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Development of a Far-SOL Unstructured Mesh Fluid Plasma Transport Solver for RF Antenna Simulations

J.D. Lore , M.L. Stowell[2], R.L. Barnett[1], D.L. Green[1], M. Kobayashi[3]

[1]Oak Ridge National Laboratory, [2]Lawrence Livermore National Laboratory, [3]National Institute for Fusion Science

lorejd@ornl.gov

OAK RIDGE National Laboratory | Lawrence Livermore National Laboratory | NIFS | RF-SciDAC

ABSTRACT

- A new fluid plasma solver MAPS (MFEM [1] Anisotropic Plasma Solver) is being developed to simulate far-SOL transport for RF antenna simulations.
- MAPS solves fluid continuity equations on an unstructured mesh using high order finite elements to overcome strongly anisotropic transport with a novel PID-based time-stepper.
- Good convergence and error properties are found for a standard anisotropic test problem and for analytic solutions in unit square and annular geometries.

MOTIVATION

- Reliable RF heating and current drive is critical for steady-state tokamak operation, but performance is limited by interaction with the SOL plasma and the reactor material surfaces.
- Modeling required to understand and predict effect of increased sheath potentials, density "pump-out" and ponderomotive force.
- Simulation is challenging due to strongly anisotropic transport, nonlinear RF-plasma interactions, 3D antenna geometries, and a wide range of spatial and temporal scales.
- Fluid plasma models can advance SOL plasma solution on equilibrium timescales and enable coupling to RF models; most advanced code in this area (EMC3-EIRENE) uses steady-state MC method on field-aligned grid.
- New MAPS code aims to enable time-dependent simulation using unstructured mesh that can conform to antenna and limiter geometry.

NUMERICAL IMPLEMENTATION

FLUID EQUATIONS

$$\frac{\partial n_n}{\partial t} = \nabla\cdot(D_n \nabla n_n) - S_{iz}, \qquad (1)$$

$$\frac{\partial n}{\partial t} = -\nabla_{||}(n v_{||}) + \nabla_\perp \cdot (D_\perp \nabla_\perp n) + S_{iz}, \qquad (2)$$

$$\frac{\partial m n v_{||}}{\partial t} = \nabla\cdot(\bar{\bar{\eta}}\cdot\nabla v_{||}) - \nabla_{||}(p + m n v_{||}^2) + \nabla_\perp\cdot(m v_{||} D_\perp \nabla n), \qquad (3)$$

$$\frac{3}{2}\frac{\partial p}{\partial t} = \nabla\cdot(n\bar{\bar{\chi}}\cdot\nabla T) + v_{||}\nabla_{||}p - \frac{5}{2}\nabla_{||} p v_{||} + \frac{5}{2}\nabla_\perp\cdot\left(n\chi_\perp\nabla_\perp T - \frac{D_\perp}{n}\nabla_\perp n\cdot\nabla_\perp p\right) - S_E, \qquad (4)$$

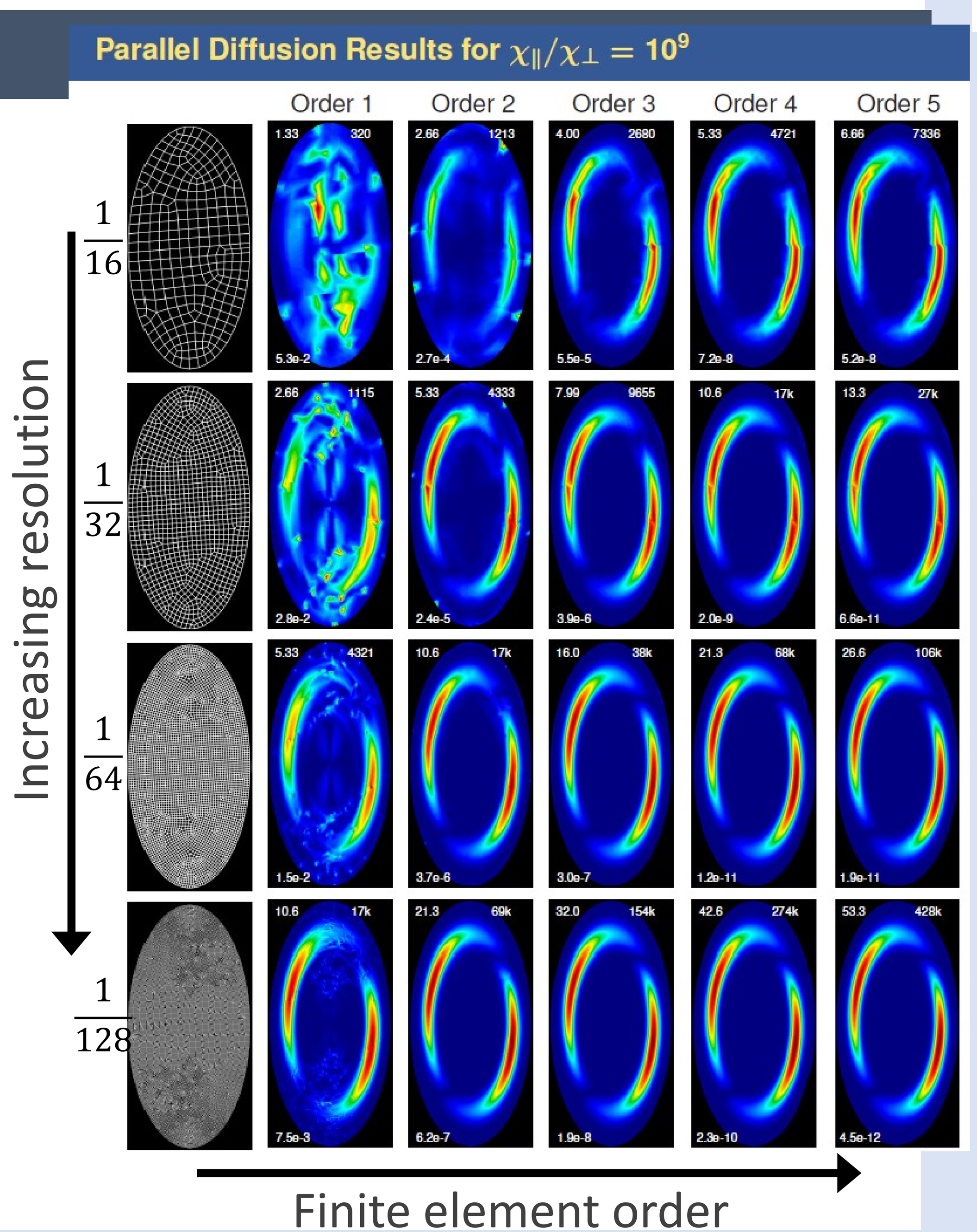
TIME INTEGRATION

$$\bar{\bar{D}} = D_\perp(\bar{\bar{I}} - \hat{b}\hat{b}) + D_{||}\hat{b}\hat{b}$$

- High-order singly-diagonally RK (SDIRK) method, with novel PID-based time step selection [Stowell, APS-DPP 2018 BP11.77]
- Balance CFL condition upper limit for advection dominated regimes with error introduced by implicit time-stepping driven selection in diffusion dominated regimes.
- Use weighted estimate of solution error and adjust time step using PID controller, with error estimate derived from embedded SDIRK solver.

ANISOTROPIC TRANSPORT

- Background temperature distribution on closed flux surfaces is perturbed by Gaussian energy source.
- Background source provides steady small magnitude cross-field flux, but perturbation produces much larger parallel flux which (flows poloidally both directions in figure).
- Acceptable levels of numerica pollution found for orders > 2 with > 16 interpolation points across the feature of interest.



BENCHMARKING

ANISOTROPIC TRANSPORT ON UNIT SQUARE

Using a standard test problem [2], acceptable error is found for a wide range of anisotropy levels using the Super-LU solver, but further tuning of the Adaptive Multigrid (AMG) solver is required

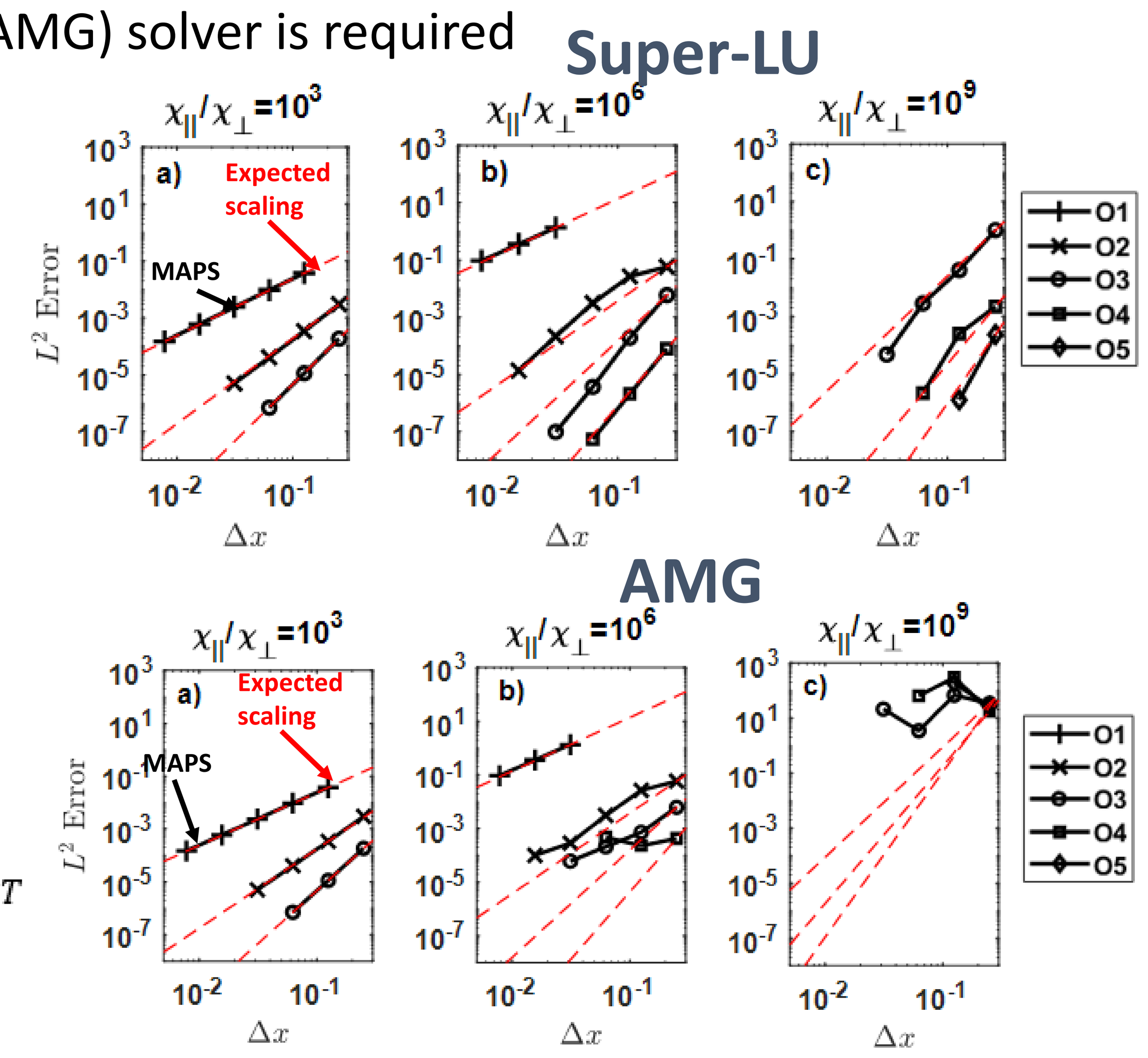
- Black lines show MAPS data, red lines are expected scalings
- Super-LU solver works well, but AMG will be required as problem size increases

$$\frac{3}{2}n\frac{\partial T}{\partial t} = \nabla\cdot\vec{q} + Q$$

$$\vec{q} = [\chi_\perp(\bar{I} - \hat{b}\hat{b}) + \chi_{||}\hat{b}\hat{b}]\cdot\nabla T$$

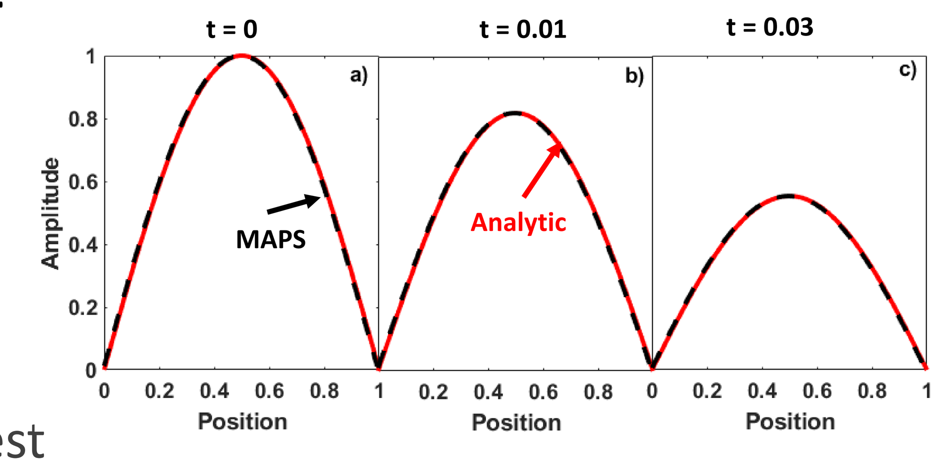
$$Q = A\sin(k_x x)\sin(k_y y)$$

$$T_{exact} = B + (1 - e^{-At})\sin(k_x x)\sin(k_y y),$$



ISOTROPIC TRANSPORT UNIT TEST

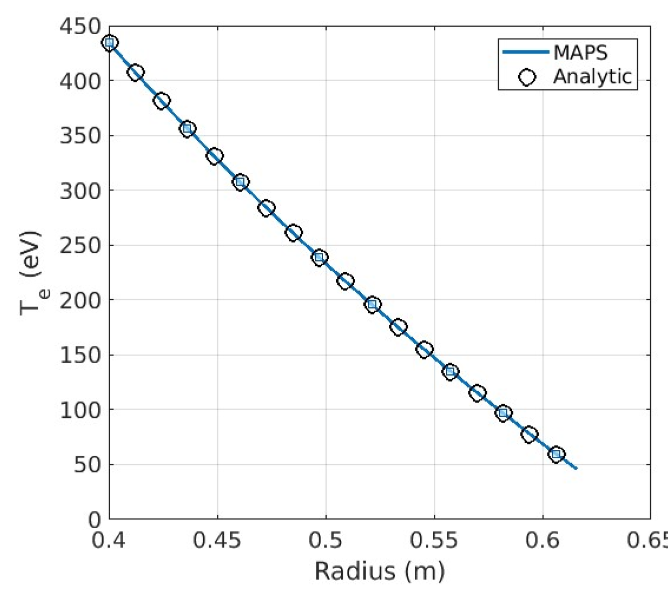
- Isotropic transport problem implemented as a unit test
- Good agreement with analytic solution (dashed black lines)
- Development will build set of test problems as coupled equation problems explored



$$u(x,y,t) = A\exp[-at]\sin(kx)\sin(ky),$$

DIFFUSION IN AN ANNULAR GEOMETRY

- Initial test uses Dirichlet BC's, indicates that numerical pollution due to anisotropy is minimal.
- Recent advancements to MAPS/MFEM allow for more complex BCs, enabling standard plasma model comparisons [3].
- Results are shown using P_{in} = 2 MW, a = 0.4 m, b = 0.64 m, n = 3 · 10^{19} m^{-3}, γ = 5.5, β = 10^{-3}, R_0 = 3.90 m, χ = 3 m^2/s, M = 1 amu, and E_{rc} = 31 eV



$$q_{surf} = \Gamma_{surf}(\gamma T + E_{rc}).$$

$$q_a = -n\chi\left.\frac{\partial T}{\partial r}\right|_a = \frac{P_{in}}{A_{surf}},$$

$$q_{surf} = \beta n\sqrt{\frac{2T_b}{M}}(\gamma T_b + E_{rc}) = -n\chi\left.\frac{\partial T}{\partial r}\right|_b.$$

$$\Gamma_{surf} = \beta\Gamma_{||} = \beta n c_s = \beta n\sqrt{(2T_b/M)}.$$

CONCLUSION

- MAPS is under development to simulate fluid plasma transport in a 3D geometry using unstructured meshes that are not aligned to the magnetic field, for the purpose of studying RF-plasma interactions in the far SOL of a tokamak
- Initial benchmarking shows acceptable error for anisotropic transport test problems, for non-aligned meshes on unit square and annular geometries
- Future extensions: Electric potential equation, cross-field drifts, coupling to MFEM-based RF solver Stix planned [Migliore, APS 2020 BO11.00005]

ACKNOWLEDGEMENTS / REFERENCES

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