Novel Tridimensional Processes in Fusion Burning Plasmas and Gained Innovative Perspectives*

B. Coppi[1,2], B. Basu[1], and A. Cardinali[2]
[1]MIT, USA; [2]CNR, Italy
coppi@mit.edu

Abstract

New tridimensional structures have been identified that can be maintained spontaneously in fusion burning plasmas and transfer energy from the emitted reaction products to the reacting nuclei populations. The involved resonant mode-particle interactions are shown to affect the initial mode spatial profile. Minimal electron temperatures of the order of the ideal ignition temperature for DT plasmas are required in order to avoid transferring energy at significant rates to the electron population by mode-particle interactions. The observed fusion reaction rates resulting from energetic neutral H-beams injection into D-plasmas confirm the validity of investigating more sophisticated and less severe ignition conditions ("cool fusion") for burning plasmas than those commonly considered. New perspectives

for fusion research can be envisioned considering that magnetically confined plasmas involve regimes where particle distributions are non-thermal and significant self-organization processes are present.

I. Introduction

Theoretical considerations and available experimental evidence indicate that envisioning meaningful fusion burning plasmas requires consideration of

i) characteristic collective modes [1] that can be excited in them

ii) the fact that as a result of the effects of these modes the distribution in momentum space of the fusing nuclei are not strictly thermal and the reaction rates can be significantly different from those evaluated with thermal distributions [2]

iii) the presence of self- organization processes [3].

In the following sections we show that the emergence of tridimensional structures in magnetically confined plasmas can significantly change the conditions under which meaningful fusion burn conditions can be reached and make the "cool fusion" approach possible. This would avoid the need of massive sustaining system to maintain burning with milder conditions than those estimated traditionally adopting the “thermonuclear” approach based on considering strictly thermal regimes.

The identified tridimensional structures can be classified as “contained ballooning modes” that have the features of being localized in the radial direction, therefore not propagating their energy outside the plasma column. Ballooning modes [4] have the attractive feature of involving an efficient coupling of waves with a spectrum of phase velocities that lends itself well to the interaction with different particle populations [5]. Therefore, the involvement of non-linear processes and the problems associated with them [6] is avoided.

II. Homogeneous Plasma Model

It is helpful to refer to a homogeneous DT plasma [1] immersed in a constant magnetic field $\mathbf{B} = B\mathbf{e}_z$. The density perturbations are of the form:

$$\hat{n} = \tilde{n}_k \exp\left(-i\omega t + ik_\perp y + ik_\parallel z\right)$$

with $\omega^2 << \Omega_{ce}^2 \sim \omega_{pe}^2$, $\Omega_{ce} = eB/\left(m_e c\right)$, $\omega_{pe}^2 = 4\pi n e^2 / m_e$.

The relevant (cold plasma) dispersion relation appropriate for low -β plasmas is

$$k_\perp^2 \simeq \left(\frac{\omega^2}{V_{AD}^2} S_A - k_\parallel^2 \right) - \frac{\Omega_D^2}{V_{AD}^2} \frac{D_A^2}{\left(S_A - k_\parallel^2 V_{AD}^2 / \omega^2 \right)} \tag{II-1}$$

for two nuclei populations (D, T) and $\omega^2 \le \Omega_T^2$, where

$$S_A \equiv \frac{\Omega_D^2}{\Omega_D^2 - \omega^2} + \frac{n_T}{n_D} \frac{m_T}{m_D} \frac{\Omega_T^2}{\Omega_T^2 - \omega^2} + \frac{V_{AD}^2}{c^2} \left(1 + \frac{\omega_{pe}^2}{\Omega_{ce}^2} \right), \tag{II-2}$$

$$D_A \equiv 1 + \frac{n_T}{n_D} - \left(\frac{\Omega_D^2}{\Omega_D^2 - \omega^2} + \frac{n_T}{n_D} \frac{\Omega_T^2}{\Omega_T^2 - \omega^2} \right), \tag{II-3}$$

$$V_{AD}^2 \equiv \frac{B^2}{4\pi n_D m_D}, \ \Omega_D \equiv \frac{eB}{m_D c}, \ \Omega_T \equiv \frac{2}{3} \Omega_D$$

with $k_\parallel^2 << k_\perp^2$. The lowest frequency modes correspond to $\omega^2 << \Omega_T^2$, $D_A \simeq 0$ and $S_A \simeq 1 + \left(3/2\right)\alpha_n$ where $\alpha_n \equiv n_T / n_D$. Thus

$$\omega^2 \simeq \left(k_\perp^2 + k_\parallel^2\right)\frac{V_{AD}^2}{S_A} \equiv \omega_A^2, \qquad \text{(II-4)}$$

that gives the frequency of the familiar compressional Alfvén waves, as $\omega^2 = \omega_A^2 + \delta\omega^2$ and $\delta\omega^2 / \Omega_T^2 \sim \omega_A^4 / \Omega_T^4$. We notice that

$$\frac{\omega_A^2}{\Omega_D^2} = k_\perp^2 d_D^2, \text{ where } d_D^2 \equiv \frac{c^2}{\omega_{pD}^2} \text{ and } \omega_{pD}^2 = \frac{4\pi n_D e^2}{m_D}.$$

Therefore, the low frequency limit is found for $k_\perp^2 d_D^2 << 1$.

The “apt frequency” modes correspond to the Ω_T resonance that is $\omega^2 = \Omega_T^2 + \delta\omega^2$. In this case the evaluation of $\delta\omega^2$ makes it necessary to find the expressions for D_A and S_A to the order $\left(\delta\omega^2 / \Omega_T^2\right)^2$. Then

$$D_A \simeq \left(\frac{\Omega_T^2}{\delta\omega^2}\right)\left[\alpha_n + \left(\alpha_n - \frac{4}{5}\right)\left(\frac{\delta\omega^2}{\Omega_T^2}\right) - \frac{36}{25}\left(\frac{\delta\omega^2}{\Omega_T^2}\right)^2\right], \qquad \text{(II-5)}$$

$$S_A \simeq -\frac{3}{2}\left(\frac{\Omega_T^2}{\delta\omega^2}\right)\left[\alpha_n - \frac{6}{5}\left(\frac{\delta\omega^2}{\Omega_T^2}\right) - \frac{24}{25}\left(\frac{\delta\omega^2}{\Omega_T^2}\right)^2\right] \tag{II-6}$$

and, neglecting $k_{\|}^2$ relative to $S_A\Omega_T^2 / V_{AD}^2 \sim \Omega_T^2 / \left(\delta\omega^2 d_D^2\right)$,

$$k_\perp^2 + k_{\|}^2 \simeq \frac{\omega^2}{V_{AD}^2} S_A - \frac{\Omega_D^2}{V_{AD}^2}\frac{D_A^2}{S_A} \simeq \frac{3}{2}\frac{\Omega_T^2}{V_{AD}^2}\left\{\left(\alpha_n + \frac{4}{5}\right) + \left[\alpha_n + \frac{4}{25}\frac{1}{\alpha_n} + \frac{26}{25}\right]\left(\frac{\delta\omega^2}{\Omega_T^2}\right)\right\}. \tag{II-7}$$

For $k^2 = k_{\perp 0}^2 + \delta k_\perp^2 + k_{\|}^2$, we obtain

$$k_{\perp 0}^2 d_D^2 \simeq \left(1 + \frac{4}{5}\frac{1}{\alpha_n}\right) \tag{II-8}$$

and

$$\left(\delta k_\perp^2 + k_{\|}^2\right) V_{AD}^2 \simeq \frac{3}{2}\left[\alpha_n + \frac{4}{25}\frac{1}{\alpha_n} + \frac{26}{25}\right]\left(\delta\omega^2\right). \tag{II-9}$$

III. Simplest Confinement Configuration

Considering the importance that the geometry of the plasmas from which relevant modes can emerge we refer, as a start, to an axisymmetric toroidal configuration with a large aspect ratio and a circular cross section of a low -β plasma column. Therefore, $n \simeq n(r)$, where $r \le a$, the minor radius, and the magnetic field is represented by

$$\mathbf{B} \simeq \frac{B_0}{1+(r/R_0)\cos\theta}\mathbf{e}_\varphi + B_\theta(r)\mathbf{e}_\theta$$

with $B_0^2 >> B_\theta^2$ and $a < R_0$. The class of ballooning modes that are analyzed [7] are tridimensional and of the "disconnected" type described as

$$\hat{n}(r,\theta,\varphi) \simeq \tilde{n}(r)A(\theta, r-r_0)\exp\left\{-i\omega t + in^0\left[\varphi - q(r)\theta\right]\right\} \qquad \text{(III-1)}$$

in the neighborhood of a rational surface $r = r_0$, where $q(r) = rB_0/(R_0 B_\theta) = m^0/n^0$ and $A(\theta, r-r_0)$ is a θ - periodicity restoring (for $r = r_0$) function that vanishes at $\theta = \pm\pi$.

The familiar ballooning modes were introduced as purely growing modes [4, 7] and with $A(\theta, r - r_0)$ as even function of θ, while here we consider modes that are oscillatory with damping or growth rates that are smaller than the relevant frequency of oscillation and $A(\theta, r - r_0)$ functions that can have either θ - parities. As will be shown, the relevant modes are contained [8] within a relatively small radial layer Δ_r such that

$$r_0^0 / m^0 < \Delta_r < r_0^0$$

where $r_0^0 \approx r_0$ and, within this layer, we take $n(r) \simeq n(r_0) + (r - r_0)n' + \left[(r - r_0)^2 / 2\right]n''$ and $B \simeq B_0\left[1 - (r_0 / R_0)\cos\theta\right]$.

The dispersion equations for the modes of interest can be inferred from the same dispersion relation as Eq. (II-1) rewritten as $k_\perp^2 + k_\parallel^2 \simeq (\omega^2 / c^2)\left[S - D^2 / \left(S - c^2 k_\parallel^2 / \omega^2\right)\right]$ where

$$S \simeq -\omega_{pD}^2 \left\{ \frac{1}{\omega^2 - \Omega_D^2} + \frac{2}{3}\frac{n_T}{n_D}\frac{1}{\omega^2 - \Omega_T^2} \right\},$$

$$D \simeq \frac{n}{n_D}\frac{\omega_{pD}^2}{\omega\Omega_D} + \frac{\omega_{pD}^2\Omega_D}{\omega}\left[\frac{1}{\omega^2-\Omega_D^2} + \frac{4}{9}\frac{n_T}{n_D}\frac{1}{\omega^2-\Omega_T^2}\right].$$

In addition, $k_\perp$ and $k_\parallel$ are replaced by the following operators

$$k_\perp^2 \to \left.k_\perp^2\right)_{op} \simeq \left(\frac{m^0}{r_0}\right)^2\left[1-\frac{2}{r_0}\left(r-r_0\right)+\frac{6}{r_0^2}\left(r-r_0\right)^2\right]-\frac{\partial^2}{\partial r^2}, \qquad \text{(III-2)}$$

$$k_\parallel^2 \to \left.k_\parallel^2\right)_{op} \simeq -\frac{\partial^2}{\partial l^2} \simeq -\frac{1}{\left(R_0 q_0\right)^2}\frac{\partial^2}{\partial\theta^2}. \qquad \text{(III-3)}$$

IV. Contained Ballooning Modes

The tridimensional fluctuating structures that have been chosen for consideration besides providing a means to transfer energy among different particle populations should have the property of not transferring the energy associated with them out of the plasma column. The simplest limit to consider is the low frequency range $\omega^2 << \Omega_T^2$ and $\left.k_\perp^0\right)^2 d_D^2 << 1$. In this case $k_\perp^0 \simeq m^0/r_0$ and

$$\omega^2 = \omega_0^2 + \delta\omega_0^2 + \left(\delta\omega^2\right)_r + \delta\omega_\theta^2,$$

where $\omega_0^2 \simeq \left(m^0 / r_0\right)^2 \left(V_{AD}^0\right)^2 / S_A$, $S_A \simeq 1 + (3/2)\alpha_n$, $\delta\omega_0^2 \sim \omega_0^4 / \Omega_T^2$, $\left(\delta\omega^2\right)_r$ is associated with the mode radial localization [8] and $\delta\omega_\theta^2$ is related to the ballooning structure in the θ variable.

Then, following the procedure given in Ref. [8] to derive the equation that is characterized by the operator [Eq. (III-2)] and the expansion of $n(r)$ around r_0 given earlier, we obtain

$$\tilde{n} \simeq \tilde{n}_0 \exp\left[-\frac{1}{2}\left(\frac{r - r_0}{\Delta_r}\right)^2\right], \tag{IV-1}$$

where $\Delta_r \sim r_0 / \sqrt{m^0}$.

The simplest θ - ballooning structure can be derived by considering modes localized over an arc $\Delta\theta$ where $\cos\theta \simeq 1 - \theta^2/2$ and $B \simeq B_0\left(1 + \bar{\varepsilon}_B / 2\right)$ with $\bar{\varepsilon}_B \equiv \varepsilon_B \theta^2$. In addition, for the sake of simplicity, we condition the analysis to the limit $\bar{\varepsilon}_B\Omega_{T0}^2 \sim \left|\delta\omega_\theta^2\right| < \delta\omega^2$. Then the relevant ballooning equation becomes

$$\frac{\left(V_{AD}^0\right)^2}{S_A\left(q_0R_0\right)^2}\frac{d^2}{d\theta^2}A_0\left(\theta\right) \simeq \left\{\frac{\left(m^0\right)^2}{S_Ar_0^2}\left(V_{AD}^0\right)^2\varepsilon_B\theta^2 - \delta\omega_\theta^2\right\}A_0\left(\theta\right), \tag{IV-2}$$

where $A_0\left(\theta\right) \equiv A\left(\theta, r-r_0\right)$ and $V_{AD}^0 \equiv B_0/\left(4\pi n_D m_D\right)^{1/2}$. The considered solution is

$$A_0\left(\theta\right) = \exp\left[-\frac{1}{2}\left(\frac{\theta}{\Delta\theta}\right)^2\right],$$

where $\Delta\theta \sim \left(r_0/R_0\right)^{1/4}/\left(m^0q\right)^{1/2}$.

The "apt frequency" modes correspond to choosing $\omega^2 = \Omega_{T0}^2 + \delta\omega^2$ where $\Omega_{T0} = eB_0/\left(m_Tc\right)$ and $\Omega_T^2 \simeq \Omega_{T0}^2\left(1+\bar{\varepsilon}_B\right)$. Then, maintaining the limit $\varepsilon_B < \delta\omega^2/\Omega_{T0}^2$ for the sake of simplicity,

$$S \simeq -\frac{\omega_{pT}^2}{\delta\omega^2}\left\{1+\frac{\Omega_{T0}^2}{\delta\omega^2}\bar{\varepsilon}_B-\frac{6}{5}\frac{n_D}{n_T}\frac{\delta\omega^2}{\Omega_{T0}^2}\left[1-\frac{9}{5}\bar{\varepsilon}_B+\frac{4}{5}\frac{\delta\omega^2}{\Omega_{T0}^2}\right]\right\}, \tag{IV-3}$$

$$D^2 \simeq \left(\frac{\omega_{pT}^2}{\delta\omega^2}\right)^2\left\{1+2\frac{\Omega_{T0}^2}{\delta\omega^2}\bar{\varepsilon}_B+\bar{\varepsilon}_B\left(1-\frac{8}{5}\frac{n_D}{n_T}\right)+\frac{\delta\omega^2}{\Omega_{T0}^2}\left(1-\frac{8}{5}\frac{n_D}{n_T}\right)\left(1-\bar{\varepsilon}_B\right)\right.$$
$$\left.-\frac{8}{25}\frac{n_D}{n_T}\left(9-2\frac{n_D}{n_T}\right)\left(\frac{\delta\omega^2}{\Omega_{T0}^2}\right)^2\right\} \tag{IV-4}$$

$$\frac{D^2}{S} \simeq -\frac{\omega_{pT}^2}{\delta\omega^2}\left\{1+\frac{\Omega_{T0}^2}{\delta\omega^2}\bar{\varepsilon}_B+\frac{\delta\omega^2}{\Omega_{T0}^2}\left[1-\frac{2}{5}\frac{n_D}{n_T}-\bar{\varepsilon}_B\left(1+\frac{4}{5}\frac{n_D}{n_T}-\frac{32}{25}\frac{n_D^2}{n_T^2}\right)\right]\right.$$
$$\left.-\frac{2}{25}\frac{n_D}{n_T}\left(9-2\frac{n_D}{n_T}\right)\left(\frac{\delta\omega^2}{\Omega_{T0}^2}\right)^2\right\} \tag{IV-5}$$

and

$$\omega^2\left(S-\frac{D^2}{S}\right) \simeq \omega_{pT}^2\left\{1+\frac{4}{5}\frac{n_D}{n_T}-\bar{\varepsilon}_B\left(1+\frac{74}{25}\frac{n_D}{n_T}-\frac{32}{25}\frac{n_D^2}{n_T^2}\right)+\left(1+\frac{26}{25}\frac{n_D}{n_T}+\frac{4}{25}\frac{n_D^2}{n_T^2}\right)\left(\frac{\delta\omega^2}{\Omega_{T0}^2}\right)\right\}. \tag{IV-6}$$

Therefore,

$$\left[k_\perp^2\big)_{op}+k_\parallel^2\big)_{op}\right]d_D^2\simeq\frac{2}{3}\frac{n_T}{n_D}\left\{1+\frac{4}{5}\frac{n_D}{n_T}-\varepsilon_B\theta^2\left(1+\frac{74}{25}\frac{n_D}{n_T}-\frac{32}{25}\frac{n_D^2}{n_T^2}\right)\right.$$
$$\left.+\left(1+\frac{26}{25}\frac{n_D}{n_T}+\frac{4}{25}\frac{n_D^2}{n_T^2}\right)\left(\frac{\delta\omega^2}{\Omega_{T0}^2}\right)\right\}\tag{IV-7}$$

where $k_\perp^2\big)_{op}$ and $k_\parallel^2\big)_{op}$ are given by Eqs. (III-2) and (III-3). The mode radial structure can be derived as in the case of the low frequency limit while the relevant (simplified) ballooning equation becomes

$$\frac{1}{\left(R_0q_0\right)^2}\frac{d^2A_0\left(\theta\right)}{d\theta^2}\simeq\frac{2}{3}\frac{\left(\omega_{pD}^0\right)^2}{c^2}\left\{\varepsilon_B\left(\alpha_n+\frac{74}{25}-\frac{32}{25}\frac{1}{\alpha_n}\right)\theta^2\right.$$
$$\left.-\left(\alpha_n+\frac{26}{25}+\frac{4}{25}\frac{1}{\alpha_n}\right)\left(\frac{\delta\omega_\theta^2}{\Omega_{T0}^2}\right)\right\}A_0\left(\theta\right)\tag{IV-8}$$

where $\omega_{pD}^{0} = \omega_{pD}\left(r = r_0\right)$. Clearly, the limit corresponding to $\delta\omega^2 / \Omega_{T0}^2 \sim \varepsilon_B$ remains to be analyzed.

V. Mode-particle Resonant Interactions

The mode-particle resonant interactions [6] that are involved in the transfer of energy among different particle populations and can be compatible with each other have to take into account the fact that ballooning modes can be viewed as composed of travelling waves with equal amplitudes propagating in opposite directions along the magnetic field. These waves have the same frequency as that, ω_0, of the ballooning modes but different phase velocities ω_0 / k_l where k_l is the parallel (to the magnetic field) wave number for each wave component. Pairs of wave numbers with opposite signs can be invoked in the set of mode-particle resonance that is considered.

The distributions in momentum space of the fusing nuclei are assumed to be symmetric in $v_{\|}$. In particular, the mode that is considered to be apt at transferring energy from the fusion reaction products to the fusing nuclei, minimizing the interaction with the electron population, is that with a frequency $\omega_0 \simeq \Omega_T$. Then the set of mode-particle resonances that involve the tritium and the α- particle populations for comparable values of k_l consists of

$$\delta\omega = \left.k_l^p\right)_T V_\parallel^T \quad \text{and} \quad \Omega_\alpha + \left.k_l^p\right)_\alpha V_\parallel^\alpha = \omega, \tag{V-1}$$

for $\omega = \omega_0 + \delta\omega$, considering that the wave energy density exchange is proportional [3] to ω. Therefore,

$$\left|\frac{\left.k_l^p\right)_T}{\left.k_l^p\right)_\alpha}\right| = 2\left|\frac{\delta\omega}{\Omega_T}\frac{V_\parallel^\alpha}{V_\parallel^T}\right|. \tag{V-2}$$

The α - particle distribution in momentum space is assumed to be isotropic while considering distributions of the fusing nuclei as symmetric in $v_\parallel$ is consistent with the symmetry of modes affecting their tails. For the sake of simplicity, we have assumed that the trapped particle population, involving the consideration [9] of relevant bounce frequencies ω_{bj}, is relatively small.

We note that the growth or the damping rates associated with mode particle resonances [6] represented by $-\omega-\Omega-k_{\|}\mathrm{v}_{\|}=0$ depend on $k_{\perp}\mathrm{v}_{\perp}/\Omega$ that is on finite gyroradius effects. In the case of the α - particle population $k_{\perp}\rho_{\alpha}\geq k_{\perp}d_{D}\sim 1$ for the modes under consideration, as

$$\mathrm{v}_{\alpha}\simeq 0.7\times 10^{9}\,\mathrm{cm/s},\; \rho_{\alpha}=\mathrm{v}_{\alpha}/\Omega_{\alpha}=\mathrm{v}_{\alpha}/\Omega_{D}\simeq 1.4/\left(B/10^{5}G\right)\,\mathrm{cm},$$

$$d_{D}\simeq 0.9\left(10^{15}cm^{-3}/n\right)\,\mathrm{cm}.$$

On the other hand, for the corresponding resonating tritons $k_{\perp}^{2}\rho_{T}^{2}<<1$, but this circumstance is compensated by the fact that the density of resonating tritons can be considerably larger than that of the α - particles resonating with the same mode. Since

$$\left.k_{l}^{p}\right)_{\alpha}\simeq\frac{1}{3\rho_{\alpha}},$$

the relevant modes will have to exclude waves with $k_{l}=0$ and the spectrum of waves composing relevant ballooning modes will need to include this value of k_{l}^{p}. Clearly, in order to minimize Landau damping on the electron population the relevant temperature T_e will have to be sufficiently high so that

$$V_\alpha^2 << V_{th,e}^2.$$

The excitation of modes [1] with $\omega \simeq \Omega_D$ and their harmonics [10] has to be examined in a different context and in particular in that of their interactions with the electron population [8].

Relevant experimental observations occurring in a different confinement configuration that that considered here is reported in Ref. [2]. These involve a drastic increase of the rate of emission of neutrons produced by D-D reactions resulting from the injection, into a Deuterium plasma, of non-reacting protons through a neutral hydrogen beam injection (15 keV) system. Consistently with our considerations, the frequency of the excited mode has been found to be ion cyclotron frequency of the target Deuterium plasma which is well below that of the proton beam.

VI. Evolution of Ballooning Modes

Referring to the combination of growth and damping resulting from the interaction of the considered modes with the reaction products and the fusing nuclei, the modes will evolve to become purely oscillatory where the growth and damping rates compensate each other. In particular, we expect that the mode radial profile will change during its evolution as shown by the following analysis referring to the case where damping prevails. Confirming the observation that an oscillatory ballooning mode can be viewed as a superposition of standing modes having the same frequencies and involving a continuous spectrum of the relevant phase velocities [4], we note that, initially,

$$\hat{n} \propto \exp\left[-\frac{\theta^2}{2\left(\Delta\theta\right)_0^2} - i\omega_0 t\right] \propto \exp\left[-\frac{l^2}{2\left(\Delta l\right)^2} - i\omega_0 t\right]. \qquad \text{(VI-1)}$$

Here $\left(\Delta\theta\right)_0^2 < 1$, $l = R_0 q_0 \theta$ represents the length along a magnetic field line and the superposed waves are represented by

$$\exp\left[-2k_l^2\left(\Delta l\right)^2 - i\left(\omega t - k_l l\right)\right]. \qquad \text{(VI-2)}$$

Clearly, each component will be subject to a specific wave-particle resonant interaction. Thus $\omega = \omega_0 - i\gamma_0\left(k_l^2\right)$ and if, for the sake of simplicity, we assume that

$$\gamma_0\left(k_l^2\right) = 2\bar{\gamma}k_l^2\left(\Delta l\right)^2 \tag{VI-3}$$

as a "crude" model, we find

$$\hat{n} \propto \frac{l}{\left(1+\bar{\gamma}t\right)^{1/2}}\exp\left[-\frac{l^2}{2\left(\Delta l\right)^2\left(1+\bar{\gamma}t\right)} - i\omega_0 t\right]. \tag{VI-4}$$

Clearly, this indicates that the resulting mode profile becomes broadened and lowered as it evolves.

Starting from an initial $\left(t=0\right)$ mode profile represented by $\exp[-l^2/2\left(\Delta l\right)^2]$ an accurate numerical analysis of the mode profile evolution has been carried out involving the relevant Landau damping with a Maxwellian ion distribution and it has confirmed qualitatively the results represented by Eq. (VI-4) as shown by the following figure.

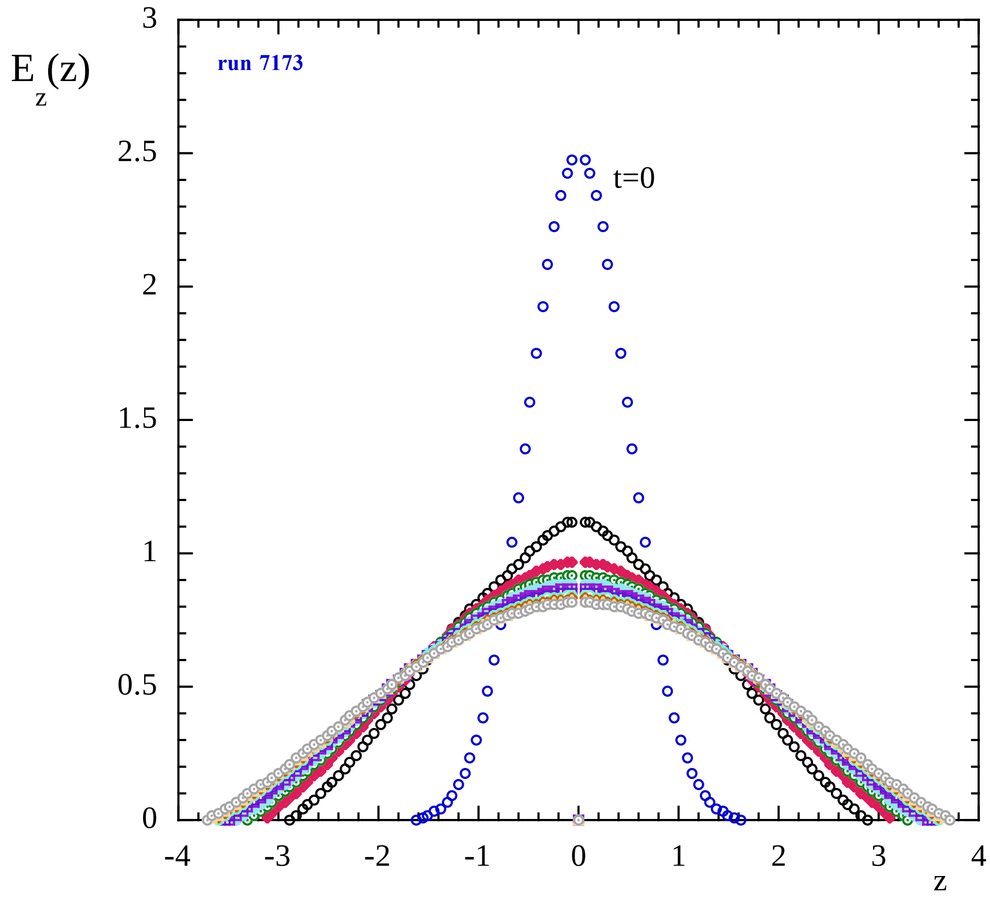

Fig.1 Evolution in time of a ballooning mode profile involving mode-particle resonances with a thermal velocity space distribution.

The evolution of a ballooning mode with the same initial profile is also being analyzed adopting a model involving resonances with two ion distributions that have different peaks in momentum space.

VII. Final Remarks

Significant fusion burning regimes are characterized by low degrees of collisionality and where the longitudinal electron thermal conductivity is relatively large. For these regimes a novel process [11] involving significant magnetic reconnection related to the electron temperature gradient has to be taken into consideration. The envisioned effect of the process is to regulate [3] the electron temperature profile affecting the global energy balance of the plasma column.

We have estimated the plasma density fluctuations that correspond to the amplitudes of modes, of the considered type, needed to extract tens of megawatts from the generated α- particle populations. The fluctuation level has been found to be sufficiently low and the relevant scale distances sufficiently short as to not significantly degrade the particle and thermal energy confinement of the reacting plasma.

Magnetically confined plasmas with "temperatures" of fusion interest involve circulating and trapped particles. When referring to the considered modes we note that

i) the relevant phase velocities and the frequencies are higher than the thermal velocities and the thermal bounce frequencies, respectively, of the reacting nuclei. Therefore, the modes "see" only circulating particles

ii) the average bounce frequencies of the trapped electrons and the transit frequencies of the circulating electrons exceed the considered mode frequencies, and the relevant mode-particle interactions are neglected

iii) for the collisionless α- particle population the formalism described in Ref. [9] that is appropriate for ballooning modes needs be applied. The adopted α- particle distribution is of the kind represented by

$$f_\alpha\left(\mathrm{v}_\perp,\mathrm{v}_\parallel\right)\propto n_\alpha\left(1+\frac{C_0\mathrm{v}_\parallel^2}{\mathrm{v}_\perp^2+\mathrm{v}_\parallel^2}\right)\exp\left[-\frac{\left(\sqrt{\mathrm{v}_\perp^2+\mathrm{v}_\parallel^2}-\mathrm{v}_\alpha\right)^2}{\Delta\mathrm{v}^2}\right]$$

with $C_0 \geq 0$.

Clearly, the injection of RF waves by systems with moderate power level deserves to be analyzed in order to investigate the possibility of coupling with the contained ballooning modes introduced earlier for the purpose of controlling their amplitudes. Moreover, given the importance of finding optimal paths to access the burn conditions with the catalyzed DD reaction chain, the presented analysis will be extended to the case where 4 nuclei populations have to be considered.

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References

[1] B. Coppi, S. Cowley, R. Kulsrud, P. Detragiache and F. Pegoraro, Phys. Fluids **12**, 4060 (1986)
[2] R. M. Magee, A. Necas, R. Clary et al., Nature **15**, 281 (2019)

[3] B. Coppi, Comments Pl. Phys. Con. Fus. Res. **5**, 261 (1980)
[4] B. Coppi, M. N. Rosenbluth and S. Yoshikawa, Phys. Rev. Lett. **20**, 190 (1968)
[5] B. Coppi, Plasma Phys. Rep. **45**, 438 (2019)
[6] B. Coppi, M. N. Rosenbluth and R. N. Sudan, Ann. Phys. **55**, 207 (1969)
[7] B. Coppi, Phys. Rev. Lett. **39**, 939 (1977)
[8] B. Coppi, Phys. Lett. A **172**, 439 (1993)
[9] B. Coppi and G. Rewoldt, *Advances in Plasma Physics*, edited by A. Simon and W. B. Thompson (Wiley, New York, 1976), Vol. 6, p. 421
[10] JET Team, Nucl. Fusion **32**, 193 (1992)
[11] B. Coppi and B. Basu, Phys. Lett. A **397**, 127265 (2021)