$E \times B$ staircase-like pattern formation in gyrokinetic simulations: a comparison with experiment and models

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I. Introduction of $E \times B$ staircase

II. Gyrokinetic simulation of $E \times B$ staircase in KSTAR plasmas

III. Gyrokinetic simulation of $E \times B$ staircase in comparison with models

IV. Summary and discussions
Staircase like structures are commonly observed in the nature: Potential vorticity staircase (D. G. Dritschel and M. E. McIntyre, J. Atmos. Sci. 2008)

Potential vorticity staircase observed in Jupiter’s weather layer based on Cassini data. Zonal wind (left), potential vorticity (PV) (middle) and PV gradient (right). Interesting cross-correlation between zonal wind and PV gradient: eastward jets are associated with large meridional PV gradients, most westward jets are associated with small meridional PV gradients.

Fig. 5 of F. J. Beron-Vera et al., J. Atmos. Sci. 65 (10):3316-3326 (2008)
I. Introduction of $E \times B$ staircase-2

Staircase like structures are commonly observed in the nature: thermohaline staircases

Observations of thermohaline staircases in Arctic Ocean. Staircase-like structures can be observed in the conductivity, temperature and depth (CTD) potential temperature profile.

Figure 5 of C. Stranne, et al., Scientific Reports 7, 15192 (2017)
I. Introduction of $E \times B$ staircase-3

In magnetic fusion plasmas: $E \times B$ staircases

**What we study here:**
The global pattern of mean $E \times B$ flow with zero toroidal and poloidal mode number $n = m = 0$ (i.e., zonal flow)

**Well known effects:**
Reduce the turbulent transport through shearing micro-turbulence and absorbing the energy. (Z. Lin et al., Science 1998; P. H. Diamond et al., PPCF 2005; L. Qi et al., NF 2017 ⋅⋅⋅)

**Importance of $E \times B$ staircases:**
The quasi-stationary and long-lived patterns of $E \times B$ flow staircase have significant effects in the regulation of mesoscopic, non-diffusive and non-local transport avalanches and form transport barriers. (P. H. Diamond and T. S. Hahm, PoP 1995; B. A. Carreras et al., PoP 1996; G. Dif-Pradalier et al., PRE 2010; T. S. Hahm and P. H. Diamond, JKPS 2018 ⋅⋅⋅)
**I. Introduction of $E \times B$ staircase-4**

Avalanches can be commonly observed in gyrokinetic simulations and tokamak experiments.

Avalanching is intrinsic to the systems exhibiting self-similarity and a consequence of self-organized criticality (SOC).

Characteristics of avalanches:
- Spectral power law scaling $S(f) \sim 1/f$
- Intermittent burst
- Mesoscopic scale (tens of $\rho_i$), non-diffusive and non-local transport events

**Self-organized criticality (SOC):**
Slowly driven, interaction-dominated systems with threshold, exhibiting self-similarity without tuning, i.e., **power law scaling $1/f$**.

*Fig. 9 in Ref. Y. Idomura et al., NF 2009*
I. Introduction of $E \times B$ staircase-5

Hurst exponent $0 < H < 1 \ (B. \ B. \ Mandelbrot \ and \ J. \ R. \ Wallis, \ Water \ Resources \ Res. \ 1968)$: (R/S analysis, B. A. Carreras et al., PRL 1998)

$1/2 < H < 1$, dynamics manifest a sustained memory, long-term persistence and positive correlations

$0 < H < 1/2$, rapid switching between high and low values, temporal anti-correlation.

$H = 1/2$ corresponds the random walk, while $H = 1$ gives $1/f$ spectrum.

Joint reflection symmetry (JRS):

Bumps move down gradient
Voids move up gradient
First experiment evidence from tokamak


From KSTAR Exp. By M. J. Choi, et al., NF 2019
The name of pattern: The pattern formation in magnetic fusion plasmas was given the name of “Zonal flow staircase” or “$E \times B$ staircase”, which is an analog to the “PV staircase” in geophysics and astrophysics (D. G. Dritschel and McIntyre M. E., J. Atmos. Sci. 2008).

Typical signatures: Staircase-like pattern occurs concurrently in both zonal flow and pressure (temperature and density) profiles. Long-lived, quasi-stationary pattern.

Characteristic scale: The staircase is typically in mesoscopic scale (~tens of $\rho_i$), i.e., several times the microturbulence correlation length (~several $\rho_i$).

Effects of the pattern: The staircase-like pattern is found to regulate the transport avalanches, and form localized transport barriers. Avalanches are prevalent in-between staircase shear layers (steps), while suppressed across.
I. Introduction of $E \times B$ staircase

Observation of Zonal flow staircase-like patterns in gyrokinetic simulations of magnetic fusion plasmas:

Flux-driven full-$f$ gyrokinetic simulations of ion temperature gradient (ITG) driven transport:
G. Dif-Pradalier et al., PRE 2010, PRL 2015, NF 2017; W. Wang NF 2018, 2020, etc..

Gradient-driven $\delta f$ gyrokinetic simulations of ITG: L. Villard et al., J. of Phys. 2014 etc..

Gradient-driven $\delta f$ gyrokinetic simulations of Trapped Electron Mode (TEM) driven turbulence: L. Qi et al., NF 2019, NF 2020 in submission.

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Figure 3 of L. Villard et al., 2014 (ORB5 code)

Figure 4 of W. Wang et al., NF 2018 (GKNET code)

Figure 6 of L. Qi et al., NF 2019 (gKPSP code)
Evidence of zonal flow staircase in tokamak plasmas:
According to our knowledge, so far it has been difficult to measure the global profile of zonal flow in experiments, while the existence of zonal flow staircase could be deduced indirectly in other ways.

Corrugations in the profile of electron temperature fluctuation in KSTAR. From Fig. 8 of M. J. Choi et al., NF 2019

Staircase-like structure in temperature and density profiles (pedestal) in DIII-D. From A. Ashourvan et al., PRL 2019
I. Introduction of $E \times B$ staircase-9

Modeling of zonal flow staircase: two approaches

**An initial value problem:** based on the dynamics of propagation of heat flux modulations.

*Traffic-jam model*

Heat avalanche ↔ traffic flow
Temperature $\delta T_e$ ↔ car density perturbation $\delta \rho$
Corrugation ↔ jam

Feedback loop: Once jamming instability is initiated, profile starts corrugating to produce $E \times B$ staircases, which can feedback to the original jam instability.

The key element is a time delay between perturbations in heat flux and gradient.

**A boundary value problem:** turbulence modulation grows, steepens to form staircase patterns.
This reduced model is based on the Hasegawa-Wakatani (HW) system of equations. Bistability and inhomogeneous turbulent (Potential Vorticity) mixing.

![Diagram](image-url)
II. Gyrokinetic simulation of $E \times B$ staircase in KSTAR plasmas
II. GK simulation of $E \times B$ staircase in KSTAR plasmas

- Electron heat **avalanches** in the *MHD-quiescent L-mode* plasmas, which is achieved in the limiter configuration.
  - $q_0 > 1 \rightarrow$ no sawtooth
  - The 2/1 TM was unstable
    - Suppressed by the ECRH
- Transport events are observed during the MHD-quiescent period
  - $R_{av} \sim R_{q=2}$

- **Zonal flow staircase** activities:
  - Corrugations in electron temperature fluctuation
  - Regulations of the electron heat avalanches

*Fig. 1 of M. J. Choi et al, NF 2019*
Gyrokinetic simulation of the above KSTAR L-mode plasmas: (Lei Qi et al., submitted to NF)

- **Tool**: gKPSP, gradient-driven $\delta f$ particle-in-cell (PIC) gyrokinetic code with bounce-averaged kinetic trapped electrons (ITG-TEM) (J. M. Kwon et al., NF 2012, CPC 2017; Lei Qi et al., PoP 2016, NF 2017). An axisymmetric heating source (only radial dependence) is applied to sustain the temperature and density profile.

- **Setup**: Realistic experimental profiles ($T_e$, $T_i$, $n_e$) and experimental configuration from EFIT, $q$-profile, plasma shape, etc.. Equilibrium $E \times B$ flow not included.

Profiles from experimental measurement as inputs

Linear simulation shows TEM dominance in this plasma $\gamma/\omega \sim 2 - 3$, not close to the linear marginality.
Comparisons between **nonlinear** GK simulation and experiment are mainly in **three** aspects:

- Power spectra scaling $S(f) \sim f^{-0.7}$ → close to $1/f$ indicating events at all scales can occur

Power spectra scaling of $|\delta T_e(f)|^2$: $S(f) \sim f^{-0.7}$
Comparisons between **nonlinear** GK simulation and experiment are mainly in **three** aspects:

- Hurst exponent factor (B. A. Carreras et al., PRL 1998) $H \sim 0.7 > 0.5 \rightarrow$ transport events can occur at all scales, $H = 1$ gives $1/f$

Hurst exponent $H = 0.83$ from GK simulation and $H = 0.73$ from the experiment.
II. GK simulation of $E \times B$ staircase in KSTAR plasmas

Comparisons in between **nonlinear** GK simulation and experiment are mainly in **three** aspects:

- Profile of electron temperature fluctuation with corrugation scale $\Delta \sim 45\rho_i$  
  → close to zonal flow staircase step size, about several times the turbulence correlation length

Profile of electron temperature fluctuation with corrugation scale $\Delta \sim 40\rho_i$ from GK simulation and $\Delta \sim 45\rho_i$ from the experiment.

\[ \]
Zonal flow staircase from the nonlinear gyrokinetic simulation is found to be responsible regulating the avalanches.

1. Leading to the electron temperature fluctuation radial corrugations.
2. Zonal flow corrugation scale $\Delta \sim 40 \rho_i$, which is about several times the micro turbulence correlation length (next slide) consistent with $\delta T_e$ radial corrugation
3. Regulations in electron heat avalanches.

(a), (b) Zonal flow $V_{E \times B}$
(c), (d) Normalized electron temperature gradient $R / L_{Te}$
(e), (f) Electron heat flux $Q_e$
Zonal flow staircase shearing effects: on the profile corrugations

(a) The long-live staircase shearing

$$\omega_{ss} \equiv \left| \frac{\partial < V_{E\times B} >}{\partial r} \right|$$

(b) Mean fluctuation of normalized electron temperature gradient, absolute values

$$|\delta(R/L_{Te})|$$

Cross correlation

$$R_{\omega_{ss}\delta(R/L_{Te})} = -0.59$$ (highly correlated with negative radial phase).

In contrast, $$R_{\omega_{ss}R/L_{Te}} = -0.062$$ (no close cross correlation between the staircase shearing and the gradient).

This indicates:

The absolute value of electron temperature gradient fluctuation is probably decreasing at a radial location with increasing staircase shearing and vice versa.
Zonal flow staircase shearing effects: on the turbulence correlation

(c) Turbulence correlation length as a function of radius

Two-point correlation

\[
C(r, \Delta r, \Delta \zeta, t) = \frac{\langle \delta \phi(r, \zeta, t) \delta \phi(r + \Delta r, \zeta + \Delta \zeta, t) \rangle \zeta}{\sqrt{\langle \delta \phi^2(r, \zeta, t) \rangle \zeta \langle \delta \phi^2(r + \Delta r, \zeta + \Delta \zeta, t) \rangle \zeta}}.
\]

By taking maximum over \( \Delta \zeta \), and full width at the half maximum of \( C(r, \Delta r, t) \), \( L_c \) is obtained.

Lower (higher) shearing corresponds to higher (lower) correlation, i.e., opposite phase relation.
II. GK simulation of $E \times B$ staircase in KSTAR plasmas

Zonal flow staircase shearing effects: on the heat avalanches

Spatial-temporal autocorrelation of flux-surface averaged electron heat flux $Q_e(r,t)$

$$C(r_0, \Delta r, \Delta t) = \frac{\langle Q_e(r_0, t)Q_e(r_0 + \Delta r, t + \Delta t) \rangle_t}{\sqrt{\langle Q^2_e(r_0, t) \rangle_t \langle Q^2_e(r_0 + \Delta r, t + \Delta t) \rangle_t}}$$

Note that $r_0 = 70\rho_i$ is where a staircase valley is located.

The figure demonstrates that large scale (mesoscopic scale at least $\sim 60\rho_i$, which covers several times the microturbulence correlation length), intermittent and inwardly propagating avalanches should have occurred but intercepted (regulated) by the zonal flow staircase.
Outlines

I. Introduction

II. Gyrokinetic simulation of $\mathbf{E} \times \mathbf{B}$ staircase in KSTAR plasmas

III. Gyrokinetic simulation of $\mathbf{E} \times \mathbf{B}$ staircase in comparison with models

IV. Summary and discussions
III. GK simulation of $E \times B$ staircase: in comparison with models-1

**Phenomena:** staircase-like structures in zonal flow and pressure profiles. Comparisons are between:

**Simulation:**
gyrokinetic ion and bounce-kinetic trapped electron; collisionless trapped electron mode (CTEM)

**Models:**

- **Reduced model** based on Hasegawa-Wakatani (HW) equations and near-adiabatic electron; collisional drift wave turbulence;
- **Traffic-jam model**

Mainly in the following aspects:

- Upward escalating propagation of staircases?
- Time delay between staircases in heat flux and gradients?

Causality among occurrence of staircases in profiles of zonal flow, pressure, transport and turbulence?

Role of potential vorticity (PV)?
Based on cyclone-like parameters: \( R_0/L_n = 2.2 \), \( R_0/L_{Ti} = 2.2 \), \( R_0/L_{Te} = 6.9 \), elongation \( \kappa = 2 \); a pure TEM case with stable ITG. In most of simulation domain, profile gradients are flat.

Eigenmode structure in poloidal cross section
Upward escalating propagation of staircases observed in the simulations:

This upward escalating is a consequence of the bistability. The propagation speed $v = 20\text{m/s}$ is estimated by the dominant non-zero frequency and radial wave number as shown in the contour plot. This speed is relatively slow comparing to our simulation domain and time, therefore not evident but observable in the zonal flow shearing contour plot. Staircase scale $\Delta \sim 10\rho_i$, about twice the microturbulence correlation in this case.
III. GK simulation of $E \times B$ staircase: in comparison with models-4

Time evolution of staircase-like structure formation:

The staircase formation process in the simulation is quite similar to the predictions from the reduced model with Neumann boundary conditions.

**Note:**
Though we only show the electron density profile here, the presented observation can also be made in other profiles, such as electron temperature (density), ion density, turbulent potential, electron (ion) heat and particle flux.

*From Fig. 3 of A. Ashourvan and P. H. Diamond, PoP 2017*
The causality: *what if we remove the zonal flow?*

We artificially remove the zonal flow in the simulation.

With zonal flow removed, staircase-like structures disappear without evident corrugations in the density (temperature) profiles and fluxes.

In other words, zonal flow causes the staircases in pressure and fluxes profiles.
III. GK simulation of $E \times B$ staircase: in comparison with models-6

Time delay in the formation of staircases in mean fluxes and gradients?

In this case, we find:
1. Staircases in zonal flow appear after the dominant zonal flow with $k_r \sim 0$, an evident time delay.
2. Staircases in zonal flow, density profile and particle flux grow up nearly simultaneously, no evident time delay.
III. GK simulation of $E \times B$ staircase: in comparison with models-7

Role of Potential Vorticity (PV):

We calculate the mean PV as $\langle q \rangle = \langle \delta n \rangle - \langle u \rangle$, the mean vorticity $\langle u \rangle = \langle \nabla^2 \delta \phi \rangle$.

Cross correlation between $\langle q \rangle$ and $\delta (R_0/L_{ne})$ is about 0.93, **very high positive correlation!**

Cross correlation between $\omega_{ss}$ and $|\delta (R_0/L_{ne})|$ is about $-0.53$, **high negative correlation!**

This shows the PV structure is highly correlated with the density profile corrugations. PV mixing has important roles in the formation of staircases.

Zonal flow staircase has shearing effects on the pressure gradient fluctuation, which seems through the PV.
We have demonstrated the role of zonal flow staircase in the regulation of electron heat avalanches in a KSTAR L-mode plasma by performing gyrokinetic simulations:

1. Prevalent avalanches by showing consistent spectrum power law scaling $S(f) \sim f^{-0.7}$ and Hurst exponent factor $H \sim 0.7$
2. Zonal flow staircase is observed with a scale $\Delta \sim 40\rho_i$, which is about several times the microturbulence correlation length $l_c \sim 10\rho_i$ in this case.
3. Avalanches are regulated by zonal flow staircase. Large scale avalanches should have occurred but intercepted by zonal flow staircase.

Gyrokinetic simulation of CTEM based on cyclone parameters demonstrate:

1. **Upward escalating** and staircase formation process are similar to predictions of the reduced model based on the Hasegawa-Wakatani equations.
2. The time delay of staircases formation in fluxes and zonal flow is not evident in this case.
3. *With zonal flow removed, staircases in pressure and flux profiles disappear.*

Shearing effects of zonal flow staircase:

1. Through shearing the microturbulence radial correlation
2. Through shearing the **gradient fluctuation**. PV mixing may play significant roles in this process. Both experimental and cyclone-like parameters exhibit this.
Remaining conundrums:

- What parameters can affect the generation of staircase and determine the scale, radial location and strength?

- Can we control the generation of staircases, the scale, radial location, strength, and thereby energy confinement?

- More…… We are looking forward to exploring more in this topic.