Extended Bounce-Kinetic Model for Trapped Particle Mode Turbulence

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ABSTRACT

• Bounce-kinetic model based on the modern nonlinear bounce-kinetic theory [B.Fong and T.S. Hahm, Phys. Plasmas 6, 188 (1999)] has been used for gKPSP gyrokinetic simulations before [J.M Kwon et al., Comp. Phys. Commun. 215, 81 (2017)]. This work reports on an extension including more accurate description of barely trapped particles, and its applications. Improvement of Collisionless Trapped Electron Mode behaviour at low magnetic shear is observed from the gyrokinetic simulations. In addition, the Rosenbluth-Hinton residual zonal flow for a bi-Maxwellian distribution has been derived.

Extended Bounce-Kinetic Model for Simulations

Modern gyrokinetic (bounce-kinetic) formalism: use Lie-transform perturbation method to systematically reduce gyro-phase (bounce-phase) dependency while maintaining conserved quantities to relevant order.

- Particle coordinates (6D) → Bounce-Gyrocenter coordinates (4D) (for $\omega \ll \omega_b \ll \omega_c$)
- Lagrangian 1-form and Hamiltonian[1] of bounce-guiding-center (before introducing turbulence) are $\Gamma = \frac{e}{c}Y_2 dY_1 + J d\Psi - H_0(Y_1, Y_2, \mu, J)dt,\ H_0 = \int \omega_b dJ.$
- Equation of motion: $\frac{dY_1}{dt} = \frac{c}{e}\frac{\partial H_0}{\partial Y_2} = 0 \qquad \frac{dY_2}{dt} = -\frac{c}{e}\frac{\partial H_0}{\partial Y_1}$
- Bounce action and bounce frequency for high aspect ratio circular tokamak:

$$J_b \simeq 2qR_0\sqrt{m\epsilon\mu B_0}\left(\kappa^2 + \frac{\kappa^4}{8} + \cdots\right), \quad \omega_b \simeq \frac{1}{qR_0}\sqrt{\frac{\epsilon\mu B_0}{m}}\left(1 - \frac{\kappa^2}{4} - \cdots\right). \qquad \kappa^2 \ll 1$$

$$J_b \simeq J_{b*}\left(1 - \frac{\kappa'^2}{4}\log\frac{16}{\kappa'^2}\right), \quad \omega_b \simeq \frac{\omega_{b*}}{\log\frac{16}{\kappa'^2}}. \qquad \kappa'^2 \equiv 1 - \kappa^2 \ll 1$$

- Approximate forms of Hamiltonian as functions of actions (of J_b and μ) used in the extended model;

$$h_{\mathrm{deeply}}(\mu, J_b) \simeq \mu B_{om} + \frac{\sqrt{\epsilon\mu B_0}}{qR_0\sqrt{m}}J_b - \frac{1}{16m(qR_0)^2}J_b^2 + \ldots$$

$$h_{\mathrm{barely}}(\mu, J_b) = \mu B_{om}(1+2\epsilon) + 32\epsilon\mu B_0\int_0^{(1/4)(1-J_b/J_{b*})}\frac{dt}{W_m(-t)}$$

Consistent with Kadomtsev and Pogutse[2] as: $\frac{dY_2}{dt} \simeq \epsilon q(r)\frac{\rho_i}{r}\frac{v_{Ti}}{r}G(\hat{s}, \kappa)$

: Used in existing gKPSP[3], $W_m(x)$ is the p=-1 branch of Lambert function: $W_p(x)e^{W_p(x)} \equiv x$[4].

- Connection formula used for gKPSP implementation: $\frac{\partial H_0}{\partial\psi} \simeq \left[1 - \left(\frac{J_b}{J_{b*}}\right)^2\right]\frac{\partial h_{\mathrm{deeply}}}{\partial\psi} + \left(\frac{J_b}{J_{b*}}\right)^2\frac{\partial h_{\mathrm{barely}}}{\partial\psi}$

GKPSP simulation using the extended bounce-kinetic model

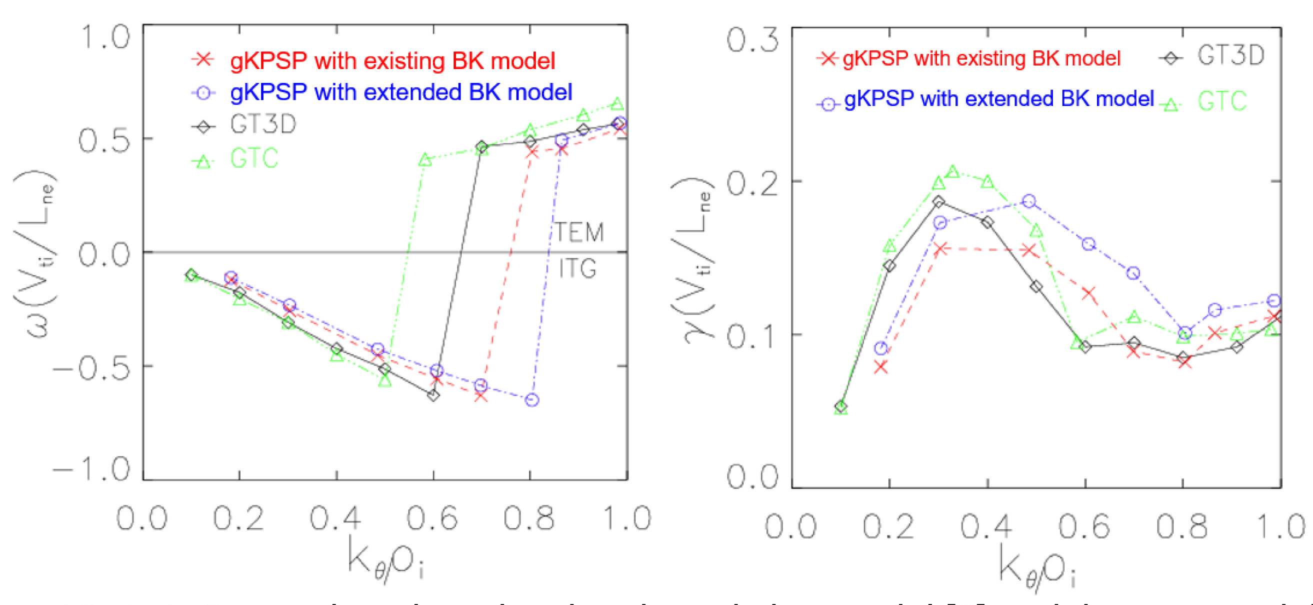

FIG. 1. ITG-TEM benchmark using the existing model [3] and the new model compared with GT3D and GTC results [5]

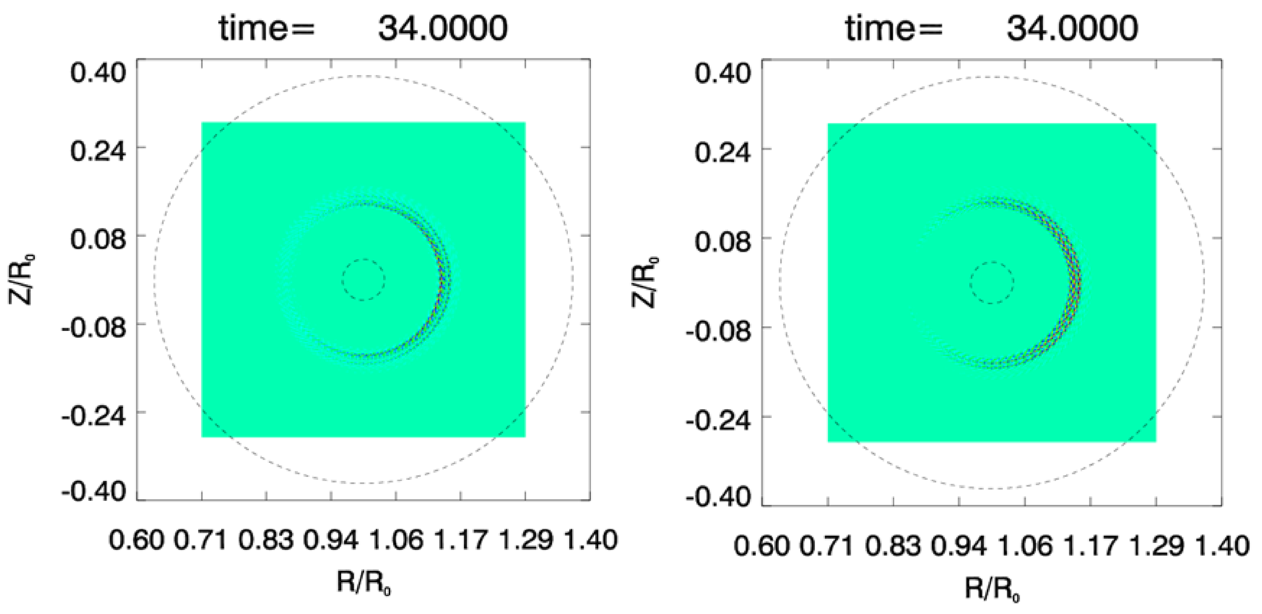

FIG. 2. CTEM mode structure using existing model (left) and CTEM mode structure using extended model (right)

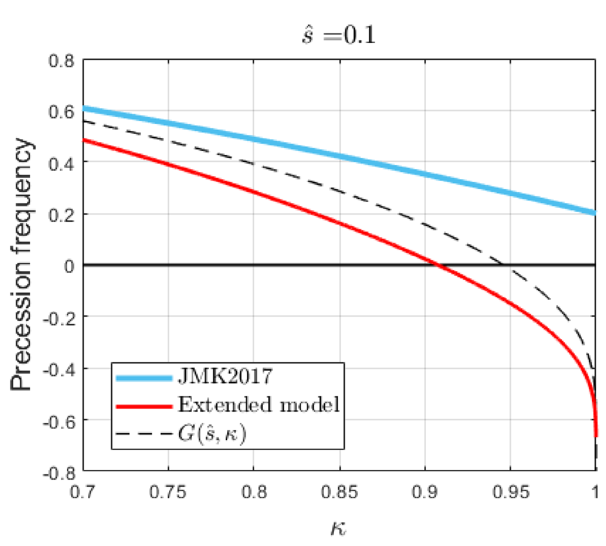

FIG. 3. Precession drift derived from existing model (blue) and extended model (red), compared with theory (dashed) [2].

→ Deeply trapped particles approximation used in the existing model does not accurately reflect precession reversal at $\hat{s}\sim 0$.

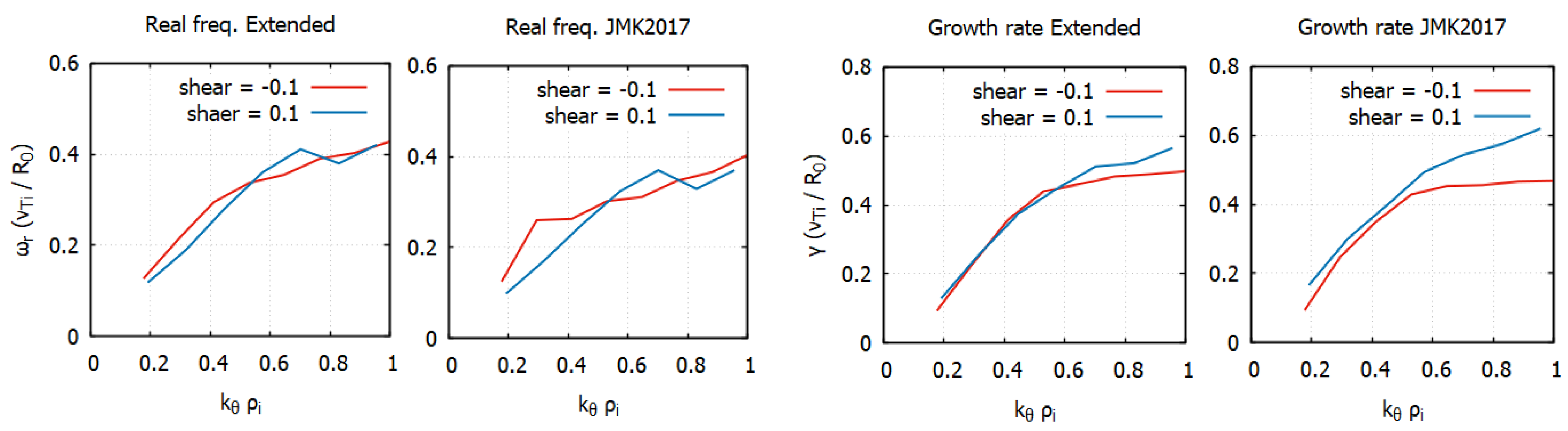

FIG. 4. Complex frequency of ITG instability at $\hat{s}$ = 0 -0.1,0.1 for the existing model and the extended model. Overall trend is similar between two model, but the extended model better reflects the marginal change in growth rate, which is expected with very small change in $\hat{s}$ from 0.1 to -0.1.

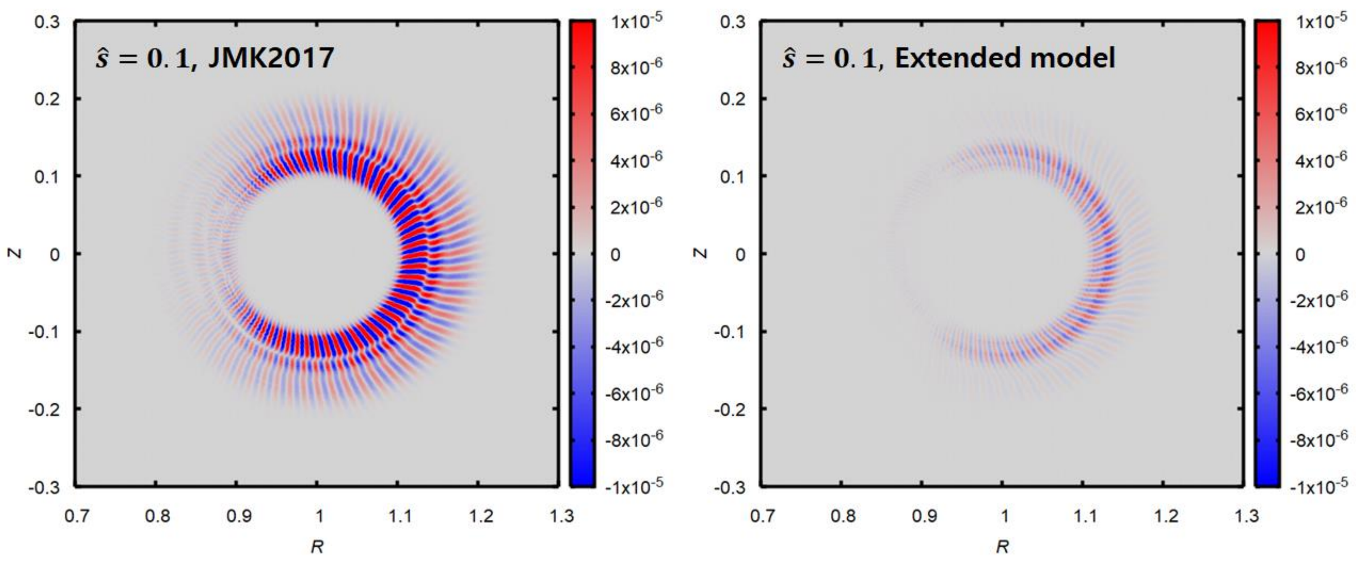

FIG. 5. Mode structure using existing model (left) and mode structure using extended model (right) with $\hat{s}$ = 0.1

Effect of Temperature Anisotropy on Residual Zonal Flow Level

- In the bounce-kinetic theory, $F_0 = F_0(J, \mu, \psi)$
- Anisotropic structure of bi-Maxwellian is most obvious in the (μ, κ) space

$$F_0(\mu, \kappa, \psi) = n_0\left(\frac{m^3}{8\pi^3 T_\parallel T_\perp^2}\right)^{1/2}\exp\left[-\left(\frac{\mu B_e}{T_\perp} + \frac{2\epsilon\mu B_0\kappa}{T_\parallel}\right)\right]$$

- Two-dimensional distribution

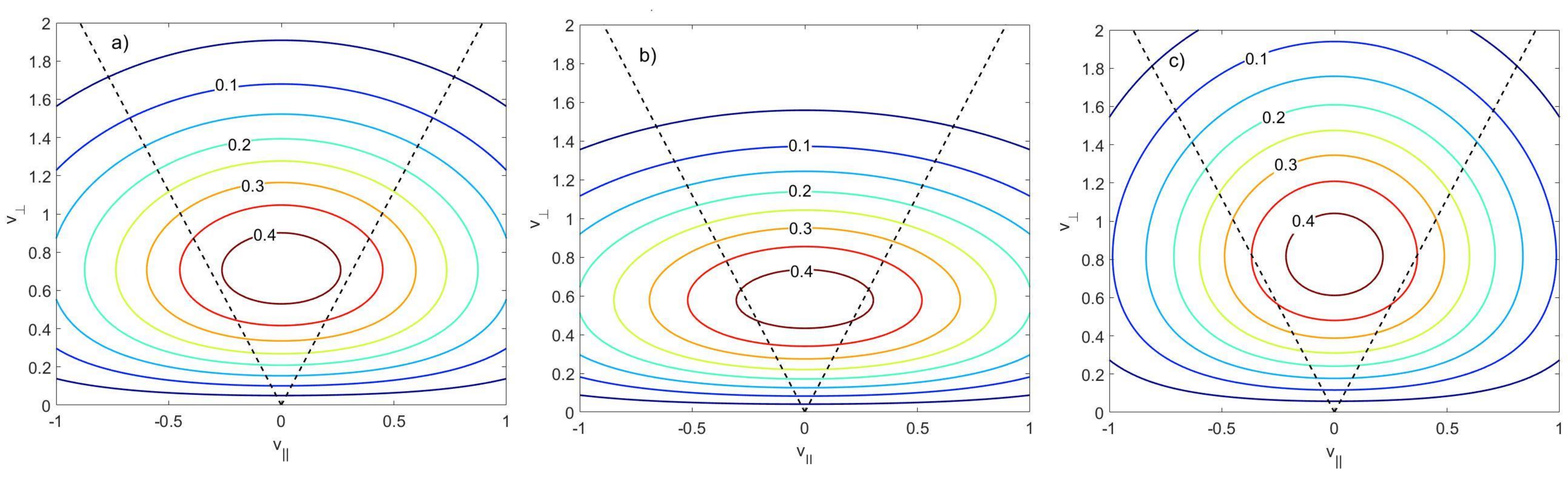

FIG. 6. Distribution of **a)** Isotropic Maxwellian **b)** Bi-Maxwellian with $T_\perp/T_\parallel = 1/2$ **c)** Bi-Maxwellian with $T_\perp/T_\parallel = 2$ Dashed line: Trapped-passing boundary for $\epsilon = 0.1$ i.e., $v_\parallel = \pm\sqrt{2\epsilon}v_\perp$

- Classical Polarization Density: $\frac{n_{cl}}{n_0} = k_r^2\rho_{i\perp}^2\frac{Z|e|\delta\phi}{T_\perp}$
- Neoclassical Polarization Density: $\frac{[n_{nc}]_\psi}{n_0} = 1.63\epsilon^{3/2}\left(\frac{T_\perp}{T_\parallel}\right)^{3/2}k_r^2\rho_{i\theta}^2\frac{Z|e|\delta\phi}{T_\perp}$
- Fraction of Trapped Particles:

$$FR = \frac{n_{tr}}{n_0} = \int_0^\infty d\mu\left(\frac{8\epsilon B_e^2 B_0}{\pi T_\parallel T_\perp^2}\right)^{1/2}\sqrt{\mu}\,exp\left(-\frac{\mu B_e}{T_\perp}\right) = \left(2\epsilon\frac{T_\perp}{T_\parallel}\frac{B_0}{B_e}\right)^{1/2} \simeq \left(2\epsilon\frac{T_\perp}{T_\parallel}\right)^{1/2}$$

- Residual Zonal Flow[6] Level in the longwave length, high aspect ratio limit:

$$R_{ZF} = \frac{n_{cl}}{n_{cl} + [n_{nc}]_\psi} = \frac{1}{1 + 1.63q^2\epsilon^{-1/2}(T_\perp/T_\parallel)^{3/2}}$$

→ Turbulence and transport are expected to get lower for $T_\perp/T_\parallel < 1$

→ In semi-quantitative agreement with H. Ren et al. [7]

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