Analysis of nonlinear mode-mode interaction using Hilbert transform on HL-2A

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ABSTRACT

• Hilbert transform algorithm is applied to the nonlinear mode-mode coupling analysis on HL-2A tokamak.
• Synthetic signal analysis is given to show the principle of phase analysis for the detection of nonlinear coupling. If the phase difference between two coherent modes is synchronized with the phase of a third mode, these three modes are nonlinearly coupled, vice versa.
• Based on the principle of phase analysis, a phase tracking flowchart is summarized for nonlinear mode-mode interaction.
• On HL-2A tokamak, with Hilbert transform, it is clearly observed that the phase difference between two Alfvén modes (AMs) \( \Delta \phi_2 \) is synchronized with the phase of a tearing mode (TM) \( \phi_{TM} \). The synchronization is confirmed with cross-correlation and histogram analysis.

INTRODUCTION

In magnetically confined fusion devices, nonlinear wave-wave interaction has been noticed to play important roles in the production of new modes. On NSTX, nonlinear interactions among low-frequency energetic particle modes (EPMs) and high-frequency toroidal Alfvén modes (TAlfs) have been reported [1]. On JET, a m/n = 3/2 neoclassical tearing mode (NTM) is stabilized through the nonlinear coupling among m/n = 3/2, 4/3 and 7/5 modes [2]. On LMD-U, quasi-two-dimensional nonlinear interactions in a drift-wave streamer is investigated in detail by the bi-phase interaction [3].

On HL-2A, high frequency coherent modes can be driven by nonlinear wave-wave coupling [4,5]. Routinely, bi-spectral analysis is applied to detect the nonlinear interaction [6]. However, a number of statistical ensembles are necessary for the bi-spectral analysis. The Hilbert transform [7] analysis does not need ensembles. It has already been applied for nonlinear mode coupling analysis on the camera data in the linear magnetized device PANTA [8].

SYNTHETIC SIGNAL ANALYSIS

• Suppose there are two waves b and c, wave b is \( \cos b = \cos(2\pi f t) \), and wave c is \( \cos c = \cos(2\pi f t) \).

• If nonlinear interaction exists between b and c, the generated new wave d is the product of the two waves, \( \cos d = \cos\theta_1 \cos\theta_2 + \sin\theta_1 \sin\theta_2 \cos(2\pi f t) \). The frequencies of \( f_d = 30 \text{ kHz}, f_b = 95 \text{ kHz} \) and \( f_c = f_b - f_c = 125 \text{ kHz} \).

NONLINEAR COUPLING EXISTING

• Suppose \( \theta_1 \) jumps at 0.5 ms, 1 ms (in the intervals of 0.5 ms) to 9.5 ms (while \( \theta_2 \) does not jump), to satisfy the phase relation \( \theta_1 \theta_2 = \theta_1 \theta_2 + \theta_1 \).\( \theta_2 \) has to jump at the same timings.

• The phase difference between wave d and wave c \( \Delta \phi_d = \theta_2 - \theta_1 \) and the phase of wave b \( \theta_1 \) for figure 1(a) are shown in figure 1(c). The jump of both \( \Delta \phi_d \) and \( \theta_2 \) at 0.5ms is clearly observed.

• Due to the phase relation, \( \Delta \phi_d \) and \( \theta_2 \) are locked together. A bright spot is observed at \( (\xi, \eta) = (95, 30) \text{ kHz} \) in the biocherence spectrogram is shown in Figure 1(d), indicating that the nonlinear coupling exists among waves b, c and d.

NONLINEAR COUPLING DOES NOT EXIST

• \( \Delta \phi_2 \) and \( \theta_2 \) are unlocked.

• As can be seen there is no bright spot but only noises are observed in bicoherence spectrogram, indicating that the nonlinear coupling does not exist.

From the synthetic signal analysis, we affirm that nonlinear wave coupling could be verified if phase relation satisfies \( \Delta \phi_d = \theta_2 - \theta_1 = \theta_1 \) when \( f_1 = f_3 + f_4 \). Therefore, to extract the phase information of the modes is the key for the experimental data analysis.

CONCLUSION

• Compared with the commonly used bispacial analysis, the Hilbert transform allows us to obtain the phase of a mode without many ensembles.

• In our work, two Alfvén modes of 139 kHz and 129 kHz and a tearing mode of 10 kHz on HL-2A, among which the nonlinear interactions were confirmed with the bispacial analysis, are checked for the presence of nonlinear interactions using phase tracking with Hilbert Transform. Results show that the phase delay between two AMs \( \Delta \phi_2 \) is approximately synchronized with the phase of TM \( \phi_{TM} \) and the maximum of the normalized cross-correlation is 0.84. The time evolution of the difference of the phase delay between two AMs and the phase of the TM \( \Delta \phi_{TM} \) stays in a small range (0, \( \pi \)), which is an advantage to the cross-correlation method for the synchronization checking.

• The nonlinear coupling analysis has been firstly applied on turbulence study, especially the analysis of the nonlinear interaction between zonal flow and the background turbulence. Nonlinear coupling among modes is a recent raised topic in energetic particle physics, for which the bispacial analysis is still the most basic tool. The application of Hilbert transform however should be brought to the attention of the energetic particle physics community that this simple analysis technique can also be used to study mode coupling.

REFERENCES


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