PROPAGATION OF RADIO FREQUENCY WAVES THROUGH TURBULENT PLASMAS

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ABSTRACT

In order to optimize the heating of plasmas, or the generation of non-inductive plasma currents, it is necessary to assess the effect of edge turbulence on the propagation of radio frequency (RF) waves in fusion devices.

In this poster, we discuss some of the consequences of Kirchhoff's approximation when applied to the propagation of RF waves through turbulent plasma. This approximation has commonly been used for studying scattering of electromagnetic and acoustic waves from rough surfaces. Coupled with the physical optics model, Kirchhoff's technique provides physical and intuitive insight into some important aspects of scattering.

We have developed a full-wave computational code ScaRF for numerical studies of the scattering of RF waves by turbulence. Beyond providing additional insight into the scattering process, ScaRF will be used to determine the limitations of Kirchhoff's approximation.

DUAL APPROACH TO SCATTERING

GEOMETRICAL (RAY) OPTICS

(Initial value formalism)

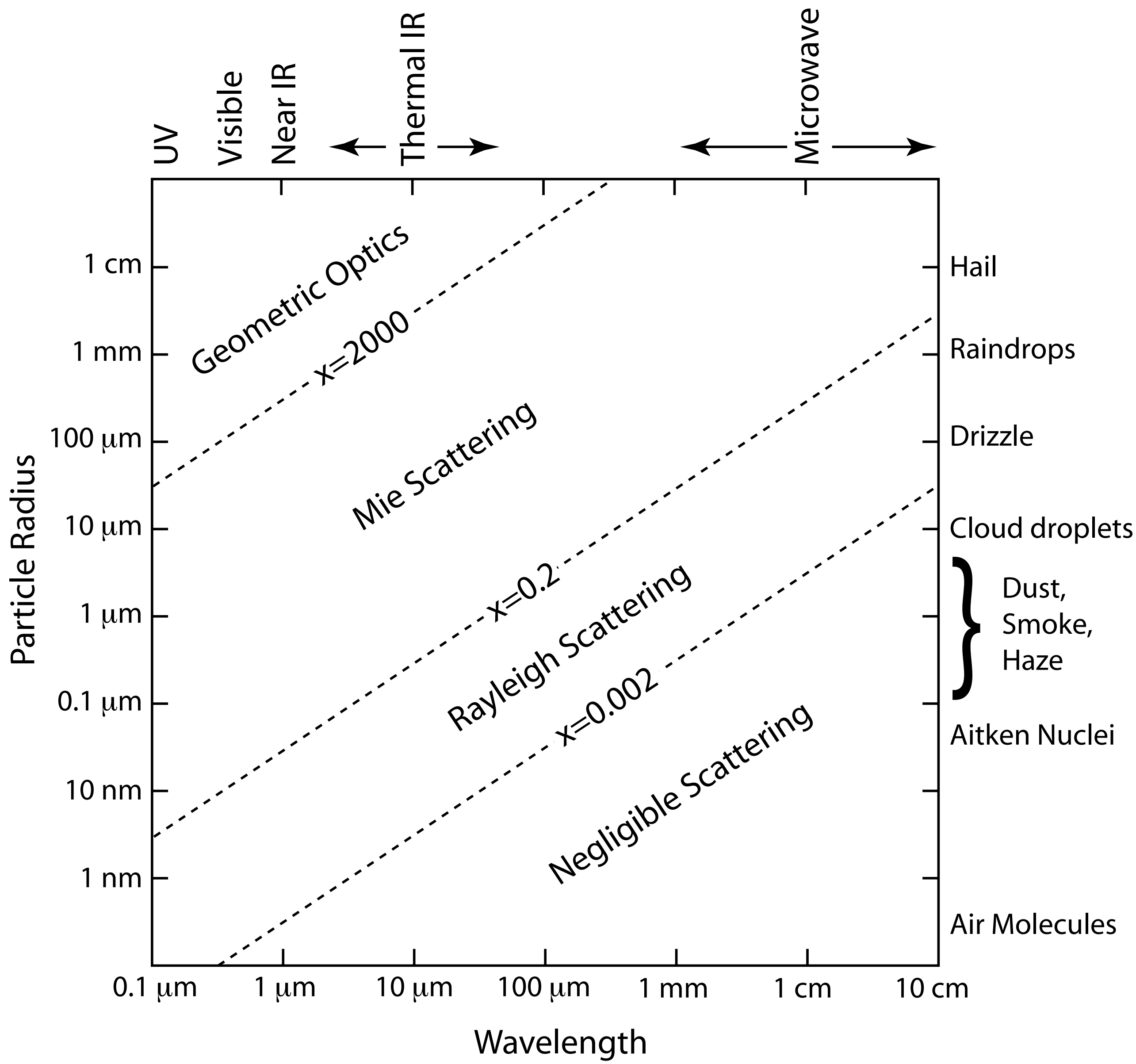
- Refraction
- Specular reflection

PHYSICAL (WAVE) OPTICS

(Boundary value formalism)

- Snell's law
- Fresnel's equations
  - reflectance
  - transmittance
- Mie scattering
- Interference
- Diffraction
- Shadowing

SCATTERING OF LIGHT BY PARTICLES



$$x = \frac{2\pi r}{\lambda}$$

G. W. Petty, *A first course in atmospheric radiation,* 2[nd] ed., Sundog Publishing, WI (2006)

SYNOPSIS AND SUMMARY

- The interaction of RF waves with planar fluctuations
  - reflection and refraction of the incident wave
  - coupling of incident wave to other plasma waves
  - changes in the wave spectrum
- The interaction of RF waves with filaments
  - side-scattering of incident wave
  - shadowing and focusing of waves
  - spatial fragmentation of power

RELEVANT APPROXIMATIONS

- Treat the fluctuations as stationary
  - for the fluctuations:

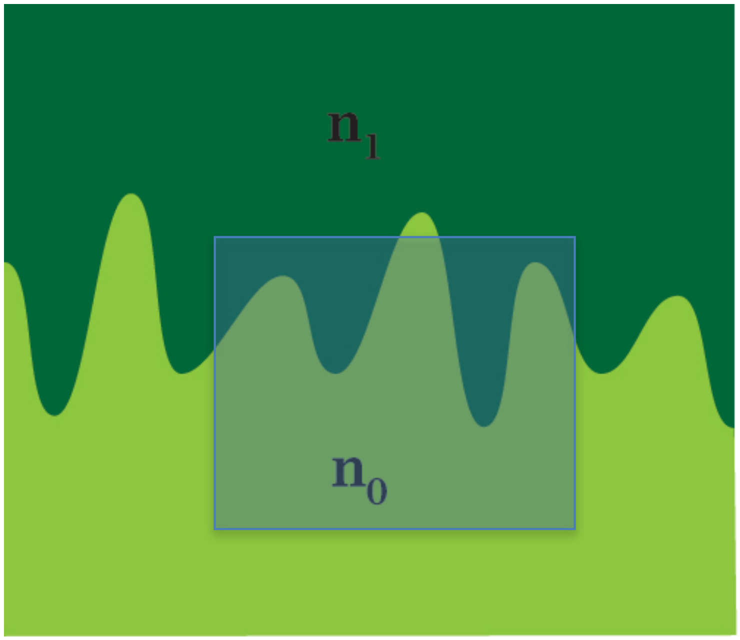
    speed $\sim$ 5×10^5 cm/sec; frequency $<$ 1 MHz
  - for the radio frequency waves:

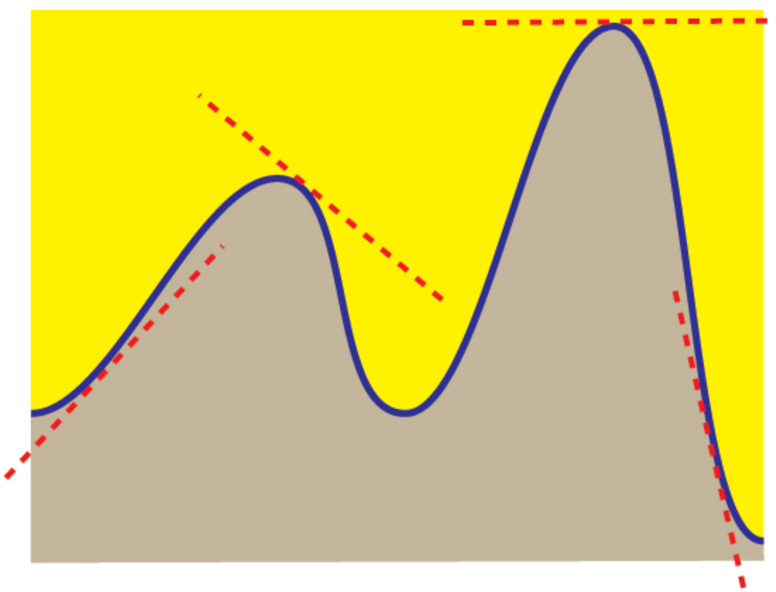
    frequencies $\sim$ $10's$ of MHz to >100 GHz

    group speed $\sim$ 10^{10} cm/sec
- Assume that the plasma is cold
- The time variation of the electromagnetic fields is

$$e^{-i\omega t}$$

MODELING SCATTERING BY PLANAR FLUCTUATIONS – KIRCHHOFF APPROXIMATION



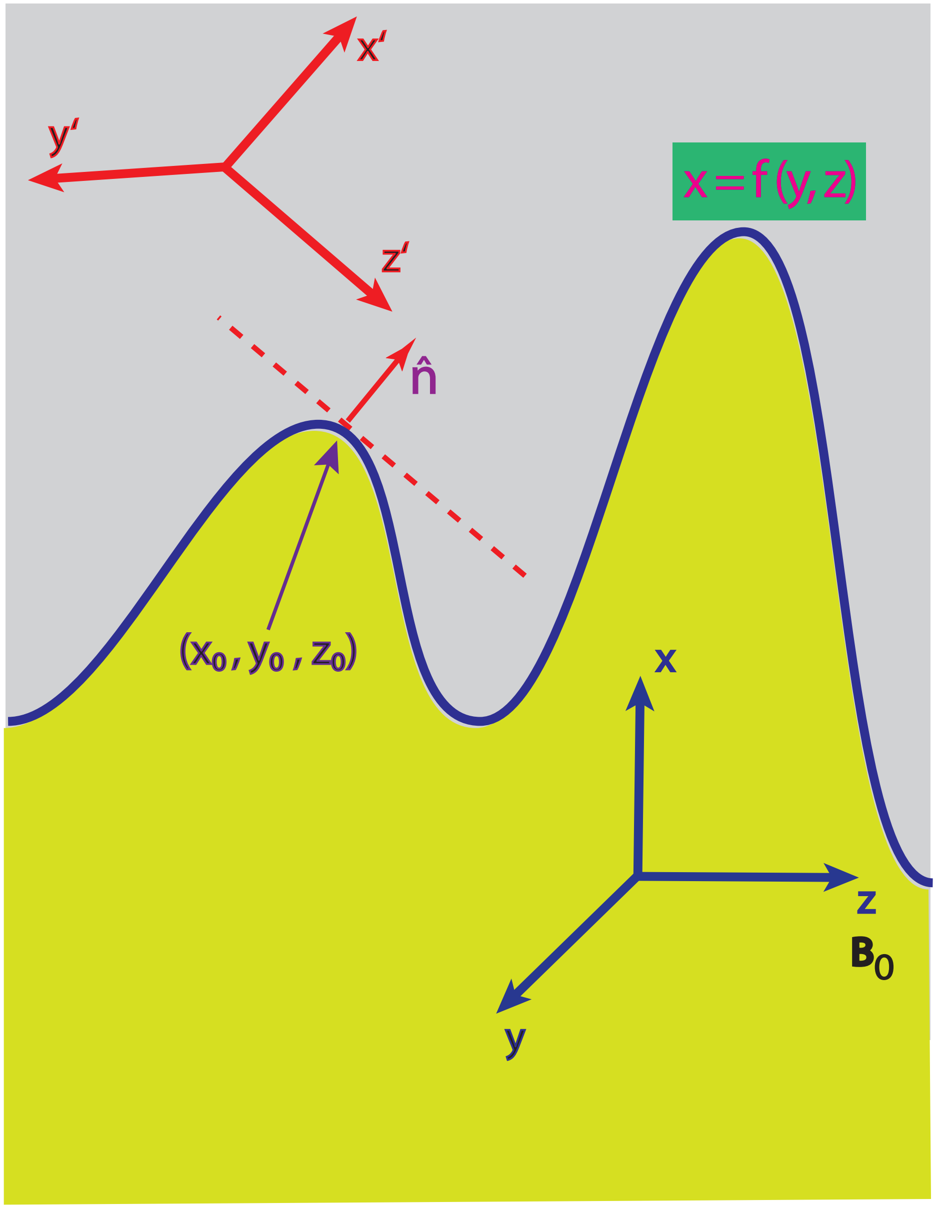


The electric field, and its normal derivative, at any point on the surface separating two different densities is the same as that on a tangent plane at that point.

(planar "micro-facets")

P. Beckmann and A. Spizzichino,
The scattering of electromagnetic waves from rough surfaces,
Artech House, MA (1987).

IMPLEMENTING KIRCHHOFF APPROXIMATION



- Let (x, y, z) define the laboratory coordinate system with

$$B_0 \parallel \hat{z}$$

- The fluctuation surface separating two different densities is

$$x = f(y, z)$$

- Consider a tangent plane at a point (x_0, y_0, z_0) on this surface with normal $\hat{n}$

- Define a new coordinate system

$$(x', y', z') \text{ with } \hat{n} \parallel \hat{x}'$$

IMPLEMENTING KIRCHHOFF APPROXIMATION

- If we define $\zeta(x,y,z) = x - f(y,z)$ then

$$\hat{n} = \frac{\nabla\zeta}{||\nabla\zeta||}\Bigg|_{(x_0,y_0,z_0)}$$

- One needs two Euler rotations to map

$$(\hat{x},\hat{y},\hat{z}) \Longrightarrow (\hat{x}',\hat{y}',\hat{z}')$$

- Rotate around the $\hat{y}$-axis through an angle ϕ and around the $\hat{z}'$-axis through an angle θ

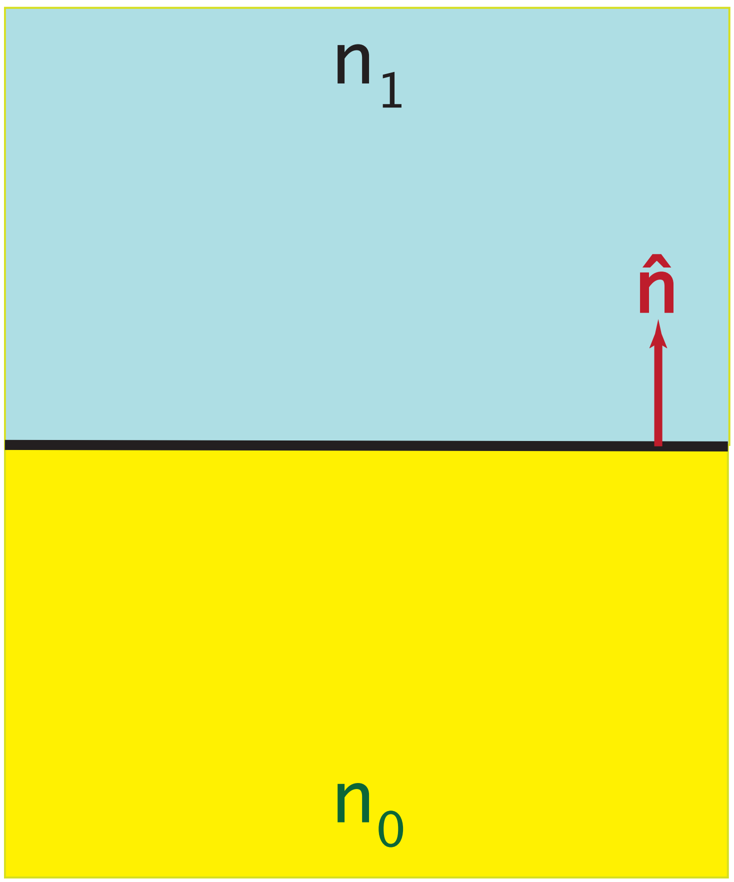
$$R_y = \begin{pmatrix} \cos\phi & 0 & -\sin\phi \\ 0 & 1 & 0 \\ \sin\phi & 0 & \cos\phi \end{pmatrix} \qquad R_{z'} = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

where $\tan\phi = -\frac{z_0}{x_0}$ and, $\tan\theta = \frac{y_0}{x_0\cos\phi - z_0\sin\phi}$

REFLECTION AND REFRACTION AT PLANAR INTERFACES – BOUNDARY CONDITIONS

- Boundary conditions
  - no free currents or charges at the interface

At the interface:



$$\Delta\left(\hat{n}\,.\,\vec{B}\right) = 0$$

$$\Delta\left(\hat{n} \times \vec{E}\right) = 0$$

$$\Delta\left(\hat{n} \times \vec{B}\right) = 0$$

$$\Delta\left(\hat{n}\,.\,\vec{D}\right) \equiv \Delta\left(\hat{n}\,.\,\overleftrightarrow{K}\,.\,\vec{E}\right) = 0$$

$$\text{Displacement field:}\quad \vec{D} = \epsilon_0\vec{E} + \vec{P}$$

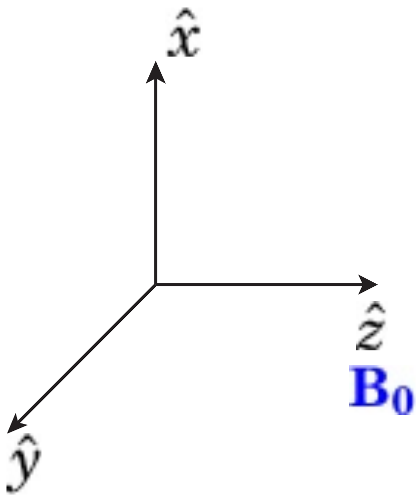
Only 4 independent conditions

WAVE DESCRIPTION IN CARTESIAN SYSTEM

- Faraday-Ampere equation:

$$\nabla \times \left(\nabla \times \vec{E}(\vec{r})\right) = \frac{\omega^2}{c^2} \overleftrightarrow{K} . \vec{E}(\vec{r})$$

$\overleftrightarrow{K}$ is the cold plasma permittivity.



$$\overleftrightarrow{K} = \begin{pmatrix} K_{\perp} & -iK_{\times} & 0 \\ iK_{\times} & K_{\perp} & 0 \\ 0 & 0 & K_{\parallel} \end{pmatrix}$$

$$K_{\perp} = 1 - \frac{\omega_{pe}^2}{\omega^2 - \omega_{ce}^2} - \sum_i \frac{\omega_{pi}^2}{\omega^2 - \omega_{ci}^2}$$

$$K_{\times} = -\frac{\omega_{ce}}{\omega} \frac{\omega_{pe}^2}{\omega^2 - \omega_{ce}^2} + \sum_i \frac{\omega_{ci}}{\omega} \frac{\omega_{pi}^2}{\omega^2 - \omega_{ci}^2}$$

$$K_{\parallel} = 1 - \frac{\omega_{pe}^2}{\omega^2} - \sum_i \frac{\omega_{pi}^2}{\omega^2}$$

WAVE DESCRIPTION IN CARTESIAN SYSTEM

- Faraday-Ampere equation:

$$\nabla \times \left(\nabla \times \vec{E}(\vec{r}) \right) = \frac{\omega^2}{c^2} \overleftrightarrow{K} . \vec{E}(\vec{r})$$

- Plane wave solutions

$$\vec{E}(\vec{r}) = \vec{E}_0 \, e^{ik_x x + ik_z z}$$

$$\vec{k} = k_x \hat{x} + k_z \hat{z} \text{ is the wave vector.}$$

- Solution to the Faraday-Ampere equation requires

$$\overleftrightarrow{D} . \vec{E}_0 = 0 \qquad \overleftrightarrow{D}\left(\vec{k}, \omega\right) = \frac{c^2}{\omega^2} \left(\vec{k}\vec{k} - k^2 \overleftrightarrow{I} \right) + \overleftrightarrow{K}$$

- Dispersion relation

$$\det \left(\overleftrightarrow{D}(k_x, k_z, \omega) \right) = 0$$

WAVE DISPERSION RELATION

- Dispersion relation

$$\det\left(\overleftrightarrow{D}(k_x, k_z, \omega)\right) = 0$$

$$\overleftrightarrow{D}\left(\vec{k}, \omega\right) = \frac{c^2}{\omega^2}\left(\vec{k}\vec{k} - k^2\overleftrightarrow{I}\right) + \overleftrightarrow{K}$$

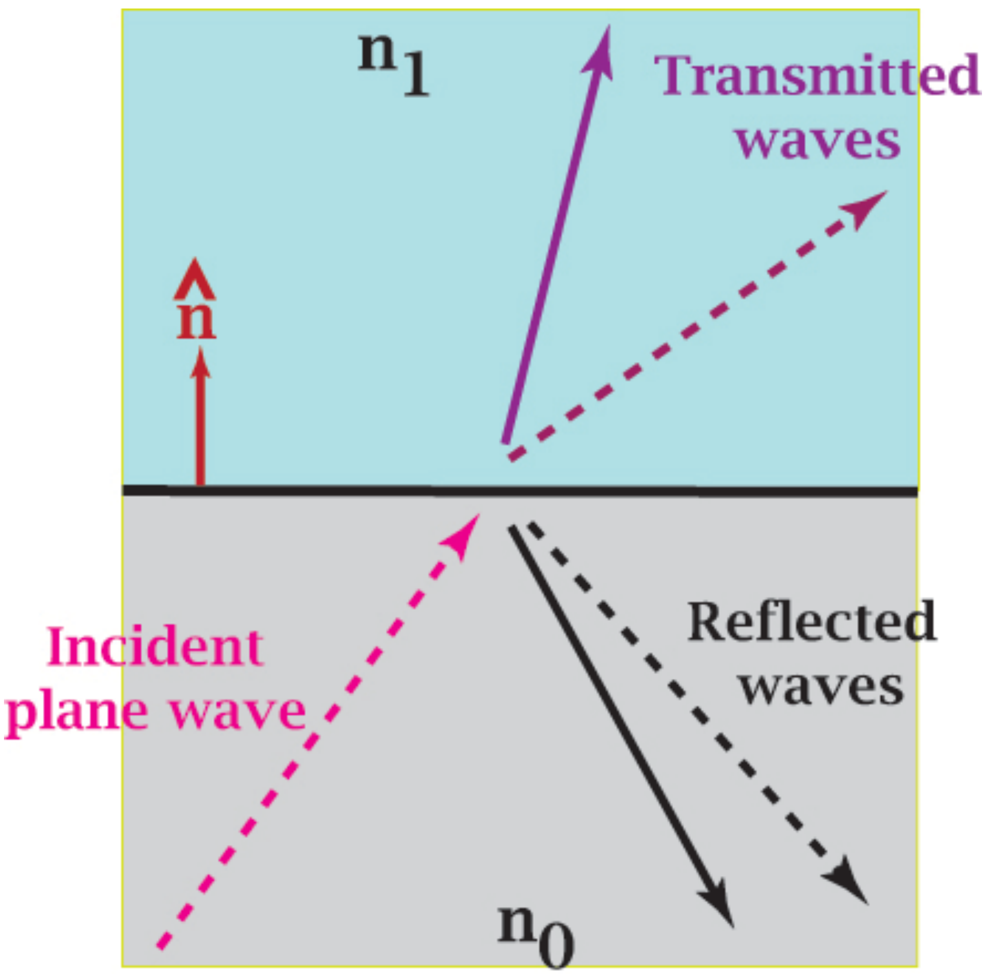
- Usually the dispersion relation is expressed as

$$k_x = k_x(k_z, \omega)$$

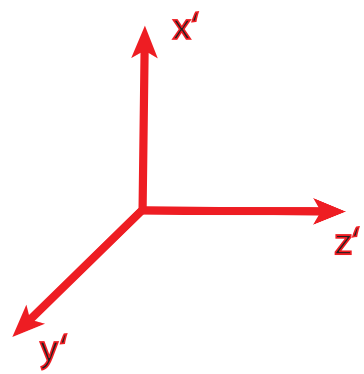
- fourth order polynomial in k_x
- only two solutions are useful at a time

REFLECTION AND REFRACTION AT PLANAR INTERFACES

Boundary conditions have to include all five waves



Incident RF power can be coupled to other normal modes of the plasma.



KINEMATIC PROPERTIES OF THE WAVES

- Assume that the normal modes of the plasma are plane waves

$$e^{i\left(k'_x x' + k'_y y' + k'_z z' - \omega t\right)}$$

- The boundary conditions at the interface $x' = 0$ must be satisfied at all points on the plane for all times. Thus,

  k'_y and k'_z are the same for all waves.

- If the incoming RF wave has the spatial phase $k_\perp x + k_\| z$, then, for all waves,

$$k'_y = -k_\perp \sin\theta \cos\phi + k_\| \sin\theta \sin\phi$$

$$k'_z = k_\perp \sin\phi + k_\| \cos\phi$$

Main results from studying planar scattering

- Fluctuations transfer some power from the launched wave to the other plasma wave

- If $\vec{B}.\nabla n_e \neq 0$ then

  the wave spectrum changes.

- If $\vec{B}.\nabla n_e = 0$ then

  the wave polarization changes and

  the wave vector normal to $\vec{B}_0$ rotates.

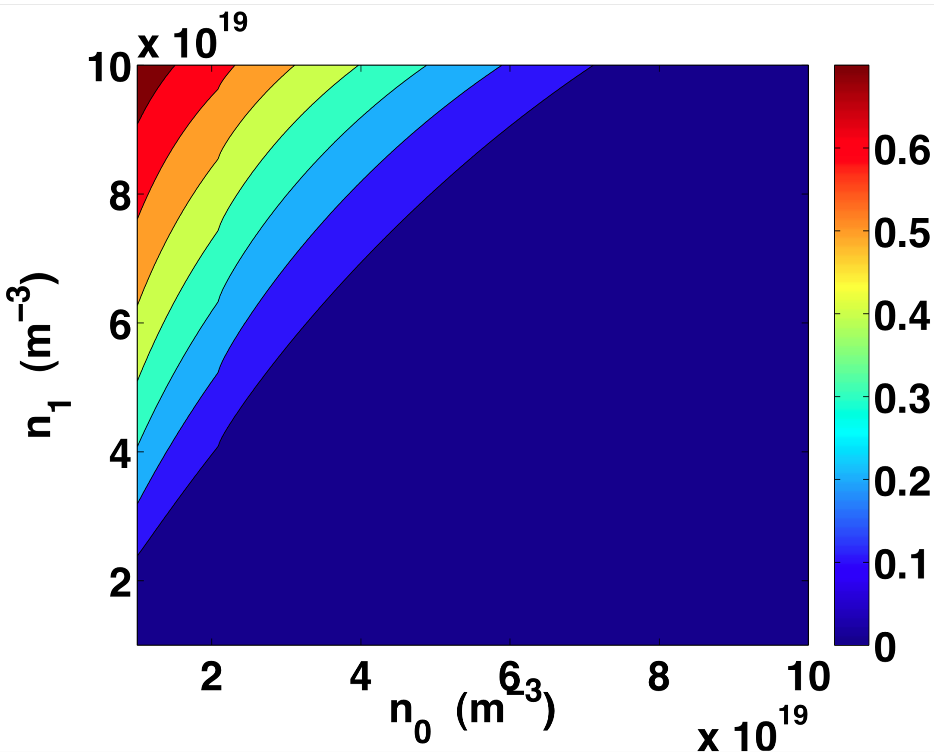
In the primed coordinate system,

$$\hat{z}^T = (-\cos\theta\sin\phi,\ \sin\theta\sin\phi,\ \cos\phi)$$

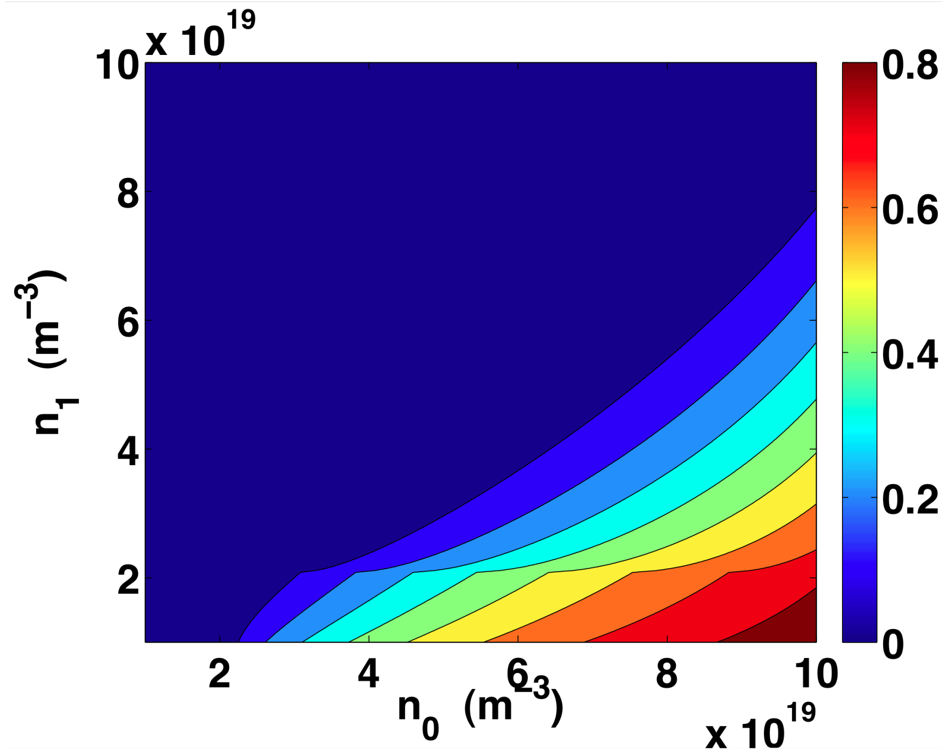
SCATTERING OF LOWER HYBRID WAVES

REFLECTED WAVE POWER (NORMALIZED TO INCOMING WAVE POWER)

SLOW WAVE



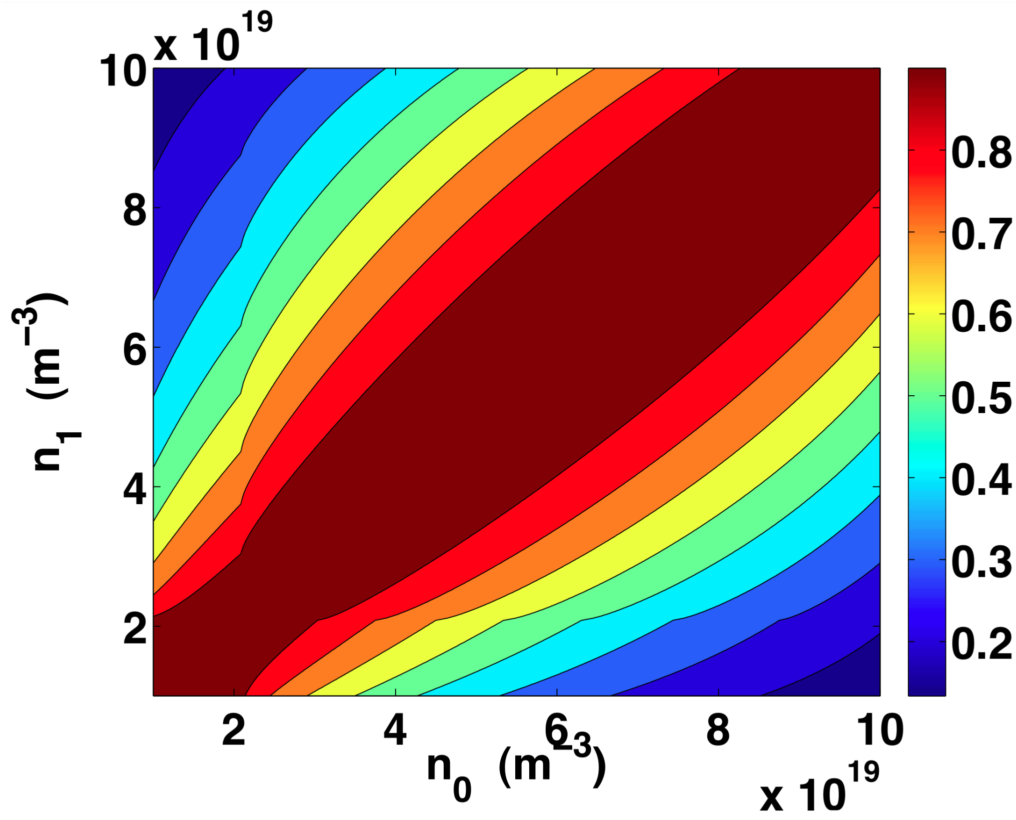
FAST WAVE



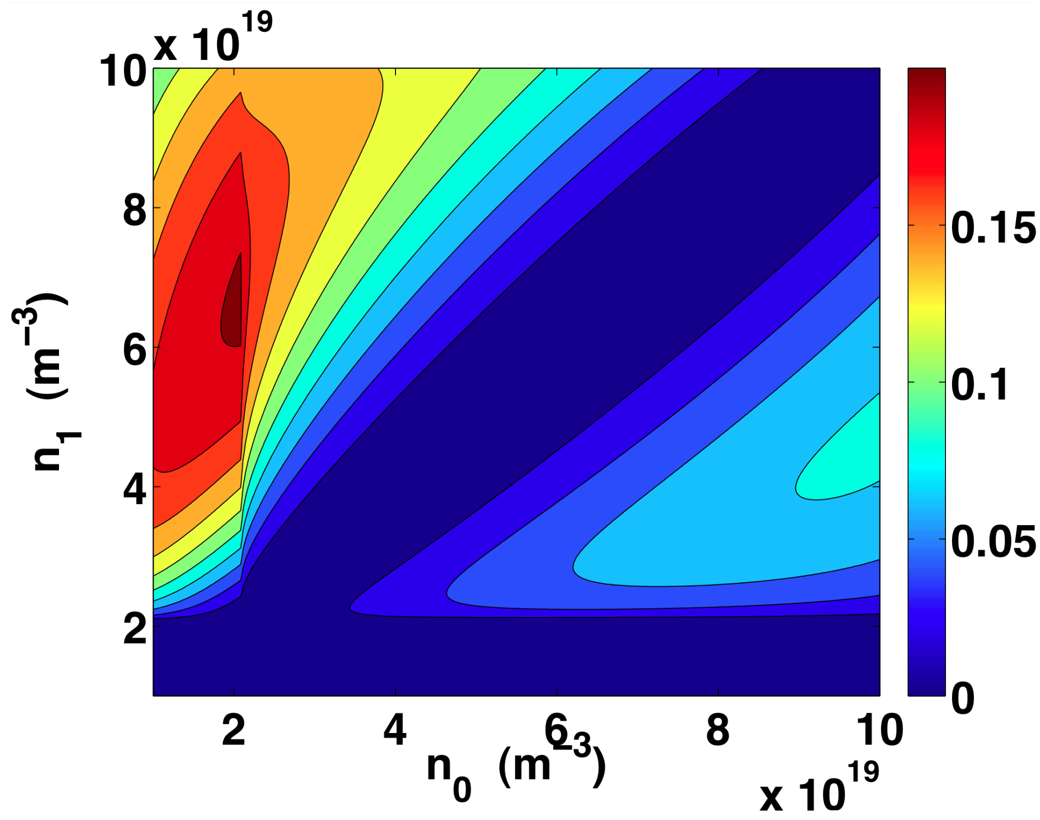
$$B_0 = 4.5 \text{ T}, \quad f = 4.6 \text{ GHz}, \quad \frac{c}{\omega} k_z = -2$$

TRANSMITTED WAVE POWER (NORMALIZED TO INCOMING WAVE POWER)

SLOW WAVE

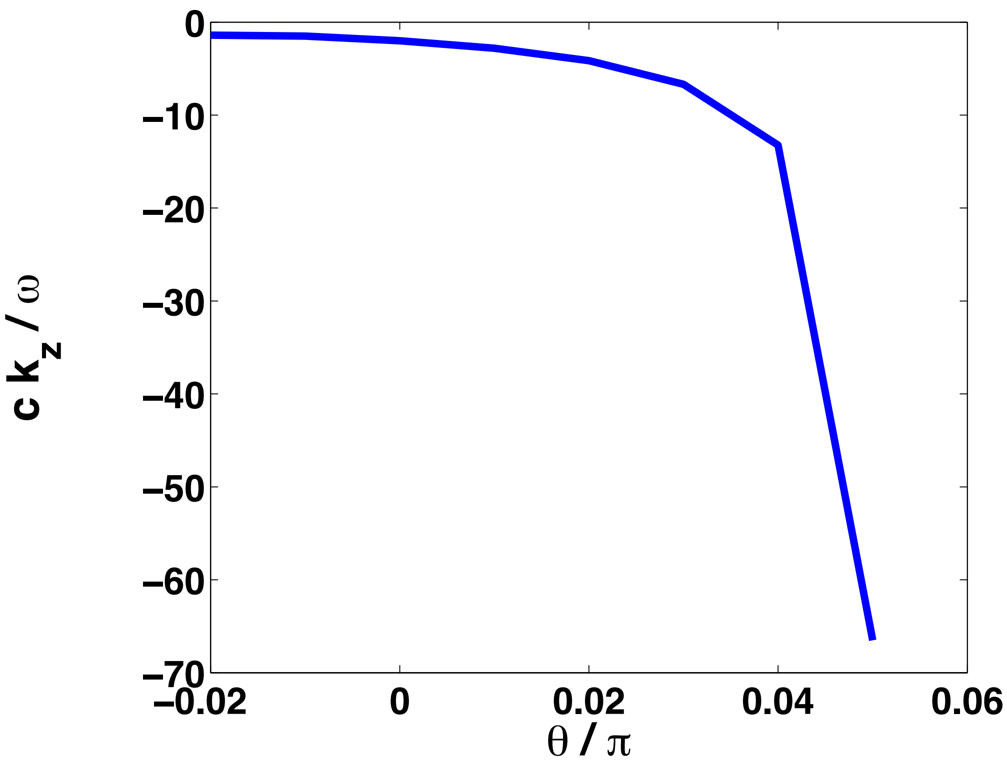


FAST WAVE

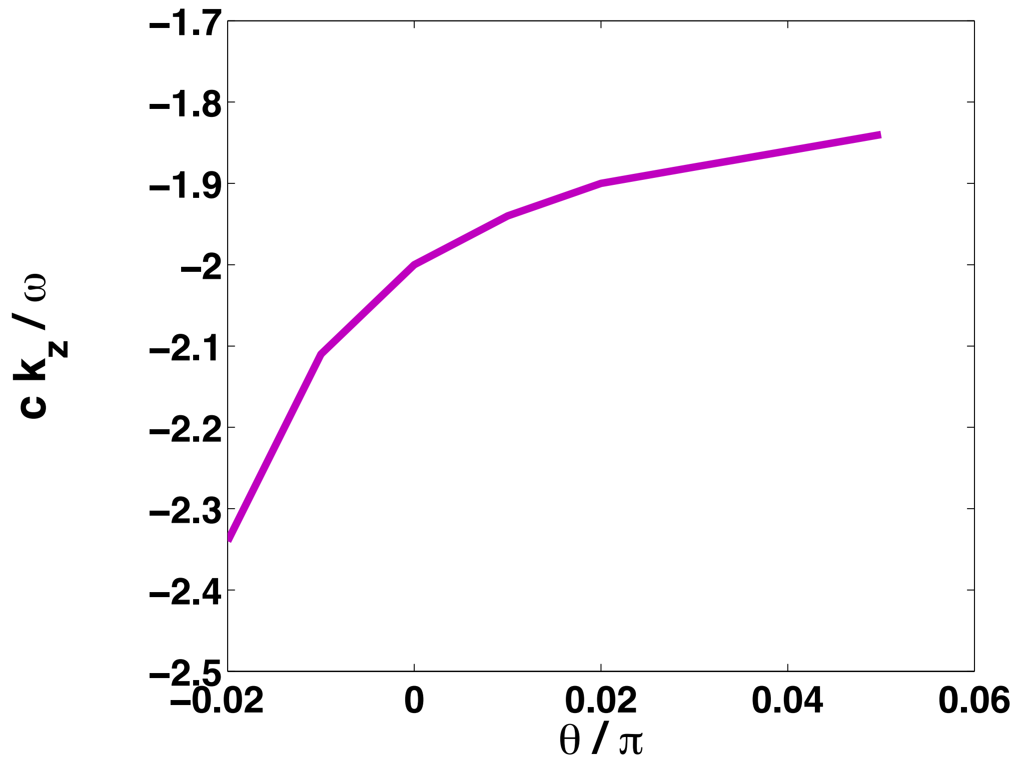


VARIATION OF THE PARALLEL WAVE VECTOR AS A FUNCTION OF THE INCLINATION ANGLE

Reflected slow wave



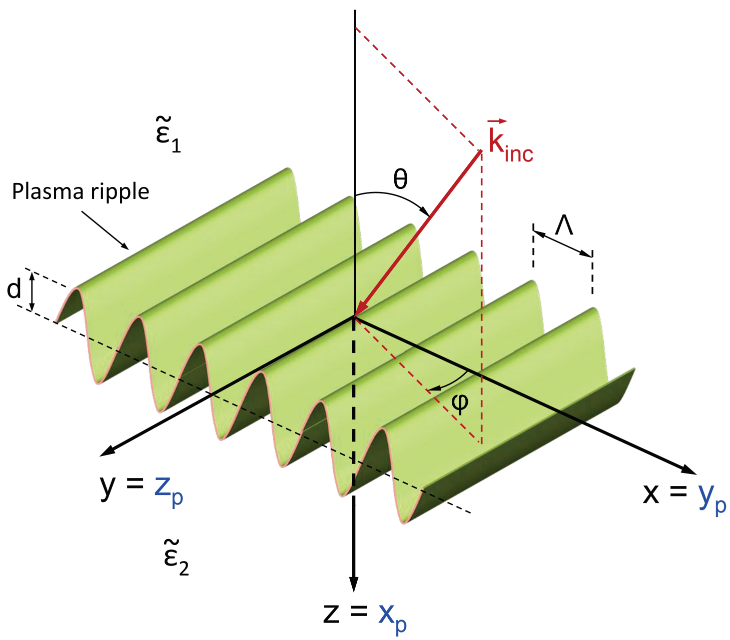
Transmitted slow wave



$$n_0 = 10^{19}\ \mathrm{m}^{-3},\ n_1 = 1.6 \times 10^{19}\ \mathrm{m}^{-3}$$

SIMULATING SCATTERING OF RF WAVES BY TURBULENT PLASMA

- We have developed a full-wave code ScaRF
  - fully electromagnetic
  - finite-difference frequency domain method
  - cold anisotropic plasma
  - allows for arbitrary density variations
  - initially used for "wave-like" fluctuations

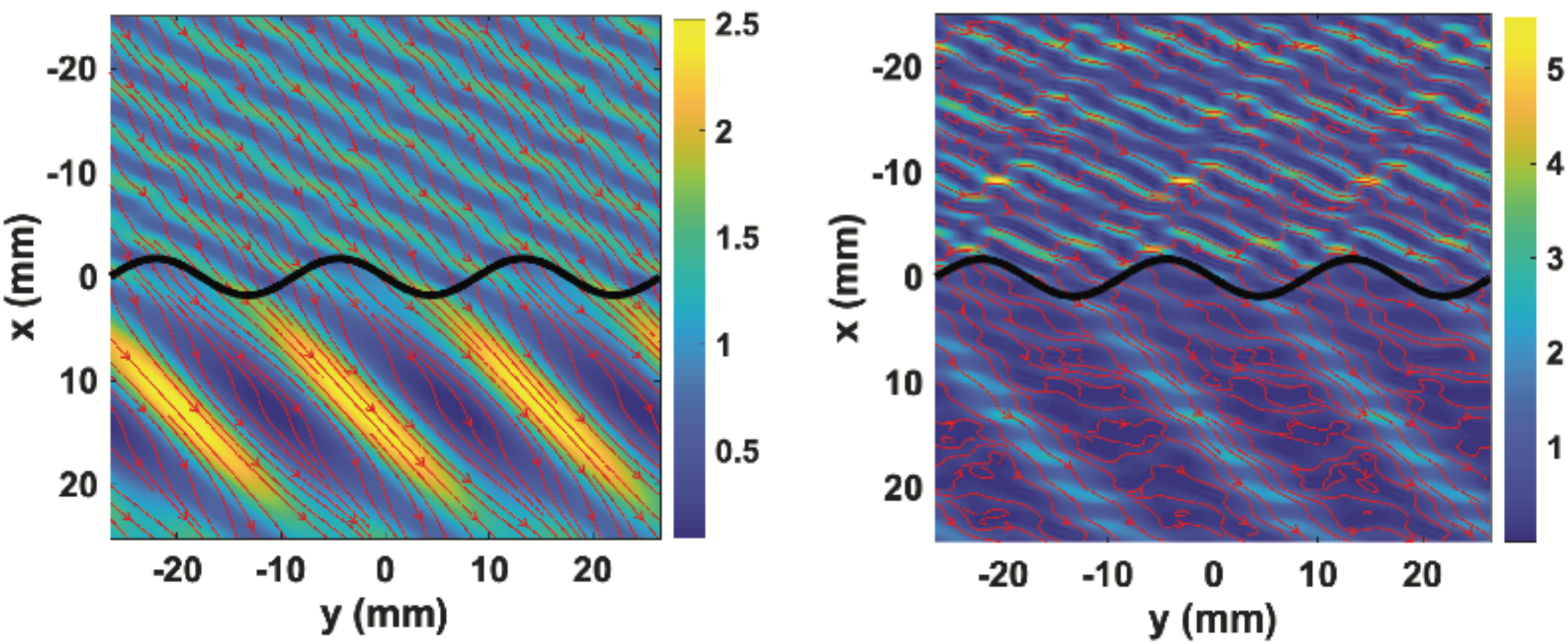


A. D. Papadopoulos et al., J. Plasma Phys. *85*(3), 905850309 (2019).

SCATTERING OF ELECTRON CYCLOTRON WAVES

plotting the magnitude of the total Poynting flux

Incident wave: Ordinary mode — Extraordinary mode

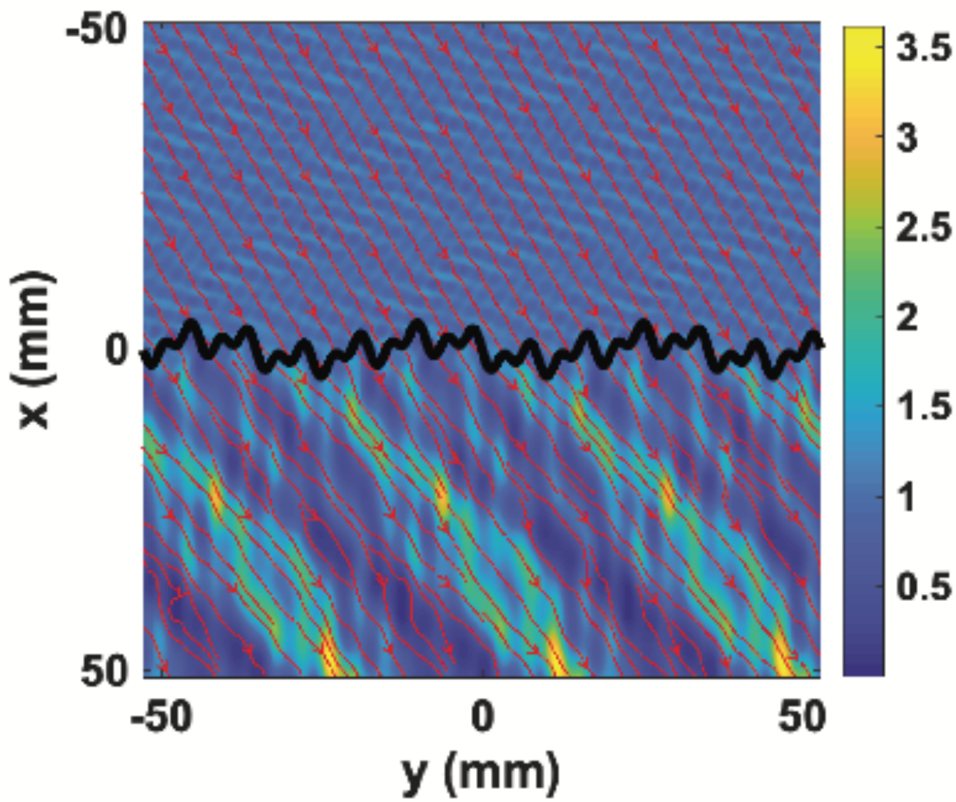


$$f = 170\ \text{GHz}, \quad B_0 = 4.5\ \text{T}, \quad n_0 = 3 \times 10^{20}\ \text{m}^{-3}, \quad n_1 = 3.2 \times 10^{20}\ \text{m}^{-3}$$

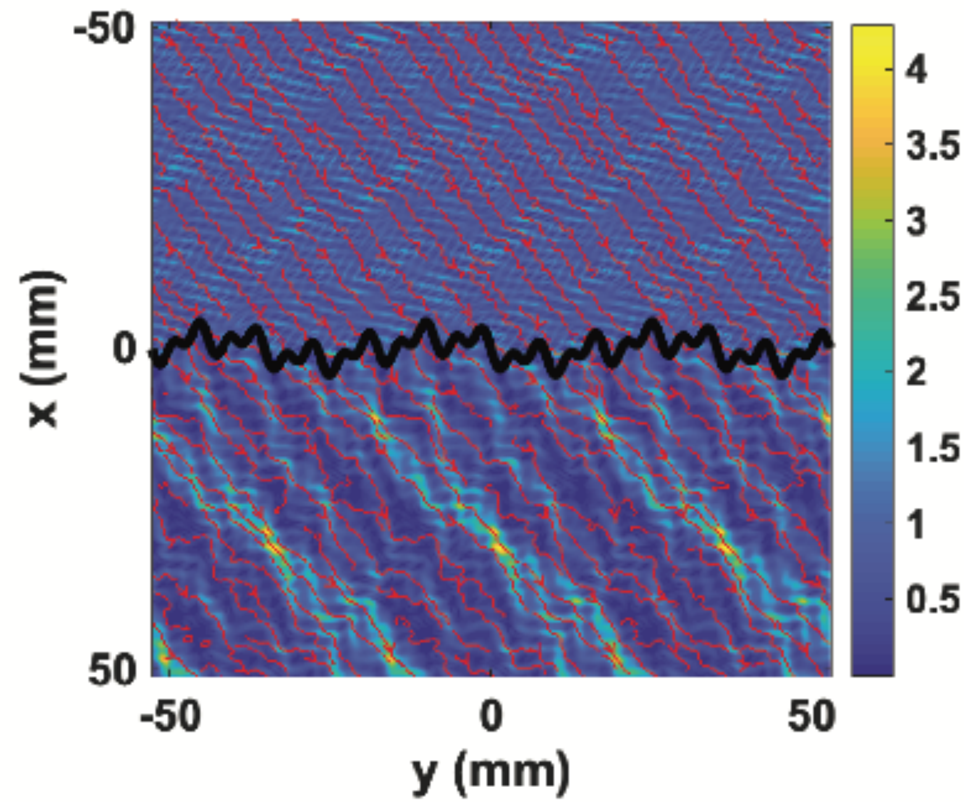
SCATTERING OF ELECTRON CYCLOTRON WAVES

plotting the magnitude of the total Poynting flux

Incident wave: Ordinary mode



Extraordinary mode



Finally, in summary,

- The Kirchhoff modeling of scattering of radio frequency waves by fluctuations shows that
  - waves are refracted, reflected, side-scattered;
  - incident wave power can couple to other plasma waves;
  - spectral changes occur in the wave characteristics.

- A simulation code ScaRF has been set up to model scattering of waves by arbitrary distribution of density density fluctuations
  - comparisons with the Kirchhoff model are underway;
  - limitations of the Kirchhoff model will be investigated;
  - studies can be extended well beyond the scope of theory.