$A(t) = \sqrt{x(t)^2 + \hat{x}(t)^2}$

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Analysis of nonlinear mode-mode interaction using Hilbert transform on HL-2A

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ABSTRACT

- Hilbert transform algorithm is applied to the nonlinear mode-mode coupling analysis on HL-2A tokamak.
- Synthetic signal analysis is given to show the principle of phase analysis for the detection of nonlinear coupling. If the phase difference between two coherent modes is synchronized with the phase of a third mode, these three modes are nonlinearly coupled, vice versa.
- Bashed on the principle of phase analysis, a phase tracking flowchart is summarized for nonlinear mode-mode interaction.
- On HL-2A tokamak, with Hilbert transform, it is clearly observed that the phase difference between two Alfvén modes (AMs) $\Delta\theta_{12}$ is synchronized with the phase of a tearing mode (TM) θ_{TM}. The synchronization is confirmed with cross-correlation and histogram analysis.

INTRODUCTION

- In magnetically confined fusion devices, nonlinear wave-wave interaction has been noticed to play important roles in the production of new modes.
- On NSTX, nonlinear interactions among low-frequency energetic particle modes (EPMs) and high-frequency toroidal Alfvén modes (TAEs) have been reported [1]. On JET, a m/n = 3/2 neoclassical tearing mode (NTM) is stabilized through the nonlinear coupling among m/n = 3/2, 4/3 and 7/5 modes [2]. On LMD-U, quasi- two-dimensional nonlinear interactions in a drift-wave streamer is investigated in detail by the bi-phase analysis [3].
- On HL-2A, high frequency coherent modes can be driven by nonlinear wave-wave coupling [4,5]. Routinely, bi-spectral analysis is applied to detect the nonlinear interaction [6]. However, a number of statistical ensembles are necessary for the bi-spectral analysis. The Hilbert transform [7] analysis does not need ensembles. It has already been applied for nonlinear mode coupling analysis on the camera data in the linear magnetized device PANTA [8].

SYNTHETIC SIGNAL ANALYSIS

- Supposing there are two waves b and c, wave b is $\cos\theta b = \cos(2\pi f_b t+\varphi_b)$, and wave c is $\cos\theta_c = \cos(2\pi f_c t+\varphi_c)$.
- If nonlinear interaction exists between b and c, the generated new wave d is the product of the two waves, $\cos\theta_d = \cos\theta_b \cos\theta_c = \cos(2\pi f_b t+\varphi_b)\cos(2\pi f_c t+\varphi_c) = (\cos(2\pi(f_b+f_c))t+\varphi_b+\varphi_c)) +\cos(2\pi(f_b-f_c)t+\varphi_b-\varphi_c))/2$. Wave d will have the frequency components of fb+fc and/or fb-fc, and the initial phases have to obey $\varphi_d = \varphi_b+\varphi_c$, $\theta_d = \theta_b+\theta_c$ and/or $\varphi_d = \varphi_b-\varphi_c$, $\theta_d = \theta_b-\theta_c$.
- A synthetic signal of 1 MHz is composed of three waves b, c, d and some random noises. Frequencies are f_b = 30 kHz, f_c = 95 kHz and $f_d = f_b+f_c$ = 125 kHz.

NONLINEAR COUPLING EXISTS

- Supposing θ_b jumps at 0.5 ms, 1 ms, (in the intervals of 0.5 ms) to 9.5 ms (while θ_c does not jump), to satisfy the phase relation $\theta_d-\theta_c = \theta_b$, θ_d has to jump at the same timings.
- The phase difference between wave d and wave c $\Delta\theta_{dc} = \theta_d-\theta_c$, and the phase of wave b θb for figure 1(a) are shown in figure 1(c). The jump of both $\Delta\theta_{dc}$ and θ_b at 0.5ms is clearly observed.
- Due to the phase relation, $\Delta\theta_{dc}$ and θ_b are locked together. A bright spot is observed at (f_c, f_b) = (95, 30) kHz in the bicoherence spectrogram is shown in Figure 1(d), indicating that the nonlinear coupling exists among waves b, c and d.

NONLINEAR COUPLING DOES NOT EXIST

- $\Delta\theta_{dc}$ and θ_b are unlocked.
- As can be seen that there is no bright spot but only noises are observed in bicoherence spectrogram, indicating that the nonlinear coupling does not exist.
- From the synthetic signal analysis, we affirm that nonlinear wave wave coupling could be verified if phase relation satisfies $\Delta\theta_{dc} = \theta_d-\theta_c = \theta_b$ when $f_d = f_b + f_c$. Therefore, to extract the phase information of the modes is the key for the experimental data analysis.

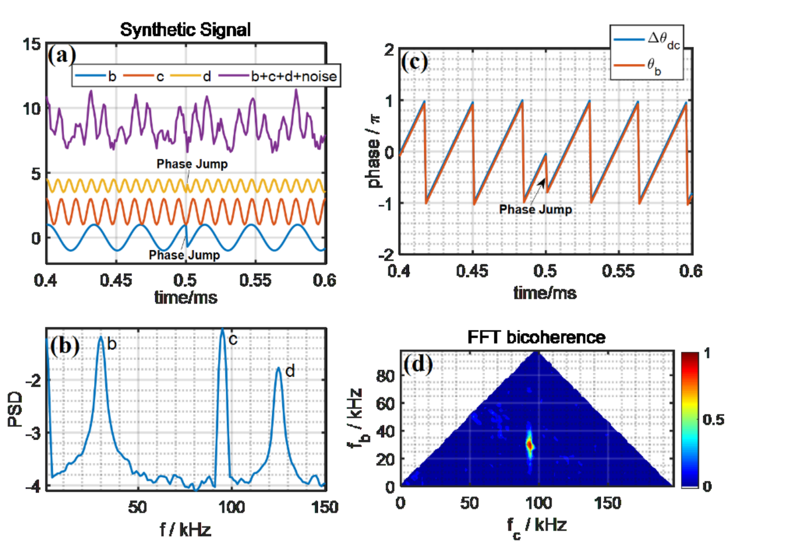

FIG. 1. (a) The synthetic signal (purple curve) is the sum of b, c, d and a random noise. (b) The power spectral density of the synthetic signal. (c) The time evolution of $\Delta\theta_{dc}$ and θ_b. Both the phases are limited to the range of [-π,π]. (d) The FFT bicoherence spectrogram of the synthetic signal.

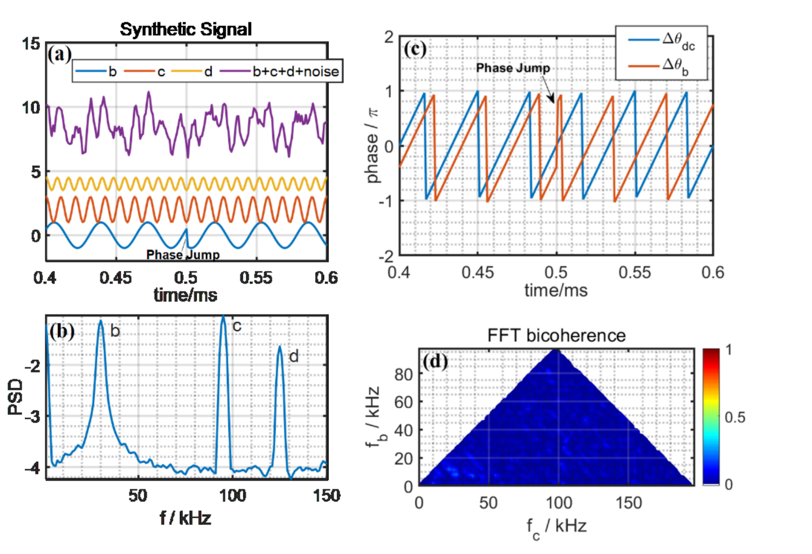

FIG. 2. (a) The synthetic signal (purple curve) is the sum of b, c, d and a random noise. (b) The power spectral density of the synthetic signal. (c) The time evolution of $\Delta\theta_{dc}$ and θ_b. Both the phases are limited to the range of [-π,π]. (d) The FFT bicoherence spectrogram of the synthetic signal.

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ANALYSIS METHOD

Supposing $x(t)$ is the signal of a mode, the definition of Hilbert transform is,

$$\hat{x}(t)=H[x(t)]=\frac{1}{\pi}\int_{-\infty}^{+\infty}\frac{x(\tau)}{t-\tau}d\tau$$

The definition of the analytic signal is,

$$z(t) = x(t)+i\hat{x}(t)$$

Equation (2) could also be written to,

$$z(t) = A(t)exp(i\theta(t))$$

where $A(t)$ and $\theta(t)$ are the instantaneous amplitude and the instantaneous phase, respectively. $A(t)$ is calculated with the following equation,

$$A(t) = \sqrt{x(t)^2 + \hat{x}(t)^2}$$

$\theta(t)$ is calculated with the following equation,

$$\theta(t) = arctan(\hat{x}(t)/x(t))$$

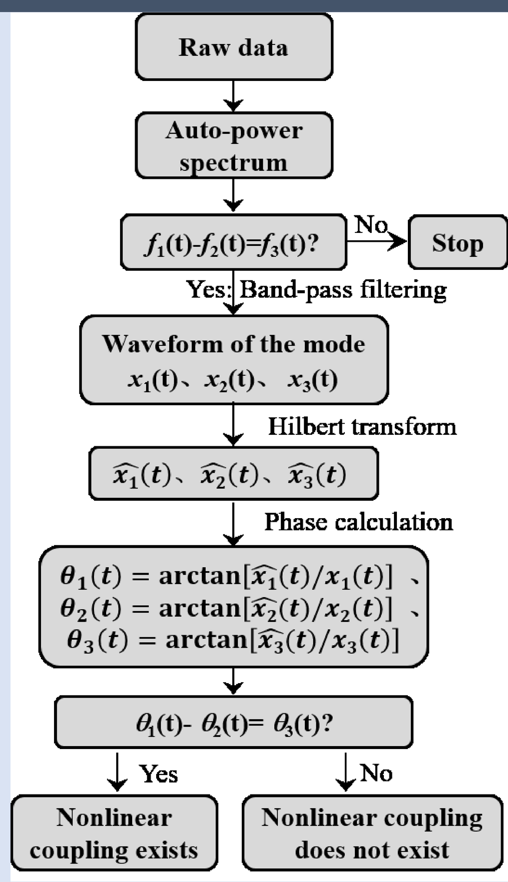

FIG. 3. Phase tracking flowchart of a mode with Hilbert transform

ANALYSIS OF EXPERIMENTAL DATA

Alfvén modes (AMs) and tearing mode (TM) on HL-2A

- Their frequencies of AMs and TM satisfy the frequency relation of nonlinear interaction, $f_{AM1} - f_{AM2} = f_{TM}$. The nonlinear coupling has been identified by observing the bright spot at (f_1, f_2) = (129 kHz, 10 kHz) and (f1, f2) = (139 kHz, 10 kHz).

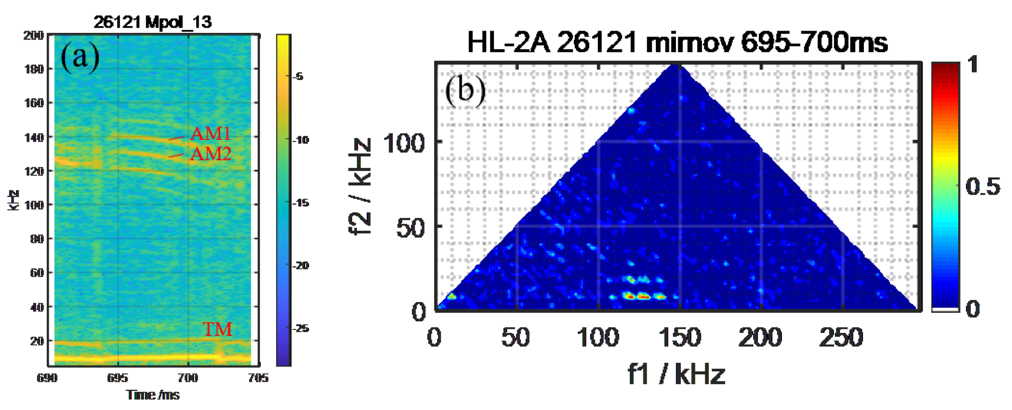

FIG. 4. (a) The power spectrogram; (b) The FFT bicoherence spectrogram

Phase analysis with Hilbert transform

- The phases of AM1, AM2 and TM with frequencies 136-141 kHz, 126-132 kHz and 7-12 kHz are calculated with the flowchart in figure 3.
- θ_{AM1}, θ_{AM2} and θ_{TM} are the instantaneous phases of AM1, AM2 and TM, respectively. In figure 5(a), the blue curve is the phase delay between AM1 and AM2, i.e. $\Delta\theta_{12} = \theta_{AM1}-\theta_{AM2}$, and the red curve is θ_{TM}. It is clear that $\Delta\theta_{12}$ and θ_{TM} are synchronized with each other, and the maximum value of cross-correlation coefficient between $\Delta\theta_{12}$ and θ_{TM} $r(\Delta\theta_{12}, \theta_{TM})$ using equation (6) reaches 0.82, as shown in figure 5(b).

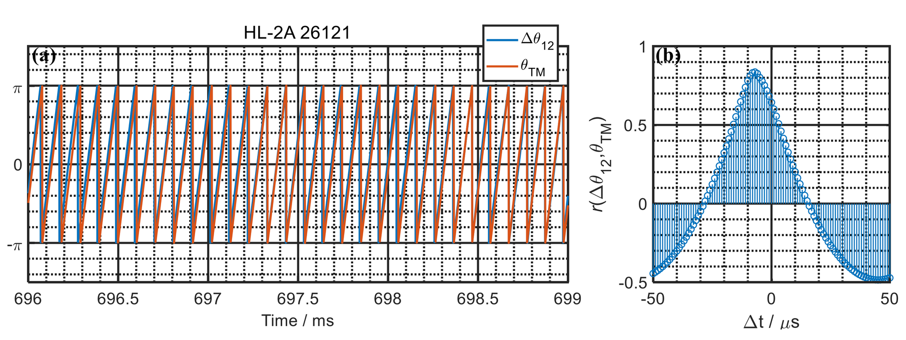

FIG. 5. (a) Time evolution of $\Delta\theta_{12}$ and θ_{TM}. (b) Cross-correlation coefficient between $\Delta\theta_{12}$ and θ_{TM}. The maximum value of $r(\Delta\theta_{12}, \theta_{TM})$ reaches 0.82.

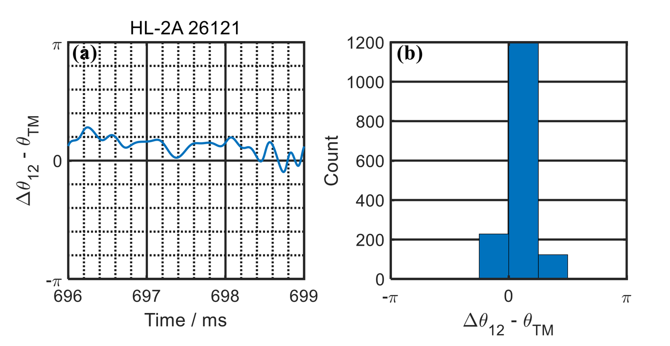

FIG. 6. (a) The time evolution of $\Delta\theta_{12}-\theta_{TM}$. (b) The histogram of the curve in FIG. 6(a).

CONCLUSION

- Compared with the commonly used bispectral analysis, the Hilbert transform allows us to obtain the phase of a mode without many ensembles.
- In our work, two Alfvén modes of 139 kHz and 129 kHz and a tearing mode of 10 kHz on HL-2A, among which the nonlinear interactions were confirmed with the bispectral analysis, are checked for the presence of nonlinear interactions using phase tracking with Hilbert Transform. Results show that the phase delay between two AMs Δθ12 is approximately synchronized with the phase of TM θTM and the maximum of the normalized cross-correlation is 0.84. The time evolution of the difference of the phase delay between the two AMs and the phase of the TM Δθ12-θTM stays in a small range (0,π), which is an alternative to the cross-correlation method for the synchronization checking.
- The nonlinear coupling analysis has been firstly applied on turbulence study, especially the analysis of the nonlinear interaction between zonal flow and the background turbulence. Nonlinear coupling among modes is a recent raised topic in energetic particle physics, for which the bispectral analysis is still the most basic tool. The application of Hilbert transform however should be brought to the attention of the energetic particle physics community that this simple analysis technique can also be used to study mode coupling.

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