Towards simpler coils for optimized stellarators

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Stellarator does not use large plasma currents

- A stellarator is a toroidal plasma confinement configuration that uses external coils to produce a non-axisymmetric magnetic field.
- Stellarator is an attractive approach to fusion energy.
    - Steady state operation
    - Low recirculating power
    - Stable to plasma currents inducing MHD instabilities
    - Free of disruptions
    - High density operation
- Conventional stellarators had relatively simple coils, but suffered bad neoclassical transport.

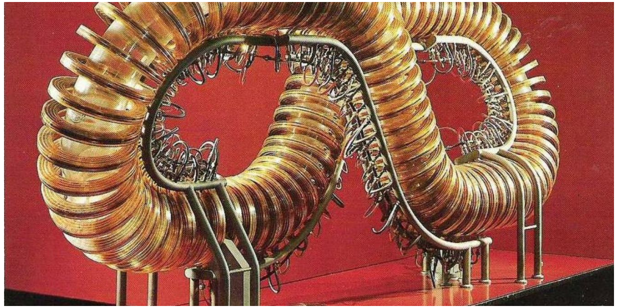
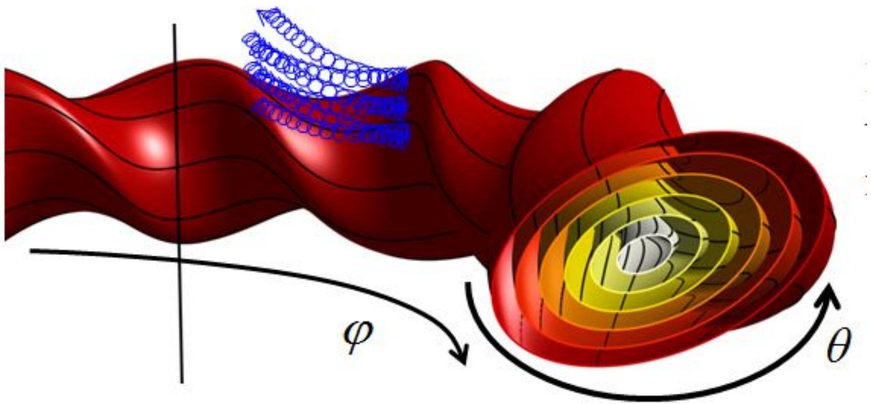
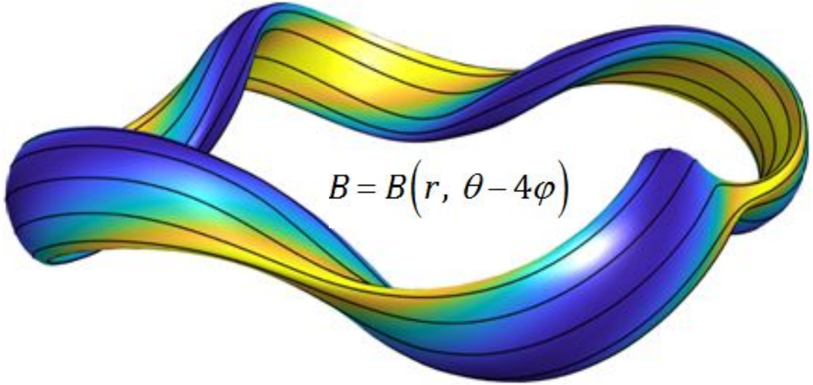


Figure-8 stellarator (1950s)

LHD stellarator (1998-present)

Trapped particles are not confined without a further condition.





Landreman, Simons Summer School, 2020

In general: trapped particles do not sample the whole surface, so cross-field drift does not average to zero. The neoclassical transport is large.

particle motion Lagrangian

Independent of ϑ and ϕ

$$\mathcal{L} = \frac{mG^2\dot{\phi}^2}{2B^2} - \mu B + q\psi\dot{\theta} - q\chi\dot{\phi}$$

Only depends on θ and ϕ through $B = |\mathbf{B}|$

Symmetry in B -> Conserved angular momentum -> zero drift average

Two stages in stellarator optimizations

Stage 1: Equilibrium optimization

Optimize the equilibrium for:

- 0D parameters (R, a, B_0, V, etc.)
- Neoclassical transport
- MHD stability
- Energetic particle confinement
- Divertors
- Turbulence transport
- ...

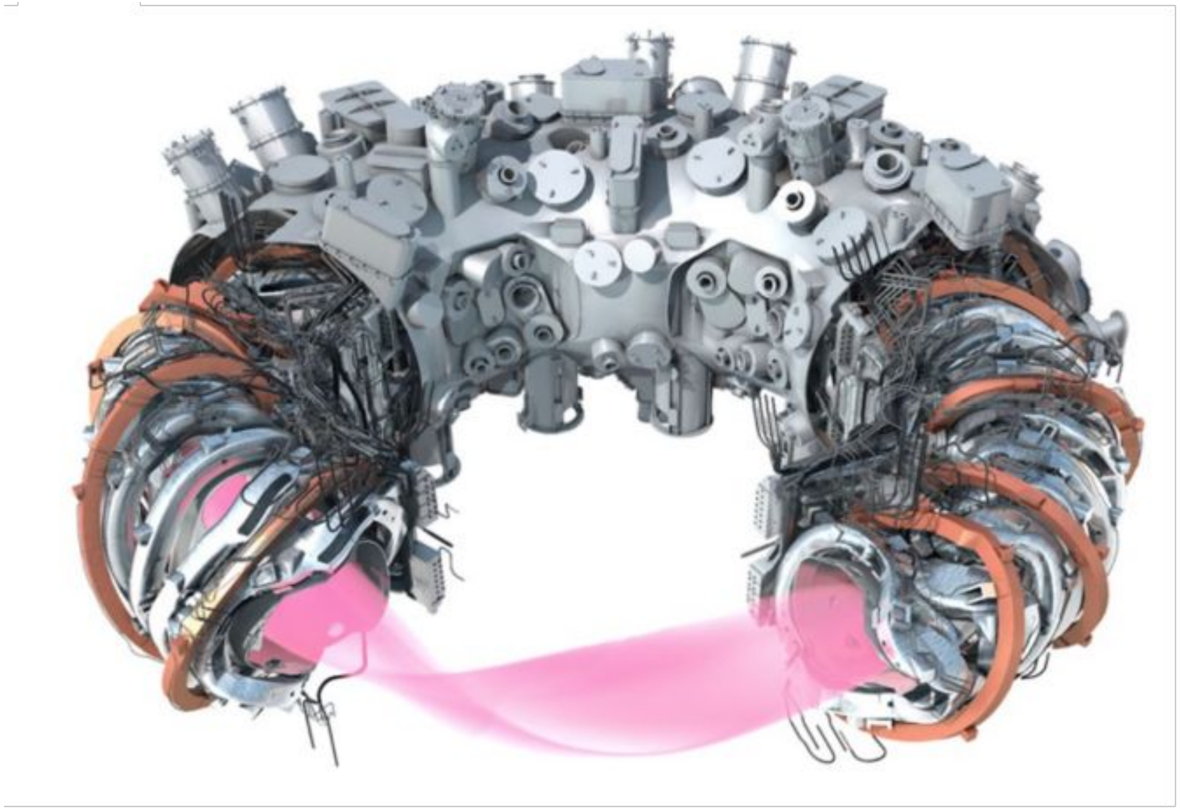
Stage 2: Coil optimization

Design buildable coils for:

- Supporting the target equilibrium
- Simple coil geometries
- Coil-to-coil space for diagnostic ports
- Coil-to-plasma space for blankets
- ...

❖ Two optimization procedures might be carried out iteratively.
❖ Coil complexity is largely determined by physics requirements.
❖ Engineering design comes after the optimizations.

Building a stellarator is not an easy project.



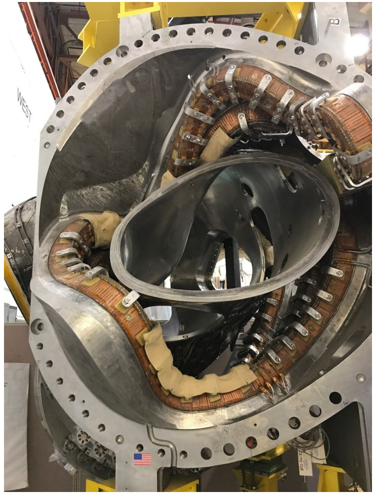
Dinklage Nature Physics 2018

The world's largest stellarator, Wendelstein 7-X, built in Germany (construction for **18** years & **€1.06 B** [wikipedia])

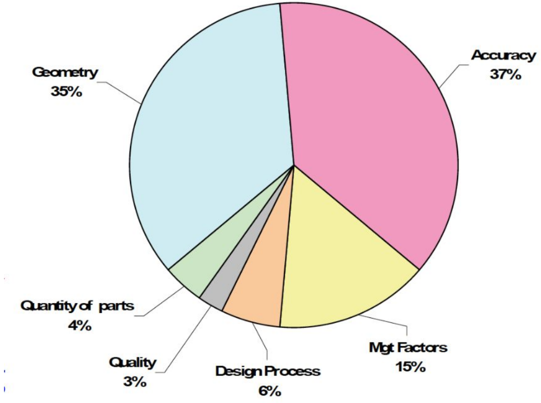
Optimized stellarator coils are generally complicated.

- ❖ Due to the 3D nature of magnetic field, optimized stellarators generally require non-planar coils.
- ❖ The challenges come from both the complicated geometry of the non-planar coils and the tight tolerance requirements.
- ❖ Difficulties in fabricating and assembling stellarator coils partly led to the cancellation of NCSX (Neilson IEEE 2010) and the delay of W7-X (Riße FED 2009).

Can we build a stellarator with simpler coils?



One modular coil of NCSX.



NCSX cost growth by category

New method to design stellarator coils

Previous coil design methods require a defined winding surface.

- ❖ For a given magnetic field, how can we find appropriate coils? (ill-posed, non-unique problem)
- ❖ Green's function to solve a continuous current potential (NESCOIL, NESVD, REGCOIL)
- ❖ Nonlinear optimization to simplify engineering complexities (ONSET, COILOPT, COILOPT++)

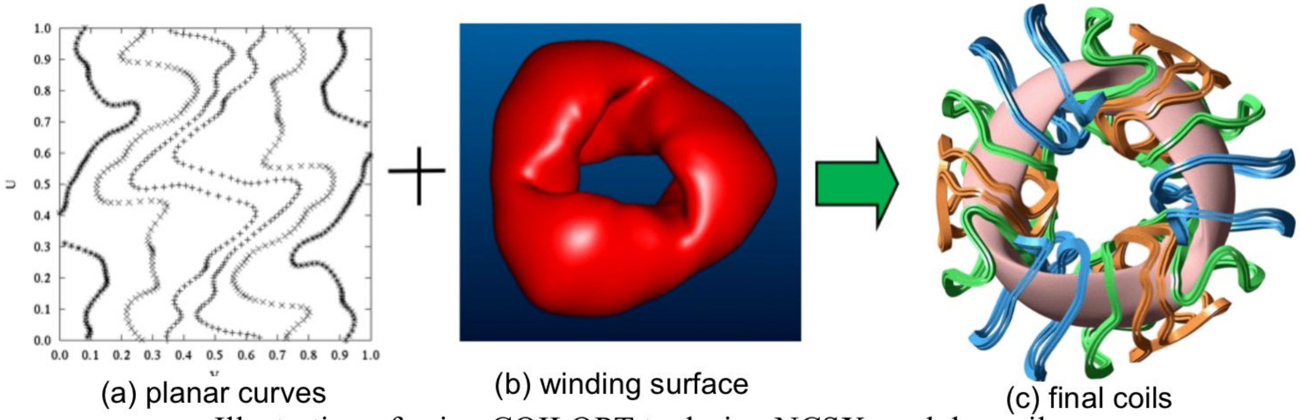


Illustration of using COILOPT to design NCSX modular coils.

- ❑ Pre-described winding surfaces limit the solution space.
- ❑ Speed and robustness are relatively poor due to not using derivatives or finite-difference.

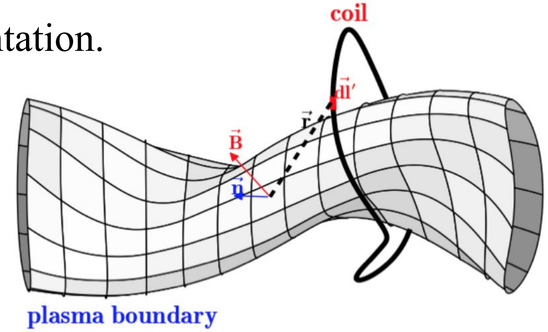
Improved methods for designing stellarator coils.

❖ Get rid of the need of winding surface by describing coils in 3D space.

- Simplify coils as single filaments (zero cross-sectional area).
- Describe coil filaments using 3D representation, like Fourier representation.

$$x(t) = X_{c,0} + \sum_{n=1}^{N_F} \left[X_{c,n} \cos(nt) + X_{s,n} \sin(nt)\right]$$

- 3D representations offer more possible designs.



❖ Analytically calculated 1st and 2nd derivatives enhance the speed and robustness.

- Derivatives are useful in optimization problems.
- Analytically calculated derivatives are faster and more accurate than finite-difference.
- Functional derivatives enable flexibility in switching parameterizations and provide shape gradient information.

$$\mathbf{B}_i(\bar{\mathbf{x}}) = I_i \oint_i \mathbf{x}_i' \times \mathbf{r}/r^3 \, d\alpha \qquad \delta\mathbf{B}(\bar{\mathbf{x}}) = \oint_i (\delta\mathbf{x}_i \times \mathbf{x}_i') \cdot \mathbf{R}_i \, d\alpha \qquad \mathbf{R} = 3\,\mathbf{r}\,\mathbf{r}/r^5 - \mathbf{I}/r^3$$

FOCUS code has been developed.

- FOCUS (Flexible Optimized Coils Using Space curves) was developed.
  - The first fully 3D stellarator coil design in the world.
  - Currently has 36 users over the world.

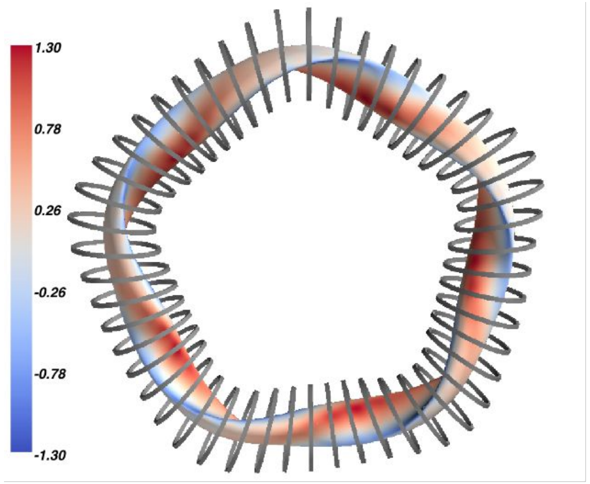
Objective functions
Normal magnetic field error
Toroidal magnetic flux
Resonant perturbation
Quasi-symmetry
Coil length
Coil curvature penalty function
Coil-surface separation

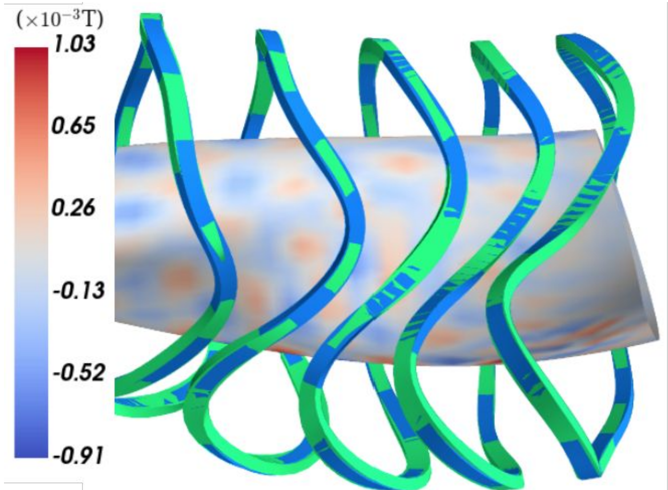
Optimization algorithms
Gradient descent
Nonlinear conjugate gradient
Levenberg-Marquardt
Modified Newton method
Scipy collections

- Zhu, *et al.* Nuclear Fusion **58** (2018) 016008
- Zhu, *et al.* Plasma Physics and Controlled Fusion **60** (2018) 065008

FOCUS obtains the same coils as the W7-X actual ones.

- Target boundary: LCFS from the W7-X standard configuration with known Bn calculated from the actual coils
- Initial guesses: circular coils (r=1.25m) equally placed surrounding the plasma;
- Objective functions: Normal field B.C. + coil length target.

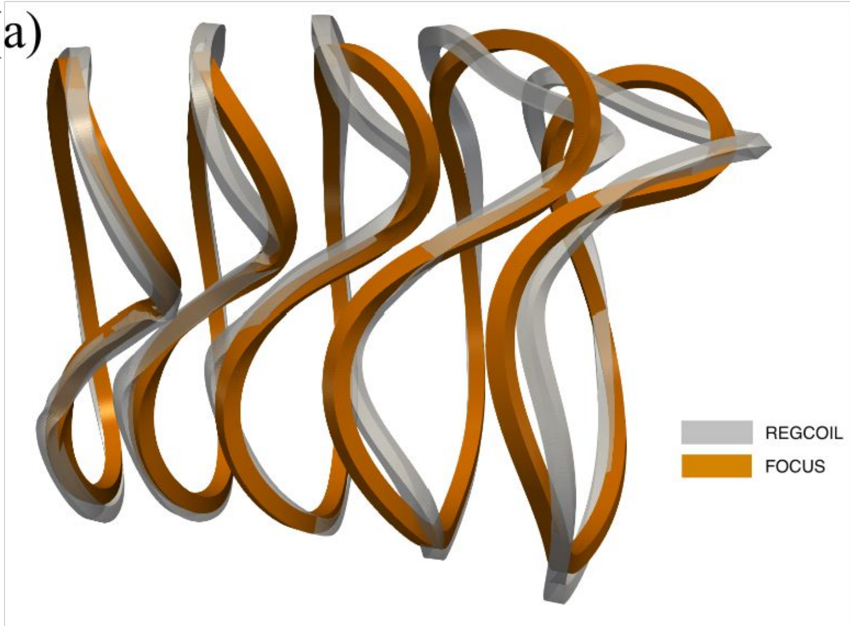




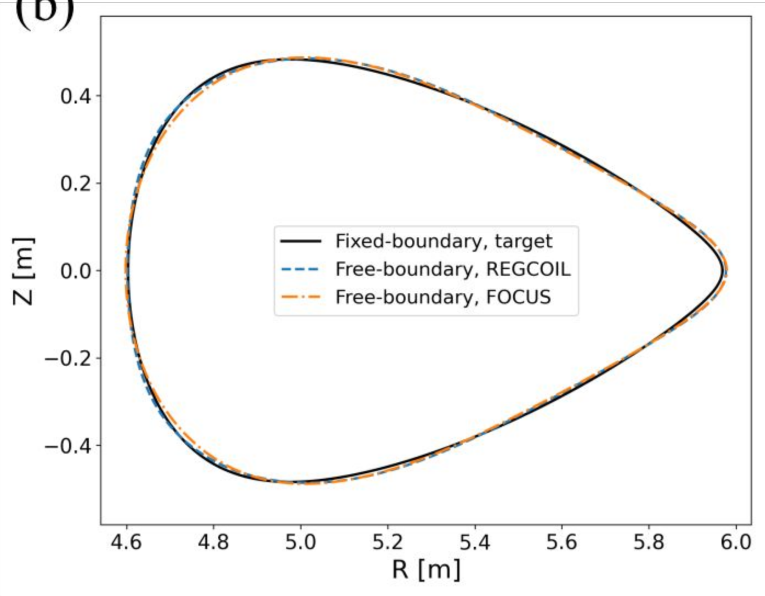
Left: Input plasma boundary, $B \cdot n - T_{Bn}$ distribution (colors) and the initial circular coil (grey). Right: Comparing optimized coils (green) and the actual coils (blue).

FOCUS can simplify stellarator coils.

(a)



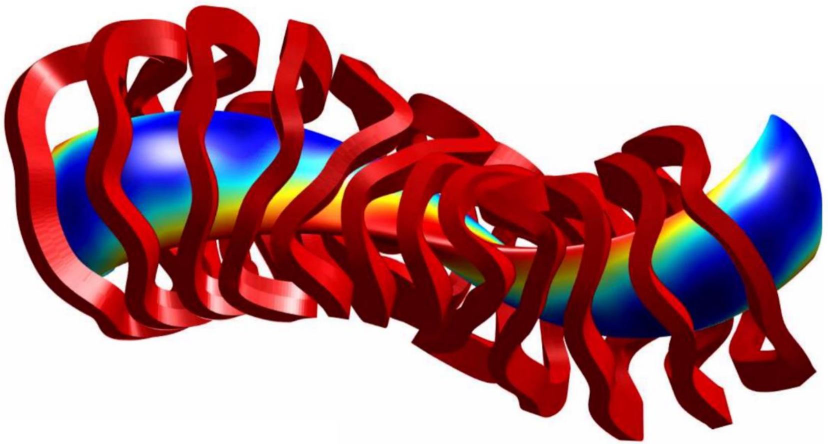
(b)

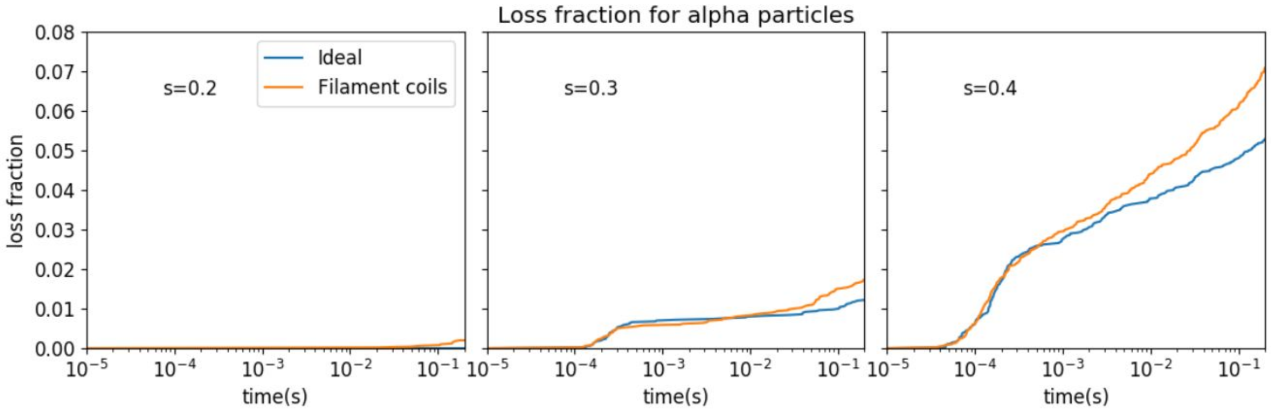


Coils	normal field error [T^2 m^2]	Average coil length [m]	Max coil curvature [m^{-1}]	Average coil curvature[m^{-1}]	Min coil-coil distance [m]	Min coil-plasma separation [m]
REGCOIL	7.71×10^{-3}	8.84	15.73	1.24	0.195	0.365
FOCUS	5.43×10^{-3}	8.19	2.38	1.13	0.173	0.352
	29.5%↓	7.35%↓	**84.9%↓**	8.9%↓	11.3%↓	3.6%↓

FOCUS designs coils for the next-generation stellarator.

- New quasi-helically symmetric configurations with good alpha-particle confinement are explored at UW-Madison.
- Modular coils designed by FOCUS have very good alpha particle confinement (Kruger JPP 2021).
- Great performance with finite coils.

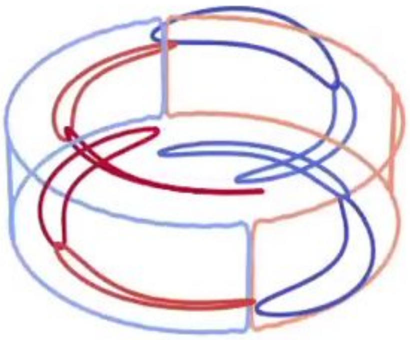


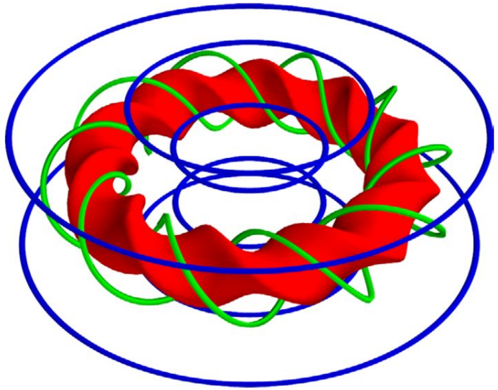


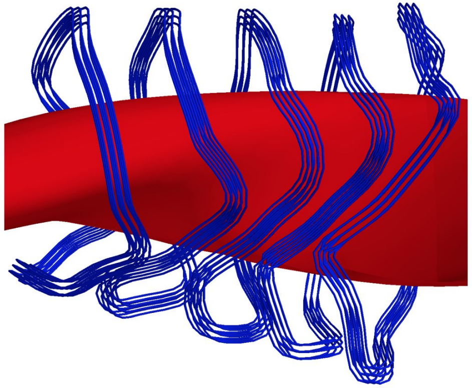
Alpha-particle losses on different flux surfaces for a reactor-size machine. (Bader IAEA-FEC 2021)

More applications and collaborations

1. Optimized stellarators using helical coils (with Columbia Univ.)
2. Novel resonant magnetic perturbation (RMP) coils (Yang NF 2020; Logan submitted to NF)
3. Optimization of finite-build coils (Singh JPP 2020)
4. Stellarator optimization using automate differentiation (McGreivy NF 2020)
5. Stochastic coil optimization (Lobsien JPP 2020)
6. Coil optimization under uncertainties (with Cornell Univ.)







Hessian matrix method for quick identification of coil sensitivity.

Accuracy requirement is crucial to stellarators.

- Plasma is sensitive to small magnetic perturbations ($\sim 10^{-4}$).
- The imperfection of magnetic field, namely error field (EF), is mainly caused by inevitable coil deviations.
- EF control is challenging for stellarators as they generally have complicated coils and the main magnetic field is provided by coils.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Confirmation of the topology of the Wendelstein 7-X magnetic field to better than 1:100,000

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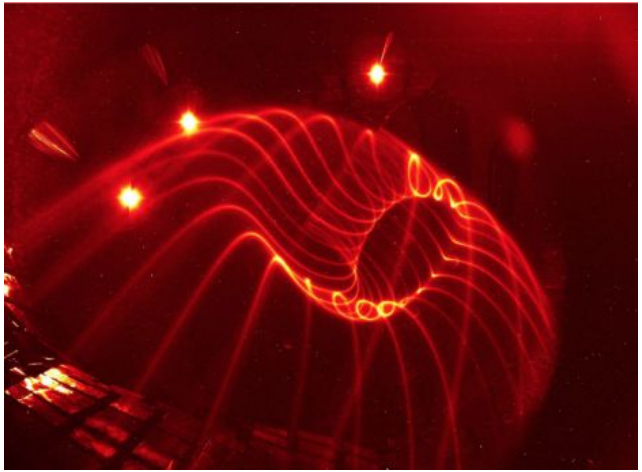


Brute-force scan of possible displacements.

- Actual coil deviations consist of combinations of local deformations, rigid shifts, module misalignments, etc.
- There exist numerous possibilities.
  - Scan a limited number of deviations;
  - Massively calculate all random schemes (*Brooks & Reiersen, SOFE, 2003*).
- Adaptive tolerance should be used for different parts of coils. (NCSX was using a uniform tolerance of 1/1000 of the major radius)

	B_{11}	B_{22}	B_{33}	B_{44}
Shift in X	0.13	0.063	0.03	0.017
Shift in Y	0.17	0.09	0.04	0.017
Shift in Z	0.12	0.043	0.027	0.017
α	2.76	1.43	0.64	0.26
β	1.22	0.45	0.27	0.18
γ	0.39	0.14	0.087	0.063

Fourier components in units $10^{-4}/B_{00}$ generated if distinct shifts (1 cm) and inclination angles (1°) are applied to the modules No. 1 in W7-X (*Kisslinger & Andreeva, FED, 2005*).

*The assembly process which took about **1,000,000** man-hours up to March 2014, was essentially dominated by the high demands on tolerances for the position of the superconducting coils.* -- H.-S. Bosch *et al* 2017 Nucl. Fusion **57** 116015

Eigenvalues provide sensitivity information.

Quadratic approximation of any arbitrary functions

$$F(\mathbf{X}) = F(\mathbf{X}_0) + (\mathbf{X} - \mathbf{X}_0)^T \cdot \mathbf{g}_0 + \frac{1}{2}(\mathbf{X} - \mathbf{X}_0)^T \cdot \mathbf{H}_0 \cdot (\mathbf{X} - \mathbf{X}_0) + \cdots$$

Deviation of the function caused by *small* perturbations near *a local minimum*

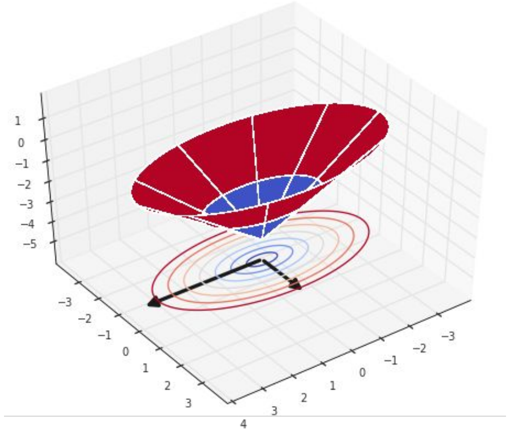
$$\delta F \approx \frac{1}{2}\delta\mathbf{x}^{\mathrm{T}} \cdot \mathbf{H}_0 \cdot \delta\mathbf{x}$$

Any arbitrary deviations can be composed in eigen-space.

$$\delta\mathbf{x} = \sum_{i=1}^{N} a_i \mathbf{v}_i$$

$$\mathbf{v}_i{}^T \mathbf{H}_0 \mathbf{v}_i = \lambda_i \,, \quad (i = 1, \cdots, N \text{ and } \lambda_i \geq 0)$$

Change in function: $$\delta F \approx \frac{1}{2}\sum_{i=1}^{N} a_i{}^2 \lambda_i$$



Eigenvectors of a 2D function.

Zhu, *et al.* Plasma Physics and Controlled Fusion **60** (2018) 054016

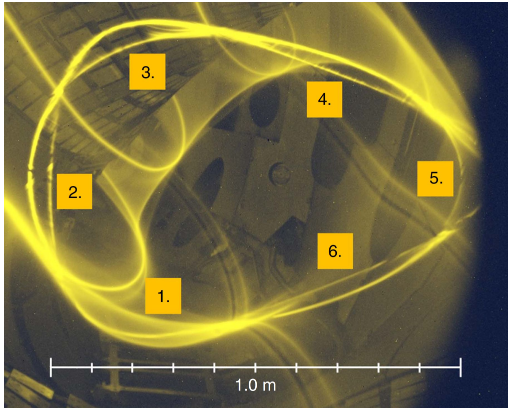
Magnetic island is one of the most import EF metric.

❖ Magnetic island is sensitive to resonant harmonics.

- Magnetic islands break flux surface and might arise instabilities.
- Manipulating resonant perturbations can control magnetic island size.

$$w = 4\sqrt{\frac{b_{mn}}{m\iota'_{mn}}} \qquad b_{mn} = \frac{1}{2\pi^2}\int_0^{2\pi}\int_0^{2\pi} \frac{\mathbf{B}\cdot\nabla\psi_t}{\mathbf{B}_0\cdot\nabla\zeta} e^{-i(m\theta - n\zeta)} \mathrm{d}\theta \mathrm{d}\zeta$$

$$F_{RP} = \sum_{m,n} (b_{mn} - b^o_{mn})^2$$



The natural n/m=5/6 island chain in W7-X (*Pedersen NC 2016*).

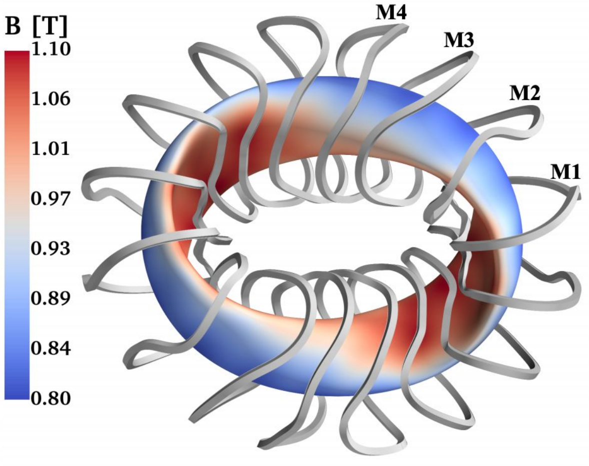
❖ Derivatives can be computed under linear approximation.

- It is difficult to accurately calculate the derivatives due to multiple nonlinear dependence.
- If assuming coordinate systems and flux surfaces don't move under *small* perturbations,

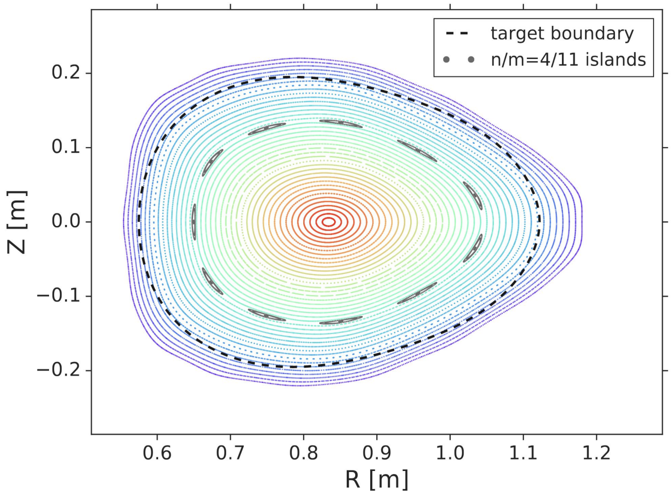
Zhu, *et al.* Nuclear Fusion **59** (2019) 126007

Application to Chinese First Quasi-axisymmetric Stellarator.

- CFQS is under construction at Southwest Jiaotong University in Chengdu, China collaborating with NIFS, Japan. (Liu NF 2020)
- The modular coils have been designed and are under fabrication.
- Lowest poloidal number rational rotational transform is n/m=4/11.



Modular coils and the target plasma boundary of CFQS.

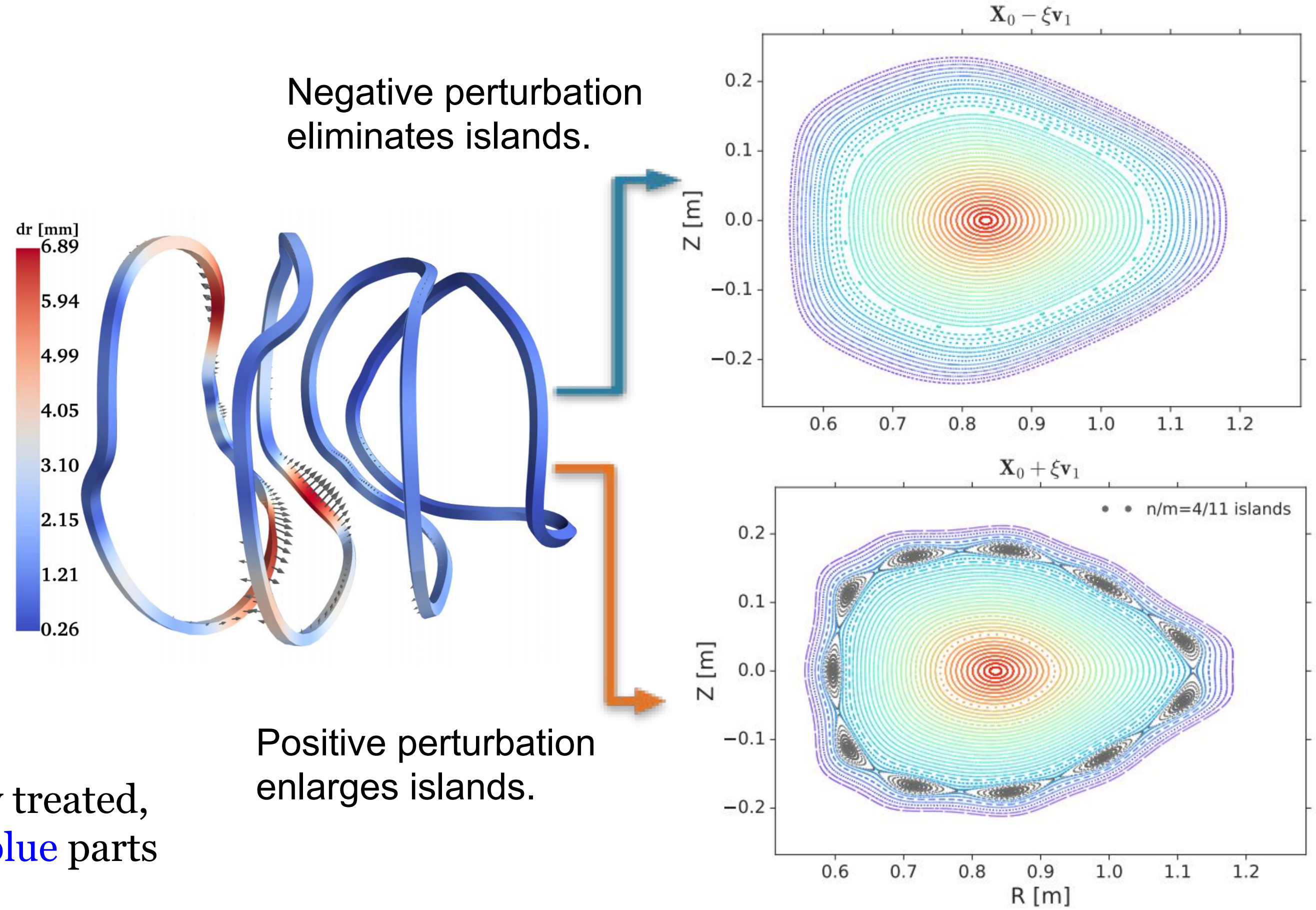


Shape of the target plasma boundary and Poincare plots of field line tracing at the bullet-shaped cross-section.

Directly identify coil deviations affecting mag. islands.

Islands are eliminated or enlarged when applying the perturbation in the direction of the first principal eigenvector (ξ=0.01; only one quarter is shown).

Negative perturbation eliminates islands.



Positive perturbation enlarges islands.

Red parts should be carefully treated, while the tight tolerance on blue parts can be relaxed.

Stellarator simplification using permanent magnets

Permanent magnets provides an alternative way to generate magnetic field.

- Rare-earth magnets are boosted to have relatively high remanent field and good coercivity.
  - Nd-Fe-B magnet has a remanent field as high as Br=1.55T (Matsuura JMM 2006).
  - It can withstand reversed background magnetic field up to H_{ci}=5.0T when cooled down.

- Special arrangements can enhance magnet efficiency (Halbach NIM 1980). A 5.16 T magnetic field was reported (Kumada IEEE 2004) using neodymium magnets.

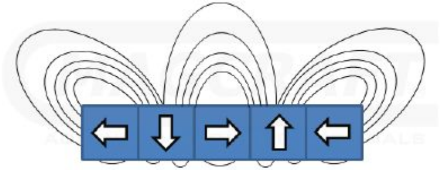


Illustration of 1D “one-sided flux” magnets.
Similar arrangements are used in fridge magnets.

- Commercially available in large amount with inexpensive price.

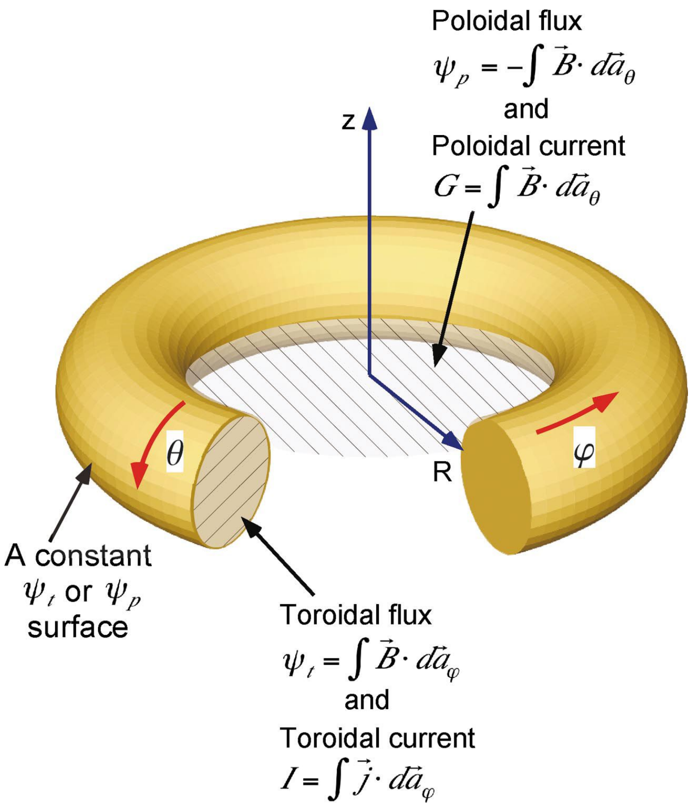
Toroidal field coils are always required.

Ampere's Law:

- Choose the loop bounded by the minimum ψ_P surface;
- No magnetization in the plasma region;
- Line integral of B is normally non-zero $\rightarrow$ poloidal current is non-zero.

$$\oint_C \mathbf{B} \cdot \mathrm{d}\mathbf{l} = \mu_0 I_{free} + \oint_C \mu_0 \mathbf{M} \cdot \mathrm{d}\mathbf{l}$$

Coils will produce the main magnetic field, while permanent magnets produce some/all shaping fied.



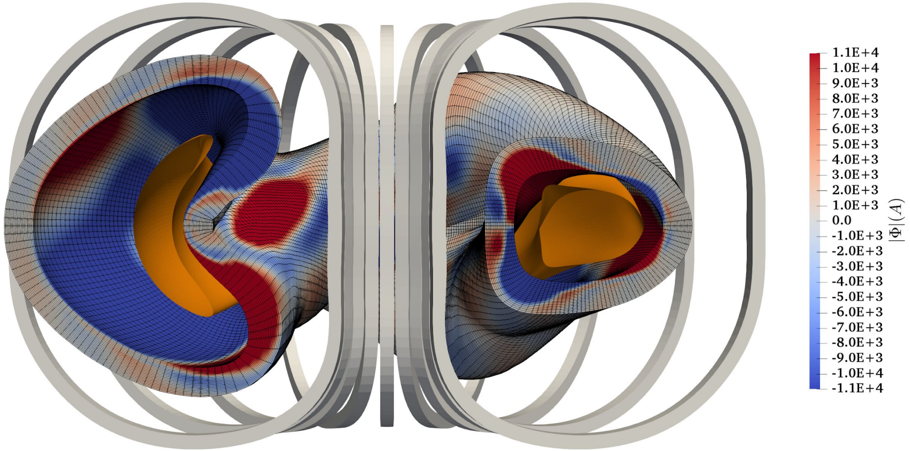
Poloidal and toroidal flux (*Boozer RMP 2005*).

Linear methods demonstrate the concept.

NESCOIL for designing coils (Merkel NF 1987): On a prescribed outer surface (winding surface) surrounding the plasma, a divergence-free surface current distribution is represented by current potential.

$$\mathbf{K} = \mathbf{n}' \times \nabla\Phi$$

The surface current distribution is chosen by linearly minimizing the normal magnetic field on the plasma surface (Neumann boundary condition).



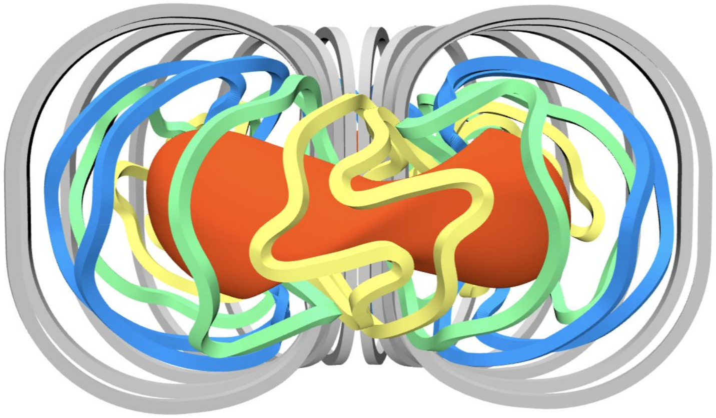
Three linear methods are proposed based on NESCOIL's theory.

- ❖ Curl-free, one-sided, tangential magnetization (Helander PRL 2020)
- ❖ Perpendicular only, multi-layer, magnetization (Zhu NF 2020a)
- ❖ Least-squares minimization, radially uniform magnetization (Landreman & Zhu, PPCF 2020)

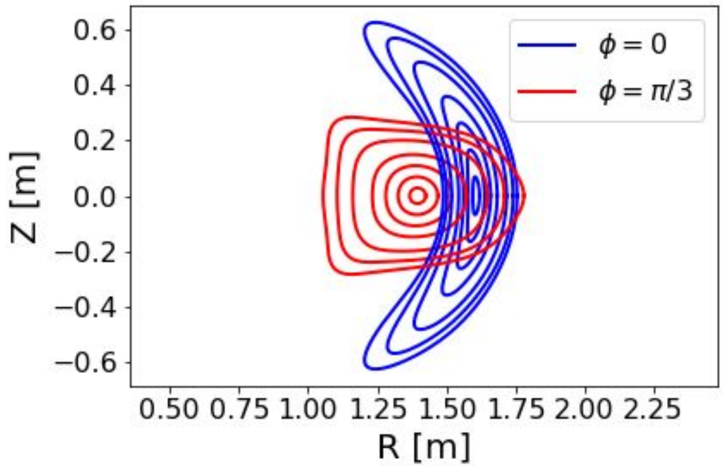
A new method varying the thickness based on Fourier and surface magnetic charges method (Xu NF 2021)

Original NCSX equilibrium is used as the target.

- NCSX was mainly optimized for quasi-axisymmetry and MHD stability (Zarnstorff PPCF 2001; Reiman PoP 2001).
- It consists of 18 modular coils (in 3 unique shapes), 18 TF coils and PF, CS coils.
- NCSX C09R00 is used as the reference equilibrium. Improved equilibria are under investigation.

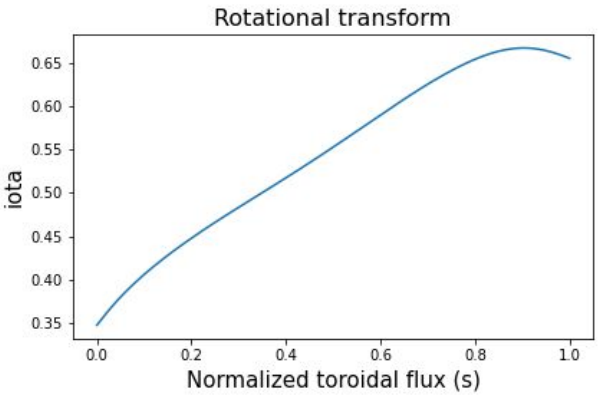


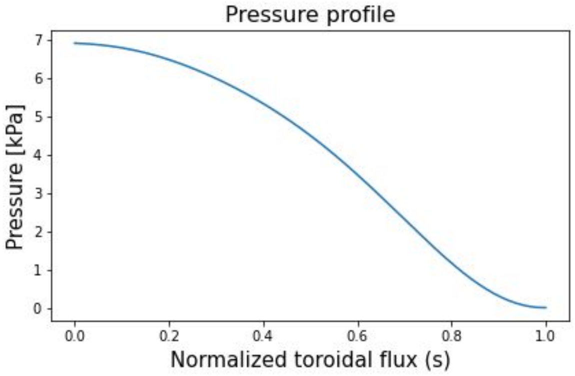
NCSX modular coils, TF coils and plasmas.



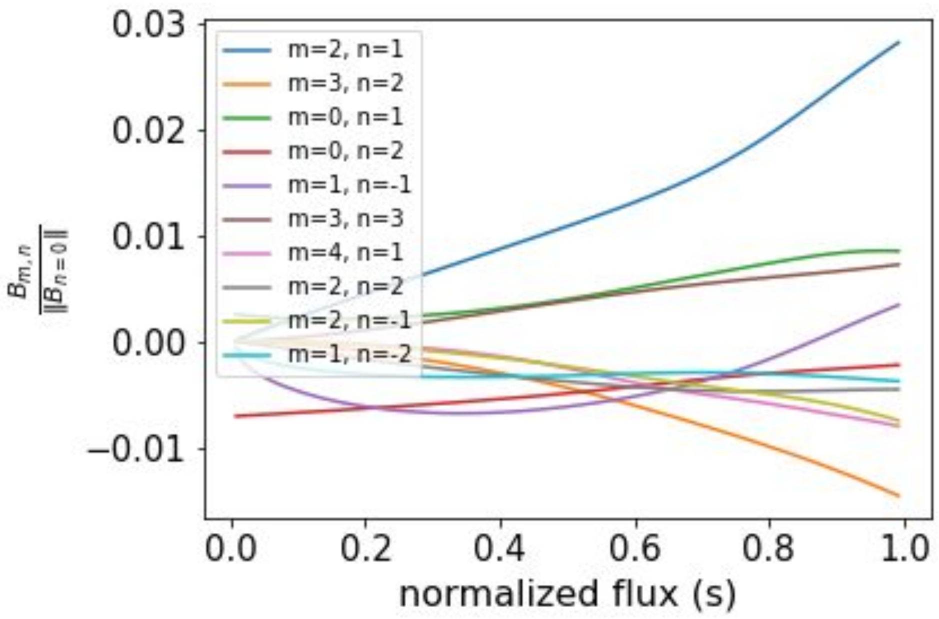
C09R00 configuration shapes at different toroidal angles.

More details on the ref. equilibrium





Scale NCSX C09R00 (<B>=1.57T, β=4.1%) to <B>=0.5T (fix the beta etc.); N_{fp}=3, R_0 = 1.44 m, a = 0.32 m, V_{plasma} = 8.89m^3



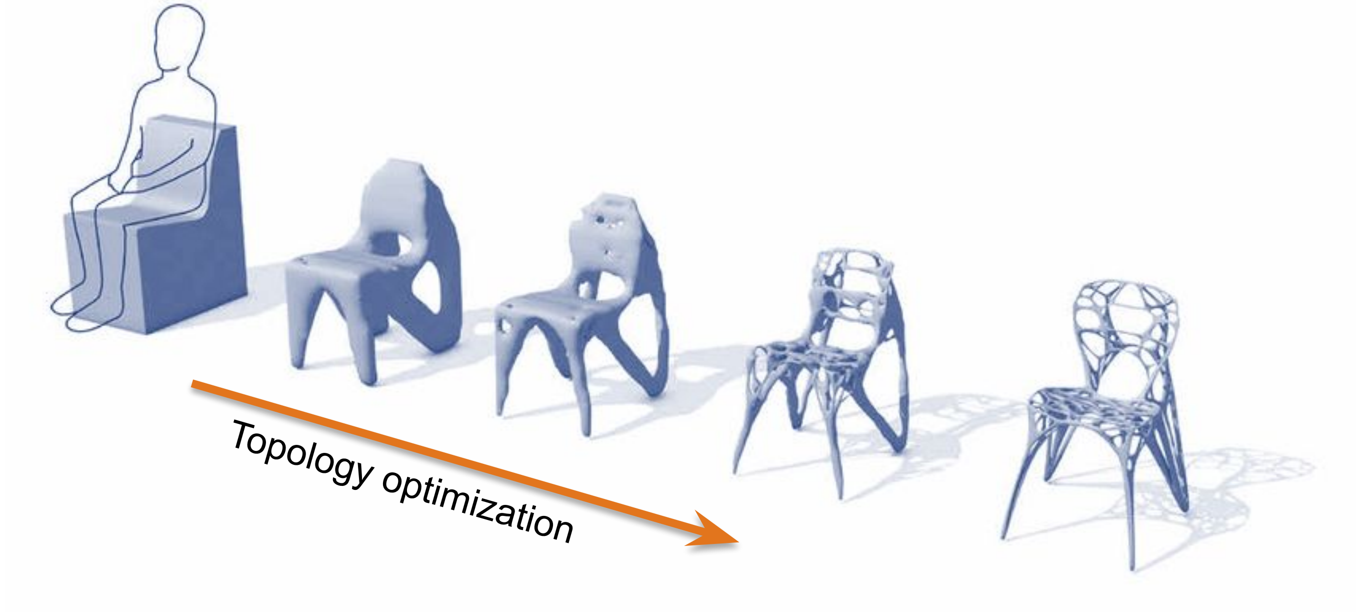
Non-axisymmetric modes of |B| in Boozer coordinates.

Re-use the TF coils and the vacuum vessel.

- ❖ Only keep TF coils for providing the toroidal field (designed to produce 0.5T as the max. field).
- ❖ Re-use the built vacuum vessel with ports opened.

Topology optimization

Topology optimization is a mathematical method that optimizes material layout within a given design space, for a given set of loads, boundary conditions and constraints with the goal of maximizing the performance of the system.



Hemmerling & Nether (2014).

Designing PM for stellarators is a topology opt. problem.

The magnets have a explicit design space: outside the vacuum vessel (avoid melting), not too far away from the plasma (~10^{-5} order at r=0.5m), and they cannot be placed over opening ports.

Given a desired magnetic field, how can we come up with an "appropriate" design for permanent magnets that is attainable with present material?

➔ The fitness of the magnetic field is normally evaluated by a boundary condition

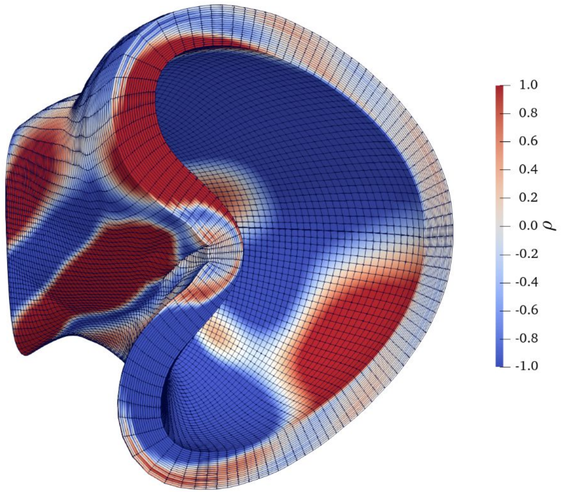
$$F_B = \oint_S \left(\mathbf{B}_M \cdot \mathbf{n} - B_n^{tgt}\right)^2 da$$

➔ Appropriate designs should embrace the engineering constraints, like using the minimum amount of magnets, easy-to-build, etc.

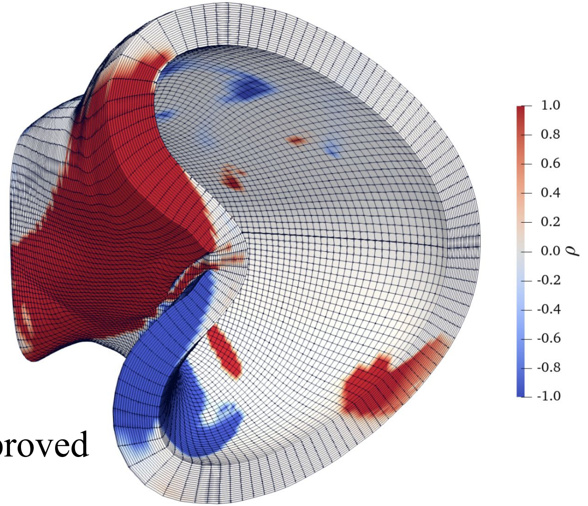
➔ Magnetization limit is determined by material grade. In this talk, we are using B_r=1.4 T with constant permeability and ignores magnetic hysteresis.

FAMUS code is developed for designing PM.

- Flexible Advanced Magnets Used for Stellarators (**FAMUS**) has been developed. (*Zhu NF 2020b*).
- Use a density method to determine the presence of material. $\rho = \begin{cases} 0 : \text{no material} \\ 1 : \text{material} \end{cases}$
- Employing L-BFGS-B (Quasi-Newton) method for constrained optimization with analytically calculated gradient.
- Meshes are provided externally. Geometry information can be imported from MAGPIE (Hammond NF 2020).

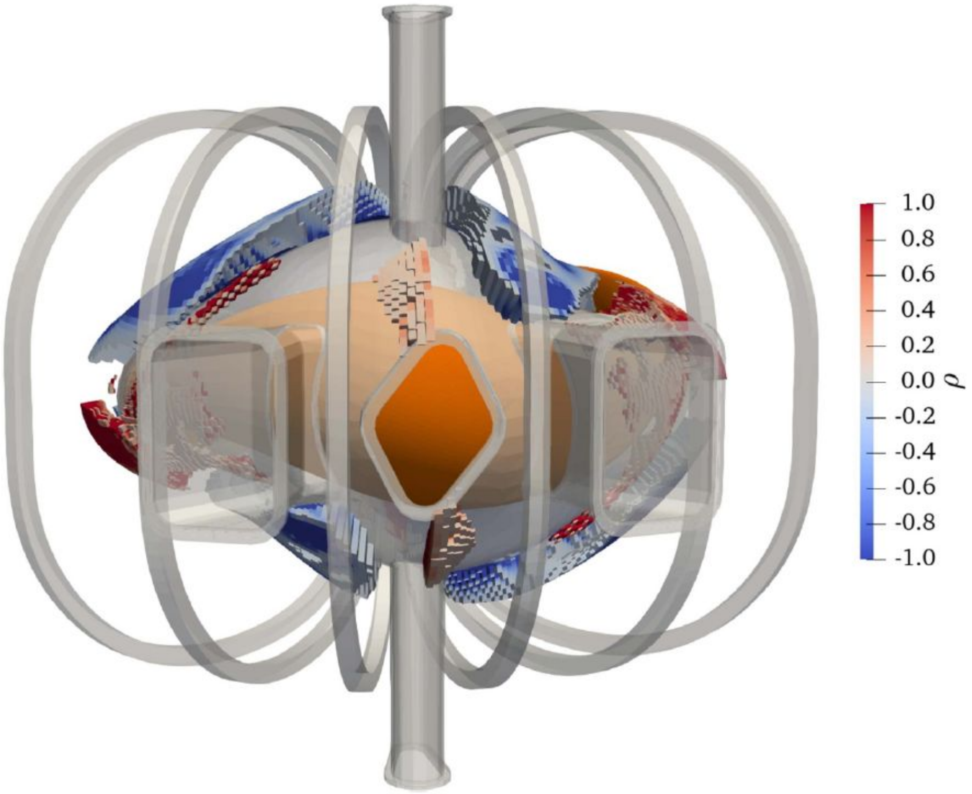


FAMUS opt.



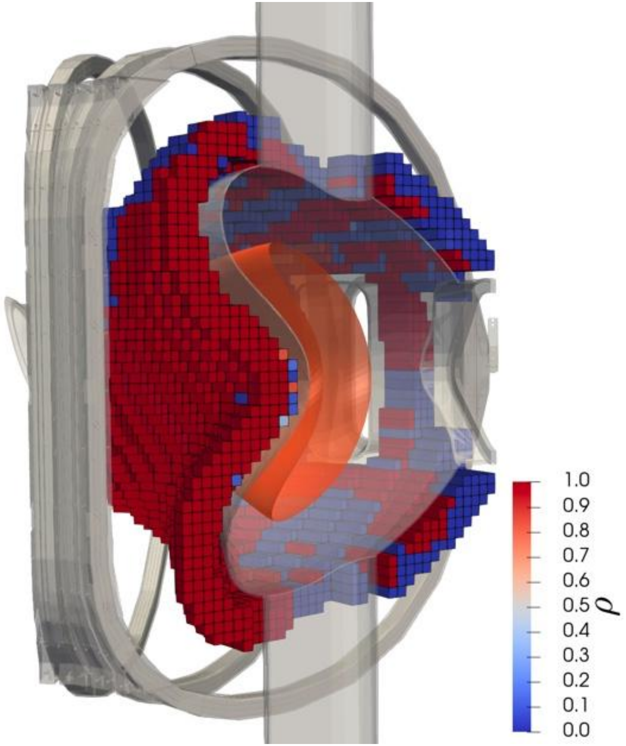
Ports are reserved; accuracy improved by x10; use 40% less magnets

Massive outboard access is attractive.

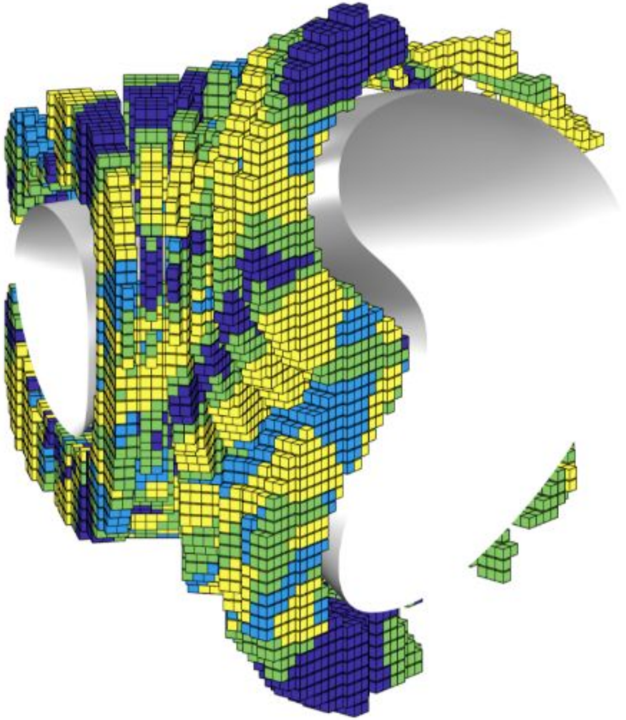


Clip the zeros (|ρ|<0.1), plotted with TF coils and vacuum vessel.

Designs with cuboidal magnets have been obtained.

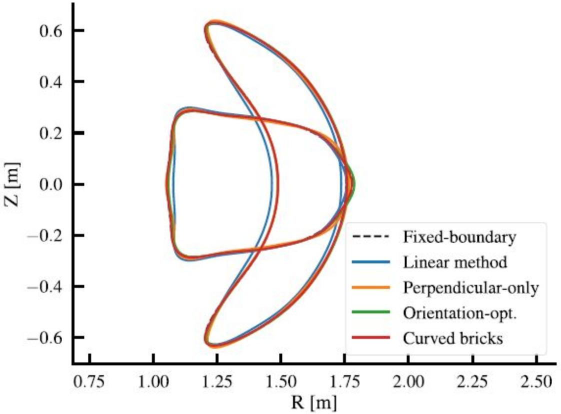


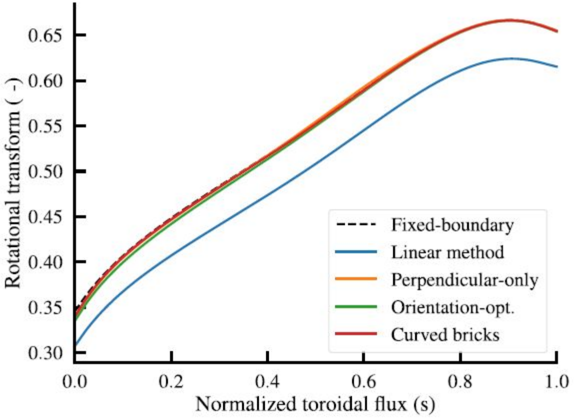
Cuboidal magnets in arbitrary orientations with almost perfect binary distribution for the magnetization.

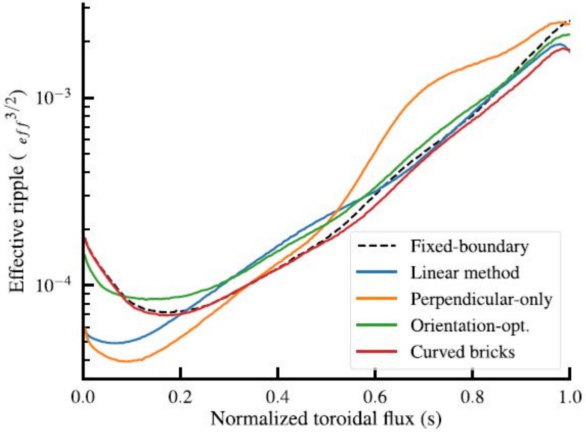


Discrete orientations using only four types of magnets (shown in different colors). All the magnets are in the same shape.

Free-boundary comparisons show good consistency.







- Free-boundary VMEC calculations are performed with the magnetic field generated by TF coils + PM.
- NEO was used to calculate the effective ripple.
- All the FAMUS solutions are having low field error and good physics properties.

Designs	Linear method	Perpendicula r	Curved bricks	Discrete orientation
Ave. |Bn/B|	1.10E-2	2.46E-3	7.26E-4	3.9E-3
Equivalent volume (m^3)	0.68	0.50	0.62	0.88

New project to build PM is funded by US DOE.

We have shown a QA stellarator design using permanent magnets along with planar TF coils. The numerical results indicate that stellarator coils can be extremely simplified by using PM.

Advantages

- ❖ Could eliminate ripple from discrete coils
- ❖ Simplify coil complexity
- ❖ Cheap in cost
- ❖ Improve access to plasma
- ❖ No power supply required
- ❖ No/less cooling needed

Disadvantages

- ❑ Limited magnetic field
- ❑ Might demagnetize
- ❑ Cannot turn off
- ❑ Challenges in forces when assembling

Funding from US DOE ARPA-E & FES awarded ($4M) to design and construct 1/6 of the permanent magnets to explore relevant technologies starting from Oct. 2020 in three years. (CDR finished in April 2021)

Summary

→ Stellarator coils are one of the most expensive part of the machine. The difficulties raise from both the complex geometry and the tight tolerance.

→ We are trying to simplify stellarator coils from different ways.

- ◆ Developed a new coil optimization code which has been widely used.
- ◆ Proposed the Hessian matrix method to quickly identify coil sensitivity.
- ◆ Demonstrated stellarator coils can be extremely simple along with permanent magnets.

→ More work can be done in the future.