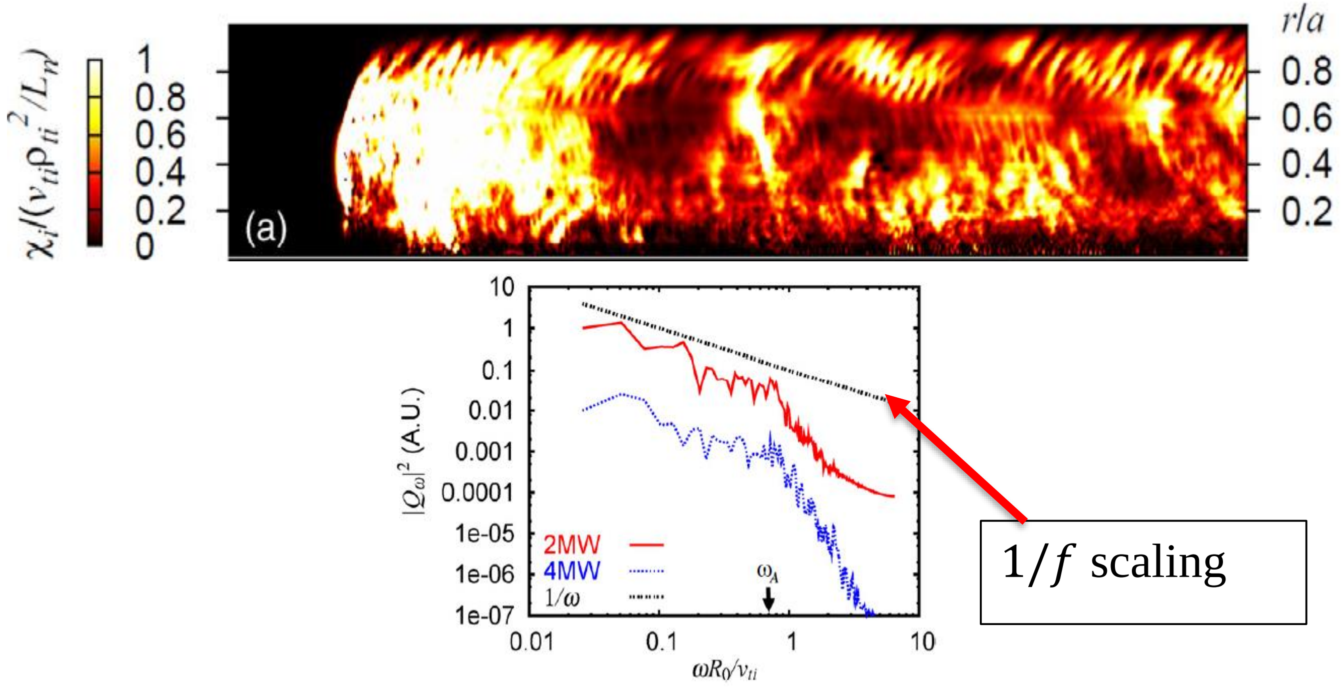


Fig. 9 in Ref. Y. Idomura et al., NF 2009

I. Introduction of $E \times B$ staircase-5

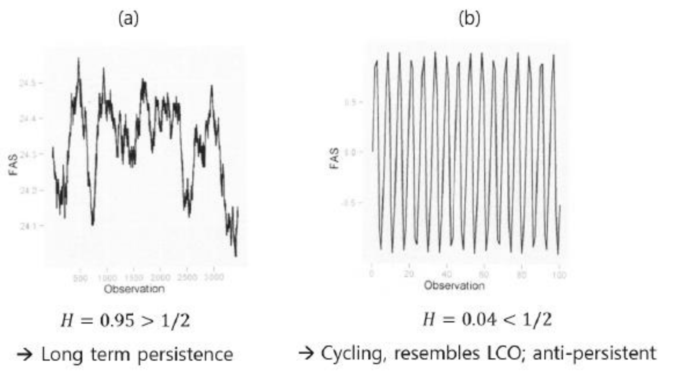
Hurst exponent $0 < H < 1$ (*B. B. Mandelbrot and J. R. Wallis, Water Resources Res. 1968*): (R/S analysis, *B. A. Carreras et al., PRL 1998*)

$1/2 < H < 1$, dynamics manifest a sustained memory, long-term persistence and positive correlations

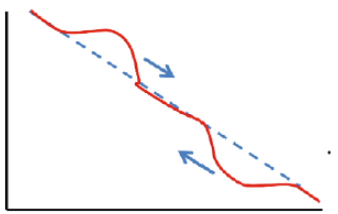
$0 < H < 1/2$, rapid switching between high and low values, temporal anti-correlation.

$H = 1/2$ corresponds the random walk,

while $\boldsymbol{H = 1}$ **gives** $\boldsymbol{1/f}$ **spectrum**.



Ref. G. Boffetta, A. Mazzino and A. Vulpiani, J. Phys. A. 2008



P. H. Diamond, presented in 6th APTWG 2016

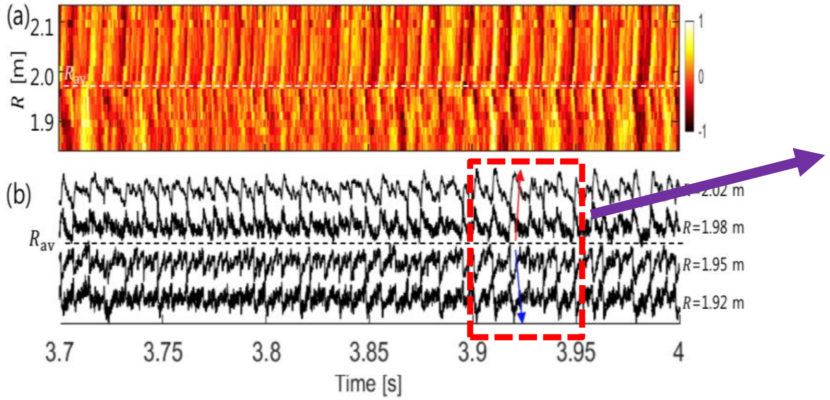
Joint reflection symmetry (JRS):

A property expected for a <u>SOC</u> system. (*T. Hwa and M. Kardar, Phys. Rev. A. 1992; P. H. Diamond and T. S. Hahm, Phys. Plasmas 1995*)

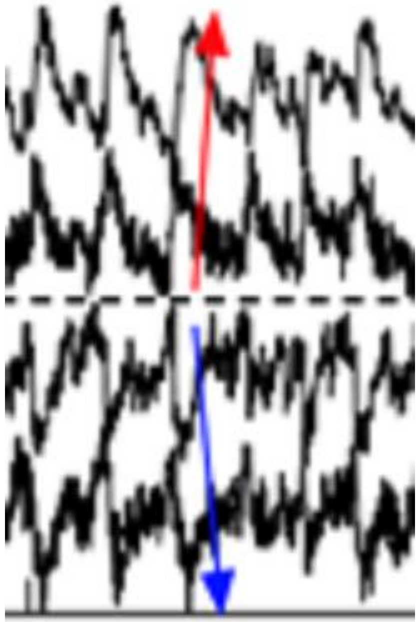
Bumps move down gradient

Voids move up gradient

First experiment evidence from tokamak



From KSTAR Exp. By M. J. Choi, et al., NF 2019



Bumps
$\delta T_e > 0$

Voids
$\delta T_e < 0$

I. Introduction of $E \times B$ staircase-6

The name of pattern: The pattern formation in magnetic fusion plasmas was given the name of "Zonal flow staircase" or "$\boldsymbol{E} \times \boldsymbol{B}$ staircase", which is an analog to the "PV staircase" in geophysics and astrophysics (*D. G. Dritschel and McIntyre M. E., J. Atmos. Sci. 2008*).

Typical signatures: Staircase-like pattern occurs concurrently in both zonal flow and pressure (temperature and density) profiles. Long-lived, quasi-stationary pattern.

Characteristic scale: The staircase is typically in mesoscopic scale (~tens of ρ_i), i.e., several times the microturbulence correlation length (~several ρ_i).

Effects of the pattern: The staircase-like pattern is found to regulate the transport avalanches, and form localized transport barriers. Avalanches are prevalent in-between staircase shear layers (steps), while suppressed across.

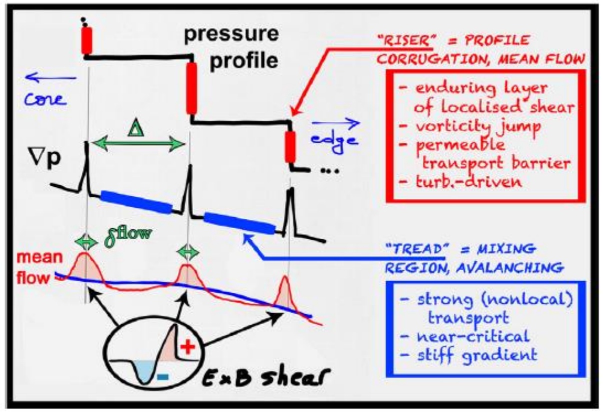


Figure 1 of G. Dif-Pradalier et al., NF 2017

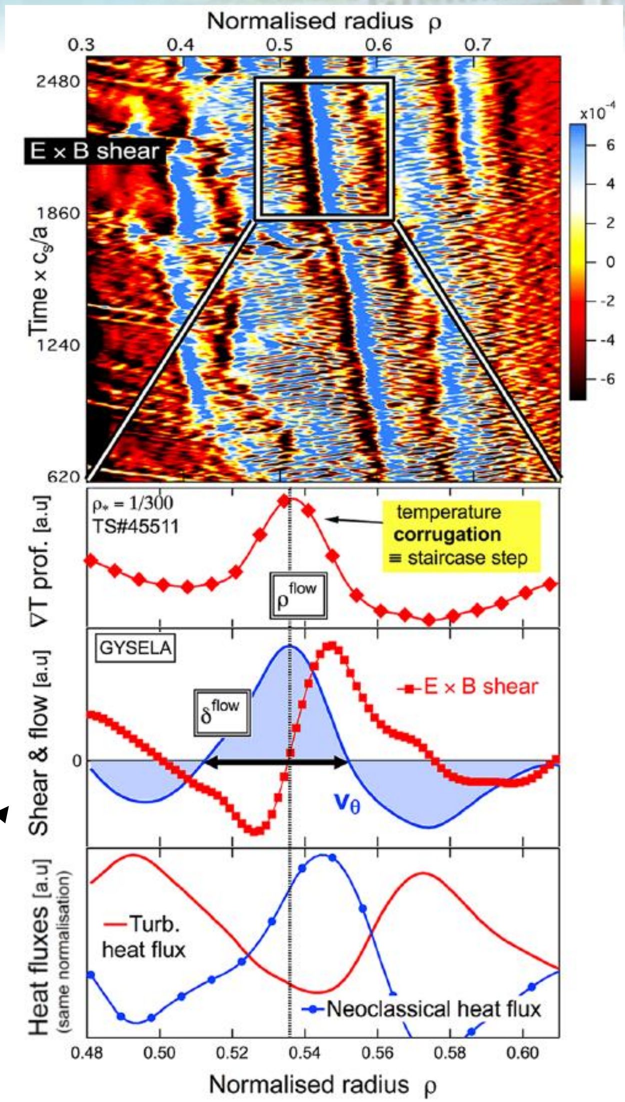


Figure 1 of G. Dif-Pradalier et al., PRL 2015 (GYSELA code)

Observation of Zonal flow staircase-like patterns in gyrokinetic simulations of magnetic fusion plasmas:

Flux-driven *full-f gyrokinetic simulations of ion temperature gradient (ITG) driven transport:*

G. Dif-Pradalier et al., PRE 2010, PRL 2015, NF 2017; W. Wang NF 2018, 2020, etc..

Gradient-driven δf gyrokinetic simulations of ITG: *L. Villard et al., J. of Phys. 2014* etc..

Gradient-driven δf gyrokinetic simulations of Trapped Electron Mode (TEM) driven turbulence: *L. Qi et al., NF 2019, NF 2020 in submission*.

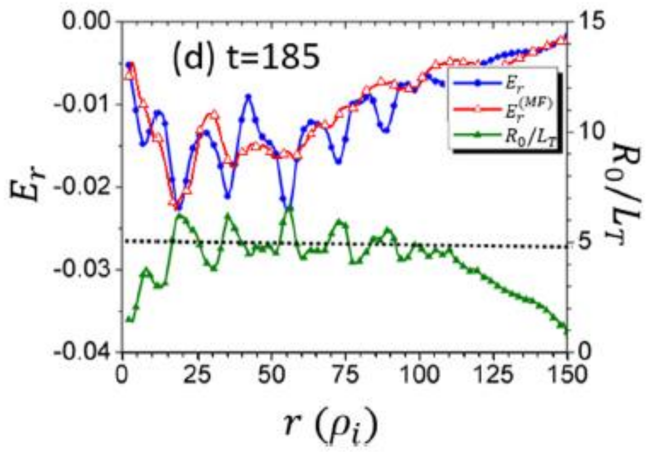


Figure 4 of W. Wang et al., NF 2018 (GKNET code)

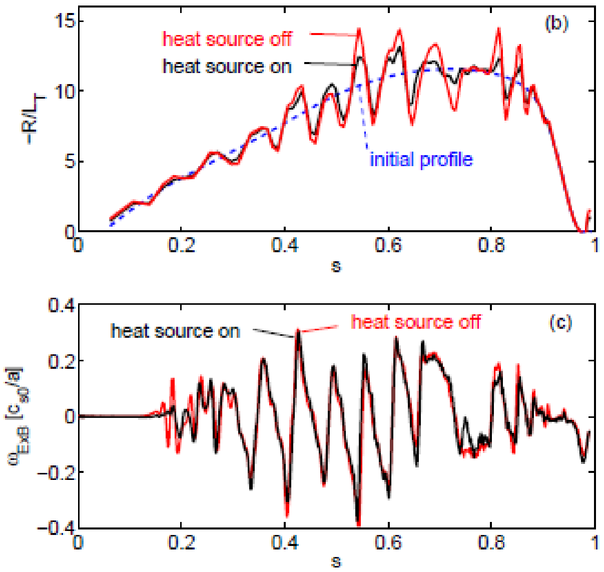


Figure 3 of L. Villard et al., 2014 (ORB5 code)

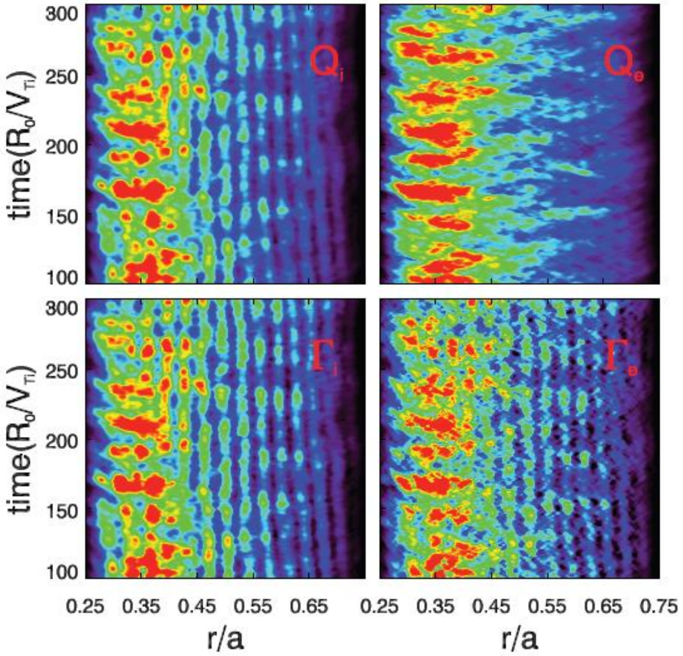
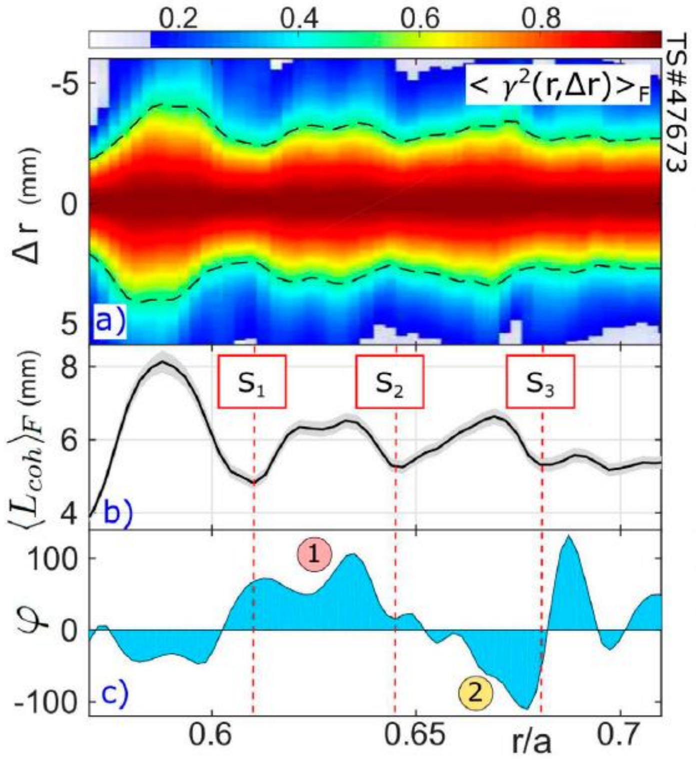
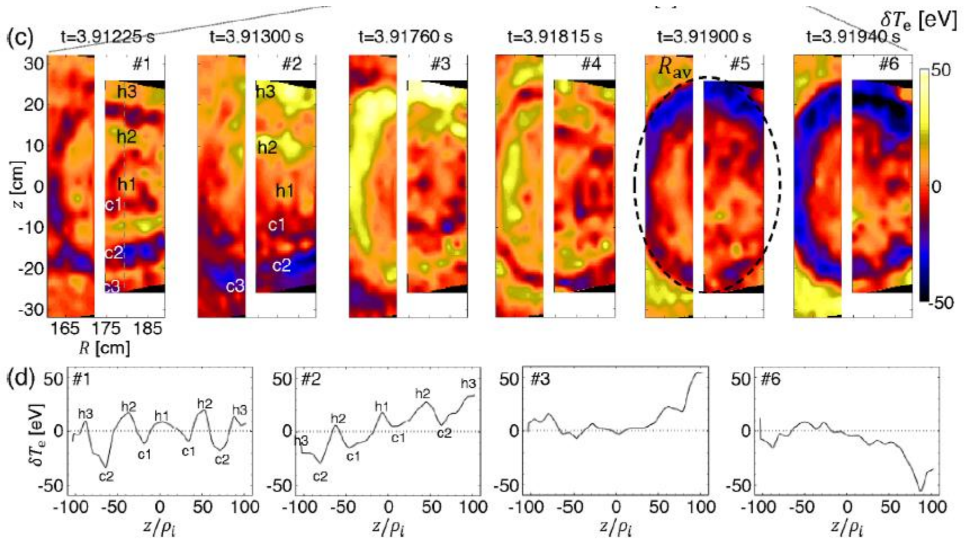


Figure 6 of L. Qi et al., NF 2019 (gKPSP code)

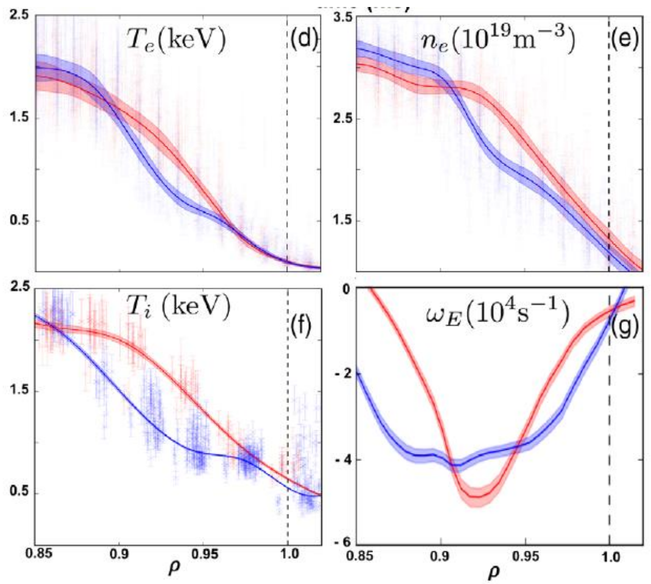
Evidence of zonal flow staircase in tokamak plasmas:
According to our knowledge, so far it has been difficult to measure the global profile of zonal flow in experiments, while the existence of zonal flow staircase could be deduced indirectly in other ways.



Turbulence coherence length measured in Tore Supra. From Fig. 1 of G. Hornung et al., NF 2017



Corrugations in the profile of electron temperature fluctuation in KSTAR. From Fig. 8 of M. J. Choi et al., NF 2019



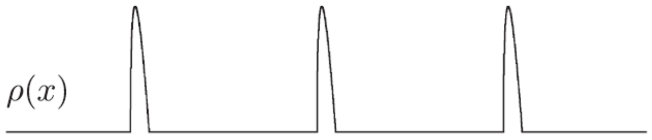
Staircase-like structure in temperature and density profiles (pedestal) in DIII-D. From A. Ashourvan et al., PRL 2019

Modeling of zonal flow staircase: two approaches

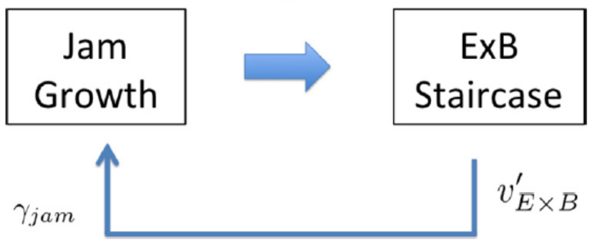
An initial value problem: based on the dynamics of propagation of heat flux modulations.

Traffic-jam model

Heat avalanche	↔	traffic flow
Temperature δT_e	↔	car density perturbation $\delta\rho$
Corrugation	↔	jam



From Fig. 2 in ref. Y. Kosuga et al., PRL 2013



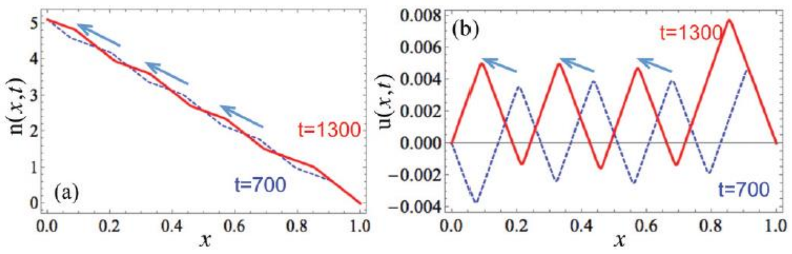
From Fig. 7 in ref. Y. Kosuga et al., PoP 2014

Feedback loop: Once jamming instability is initiated, profile starts corrugating to produce $E \times B$ staircases, which can feedback to the original jam instability.

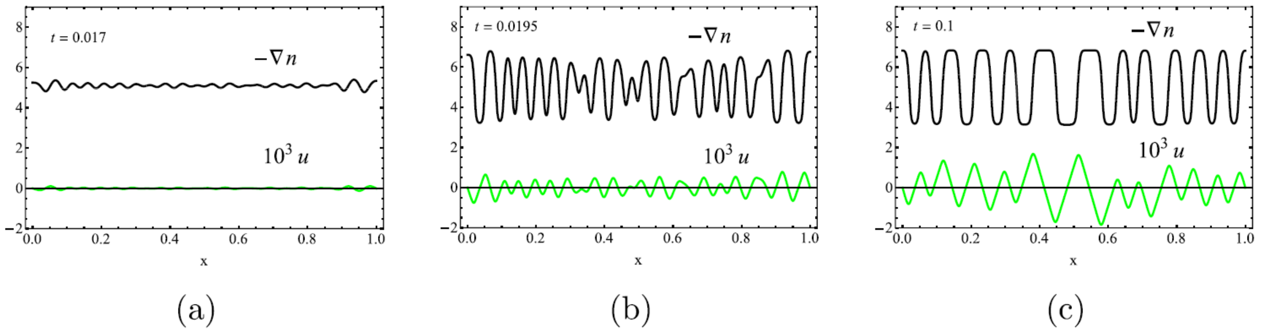
The key element is a time delay between perturbations in heat flux and gradient.

A boundary value problem: turbulence modulation grows, steepens to form staircase patterns.

This reduced model is based on the Hasegawa-Wakatani (HW) system of equations. Bistability and inhomogeneous turbulent (Potential Vorticity) mixing.



From Fig. 2 in ref. A. Ashourvan and P. H. Diamond, PRE 2016
Similar figure could also be found in ref. Weixing Guo et al., PPCF 2019



From Fig. 3 in ref. A. Ashourvan and P. H. Diamond, PoP 2017

Outlines

II. Gyrokinetic simulation of $\boldsymbol{E} \times \boldsymbol{B}$ staircase in KSTAR plasmas

II. GK simulation of $E \times B$ staircase in KSTAR plasmas-1

- Electron heat **avalanches** in the *MHD-quiescent L-mode* plasmas, which is achieved in the limiter configuration.
  - $q_0 > 1 \rightarrow$ no sawtooth
  - The 2/1 TM was unstable
    - ✓ Suppressed by the ECRH
- Transport events are observed during the MHD-quiescent period
  - $R_{\text{av}} \sim R_{q=2}$
- **Zonal flow staircase** activities:
  - Corrugations in electron temperature fluctuation
  - Regulations of the electron heat avalanches

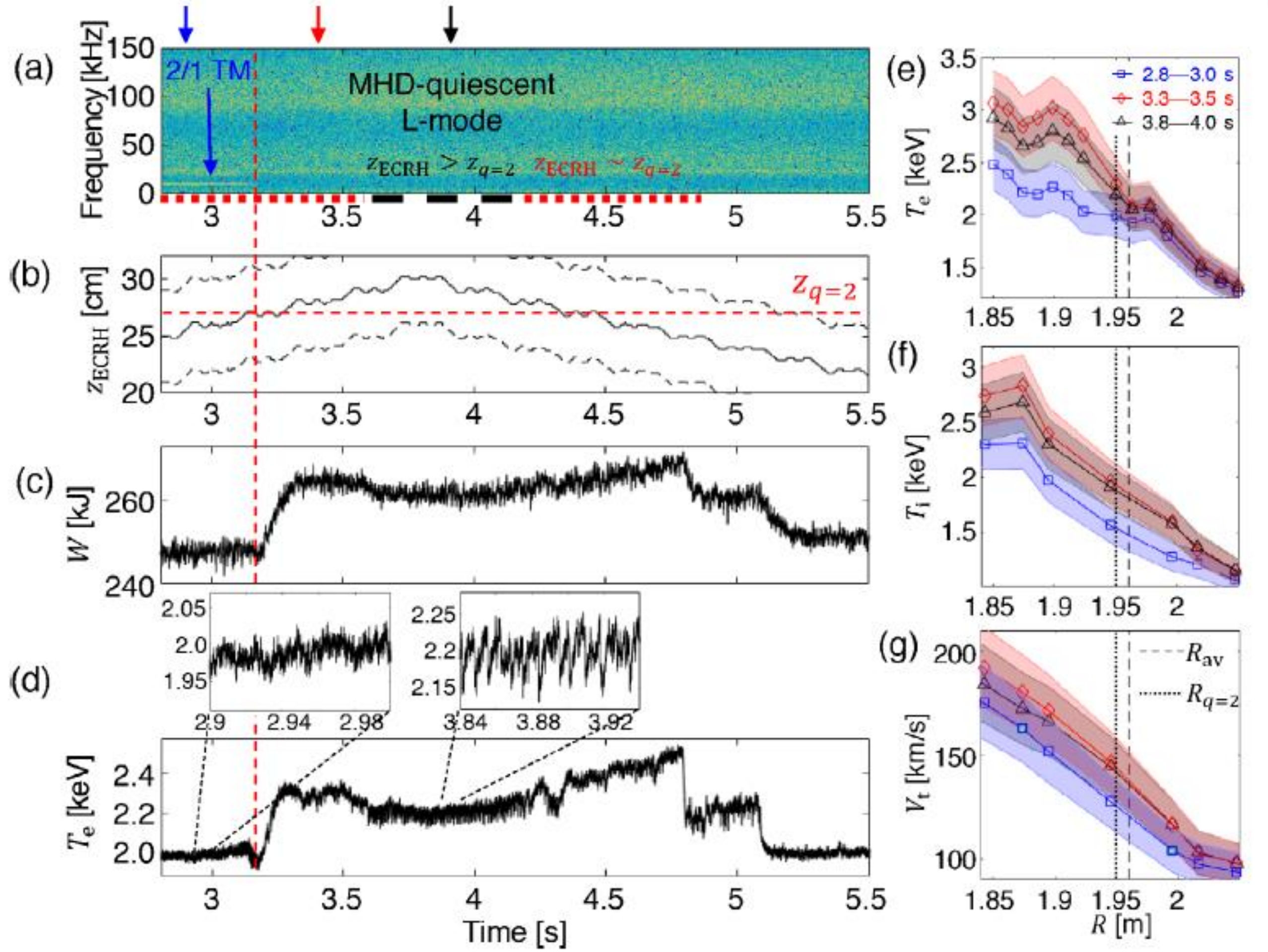
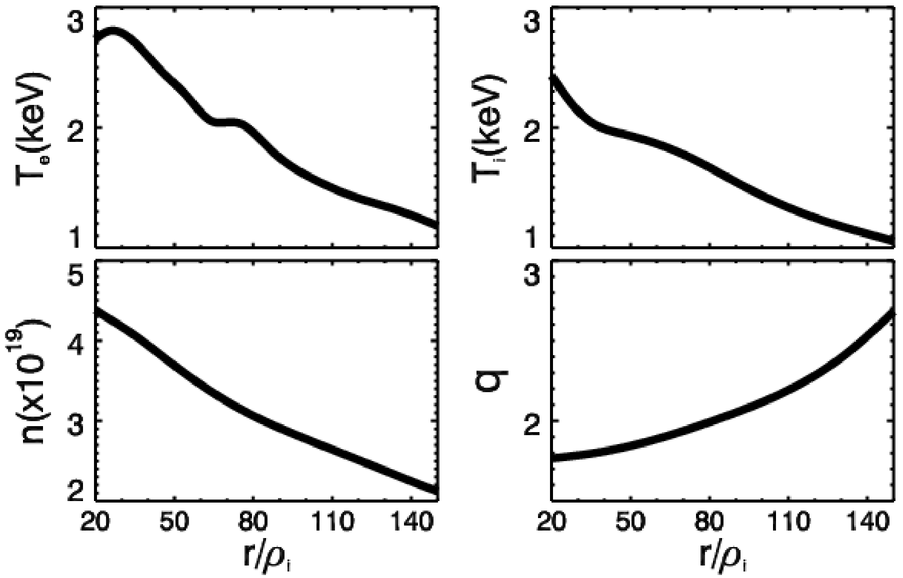


Fig. 1 of M. J. Choi et al, NF 2019

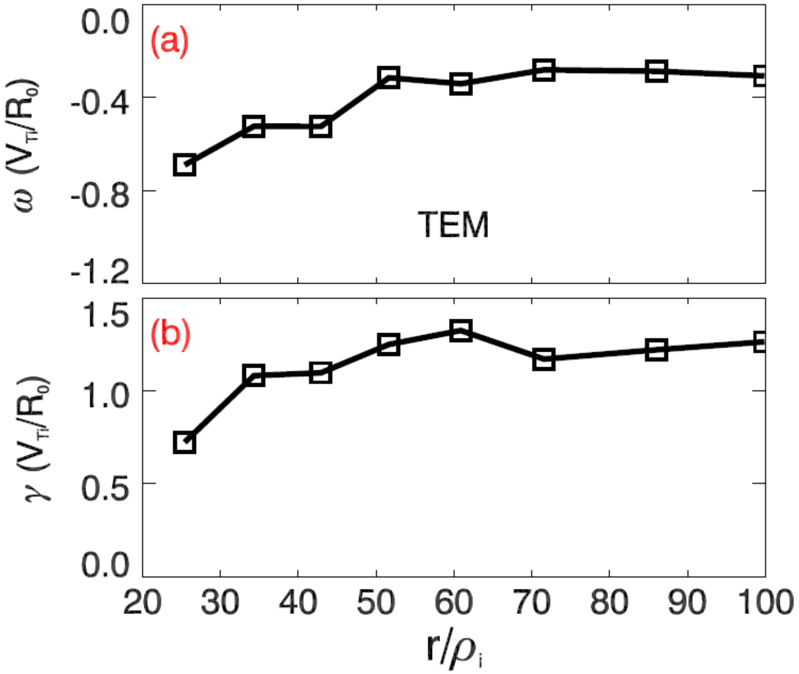
II. GK simulation of $E \times B$ staircase in KSTAR plasmas-2

Gyrokinetic simulation of the above KSTAR L-mode plasmas: (*Lei Qi et al., submitted to NF*)

- **Tool**: gKPSP, gradient-driven δf particle-in-cell (PIC) gyrokinetic code with bounce-averaged kinetic trapped electrons (ITG-TEM) (*J. M. Kwon et al., NF 2012, CPC 2017; Lei Qi et al., PoP 2016, NF 2017*). An axisymmetric heating source (only radial dependence) is applied to sustain the temperature and density profile.
- **Setup**: Realistic experimental profiles (T_e, T_i, n_e) and experimental configuration from EFIT, q-profile, plasma shape, etc.. Equilibrium $E \times B$ flow not included.



Profiles from experimental measurement as inputs



Linear simulation shows TEM dominance in this plasma
$\gamma/\omega \sim 2-3$, not close to the linear marginality.

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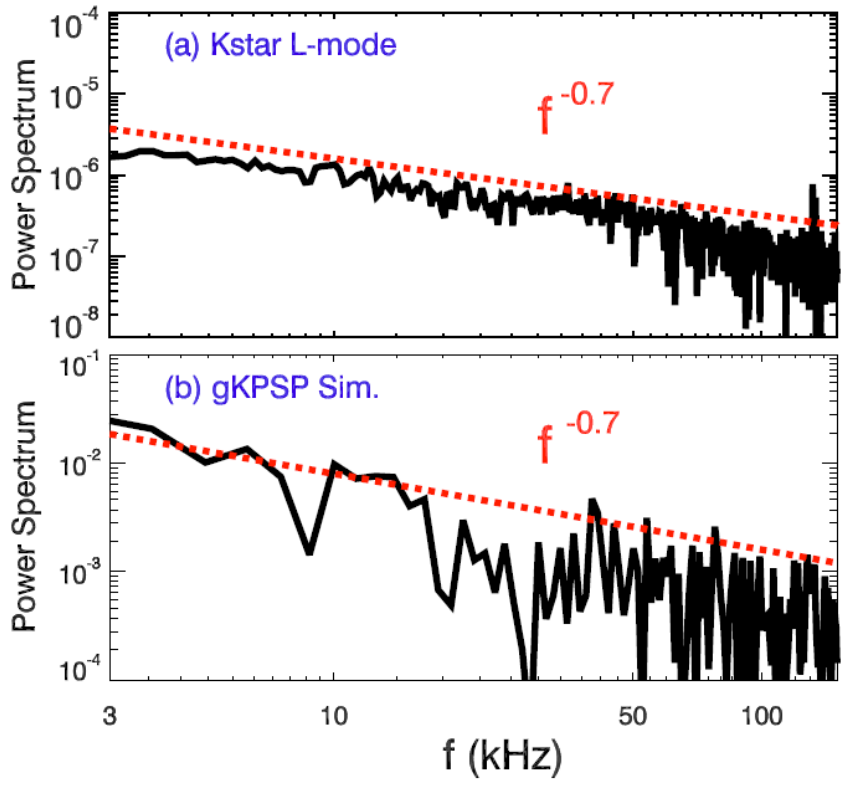
II. GK simulation of $E \times B$ staircase in KSTAR plasmas-3

Comparisons between **nonlinear** GK simulation and experiment are mainly in **three** aspects:

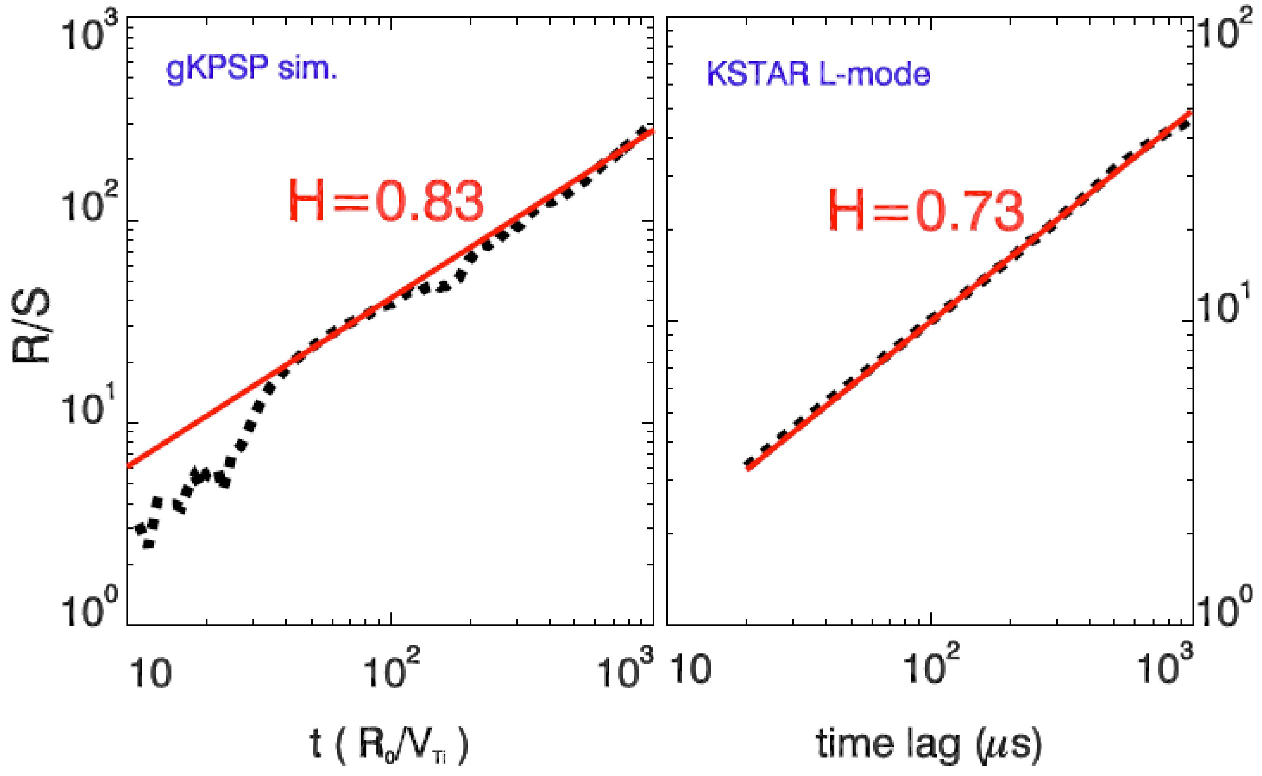
- Power spectra scaling $S(f) \sim f^{-0.7}$ → close to $1/f$ indicating events at all scales can occur



Power spectra scaling of $|\delta T_e(f)|^2$: $S(f) \sim f^{-0.7}$

Comparisons between **nonlinear** GK simulation and experiment are mainly in **three** aspects:

- Hurst exponent factor (*B. A. Carreras et al., PRL 1998*) $H \sim 0.7 > 0.5$ →transport events can occur at all scales, $H = 1$ gives $1/f$



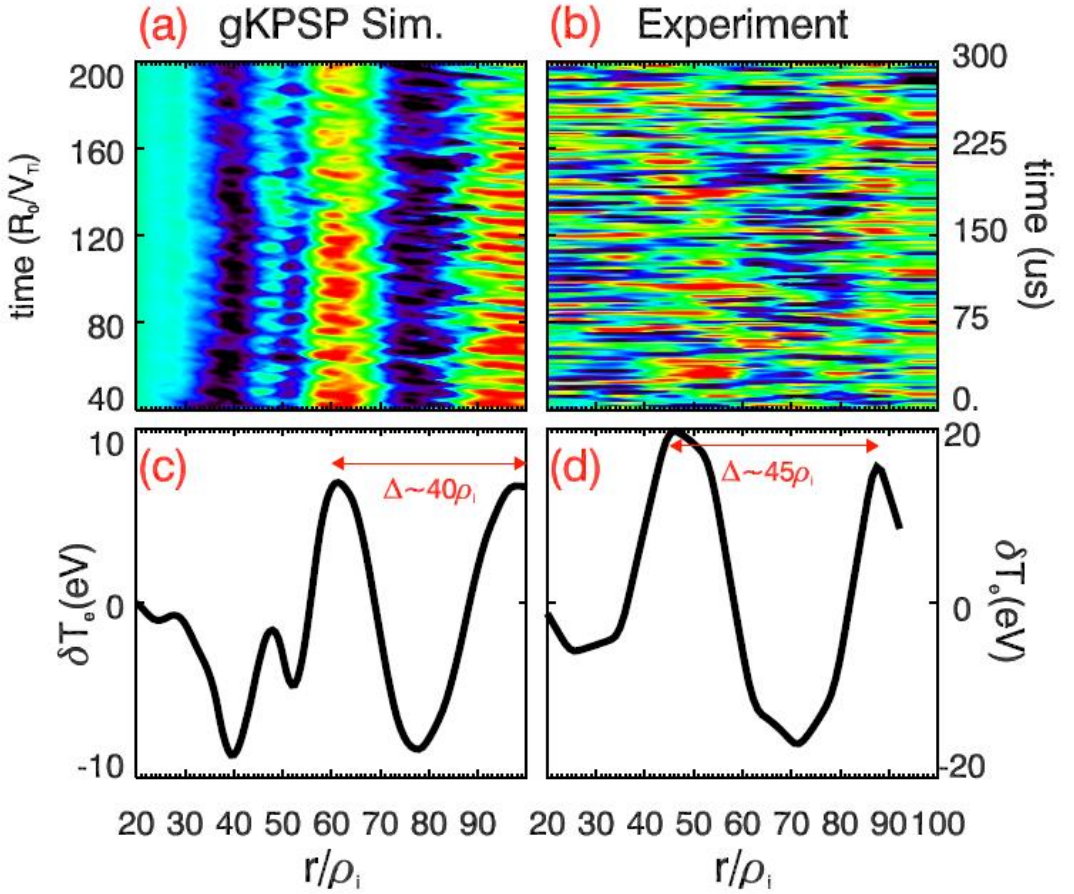
Hurst exponent $H = 0.83$ from GK simulation and $H = 0.73$ from the experiment.

II. GK simulation of $E \times B$ staircase in KSTAR plasmas-5

Comparisons in between **nonlinear** GK simulation and experiment are mainly in **three** aspects:

- Profile of electron temperature fluctuation with corrugation scale $\Delta \sim 45\rho_i$
  → close to zonal flow staircase step size, about several times the turbulence correlation length

Profile of electron temperature fluctuation with corrugation scale $\Delta \sim 40\rho_i$ from GK simulation and $\Delta \sim 45\rho_i$ from the experiment.



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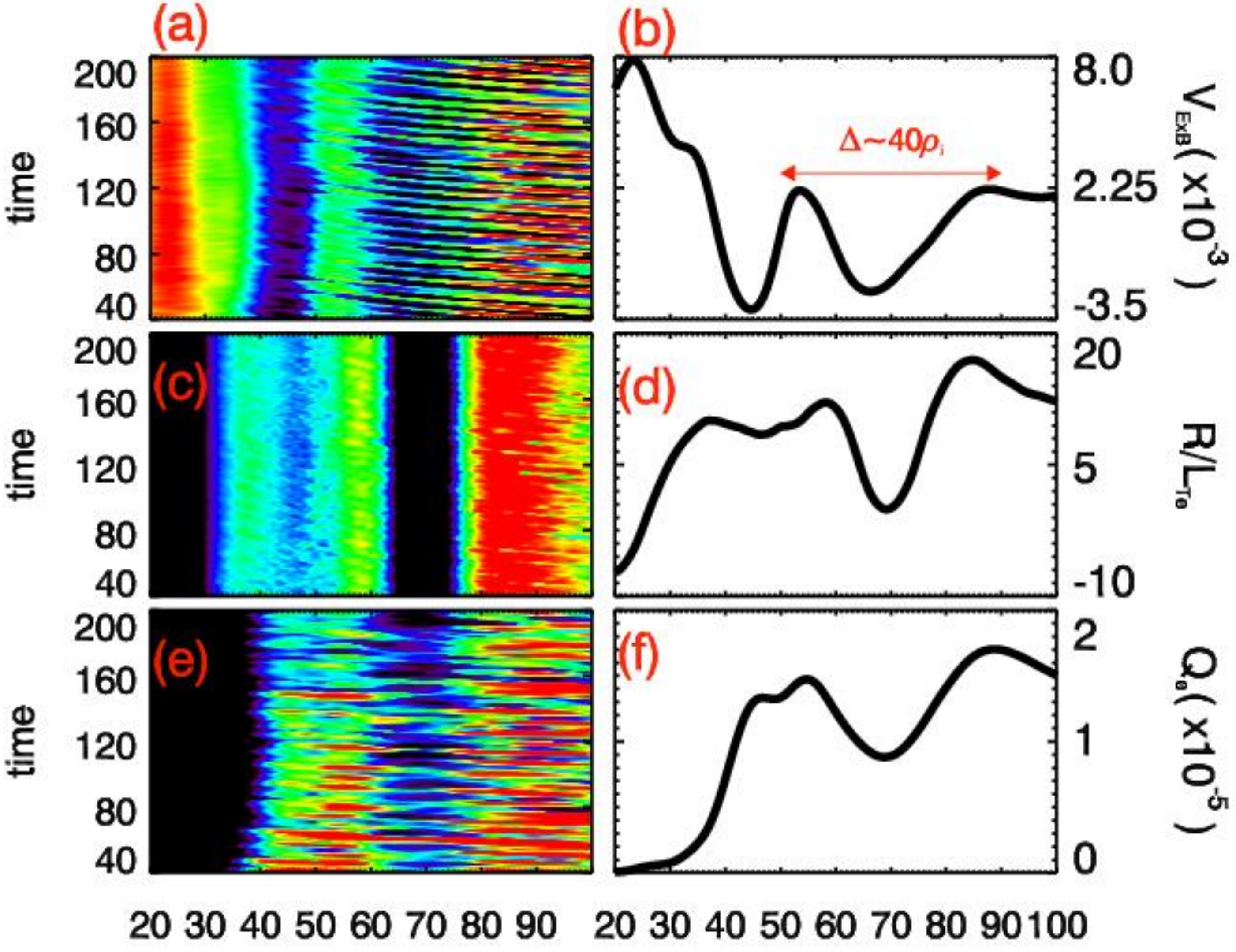
II. GK simulation of $E \times B$ staircase in KSTAR plasmas-6

Zonal flow staircase from the nonlinear gyrokinetic simulation is found to be **responsible** regulating the avalanches.

1. Leading to the electron temperature fluctuation radial corrugations.
2. Zonal flow corrugation scale $\boldsymbol{\Delta \sim 40\rho_i}$, which is about several times the micro turbulence correlation length (next slide) consistent with $\boldsymbol{\delta T_e}$ radial corrugation
3. Regulations in electron heat avalanches.



(a), (b) Zonal flow $V_{E\times B}$
(c),(d) Normalized electron temperature gradient R/L_{Te}
(e), (f) Electron heat flux Q_e

Zonal flow staircase shearing effects: on the profile corrugations

(a) The **long-live** staircase shearing

$$\omega_{ss} \equiv \left| \frac{\partial < V_{E\times B} >_t}{\partial r} \right|$$

(b) Mean fluctuation of normalized electron temperature gradient, absolute values

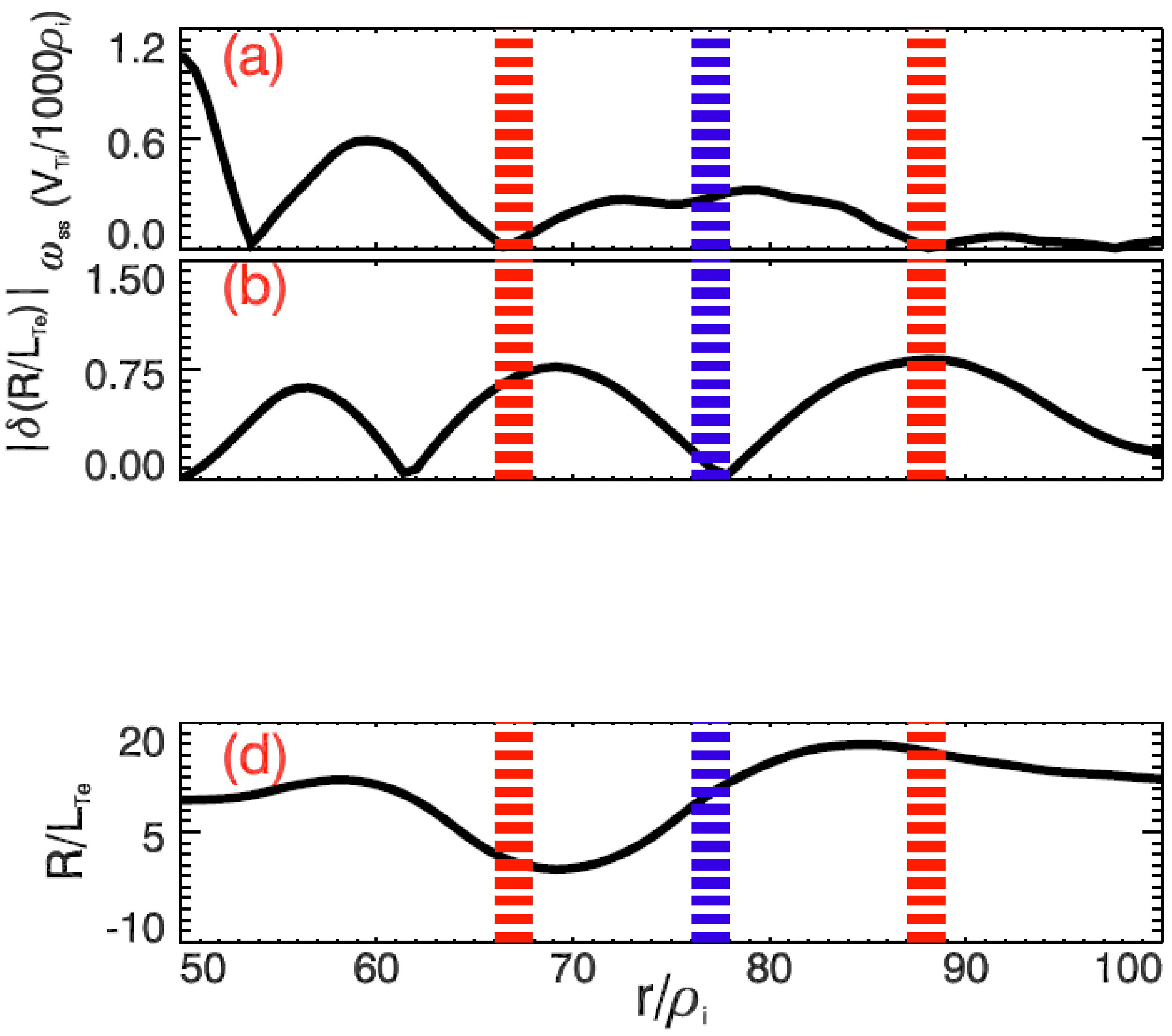
$$|\delta(R/L_{Te})|$$

Cross correlation $R_{\omega_{ss}|\delta(R/L_{Te})|} = -0.59$ (highly correlated with negative radial phase).

In contrast, $R_{\omega_{ss}R/L_{Te}} = -0.062$(no close cross correlation between the staircase shearing and the gradient).

This indicates:

The absolute value of electron temperature gradient fluctuation is probably decreasing at a radial location with increasing staircase shearing and vice versa.



Zonal flow staircase shearing effects: on the turbulence correlation

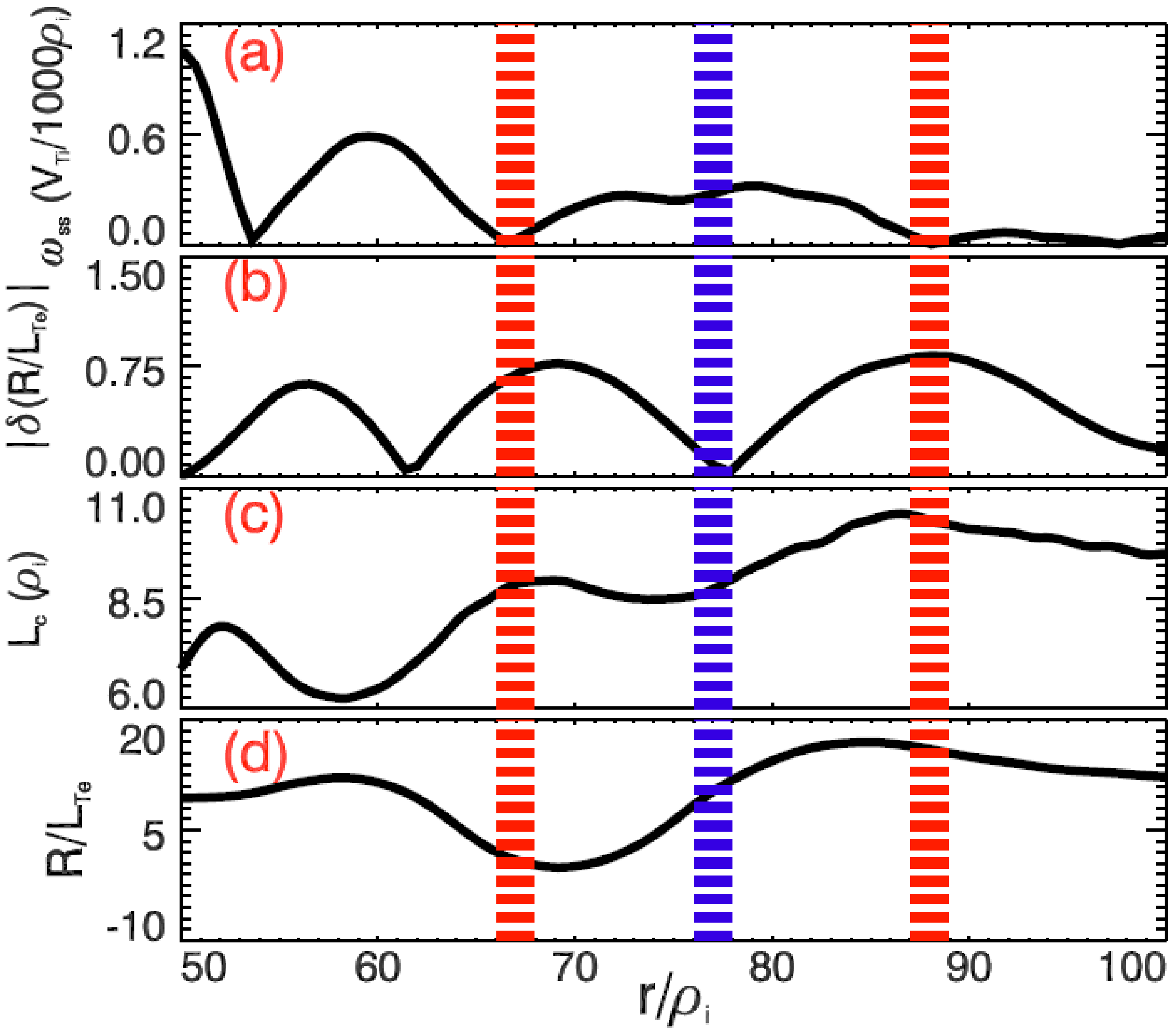
(c) Turbulence correlation length as a function of radius

Two-point correlation

$$C(r,\Delta r,\Delta\zeta,t)=\frac{\langle\delta\phi(r,\zeta,t)\delta\phi(r+\delta r,\zeta+\Delta\zeta,t)\rangle_\zeta}{\sqrt{\langle\delta\phi^2(r,\zeta,t)\rangle_\zeta\langle\delta\phi^2(r+\delta r,\zeta+\Delta\zeta,t)\rangle_\zeta}}.$$

By taking maximum over $\Delta\zeta$, and full width at the half maximum of $C(r,\Delta r,t)$, L_c is obtained.

Lower (higher) shearing corresponds to higher (lower) correlation, i.e., opposite phase relation.



II. GK simulation of $E\times B$ staircase in KSTAR plasmas-9

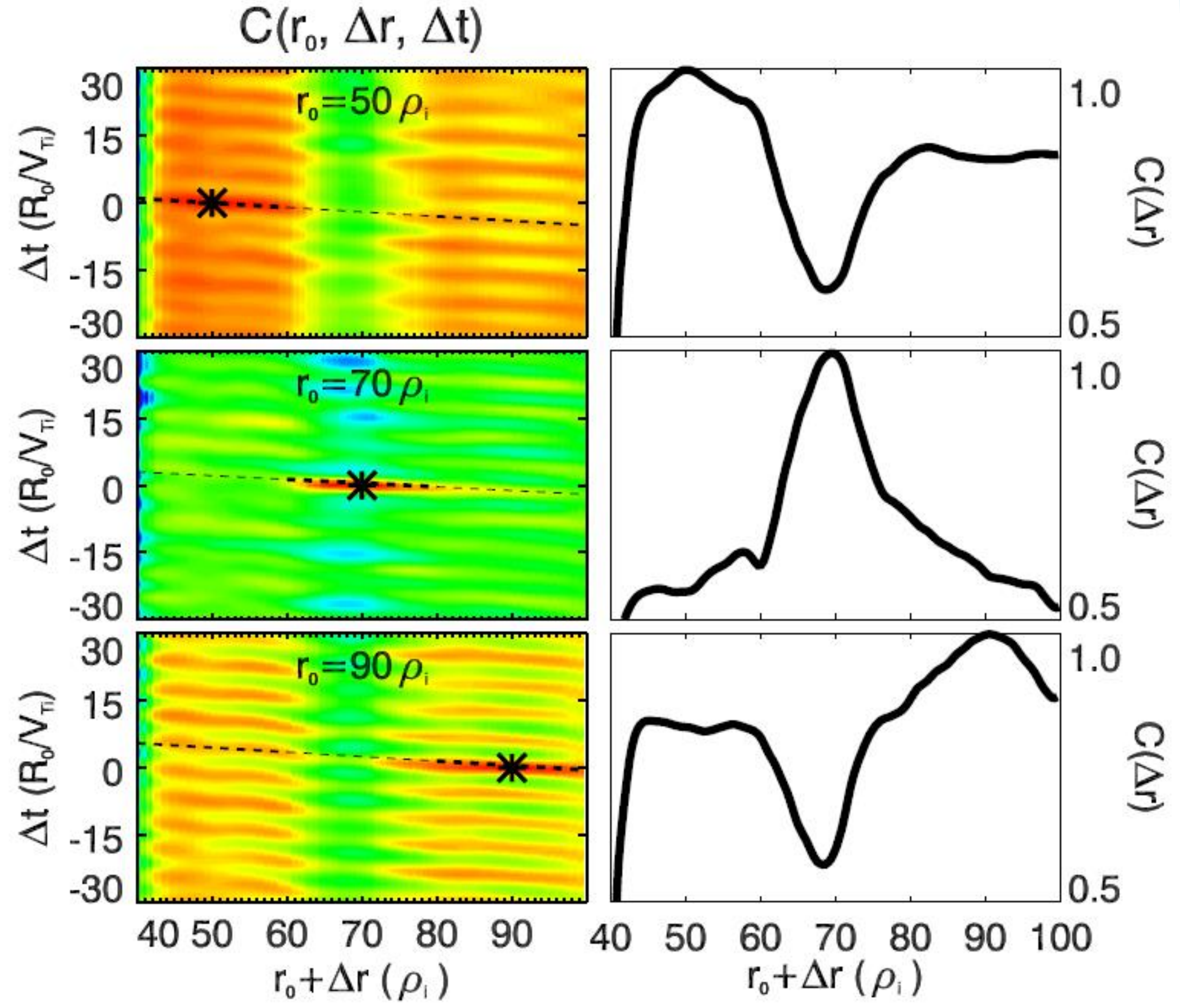
Zonal flow staircase shearing effects: on the heat avalanches

Spatial-temporal autocorrelation of flux-surface averaged electron heat flux $Q_e(r,t)$

$$C(r_0,\Delta r,\Delta t)=\frac{\langle Q_e(r_0,t)Q_e(r_0+\Delta r,t+\Delta t)\rangle_t}{\sqrt{\langle Q_e^2(r_0,t)\rangle_t\langle Q_e^2(r_0+\Delta r,t+\Delta t)\rangle_t}}$$

Note that $\boldsymbol{r_0=70\rho_i}$ **is where a staircase valley is located**.

The figure demonstrates that large scale (**mesoscopic** scale at least $\sim 60\rho_i$, which covers several times the microturbulence correlation length), **intermittent** and **inwardly propagating** avalanches should have occurred but intercepted (regulated) by the zonal flow staircase.



Outlines

III. Gyrokinetic simulation of $\boldsymbol{E} \times \boldsymbol{B}$ staircase in comparison with models

III. GK simulation of $E \times B$ staircase: in comparison with models-1

Phenomena: staircase-like structures in zonal flow and pressure profiles. Comparisons are between:
Simulation:
gyrokinetic ion and bounce-kinetic trapped electron; collisionless trapped electron mode (CTEM)
Models:
- ✓ *Reduced model* based on Hasegawa-Wakatani (HW) equations and near-adiabatic electron; collisional drift wave turbulence;
- ✓ *Traffic-jam model*

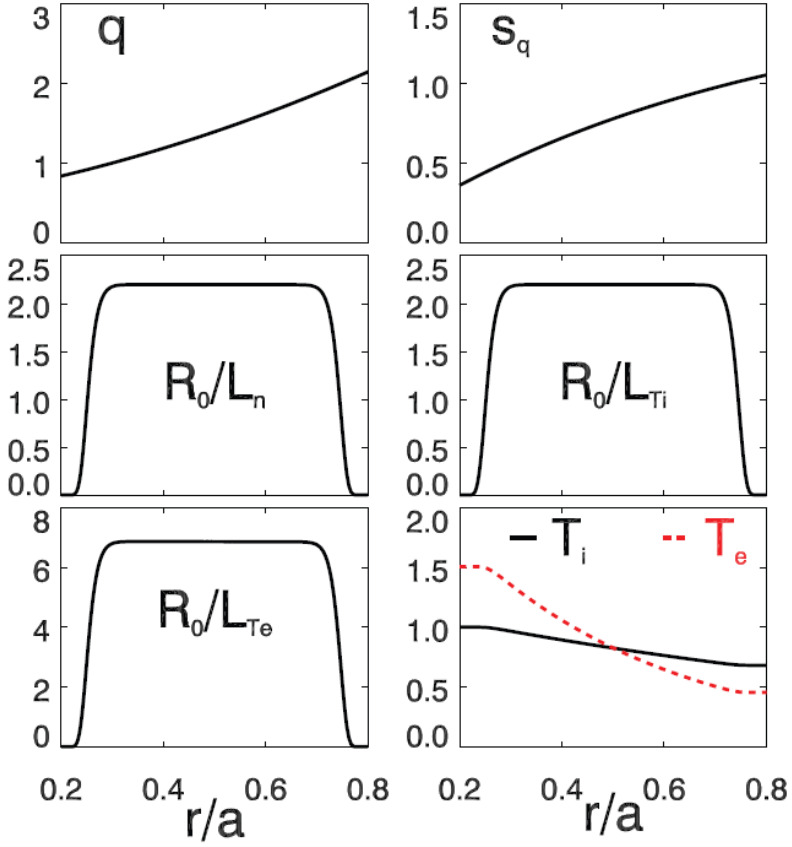
Mainly in the following aspects:
- ✓ Upward escalating propagation of staircases?
- ✓ Time delay between staircases in heat flux and gradients?

Causality among occurrence of staircases in profiles of zonal flow, pressure, transport and turbulence?

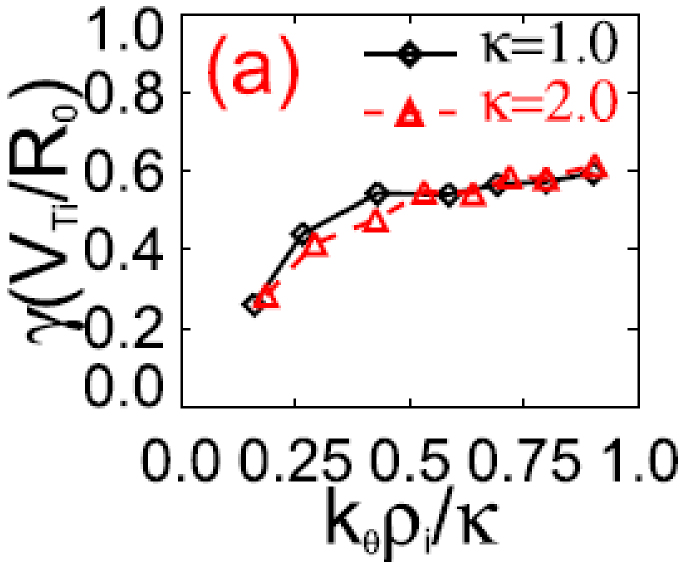
Role of potential vorticity (PV)?

III. GK simulation of $E\times B$ staircase: in comparison with models-2

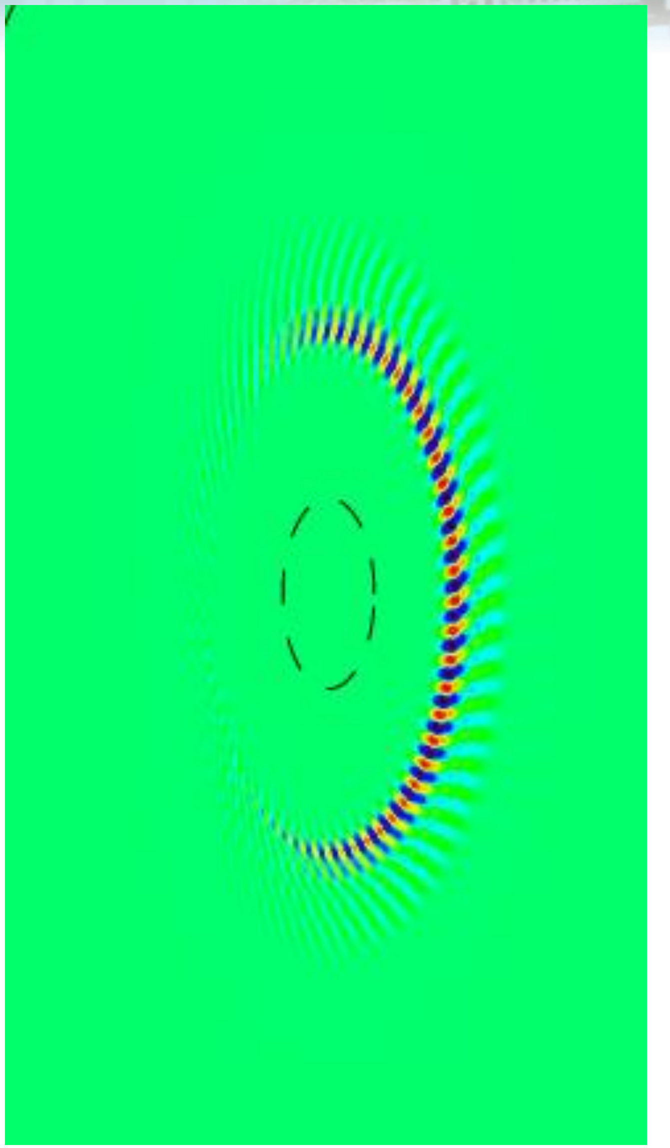
Based on cyclone-like parameters: $R_0/L_n = 2.2, R_0/L_{Ti} = 2.2, R_0/L_{Te} = 6.9$, elongation $\kappa = 2$; a pure TEM case with stable ITG. In most of simulation domain, profile gradients are flat.



Simulation profiles



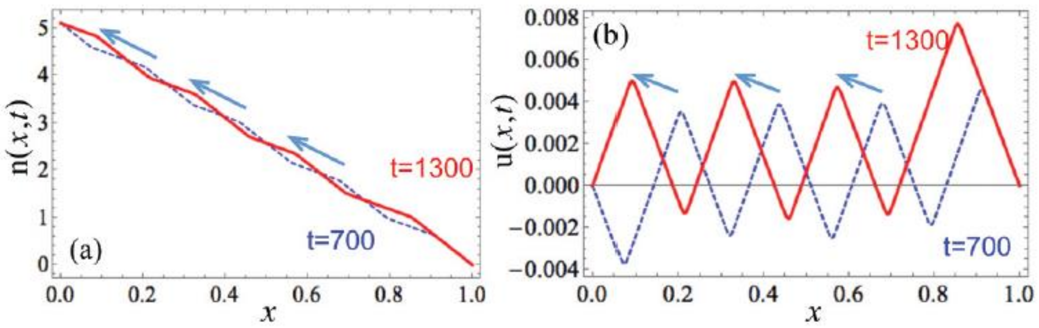
Linear growth rate as a function of $k_\theta \rho_i$



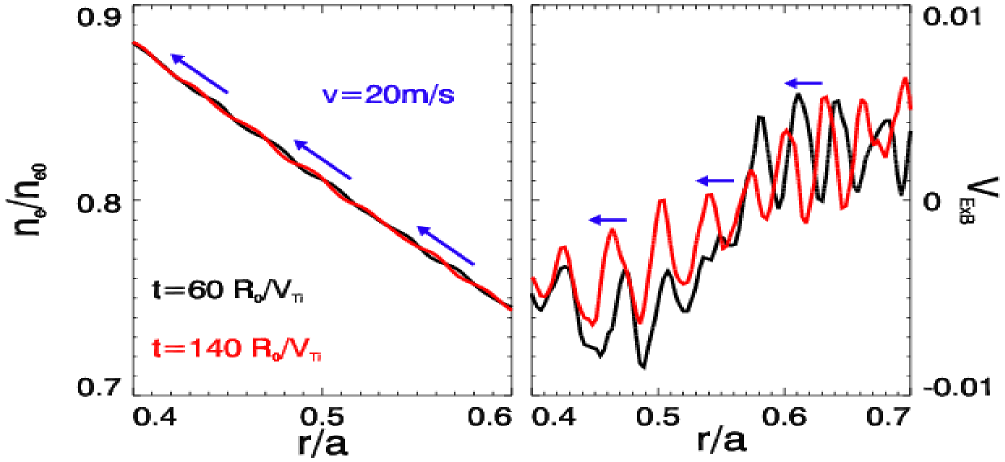
Eigenmode structure in poloidal cross section

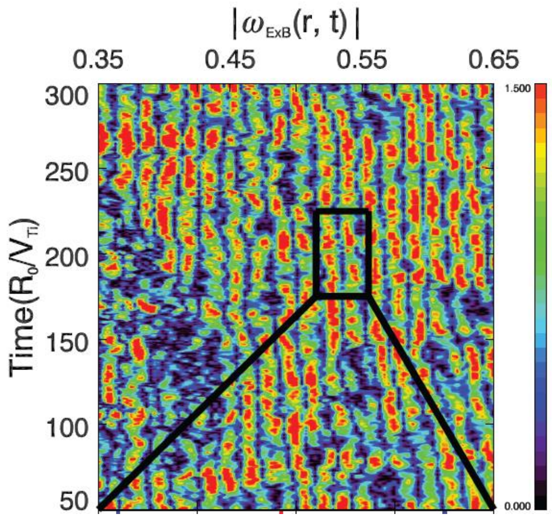
III. GK simulation of $E \times B$ staircase: in comparison with models-3

Upward escalating propagation of staircases observed in the simulations:



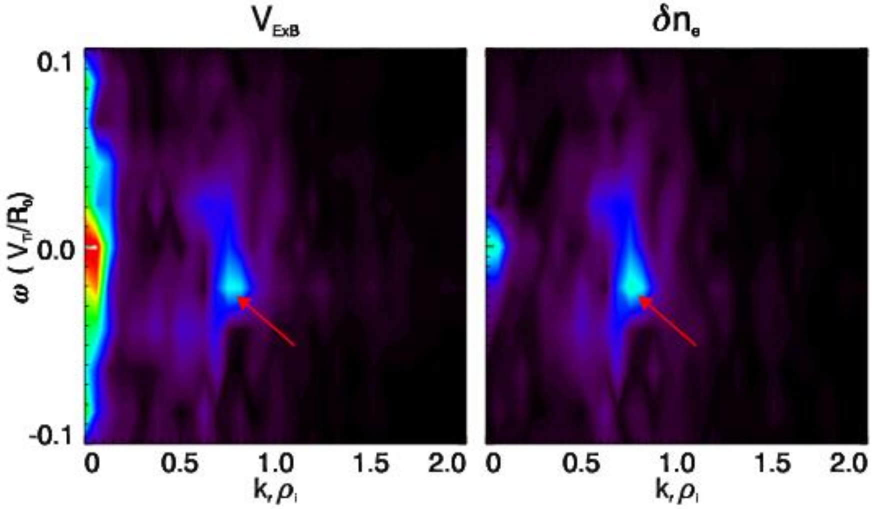
Prediction from the reduced model (Fig. 2 of A. Ashourvan and P. H. Diamond, PRE 2016)





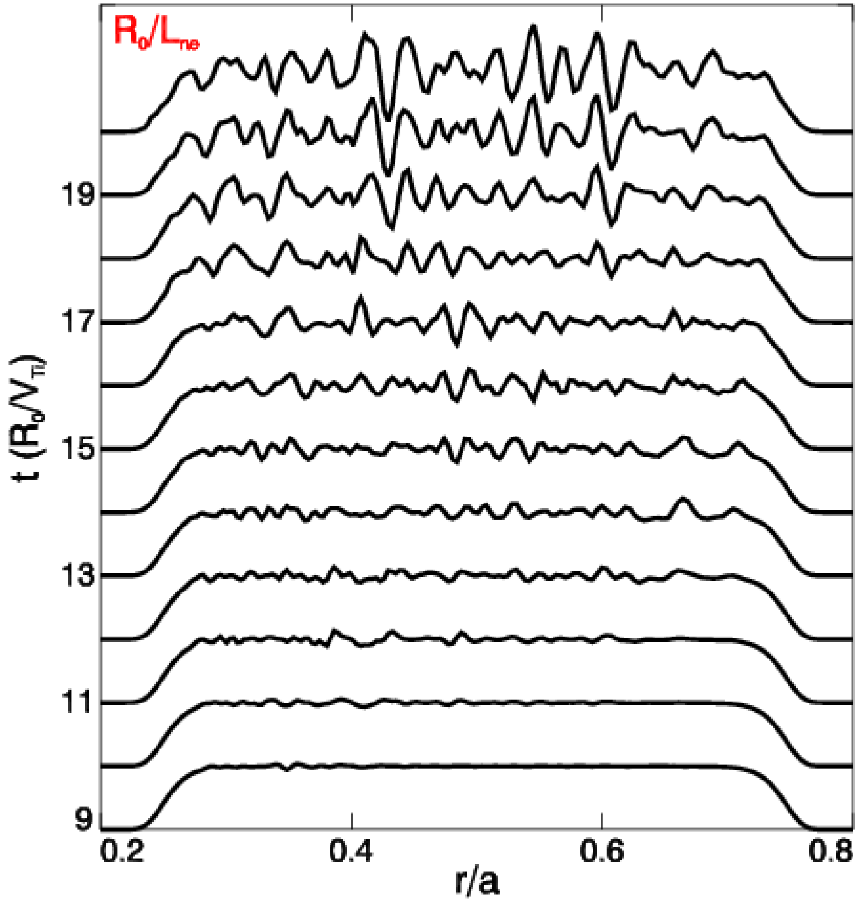
Zonal flow shearing in radius-time domain

This upward escalating is a consequence of the bistability. The propagation speed $v = 20\mathrm{m/s}$ is estimated by the dominant non-zero frequency and radial wave number as shown in the contour plot. This speed is relatively slow comparing to our simulation domain and time, therefore not evident but observable in the zonal flow shearing contour plot. Staircase scale $\Delta \sim 10\rho_i$, about twice the microturbulence correlation in this case.

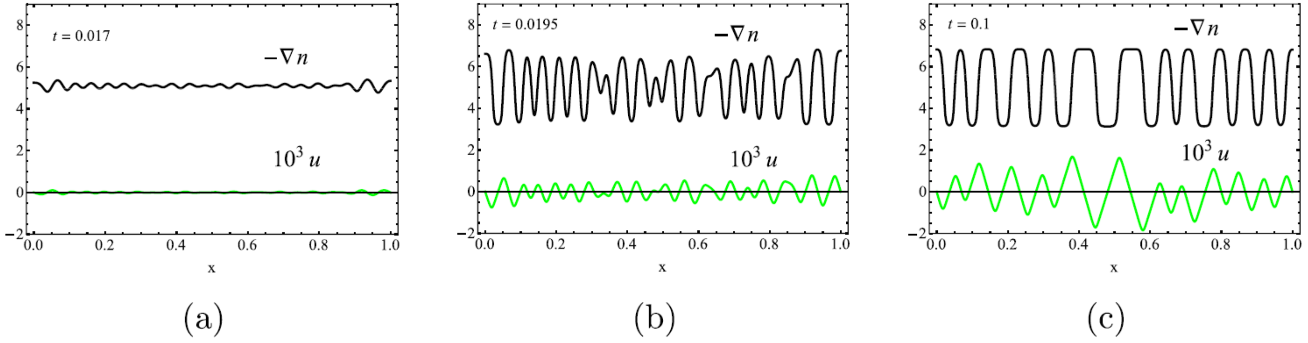


gKPSP simulation result

Time evolution of staircase-like structure formation:



gKPSP simulation result: time evolution of the profile of normalized electron density gradient



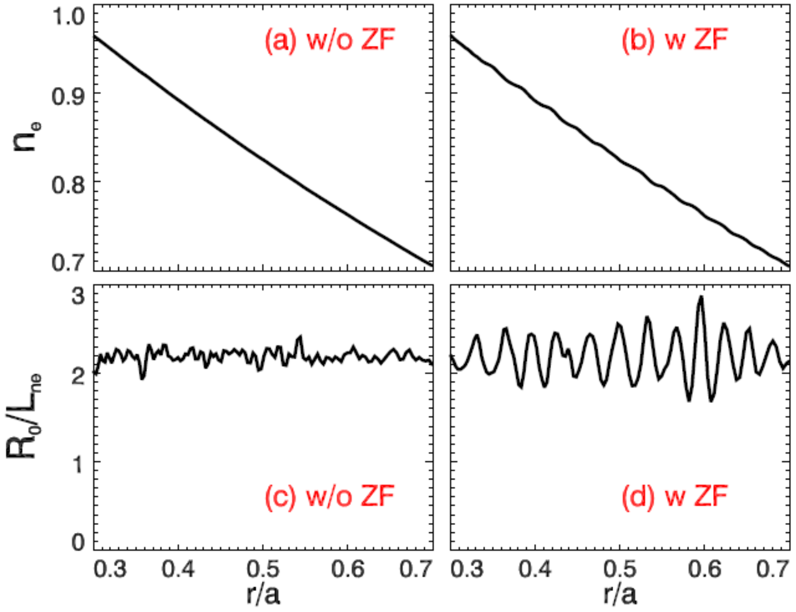
From Fig. 3 of A. Ashourvan and P. H. Diamond, PoP 2017

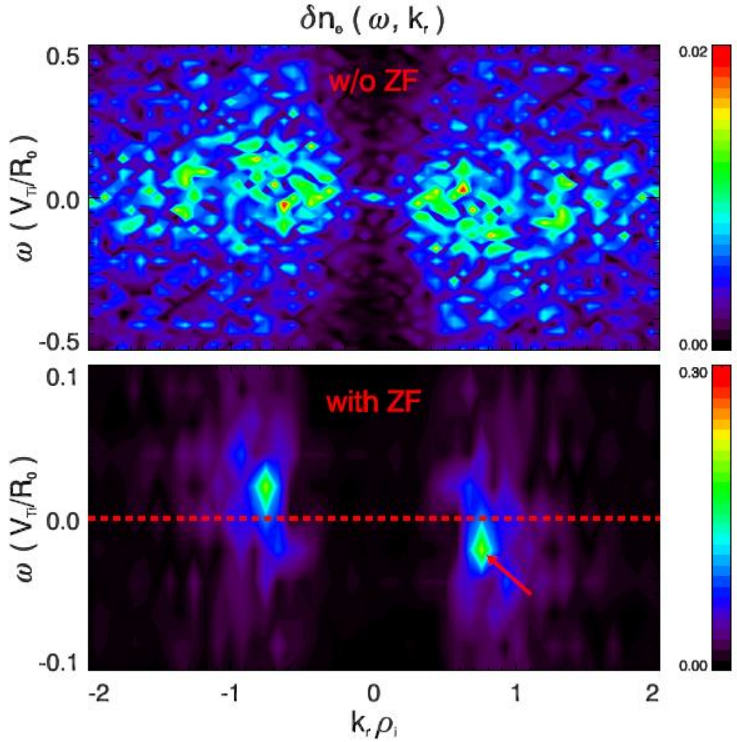
The staircase formation process in the simulation is quite similar to the predictions from the reduced model with **Neumann** boundary conditions.

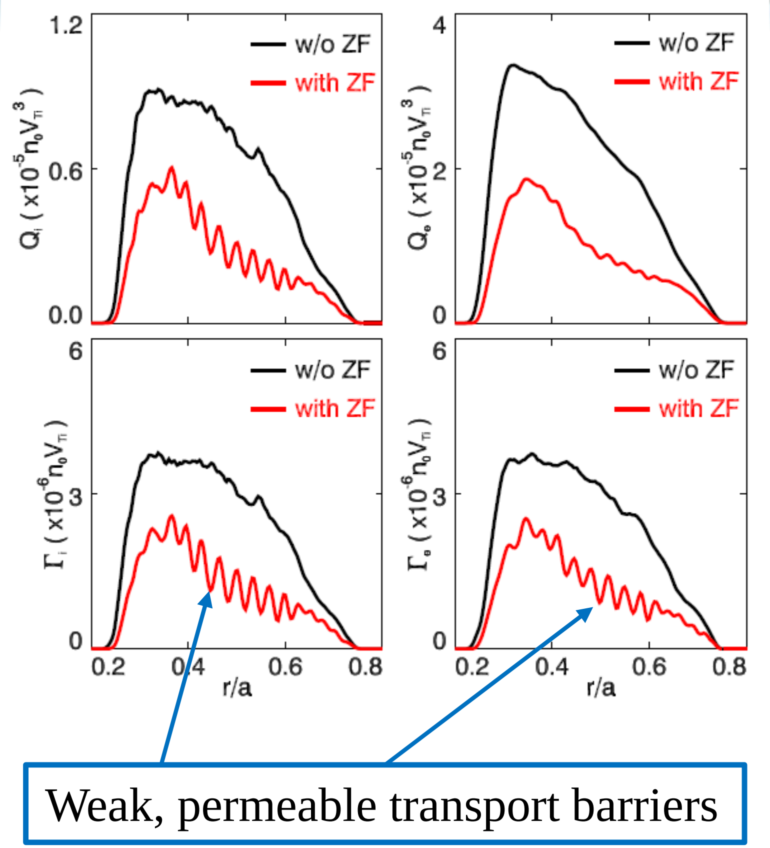
Note:
Though we only show the electron density profile here, the presented observation can also be made in other profiles, such as electron temperature (density), ion density, turbulent potential, electron (ion) heat and particle flux.

The causality: *what if we remove the zonal flow?*

We artificially remove the zonal flow in the simulation.





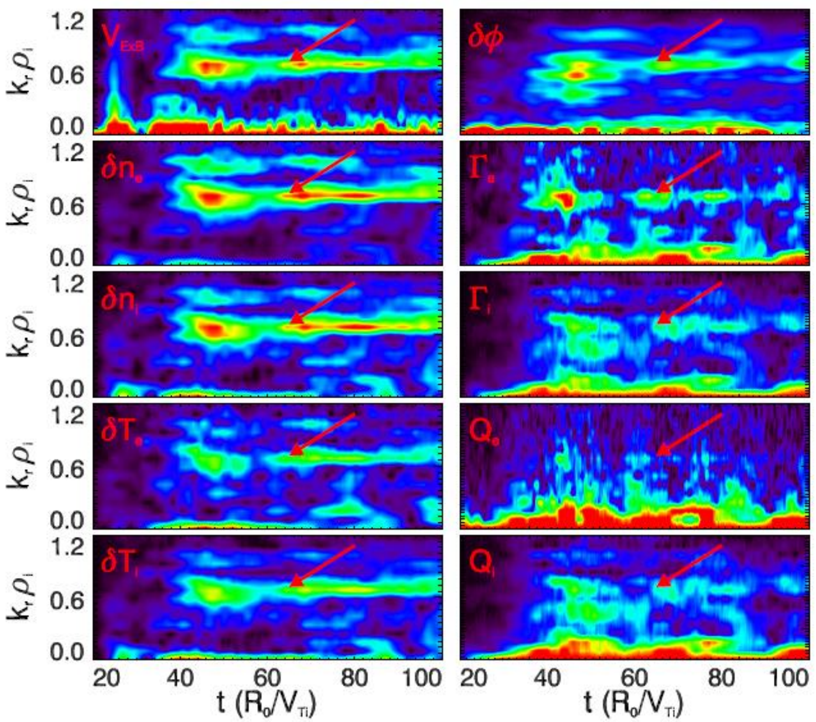


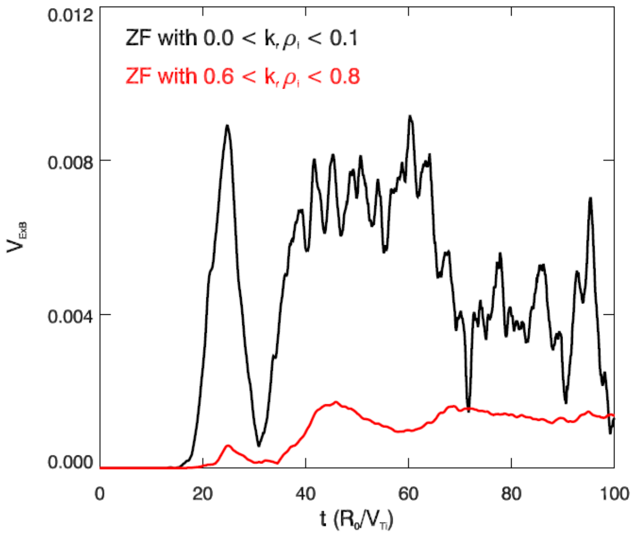
With zonal flow removed, staircase-like structures disappear without evident corrugations in the density (temperature) profiles and fluxes.

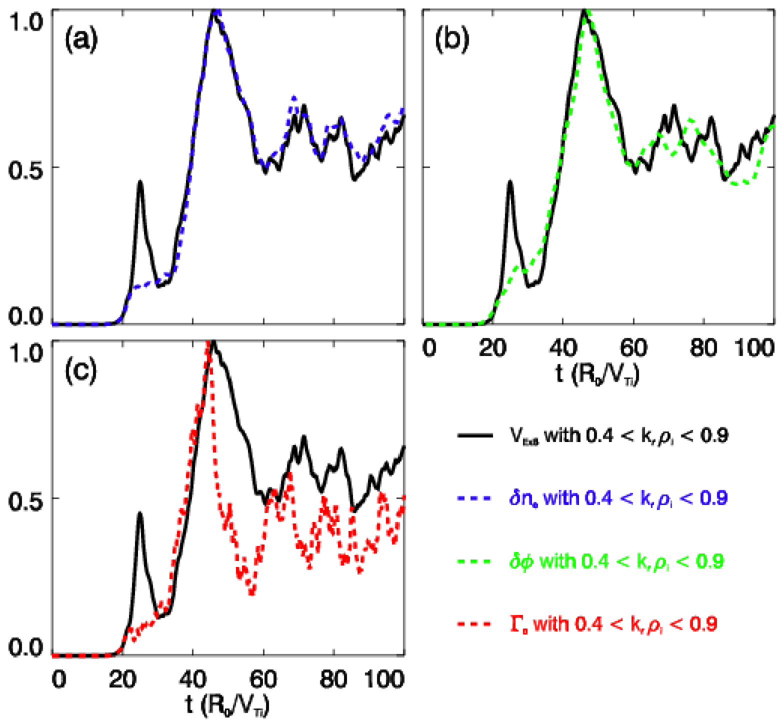
In other words, zonal flow causes the staircases in pressure and fluxes profiles.

III. GK simulation of $E\times B$ staircase: in comparison with models-6

Time delay in the formation of staircases in mean fluxes and gradients?





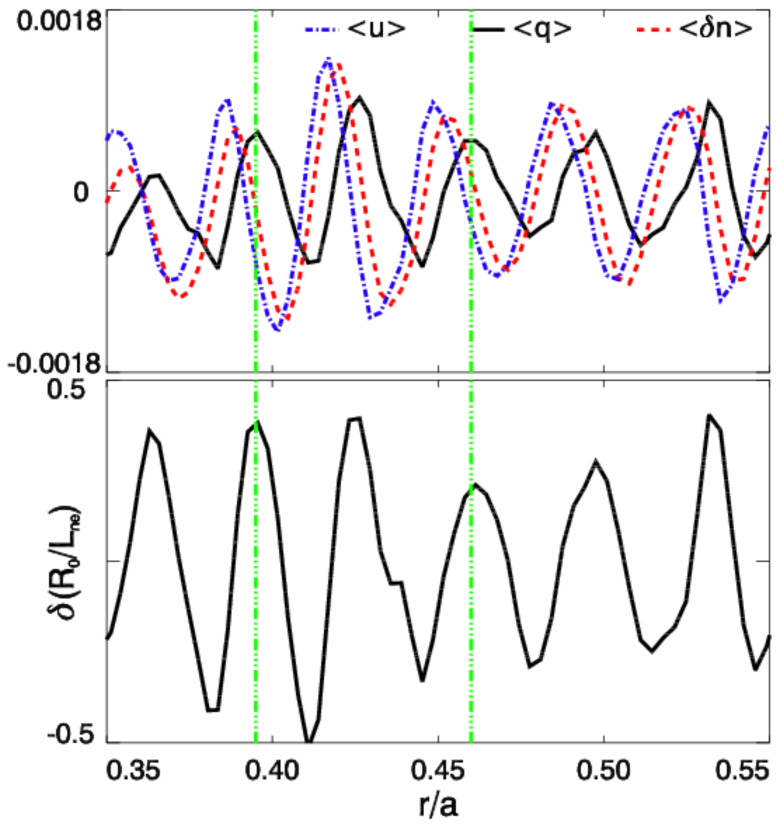


In this case, we find:

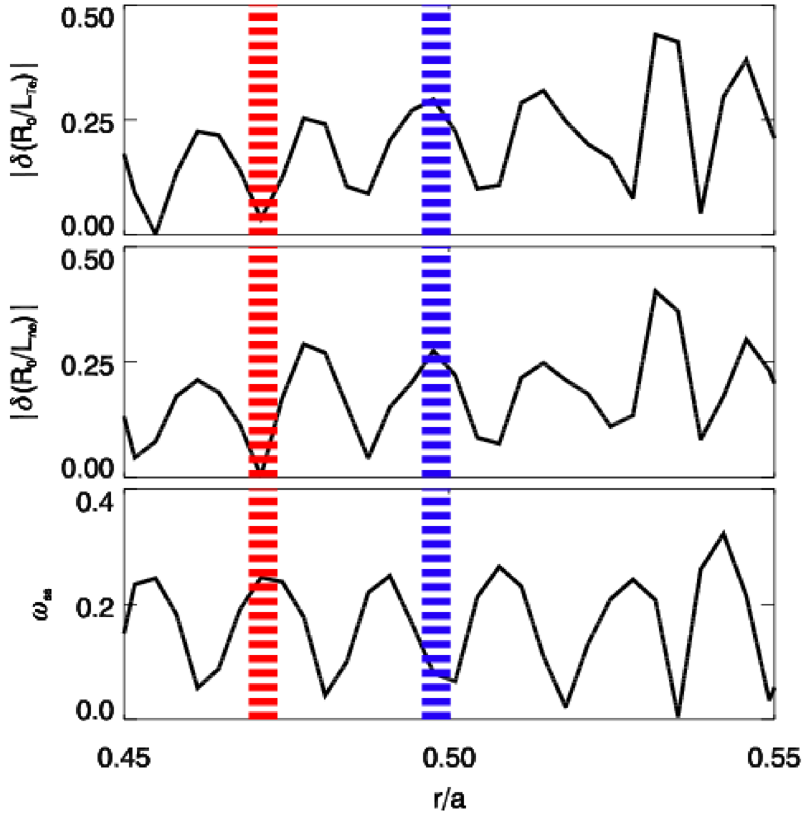
1. Staircases in zonal flow appear after the dominant zonal flow with $k_r\sim 0$, an evident time delay.
2. Staircases in zonal flow, density profile and particle flux grow up nearly simultaneously, no evident time delay.

Role of Potential Vorticity (PV):

We calculate the mean PV as $\langle q\rangle = \langle\delta n\rangle$-$\langle u\rangle$, the mean vorticity $\langle u\rangle$=$\langle\nabla_\perp^2\delta\phi\rangle$



Cross correlation between $\langle q\rangle$ and $\delta(R_0/L_{ne})$ is about 0.93, **very high positive correlation!**



Cross correlation between ω_{ss} and $|\delta(R_0/L_{ne})|$ is about -0.53, **high negative correlation!**

This shows the PV structure is highly correlated with the density profile corrugations. PV mixing has important roles in the formation of staircases.

Zonal flow staircase has shearing effects on the pressure gradient fluctuation, which seems through the PV.

IV. Summary and discussions-1

We have demonstrated the role of zonal flow staircase in the regulation of electron heat avalanches in a **KSTAR L-mode plasma** by performing gyrokinetic simulations:

1. Prevalent avalanches by showing consistent spectrum power law scaling $S(f)\sim f^{-0.7}$ and Hurst exponent factor $H\sim 0.7$
2. Zonal flow staircase is observed with a scale $\Delta\sim 40\rho_i$, which is about several times the microturbulence correlation length $l_c\sim 10\rho_i$ in this case.
3. Avalanches are regulated by zonal flow staircase. Large scale avalanches should have occurred but intercepted by zonal flow staircase.

Gyrokinetic simulation of **CTEM based on cyclone parameters** demonstrate:

1. **Upward escalating** and staircase formation process are similar to predictions of the reduced model based on the Hasegawa-Wakatani equations.
2. The time delay of staircases formation in fluxes and zonal flow is not evident in this case.
3. With zonal flow removed, staircases in pressure and flux profiles disappear.

Shearing effects of zonal flow staircase:

1. Through shearing the microturbulence radial correlation
2. Through shearing the **gradient fluctuation**. PV mixing may play significant roles in this process. Both experimental and cyclone-like parameters exhibit this.

IV. Summary and discussions-2

Remaining conundrums:

- What parameters can affect the generation of staircase and determine the scale, radial location and strength?
- Can we control the generation of staircases, the scale, radial location, strength, and thereby energy confinement?
- More…… We are looking forward to exploring more in this topic.