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Transport Physics of the Density Limit

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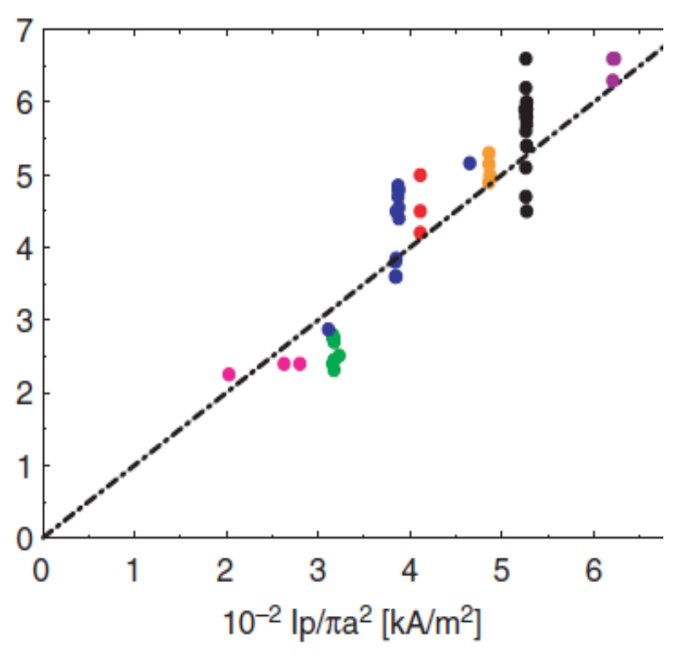
Summary

- This work presents a theory of edge shear layer collapse for $n \to n_g$, the Greenwald density limit. This novel theory of shear layer collapse, in contrast to the earlier work, applies to the adiabatic regime which is relevant for present day hot tokamaks.
- The zonal shear flow screening length is calculated for plateau and Pfirsch - Schluter regimes. F*avorable poloidal magnetic field (B_θ) scaling persists in the plateau regime*.
- Neoclassical screening and drift wave - zonal flow dynamics are combined in a theory, which is then reduced to a novel predator - prey model. Zonal noise, due incoherent mode coupling, is retained.
- *The threshold condition for edge shear layer collapse is computed, and linked to a critical value of the dimensionless parameter* $\rho_s/\sqrt{\rho_{sc}L_n}$ which underpins the density limit $n_g \sim I_p$. Here, ρ_s is ion sound radius, ρ_{sc} is screening length and L_n is density scale length. Zonal flows collapse when $\rho_s/\sqrt{\rho_{sc}L_n}$ falls below a critical value, determined by the zonal flow damping rate, turbulence nonlinear damping rate, triad interaction time and adiabaticity parameter. *Smaller ρ_{sc} i.e., higher B_θ expands the regime of zonal flow persistence.*
- Zonal flows collapse when the integrated particle source S falls below a critical value $S_c \sim B_\theta^{-3}$. That means the particle source, required to hold the shear layer, decreases with increasing current.
- The limiting initial edge density for shear layer collapse is derived and shown to scale favorably with plasma current. In a viscosity dominated regime, the critical edge density $n_c \sim B_\theta$, whereas in charge exchange friction dominated regime $n_c \sim B_\theta^2$.
- Zonal shear collapse, beyond $n > n_c$, can lead to edge cooling by a sequence of shear layer collapse → increased edge transport → edge cooling → onset of radiative condensation and/or radiation induced island growth. In this scenario, the radiative cooling is secondary (i.e., a consequence of) to the transport bifurcation. Thus, a transport bifurcation -i.e., edge shear layer collapse may trigger undesired macroscopic phenomena in the discharge. These results encapsulate the key transport physics underpinning the Greenwald limit.

Density limit basics

- Discharge terminates when line integrated density exceeds a critical value $\overline{n}_g = \frac{I_p}{\pi a^2}$.
- Why care? Fusion power $\propto n^2$.
- A fundamental limit on performance.
- Not a dimensionless number
——more physics involved.
- Still begging the origin of current I_p scaling!?



Greenwald PPCF 2002

Often associated with macroscopic phenomena

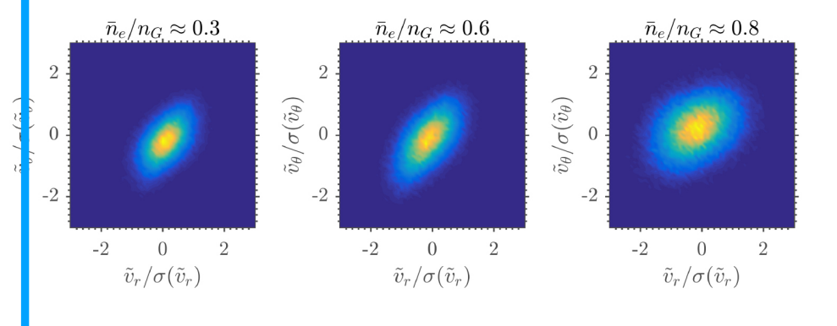
- Global thermal collapse, Radiative condensation / MARFEs
- Poloidal detachment, Divertor detachment
- MHD activity -radiation driven islands

Connections with transport physics

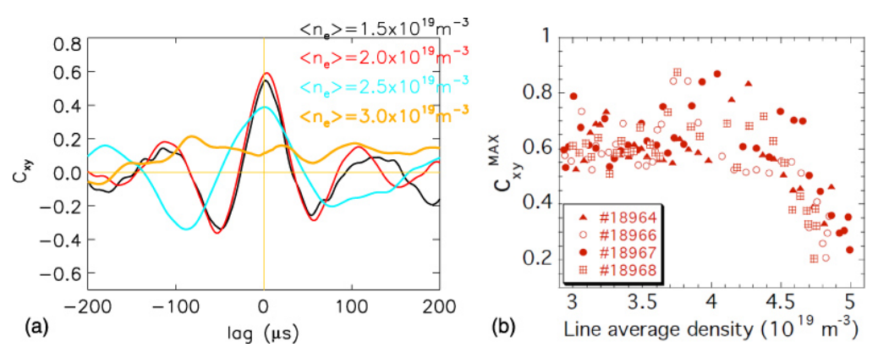
Role of particle transport?

- A SOFT limit: Shallow pellet injection in plasma with $\overline{n} = n_G$ triggered transient particle increased relaxation to n_G by transport rather than by disruption! [Greenwald NF 1988]
- Shallow pellet injection avoids excessive edge cooling— No MARFEs, disruptions!
- Pellet in DIII-D beat Greenwald limit by peaked density profiles <—> enhanced core confinement. Accumulation of impurities—-> increase in radiation—-> disruption.[Mahdavi *et al* 1997]
- Disruption ensuing as a secondary consequence of strong edge cooling due to gas fueling/ radiative cooling.
- Recent experiments and theory suggest that density limit phenomenology emerge from the collapse of edge shear layer leading to increased turbulence, transport and edge cooling, et seq. [Hong *et al* NF 2018, Hajjar *et al* PoP 2018]

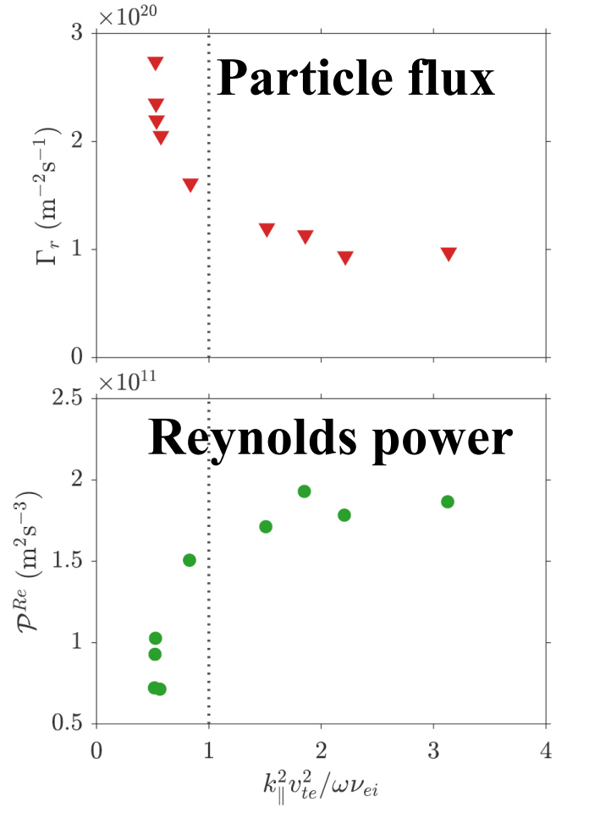
Recent experiments



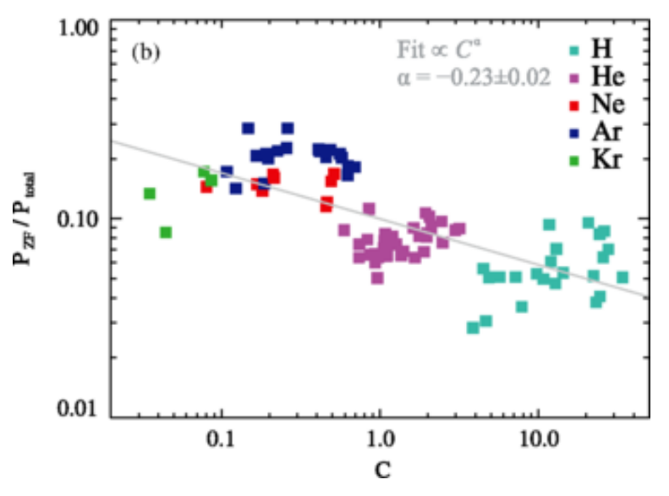
Radial and poloidal velocity correlations drops as Hong *et al* NF 2018
$n \to n_g$



Long range correlations (LRC) decrease as the line averaged density increases in both TEXTOR and TJ-II. LRC↔ZF strength Y. Xu *et al* NF 2011



Reynolds power and particle flux as drops below 1.

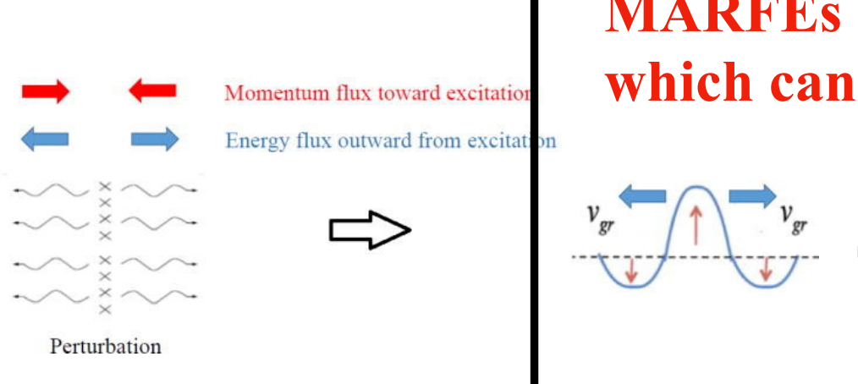


Zonal flow contribution to the total turbulent spectrum decreases with collisionality . Schmid *et al* PRL 2017

Shear layer collapse with hydrodynamic electrons: HDM theory and its limitations

- Clearly, shear layer collapse, increased turbulence and transport as $n \to n_g$!
- Plasma response for Hasegawa - Wakatani :- HDM Theory[Hajjar, Diamond, Malkov 2018]

Response		Adiabatic	Hydro-dynamic
Particle flux	Γ_n	$\sim \alpha^{-1}$	$\sim \alpha^{-1/2}$
Turbulent viscosity	χ_y	$\sim \alpha^{-1}$	$\sim \alpha^{-1/2}$
Residual vorticity flux	Π^{res}	$\sim \alpha^{-1}$	$\sim \alpha^{1/2}$
Vorticity gradient	$\nabla_r^3\overline{\phi} = \frac{\Pi^{res}}{\chi}$	$\sim \alpha^0$	$\sim \alpha^1$



In hydro-regime the link of wave energy flux to Reynolds stress is broken !

- $\Gamma_n, \chi \uparrow$ and Π^{res}, $\nabla_\perp^2\overline{\phi} \downarrow$ as the electron response passes from adiabatic to hydrodynamic regime.
- Weak zonal flow production for $\alpha \ll 1 \to$ weak regulation of turbulence and enhancement of particle transport and turbulence.

Limitations

- Shear layer collapse in hydro regime only. Not relevant for present day **hot** tokamaks.
- Can not explain the origin of plasma current associated with shear layer collapse.
- Connection of shear layer collapse scenario with Greenwald scaling $\overline{n}_g \sim I_p$?

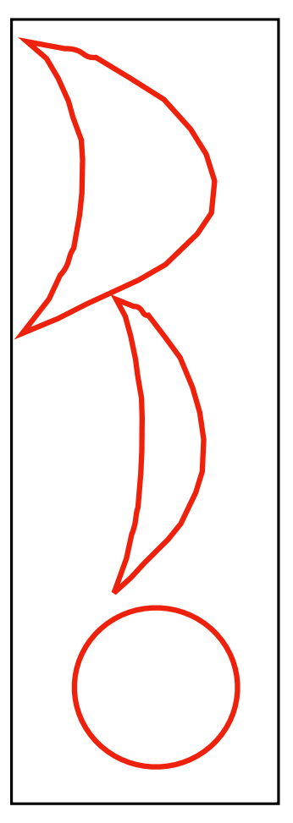
Origin of current scaling

- Key physics: zonal flow drive is "screened" by neoclassical dielectric [Rosenbluth - Hinton 1998]. $\frac{\partial}{\partial t}\left\langle\left|\phi_k\right|^2\right\rangle = \frac{2\tau_c\left\langle\left|S_k\right|^2\right\rangle}{\left|\varepsilon(q)\right|^2}$; $\varepsilon = \varepsilon_{cl} + \varepsilon_{neo} = \frac{\omega_{pi}^2}{\omega_{ci}^2}\left\{1 + \frac{q^2}{\epsilon^2}\right\}k_r^2\rho_i^2$ banana regime

Emission from polarization interaction
Neoclassical response
Zonal wave #

- Poloidal gyro-radius ρ_θ emerges as screening length ! Effective ZF inertia ↓ as $I_p \uparrow \to$ ZF strength increases with I_p
- But edge region is most likely in Plateau regime.[T Long *et al* NF 2019]→Need revisit R-H screening calculations.

Collisionality regimes	Screening length ρ_{sc}	Residual level $\frac{\phi_k(\infty)}{\phi_k(0)}$	B_θ dependence
Banana	$= \sqrt{\rho_s^2 + \rho_\theta^2} \approx \rho_\theta$	$\approx \left(\frac{B_\theta}{B_T}\right)^2$	Favorable
Plateau	$= \sqrt{\rho_s^2 + \mathscr{L}\rho_\theta^2} \approx \mathscr{L}^{1/2}\rho_\theta$	$\approx \frac{1}{\mathscr{L}}\left(\frac{B_\theta}{B_T}\right)^2$	Favorable
P-S	$= \rho_s$	$= 1$	None



Here $\mathscr{L} = 1 - \frac{4}{3\pi}(2\epsilon)^{3/2} < 1$. Favorable I_p scaling persist in plateau regime. Robust trend! No I_p scaling in P-S regime. Effective inertia minimum in P-S

Feedback loop with nonlinear zonal noise

Neoclassical screening and drift wave - zonal flow dynamics are combined, reduced to predator prey model.
Turbulence energy E_t evolves as

$$\frac{\partial E_t}{\partial t} = \gamma E_t - \sigma E_v E_t - \eta E_t^2$$

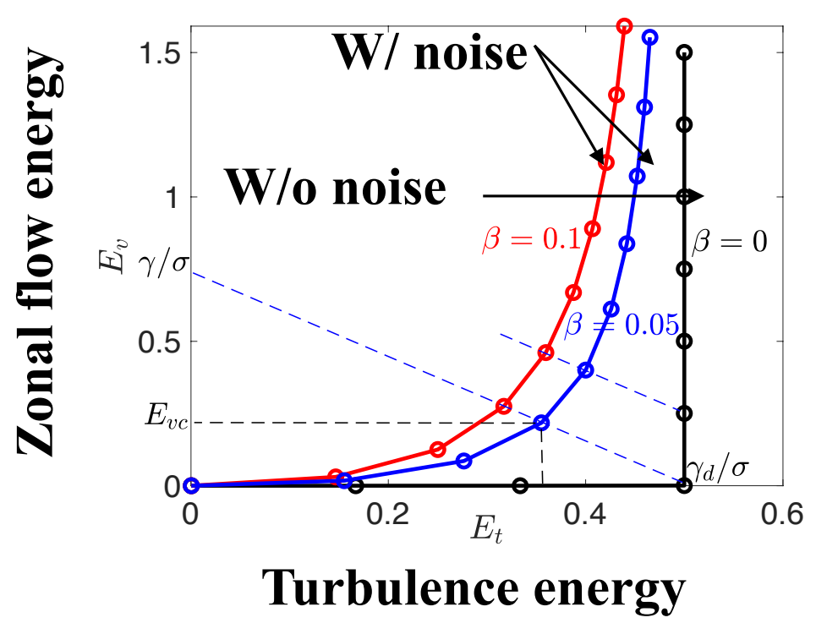
Induced diffusion
Nonlinear damping

Zonal flow energy E_v evolves as

$$\frac{\partial E_v}{\partial t} = \sigma E_t E_v - \gamma_d E_v + \beta E_t^2$$

Modulational growth
Zonal noise

Notice, $\sigma \sim \varepsilon^{-1} \sim B_\theta^2 \sim I_p^2$ and $\beta \sim \varepsilon^{-2} \sim B_\theta^4 \sim I_p^4$
[See details in paper]



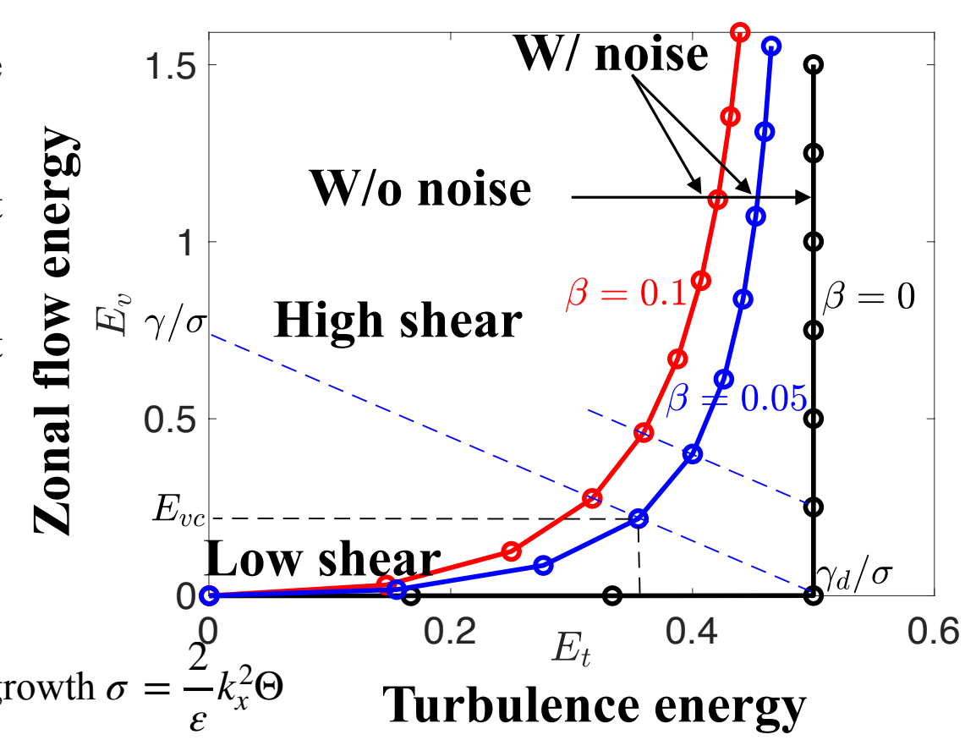
$\Longrightarrow I_p$ jacks up modulational growth and zonal noise → stronger feedback on turbulence.

With noise:

- **Both zonal flow and turbulence co-exist at any growth rate: - No threshold in growth rate for zonal flow excitation.**
- **Zonal flow energy is related to turbulence energy as $E_v = \beta E_t^2/\left(\gamma_d - \sigma E_t\right)\uparrow$ with I_p.**
- **Turbulence energy never hits the modulational instability threshold, absent noise!**
- **Turbulence energy ↓ and zonal flow energy ↑:- Noise feeds energy into zonal flow!**

Shear layer collapse in adiabatic regime

- Zonal flow collapse is continuous evolution from state of strong shear to state of weak shear
- Zonal flow decay occurs when $E_v < E_{vc}$. E_{vc} is upshift of zero zonal energy state due to noise
- Criterion for zonal flow collapse with noise tracks that for collapse of zonal flow without noise i.e., $E_{v0} < 0$



$$E_{v0} < 0 \implies \gamma < \eta\frac{\gamma_d}{\sigma}.$$

Using the linear growth $\gamma = \frac{q_\perp^2}{\hat{\alpha}}\frac{\omega_{\star e}^2}{\left(1+q_\perp^2\right)^3}$ and zonal growth $\sigma = \frac{2}{\varepsilon}k_x^2\Theta$

$$\Longrightarrow \frac{\rho_s}{\sqrt{\rho_{sc}L_n}} < \left[\frac{\eta}{\Omega_i}\frac{\gamma_d}{2k_x^2\rho_s^2\Theta\Omega_i^2}\frac{\hat{\alpha}}{q_\perp^2\rho_s^2}\frac{\left(1+q_\perp^2\rho_s^2\right)^3}{q_y^2\rho_s^2}\right]^{1/4}$$

- Zonal flows collapse when the dimensionless scale length ratio $\rho_s/\sqrt{\rho_{sc}L_n}$ falls below a critical value determined by, determined by the zonal flow damping rate γ_d, triad interaction time Θ, and the adiabaticity parameter $\hat{\alpha}$.
- Note that smaller ρ_{sc} i.e., higher B_θ enlarges the regime of zonal flow persistence.

Critical particle source and critical density for shear layer persistence

- The criterion for zonal flow collapse based on the dimensionless parameter $\rho_s/\sqrt{\rho_{sc}L_n}$ can be cast into criterion based on particle source $S = \int dx\,\langle\nu\sigma\rangle_i n_n n$ using the particle balance. [details in Appendix2]

$$\frac{S}{nc_s} > \frac{\rho_{sc}^3}{\rho_s^3}\left[\frac{\eta}{\Omega_i}\right]^{1/2}\left[\frac{\gamma_d}{2k_x^2\rho_s^2\Theta\Omega_i^2}\right]^{3/2}\left[\frac{q_\perp^2\rho_s^2}{\hat{\alpha}}\right]^{1/2}\left[\frac{\left(1+q_\perp^2\rho_s^2\right)}{q_y^2\rho_s^2}\right]^{1/2}$$

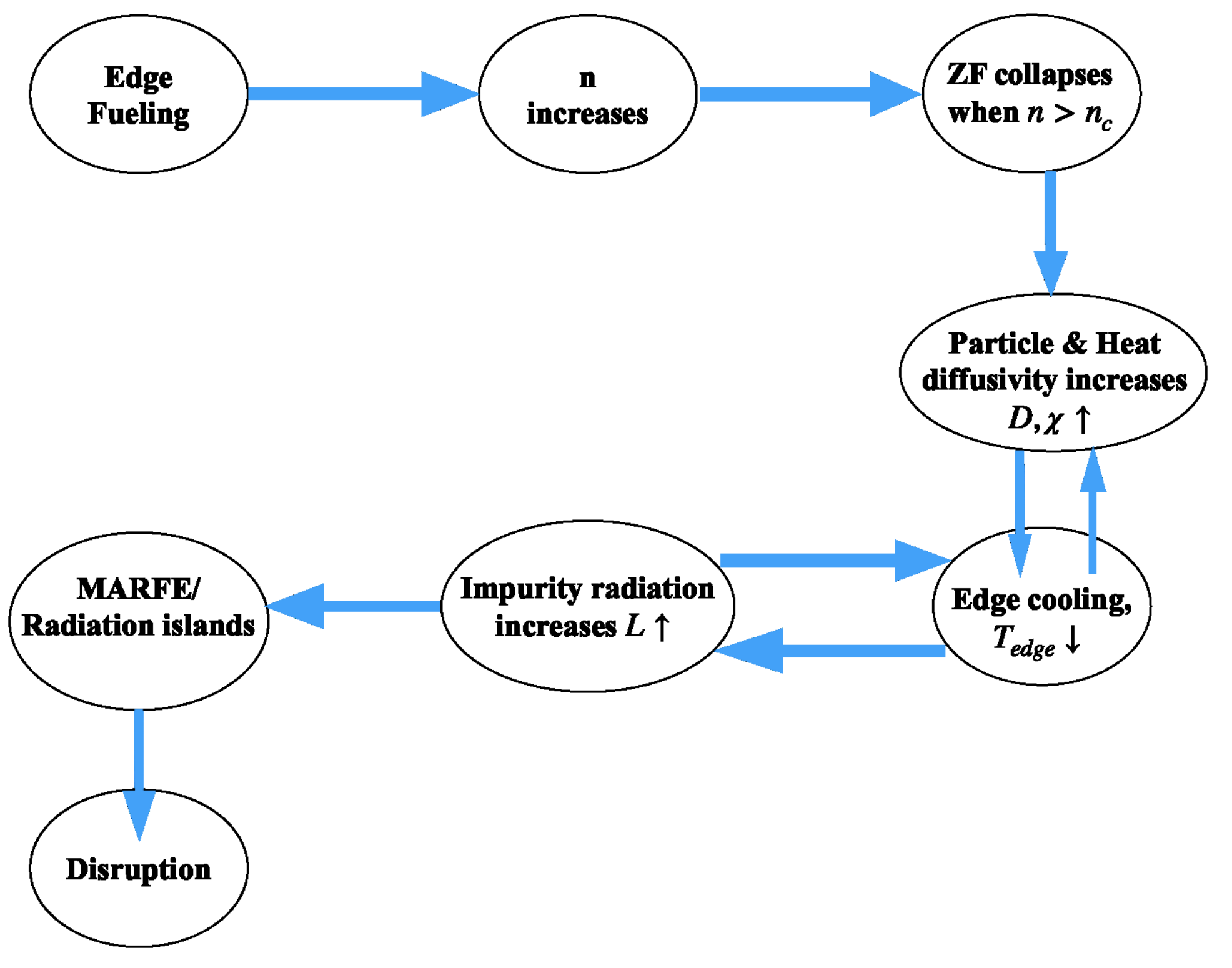
- Critical particle source required to hold shear layer decreases with plasma current $S_{crit} \sim \frac{\rho_{sc}^3}{\rho_s^3} \sim B_\theta^{-3}$.
- This inequality can be converted into limit on local edge density. For viscosity dominated regime $\gamma_d = \gamma_{visc}$, the allowed density range for zonal flow persistence becomes

$$n < \frac{\rho_s}{\rho_{sc}}\left(\frac{S}{c_s}\right)^{1/3}\left(\frac{n\hat{\alpha}}{q_\perp^2\rho_s^2}\right)^{1/6}\left[\frac{2\Theta\Omega_i^2}{\rho_i^2\nu_{ii}/\rho_s^2 n}\right]^{1/2}\left[\frac{\Omega_i}{\eta}\right]^{1/6}\left[\frac{q_y^2\rho_s^2}{\left(1+q_\perp^2\rho_s^2\right)}\right]^{1/6}$$

- In viscosity dominated regime, zonal flows collapse when the local density $n > n_{crit} \sim \frac{\rho_s}{\rho_{sc}} \sim B_\theta \sim I_p$.
- For charge exchange friction dominated regime $\gamma_d = \gamma_{icx}$, the allowed density range for zonal flow persistence becomes$n < \frac{\rho_s^2}{\rho_{sc}^2}\left(\frac{S}{c_s}\right)^{2/3}\left(\frac{n\hat{\alpha}}{q_\perp^2\rho_s^2}\right)^{1/3}\left[\frac{2k_x^2\rho_s^2\Theta\Omega_i^2}{\gamma_{icx}}\right]\left[\frac{\Omega_i}{\eta}\right]^{1/3}\left[\frac{q_y^2\rho_s^2}{\left(1+q_\perp^2\rho_s^2\right)}\right]^{1/3}$
- In charge exchange friction dominated regime, the critical density for zonal flow collapse has somewhat stronger dependence on poloidal field i.e., $n_{crit} \sim \frac{\rho_s^2}{\rho_{sc}^2} \sim B_\theta^2 \sim I_p^2$.

Consequences of shear layer collapse

Enhanced transport due to shear layer collapse can aggravate excitation of MARFEs and /or radiation driven islands due to enhanced edge cooling, which can lead to disruption of discharge.



Conclusions

- We presented a theory of zonal flow collapse for $n \to n_g$, valid for adiabatic regime which is relevant for present day hot tokamaks.
- Zonal flow screening lengths compare as $\rho_{sc}^{PS} \ll \rho_{sc}^{plateau} < \rho_{sc}^{banana}$. Favorable B_θ scaling persist in the plateau regime, which is a relevant regime for present day tokamak.
- Neoclassical screening and drift wave - zonal flow dynamics unified, then reduced to a novel predator prey model. The threshold condition for zonal flow collapse calculated.
- Modulational growth of zonal flow $\sim B_\theta^2 \sim I_p^2$ and zonal noise $\sim B_\theta^4 \sim I_p^4$. Large I_p favors stronger zonal flow production and stronger feedback on turbulence.

Shear layer collapse criterion

- $\rho_s/\sqrt{\rho_{sc}L_n}$ emerges as the key dimensionless ratio which underpins the density limit. Zonal flows collapse when $\rho_s/\sqrt{\rho_{sc}L_n} < \left(\rho_s/\sqrt{\rho_{sc}L_n}\right)_{crit}$. Smaller ρ_{sc} i.e., higher B_θ expands the regime of zonal flow persistence.
- Relating density gradient to source S through particle balance yields a critical particle source S_{crit} for zonal flow collapse. Zonal flows collapse when $S < S_{crit} \sim B_\theta^{-3}$. $\Longrightarrow$ Particle source required to hold the shear layer decreases with increasing current.
- In terms of local edge density, zonal flows collapse when $n > n_{crit}$. In a viscosity dominated regime $n_{crit} \sim B_\theta$ and in charge exchange friction dominated regime $n_{crit} \sim B_\theta^2$.

After collapse

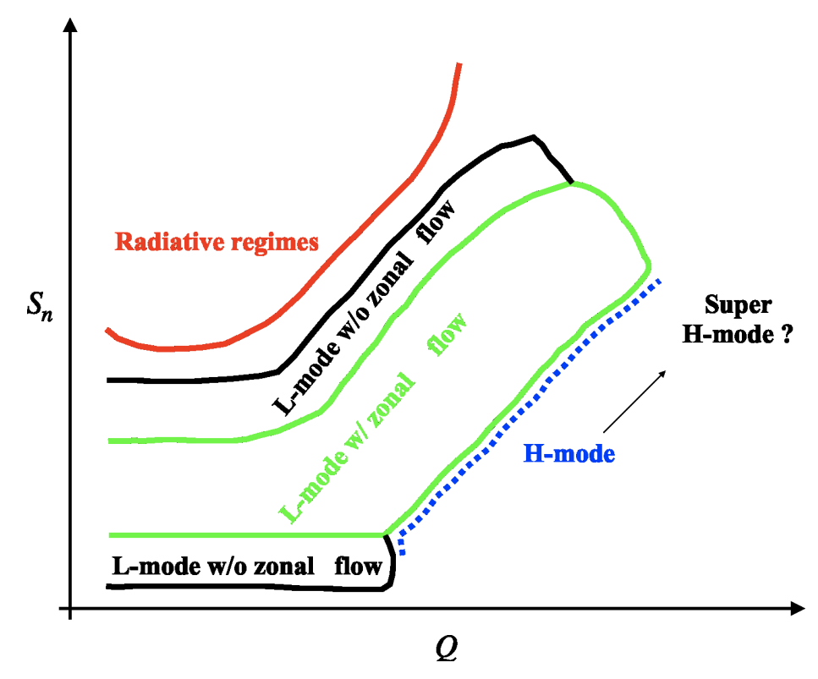
- Zonal flow collapse can lead to edge cooling by a sequence of shear layer collapse→ increased edge transport→ edge cooling→ onset of radiative condensation and/or radiation - induced island growth.
- Thus, the radiative cooling is secondary (i.e., a consequence of) to the transport bifurcation.

Implications for other devices

- Density limit is linked to shear layer collapse which is (in part) controlled by neoclassical screening response.
- In stellarators, the principal correction to classical screening is due to helically trapped particles.
- This has no obvious length scale other than ρ_i the zonal flow screening is classical. Thus the "effective inertia" for zonal flows in stellarators is lower than that for tokamaks.

Towards the big picture

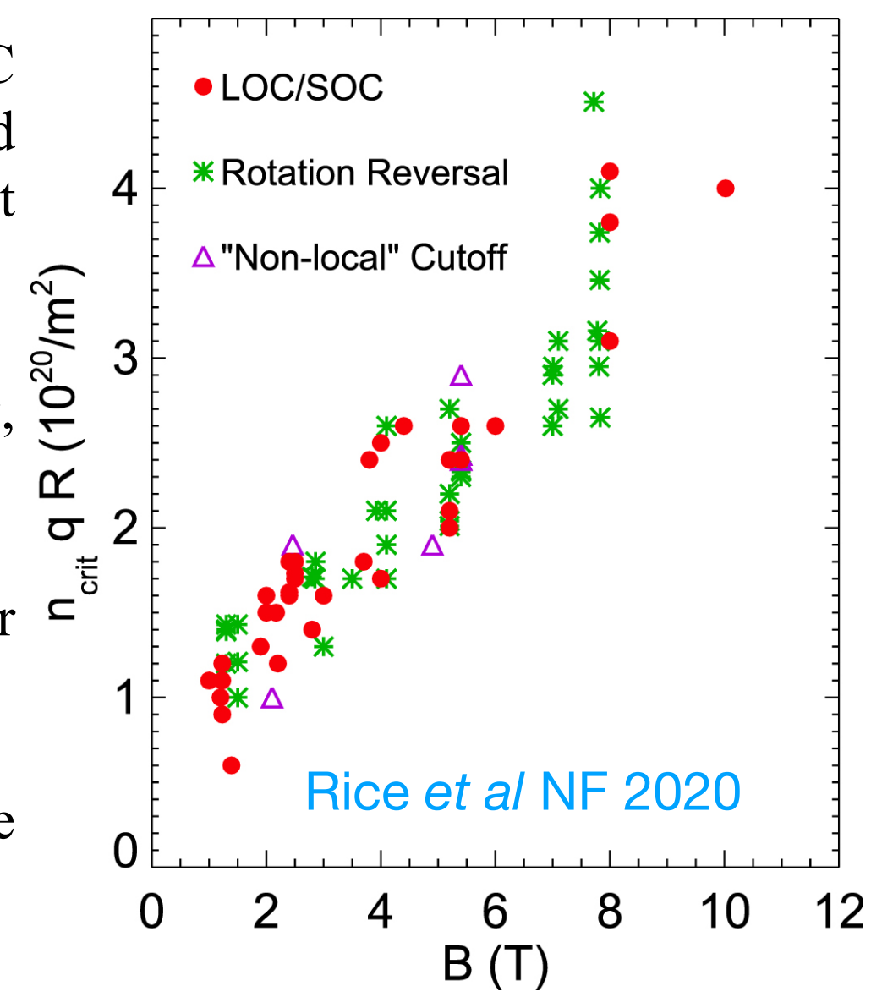
All of the L→ H transition, L-mode with and without shear layer and the density limit regime can be unified in a space of edge fueling S_n and core heating Q in this way.



Sketch of the 'phase diagram' characterizing the states of the tokamak edge in $S_n - Q$ space.

Connection with Ohmic confinement phenomenology and L-H threshold power minimum

- Ohmic phenomenology (i.e., LOC-SOC phenomenology, rotation reversal etc) may be unified by the scaling relation $n_{crit}qR = B_T$. This is equivalent to $n/n_G = \mathcal{O}(1/2)$.
- Greenwald limit phenomenology-i.e., radiative cooling, Marfes, disruption etc., - set in for $n/n_G = \mathcal{O}(1)$
- Why the similarity to the Greenwald scalings, but for phenomena which occur at lower density?
- These occur for $n/n_G = < 1$→ a precursor to the more violent phenomena associated with the density limit.



Rice *et al* NF 2020

- $n_{crit}qR = B_T$ appears to be related to the minimum in the power threshold for L→ H transition.
- This may be due to the onset of pre-transition shear layer decay for $n/n_G = < 1$.
- This in turn weakens the 'seed' shear which initiates the L→ H transition, and thus necessitates an increase in the power required for the transition.

Suggestions for experiments

- Scale of shear layer collapse and its relation with screening length.
- Pellet/SMBI experiment of Greenwald with relevant fluctuations measurements. Relate the density relaxation time to the predictions based on transport dynamics.
- Is the critical edge density with RMP lower than without? Is this because zonal shears are already weekend by the RMP?
- Can edge biasing sustain $n > n_g$ by driving the edge shear layer, externally?
- Is the shear layer collapse transport bifurcation hysteretic or not?

Future directions in theory

- Interplay of B_θ scaling via ρ_{sc} with B_θ scaling from $k_\parallel = 1/qR$ (i.e., from Landau damping!?)
- H-mode density limit: H-L back transition by mean ExB shear collapse? Mechanism of mean shear collapse at high density?
- How flux surface shaping effects the shear layer collapse criterion? Can negative triangularity sustain $n > n_g$?