Drift-Kinetic Theory of Neoclassical Tearing Modes close to the Threshold in Tokamak Plasmas

H.R. Wilson[1], J.W. Connor[2], D. Dickinson[1], A.V. Dudkovskaia[1], P. Hill[1], K. Imada[1] and S. Leigh[1]

[1]York Plasma Institute, Dept of Physics, University of York, Heslington, York YO10 5DD UK

[2]CCFE-UKAEA, Culham Science Centre, Abingdon, Oxon OX14 3DB UK

howard.wilson@york.ac.uk

ABSTRACT

- Drift-kinetic theory presented for particle response to narrow magnetic islands, $w \sim \rho_{\theta i}$, in a low collisionality plasma
- Particle streamlines lie on "drift surfaces", forming "drift islands" for passing particles - similar to magnetic islands, but shifted radially by $\sim\rho_{\theta i}$
- Consequence is a gradient across small islands, suppressing bootstrap current perturbation, resulting in a threshold width for island growth ~few ion banana widths, qualitatively consistent with experiment

BACKGROUND

- Neoclassical Tearing Modes (NTMs) arise from a filamentation of the current density in the vicinity of a rational surface of a tokamak plasma
- Creates magnetic island structures, that short-circuit transport, thus removing pressure gradient inside island:
  - Perturbs bootstrap current, reinforcing filamentation and driving island to large widths
  - Degrades confinement
- Experiment identifies a threshold island width, comparable to ion poloidal Larmor radius, that must be exceeded for island growth
- Quantifying NTM control or avoidance strategies for ITER will benefit from a prediction of the threshold
- We present a drift-kinetic theory for the particle response to small islands, to quantify the current filamentation and predict the threshold

THEORETICAL MODEL

SMALL ISLAND EXPANSION

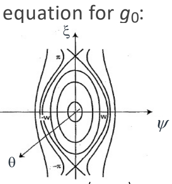
- Small islands are considered, so we expand drift-kinetic eqn for non-adiabatic distribution function, g, in powers of $\delta \sim w/r \sim \rho_{\theta i}/r << 1$
- Toroidal, canonical angular momentum, $p_\phi=\psi - Iv_\parallel/\Omega_c$ is conserved to leading order ($I=RB_\phi$, Ω_c=cyclotron frequency)

$$\frac{v_\parallel}{Rq}\frac{\partial g_0}{\partial\theta}\bigg|_{p_\phi} = 0 \quad \Rightarrow \quad g_0(\boldsymbol{x},\boldsymbol{v}) = g_0(p_\phi,\xi,\lambda,v;\sigma)$$

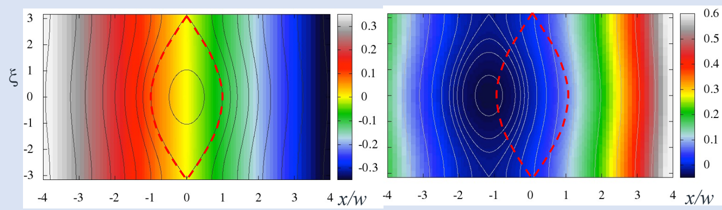
- Averaging next order over particle orbits, $\langle\cdots\rangle$, provides equation for g_0:

$$\frac{v_\parallel}{Rq}\frac{\partial g_1}{\partial\theta}\bigg|_{p_\phi} = \mathcal{L}g_0 \quad \Rightarrow \quad \langle\mathcal{L}\rangle g_0 = 0$$

$$\left[\frac{p_\phi}{L_q}\Theta(\lambda_c-\lambda) - \langle\omega_D\rangle - \frac{1}{2}\left\langle\frac{\rho_\theta}{v_\parallel}\frac{\partial\Phi}{\partial\psi}\right\rangle\right]\frac{\partial g_0}{\partial\xi}\bigg|_S = \left\langle\frac{C(g_0)}{v_\parallel}\right\rangle$$



$$S = \left[2\frac{(p_\phi - \langle\omega_D\rangle L_q)^2}{w^2} - \cos\xi\right]\Theta(\lambda_c-\lambda) - 4L_q\langle\omega_D\rangle p_\phi\Theta(\lambda-\lambda_c) - 2L_q\left\langle\frac{\rho_\theta\Phi}{v_\parallel}\right\rangle$$

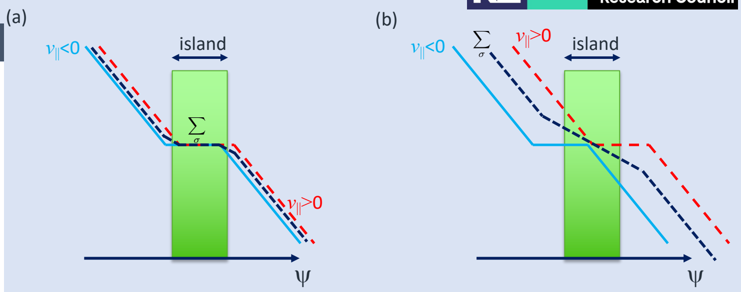


Constant S colour contour plot compared to magnetic flux surfaces for trapped (left) and passing (right) particles (magnetic island separatrix is red dashed curve)

Low collision frequency

- Reduces to a 3-D system $g_0(p_\phi,\xi,\lambda,v;\sigma)=g_0(S,\lambda,v;\sigma)$
- Satisfies a collisional constraint equation, averaged over S contours $\langle\cdots\rangle_S$

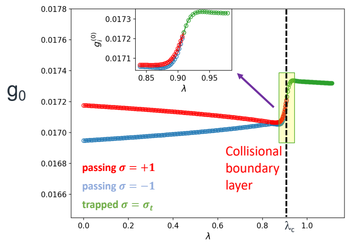
$$\frac{2\hat{\nu}_i}{v}\left\{\left\langle A^{-1}\right\rangle_S\left[\frac{\partial}{\partial\lambda}\left(\lambda\left\langle(1-\lambda B)^{1/2}\right\rangle\frac{\partial g_0}{\partial\lambda}\right) + \left\langle\overline{u}_{\parallel,i}e^{-v^2}\right\rangle\Theta(\lambda_c-\lambda)\right] + C_{\lambda,S}\frac{\partial^2 g_0}{\partial\lambda\partial S} + C_S\frac{\partial g_0}{\partial S} + C_{S,S}\frac{\partial^2 g_0}{\partial S^2}\right\} = 0$$



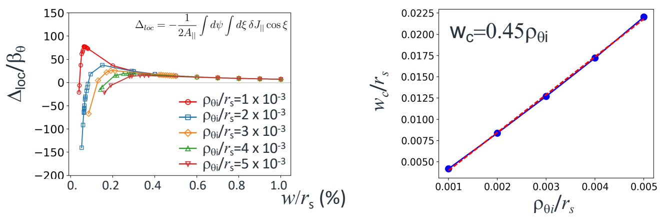
Distribution function across the magnetic island region for $v_{||}>0$ (red curve), $v_{||}<0$ (blue curve) and the average over the two streams (black curve), which provides a measure of density for (a) $\rho_\theta << w$, (electrons), and (b) $\rho_\theta \sim w$, (ions). Quasi-neutrality restored by electrostatic potential

OUTCOME

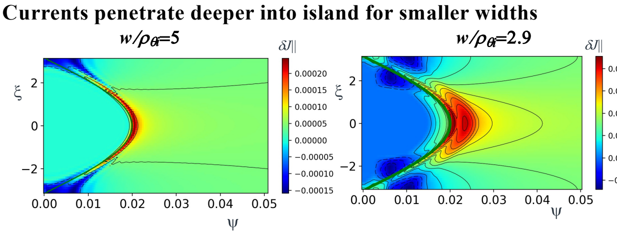
Collisional layer connecting trapped and passing phase space needs special treatment



Layer current (Δ_{loc}) is stabilising for $w<w_c$~1.5 ion banana widths



Currents penetrate deeper into island for smaller widths



Parameters: ε=0.1, L_q/w=1, $1/L_B$=0, L_n/L_T=1, ω=0

CONCLUSION AND FUTURE WORK

- Finite ion banana width effects are a key part of the NTM threshold physics, predicting a critical island width similar to experiment
- Motivates additional extensions to the theory:
  - Treatment of "Transport" boundary layer about drift island separatrix
  - Torque balance and mode propagation frequency effects

ACKNOWLEDGEMENTS / REFERENCES

Relevant references are: **IMADA, K.**, et al, Phys. Rev. Letts. 121 175001 (2018); **IMADA, K.**, et al, Nucl. Fusion **59** 046016 (2019); **DUDKOVSKAIA, A.**, et al, Plas. Phys. Contr. Fus **63** 054001 (2021)

This work was supported by the UK Engineering and Physical Sciences Research Council, grant number EP/N009363/1