Analysis of the inter-species power balance in JET plasmas
(post material … adapted to circumstances)

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view on youtube: https://www.youtube.com/watch?v=He1E_Dgz6_Q

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Overview

Motivation for this study of power balance:

● $T_i$ from CXRS in JET-ILW (most often) difficult to analyse, long delays, often unavailable and sometimes inconsistencies (W nuisance lines) ⇒ need consistency checks

● $T_i < T_e$ at low power, $T_i > T_e$ in high power ‘DT scenarios’ Neutron rates depend strongly on $T_i$ ⇒ need better surrogates than simply assuming $T_i = T_e$

● Power balance is an essential part of any transport study, e.g. for comparison with GK modelling

Contents :

● Ion-electron power balance in JET
● “Equipartition temperatures”
● Fraction of ion heating source going to electrons by equipartition
● Main ion - impurity power balance
● Comparison with TRANSP
● Application of power balance for $T_i$ profile reconstruction
Stationary heat balance of a main (i) and an impurity (z) species:

\[ Q_i = Q_{is} - Q_{ie} - Q_{iz} \quad Q_z = Q_{zs} - Q_{ze} + Q_{iz} \]

net → source → i-e and i-z equipartition

Equipartition power density in W/m³ for any two species i & j:

\[ p_{ij} = c_{eq} \Lambda_{ij} \frac{(A_i A_j)^{1/2}}{(T_j A_i + T_i A_j)^{3/2}} Z_i^2 Z_j^2 n_i n_j (T_j - T_i) \]

\[ c_{eq} = 3.2542 \times 10^{-32} \text{ W eV}^{1/2} \text{ m}^3 \]

(from Wesson, also NRL formulary)

Example:
Electron-ion PB based on measurements of Te, Ti and heat deposition from PENCIL
(no distinction between impurities and main ions here)
\( \Delta Q_i / Q_i \) was evaluated assuming \( \Delta T_i / T_i = 15\% \)
ref: H. Weisen, NF 2020
“Equipartition temperature”

- In JET $T_e(\rho)$ and $n_e(\rho)$ are reliably measured using two Thomson scattering systems.
- The evaluation of $T_i(\rho)$ from CXRS in JET-ILW remains slow and error-prone because of the presence of W nuisance lines.
- Alternative way at looking at the electron-ion power balance: “What would be $T_i(\rho)$ be if a given fraction of the ion/electron deposited power was transferred to/from the electrons by thermal equipartition?”
- Answer defines two families of ‘equipartition temperatures’ with examples shown for different $f_i(\rho)=Q_{ie}(\rho)/Q_{is}(\rho)$ and $f_e(\rho)=Q_{ei}(\rho)/Q_{es}(\rho)$. 
JET overview: typical ITG situation

- Very wide range of conditions from JETPEAK
- For medium to high power NBI: $1 < Q_{ie}/Q_{is} < 0.4$
  cases with most accurate PB: $1 < Q_{ie}/Q_{is} < 0.25$
- $Q_i/(Q_i + Q_e)$ in range 0.5-0.7 for $T_i > T_e$
- $Q_i/(Q_i + Q_e)$ mostly in range 0.5-0.8 for $T_e > T_i$

$⇒$ Typical for ITG dominated discharges
Ion-impurity power balance

- The usual assumption $T_{\text{main}} = T_{\text{imp}}$ is not warranted at high power/low density!
- The energy exchange by collisional heating (NBI, ICRH, even ECH) scales as $Z^2/A$ and is hence 10 larger for Neon ions (used for CXRS in JET-ILW) than for D ions!
- The stronger heating for higher $Z$ is strongly counteracted by thermal exchange between impurities and main ions
- Additionally, main ions and impurities undergo transport
- Solution for $T_z/T_i$ with explicit species dependence in H. Weisen, NF 2020

$$T_z/T_i \approx 1 + \frac{Q_i}{Q_{110}} \frac{A_i}{2^{3/2} Z_i^4 A_z \hat{A}}$$

$$1 + \alpha \frac{Q_i}{Q_{110}} \frac{1}{2^{3/2} Z_i^2 Z_z^2 \hat{A} \Lambda_{11}}$$

where $\hat{A} = \frac{(A_i A_z)^{1/2}}{(A_i + A_z)^{3/2}}$

$$Q_{110} = c_{eq} \Lambda_{11} 2^{-3/2} \int \frac{n_i^2}{T_i^{1/2}} dV \quad \alpha = \chi_z / \chi_i = 1$$

- $Q_{110}$ is a reference ‘equipartitionality’
- $T_z/T_i$ scales nearly linearly with $Q_i/Q_{110}$
- No significant species dependence for $Z>3$ ⇒ simplifies composite $T_z$ profiles from several impurities. Mostly $Q_i/Q_{110}<0.1$
Species dependencies for $Q_i/Q_{110} = 0.1$

**Histograms for $Q_i/Q_{110}$**

- $Q_i/Q_{110} < 0.1$ for majority of cases
- $Q_i/Q_{110} \sim 0.1$ is obtained at high $P_{NBI}$, low density, e.g. AT, hybrid

Species-resolved for $Q_i/Q_{110} = 0.1$ and 4 main species, $\alpha = \chi_z/\chi_i = 1$

- All commonly used impurities for diagnostics have $T_z$ within 1%
  B, N, C, Ne, Ar, Ni are especially close to each other
- Convenient, as no need to distinguish between $T_{imp}$ from different species
- Highest $T_H/T_T$ for hydrogen impurity in tritium plasma (~1.28!)

All data JET-ILW only
$T_Z/T_D$ for $Q_i/Q_{110} = 0.1$ in D/T mixture

- Calculation for mixed main species technically similar to single main species, see H. Weisen, NF 2020
- Unsurprisingly, $T_Z/T_m$ in mixed isotope plasmas assumes values intermediate between those of single isotope plasmas.
- $T_T/T_D \approx 0.95$ for $Q_i/Q_{110} = 0.1$ (hybrids, AT)
- Inconsequential for DT operation
Comparison with TRANSP-SLVTX

- Dataset of over 300 TRANSP JET-C runs “Neutron deficit” H. Weisen NF 2018]
- Implemented a stationary state Matlab version (\(\partial/\partial t=0\)) of the procedure intended for TRANSP routine SLVTX (S.D. Scott, PPPL document, 4.2.2003)
- SLVTX is based on the idea of a local confinement time and applied it to the same input data as in our own calculations (analytical and iterative).
- The SLVTX (matlab) and analytical methods are virtually indistinguishable. Both calculate a slightly larger than the fully iterative solution presented H. Weisen NF 2020. In all cases \(\alpha=\chi_z/\chi_i=1\) is assumed.

Fig. 7. Comparison of \(\theta=\frac{T_C}{T_D}\) obtained with the analytical procedure (eq. 4, left) and the iterative procedure ([2], right) with the stationary-state Matlab version of SLVTX (vertical axis) for \(\rho=0.2\).
Likely error of sign in TRANSP-SLVTX

- While the intended algorithm behind the SLVTX routine in TRANSP is correct, the code implementation is not.
- At high $Q_i/Q_{110}$ it predicts $T_i/T_D$ twice as high as the Matlab implementation and as the analytical and iterative methods.
- The difference is not a different assumption for $\alpha = \chi_Z/\chi_i$. However, setting $\alpha = \chi_Z/\chi_i = -1$ brings “agreement” suggesting a sign error SLVTX for a term designating $\alpha = \chi_Z/\chi_i$ or $Q_Z$.
- Despite simplifications ($T_Z/T_i = \nabla T_Z/T_i$) the iterative solution mostly provides $T_i$ profiles close to satisfying $\chi_Z/\chi_i = 1$, the analytical solution falls short (although $T_Z$ is very similar).
- TRANSP is far from satisfying $\chi_Z/\chi_i = 1$.
- TRANSP also produces many cases where $Q_{CD}/Q_{Cs} > 1$, i.e. the impurity power balance is crassly unphysical. (For the iterative method $Q_{CD}/Q_{Cs} \approx 0.77$)
Being creative

1. You are desperate for $T_D$ profiles (e.g. P. Sirén et al, IAEA FEC 2020) & no $T_Z$ from CXRS
2. $<T_{\text{Ni}^{26+}}>$ from X-ray crystal spectroscopy always available, but line averaged, rotationally shear-smeared and systematically above available and believable $T_Z$ from CXRS

ANSWER

1. Regress $T_Z$ CXRS with $<T_{\text{Ni}^{26+}}>$ and $<\Omega_{\text{Ni}^{26+}}>$ to std=250eV (left figure)
2. Infer equivalent $T_Z$ CXRS ★ for any plasma with good Ni26+ data at that point (right figure)
3. Extrapolate $T_Z$ to whole profile vie equipartition temperature (black —)
4. Get $T_{\text{main}}$ from ion-impurity power balance calculations (grey--). Note: doesn’t work for ion-ITBs!

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one of the few high $R_{DD}$ hybrids that has Ti from CXRS
Summary

- Equipartition limits show far the temperatures of plasmas species can go apart.
- In most chiefly ion heated plasmas in JET ($T_i > T_e$, e.g. NBI) the ions lose up to ~30% of their input power to the electrons by thermal transfer.
- The fraction of power lost to electrons is found to be close to constant for most of the plasma cross section.
- In a wide range of JET plasmas, whether $T_i > T_e$ or $T_i < T_e$, we find $Q_i/(Q_i+Q_e) \geq 0.5$, which is a hallmark of ITG-dominated micro-turbulent transport.
- Main hydrogenic ion temperatures near the magnetic axis are typically a few % lower than impurities, exceptionally up to 10% at high power and low density. The tritium temperature is predicted to be a few % below the deuterium temperature.
- For $Z>3$ impurity temperatures are virtually species-independent.
- The analytical method and a Matlab implementation of SLVTX produce virtually indistinguishable results. Both predict slightly larger $T_C/T_D$ than the iterative method.
- Someone should shoulder the ungrateful task of revising the SLVTX routine’s vintage Fortran code in TRANSP.
- The power balance calculation is made good use of at JET by allowing to infer an entire ion temperature profile from a single local or line-integrated measurement, e.g. from $\text{Ni}^{26+}$ emission, when charge exchange profile measurements are unavailable.