Burning Plasma Transport Simulation for Axisymmetric Tokamaks with Alpha-Particle Heating

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Udaya Maurya[1,a], Sumit Bainjwan[2], R. Srinivasan[1]
[1]Institute For Plasma Research, Bhat, Gandhinagar, India
[2]Pandit Deendayal Petroleum University, Gandhinagar, India
[a]e-mail: udaya.maurya@ipr.res.in

INTRODUCTION

- The primary goal of Tokamak based fusion reactors is to achieve a self-sustained plasma, which requires understanding and controlling of specific plasma profiles with fusion alpha-particles as main source of heating.
- In this study, we have developed a model consisting of 1.5D transport [1] and alpha-particle heating [2]. Using this model we study the burning plasma performance and find plasma profiles in an ITER-like case.
- Transport simulation is performed in flux coordinates, which are obtained by solving Grad-Shafranov equation using 2D VMOMS code [3].
- Alpha-particle heating is modeled with Fokker-Planck equation, where collisions with ions and electrons is taken as energy transfer mechanism.

THEORY

- 1.5D transport simulation requires to solve 1D fluid equations in radial flux coordinate ρ coupled with a 2D MHD equilibrium solver.
- Assuming slow evolution of flux coordinates, the fluid equations [1] solved are:

$$\frac{\partial \langle n_i \rangle}{\partial t} = -\frac{1}{V'}\frac{\partial}{\partial \rho}\left(V'\langle \Gamma_i \rangle\right) - \left[\frac{dn}{dt}\right]_{fus}$$

$$\frac{\partial p_i}{\partial t} = -\frac{2}{3V'}\frac{\partial}{\partial \rho}\left(V'\left\langle q_i + \frac{5}{2}T_i\Gamma_i\right\rangle\right) + \mathcal{P}_{\alpha i} + \mathcal{P}_{coll}$$

$$\frac{\partial p_e}{\partial t} = -\frac{2}{3V'}\frac{\partial}{\partial \rho}\left(V'\left\langle q_e + \frac{5}{2}T_e\Gamma_e\right\rangle\right) + \mathcal{P}_{\alpha e} - \mathcal{P}_{coll} + \mathcal{P}_{ohm} - \mathcal{P}_{rad} + \mathcal{P}_{aux}$$

- Poloidal flux ψ, toroidal flux ϕ and other relevant electromagnetic quantities are obtained by solving Maxwell's equations with generalized Ohm's Law [1].
- The Jacobian and relevant geometric quantities are obtained by solving Grad-Shafranov Equation (where $\mu_0 F = B_\zeta$):

$$\frac{\partial^2 \psi}{\partial Z^2} + R\frac{\partial}{\partial R}\left[\frac{1}{R}\frac{\partial \psi}{\partial R}\right] = -\mu_0 R^2 \frac{dp}{d\psi} - \mu_0^2 F\frac{dF}{d\psi}$$

- Equation for alpha-particle heating is obtained by solving the Fokker-Planck equation [2] (Φ is error-function, v is speed of α-particles and μ is reduced mass):

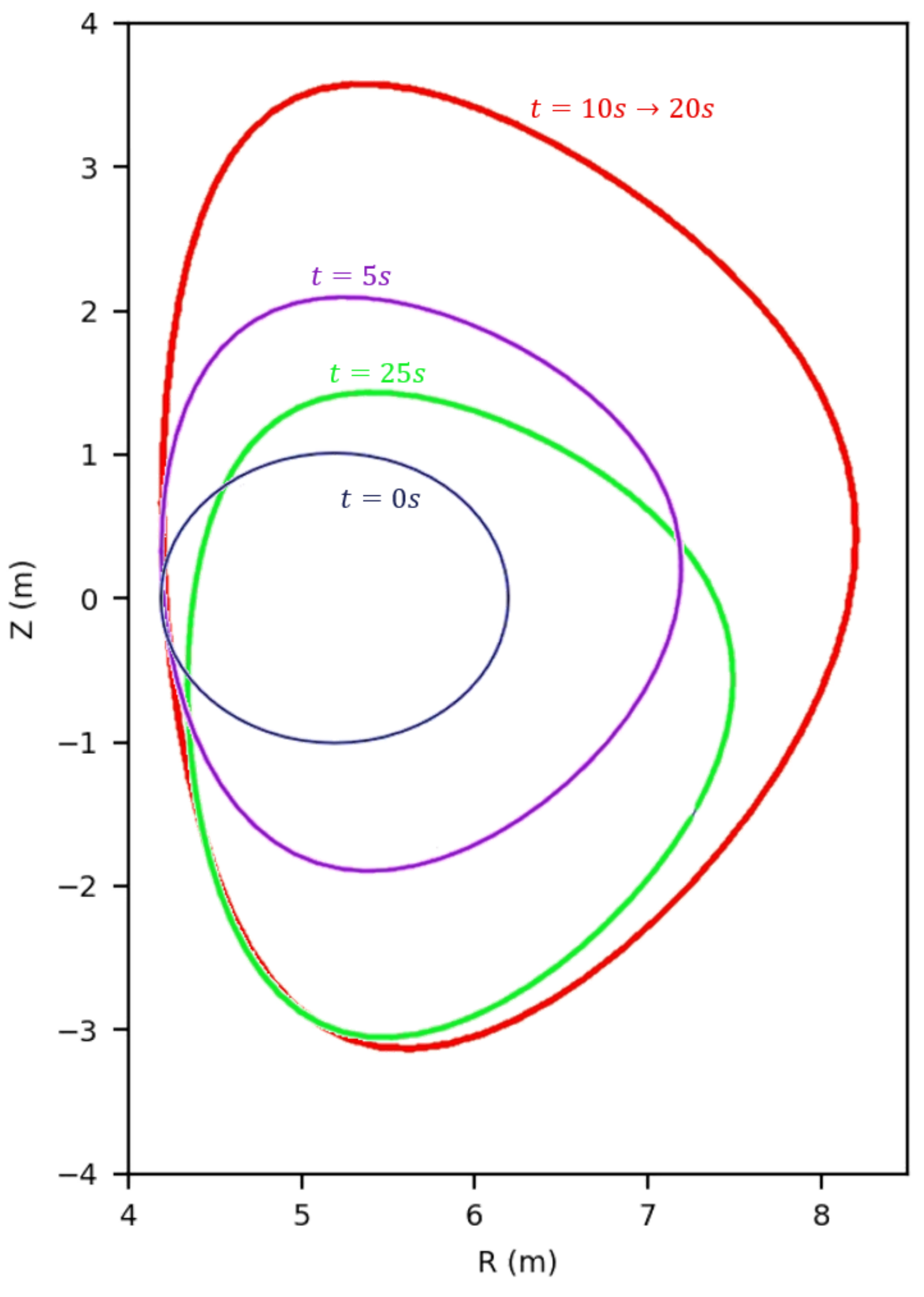
$$\mathcal{P}_{\alpha(e,i)} = \frac{n_{e,i} q_\alpha^2 q_{e,i}^2 \log\Lambda}{4\pi\epsilon_0^2}\left[\frac{2\exp\left(-v^2/v_{th(e,i)}^2\right)}{\sqrt{\pi}\, v_{th(e,i)} \mu_{\alpha(e,i)}} - \frac{1}{m_{e,i} v}\Phi\left(\frac{v}{v_{th(e,i)}}\right)\right]$$

IMPLEMENTATION

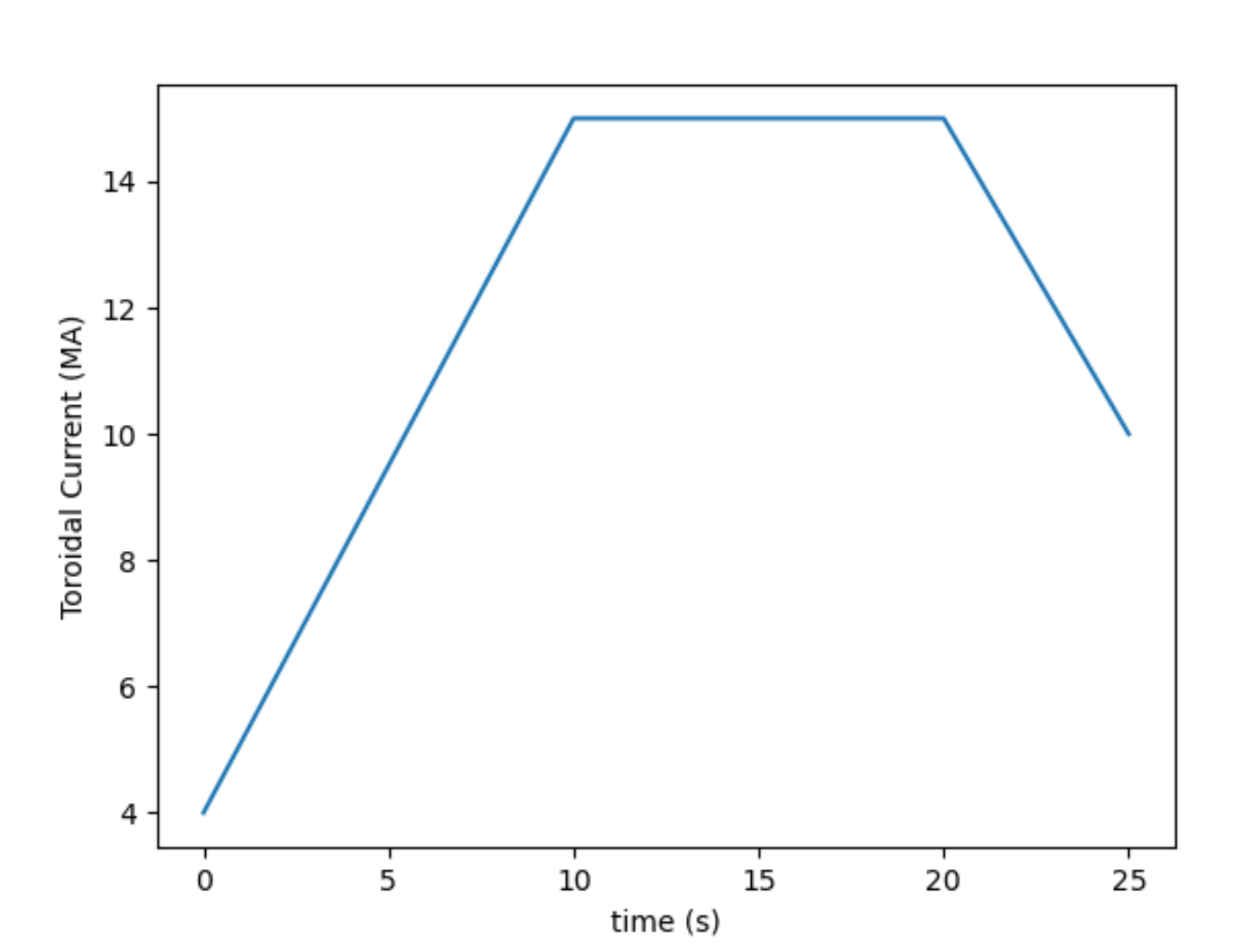
- Fluid equations are solved using LCPFCT [4], which solves coupled generalized continuity equations in 1D. LCPFCT is an explicit finite-difference algorithm that works on the principle of flux-corrected transport, which is conservative and maintains monotonicity and positivity.
- Particle and thermal diffusion coefficients are calculated using mixed Bohm/gyro-Bohm model [5]. Neoclassical resistivity and bootstrap current are calculated using Sauter's model [6].
- LCPFCT natively supports Cartesian, cylindrical and spherical coordinates. We modified LCPFCT to work with flux coordinates.
- Grad-Shafranov equation is solved using VMOMS [3], which assumes a Fourier series transformation between coordinates (R, Z) and flux coordinates (ρ, θ), then minimizes the action corresponding to Grad-Shafranov equation to find the said transformation. The resultant coupled second-order differential equations are solved using Shooting method for a closed boundary setting. We also modified VMOMS to work with up-down asymmetry.
- To include alpha-particle heating, we keep track of alpha-particle density and energy profile and find energy-deposited to ions and electrons based on equation of $\mathcal{P}_{\alpha(e,i)}$.

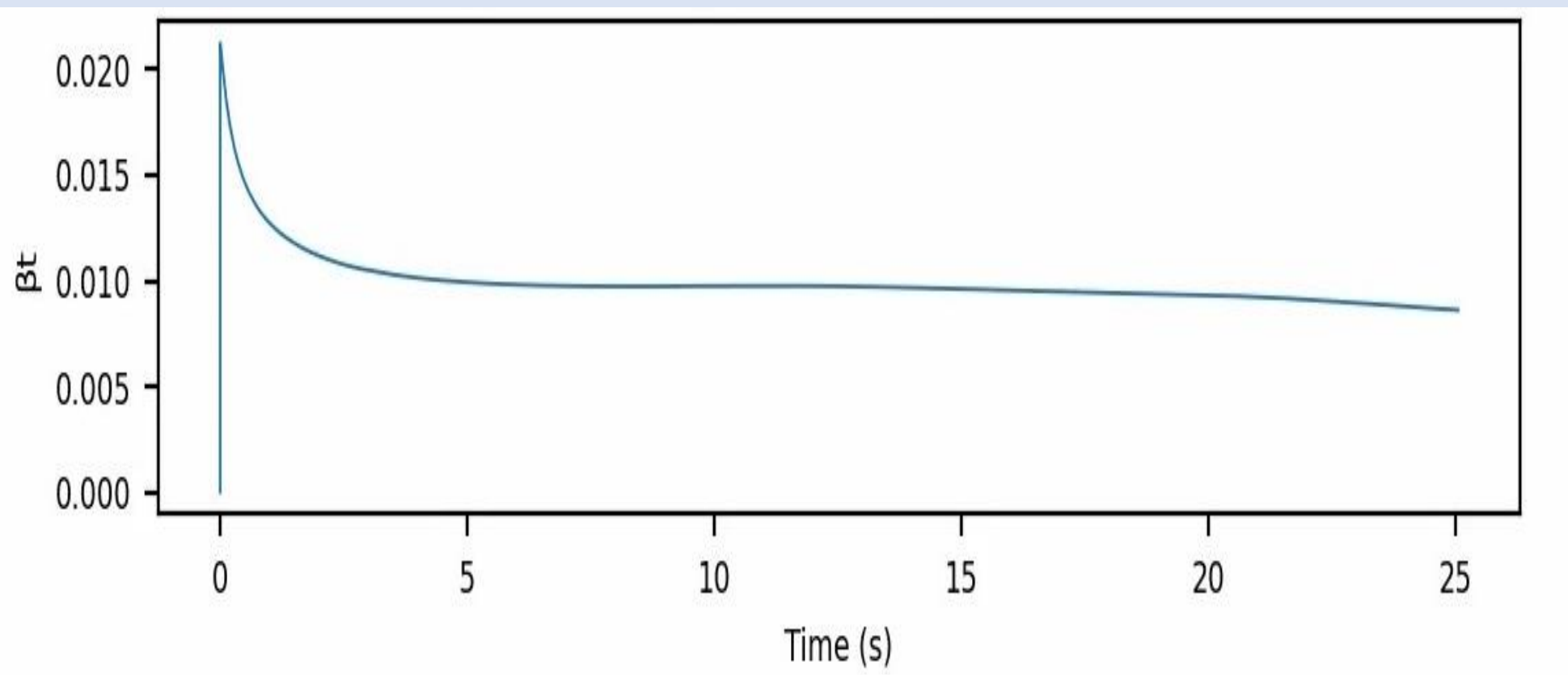
SIMULATION & RESULTS

- We performed a simulation for ITER-like configuration for a total runtime of 25 seconds including ramp-up of 10 seconds and ramp-down of 5 seconds.
- The evolution of toroidal current and LCFS are as shown in figures below. Density and pressure had fixed boundary value at edge as shown in figure.
- Tokamak parameters used for simulation are major radius = $6.2m$, minor radius = $2m$, axis toroidal magnetic field = $5.3T$.
- Cumulative alpha-power deposited is found to be $\sim 30MW$ in plateau region.
- Normalized beta during plateau region was found to be $\sim 1\%$, which suggests a good confinement.
- Safety factor ranges between 2-3.5, consistent with other parameters.

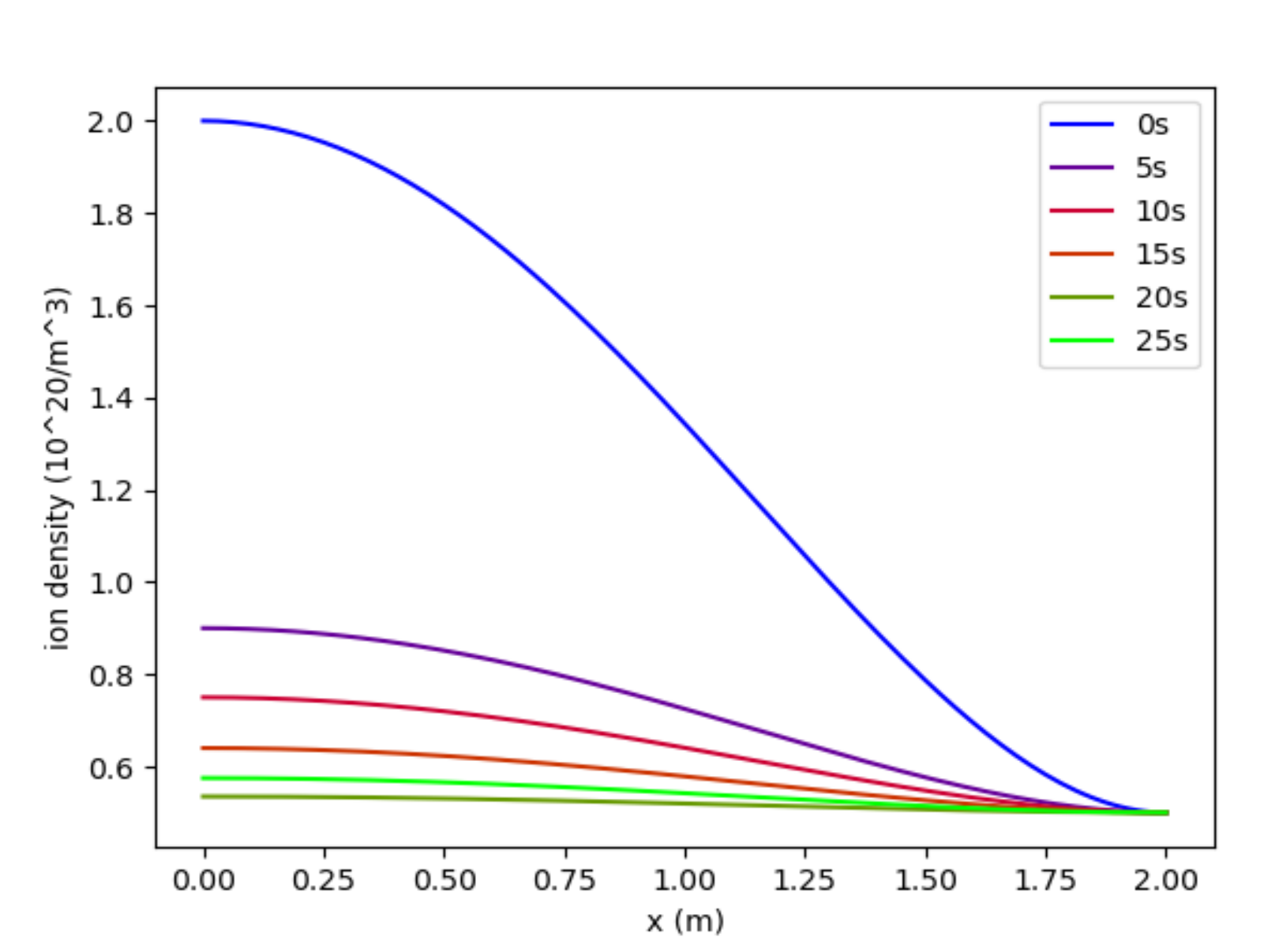


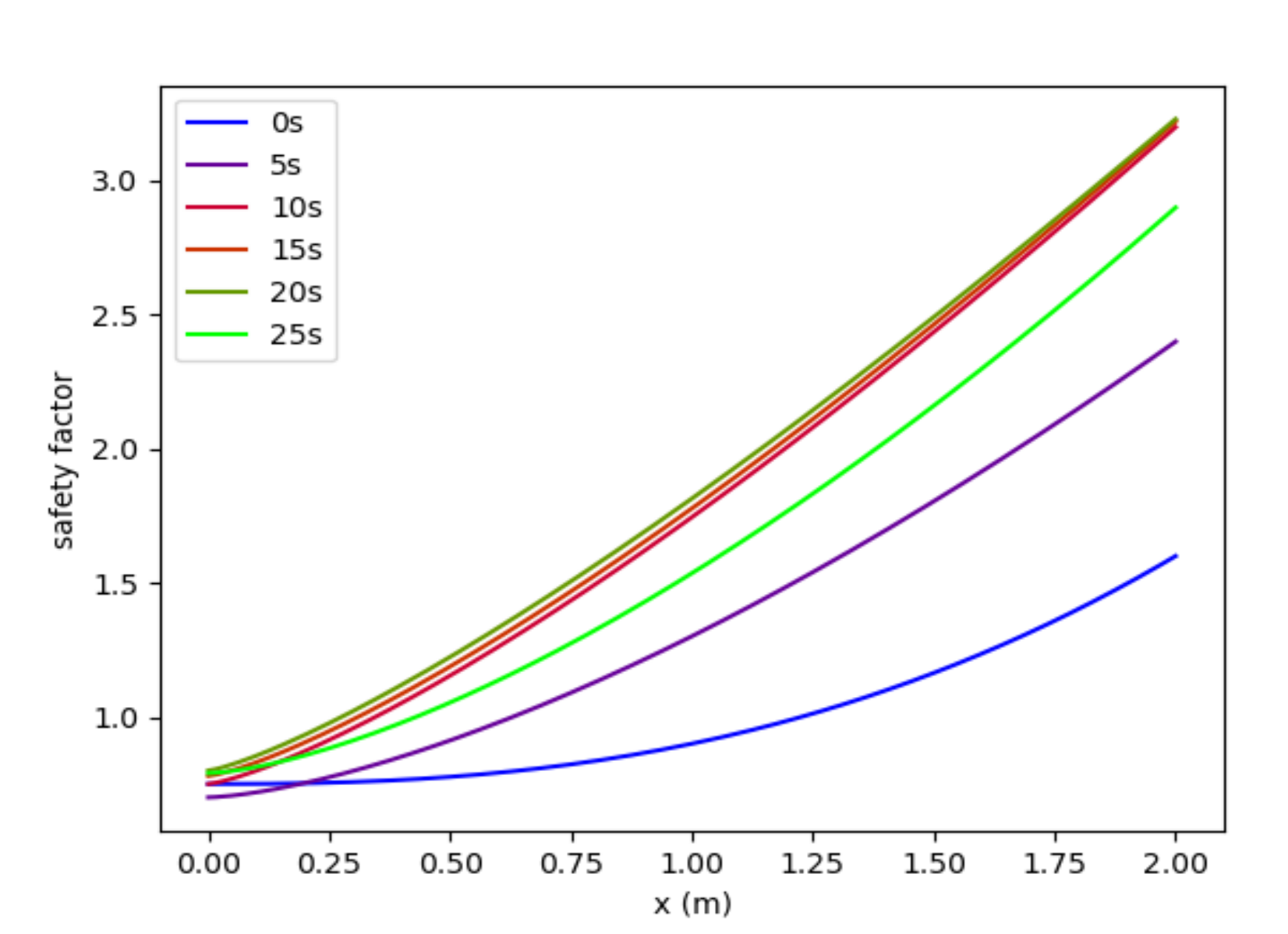
Evolution of LCFS during simulation

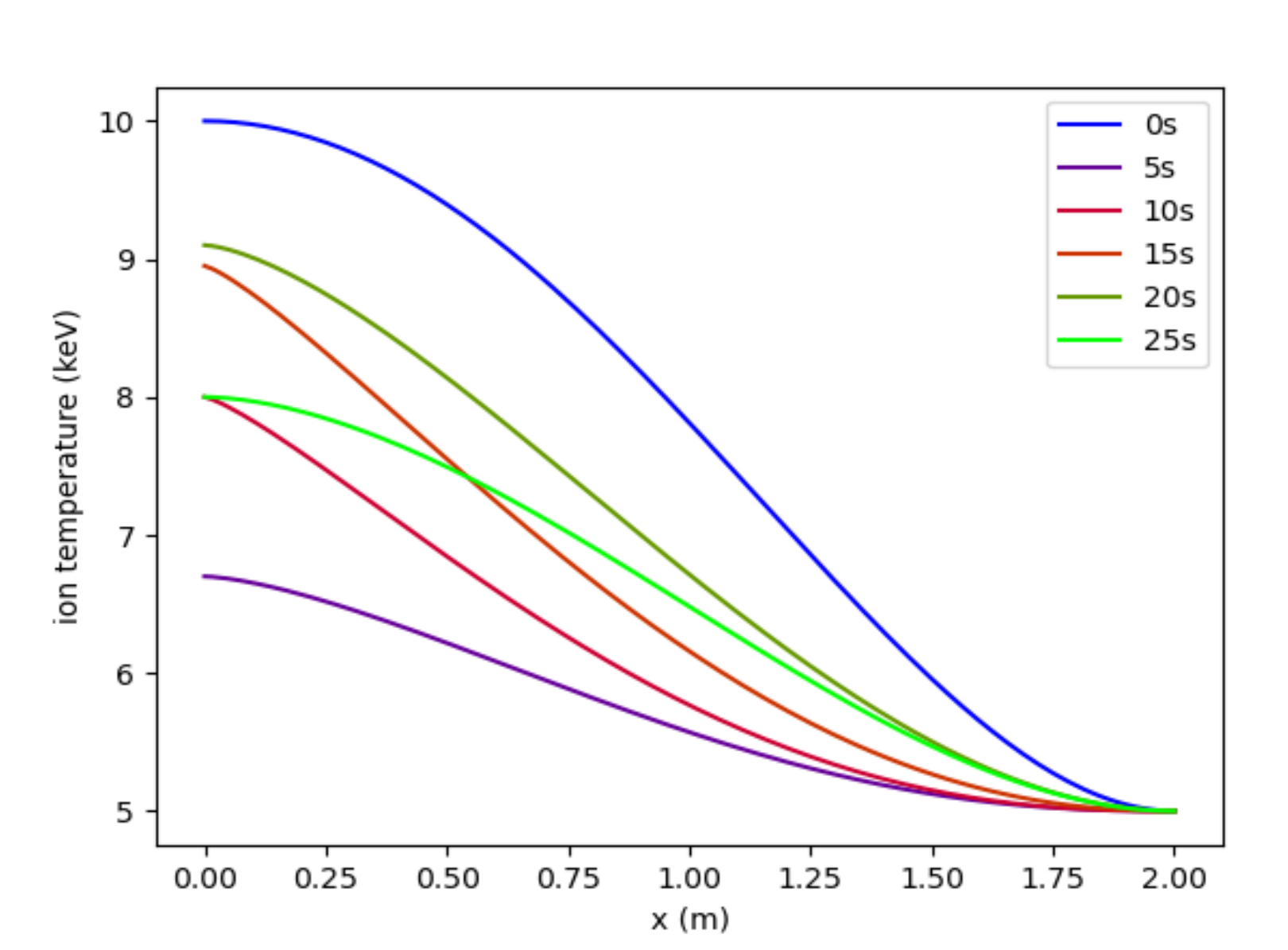


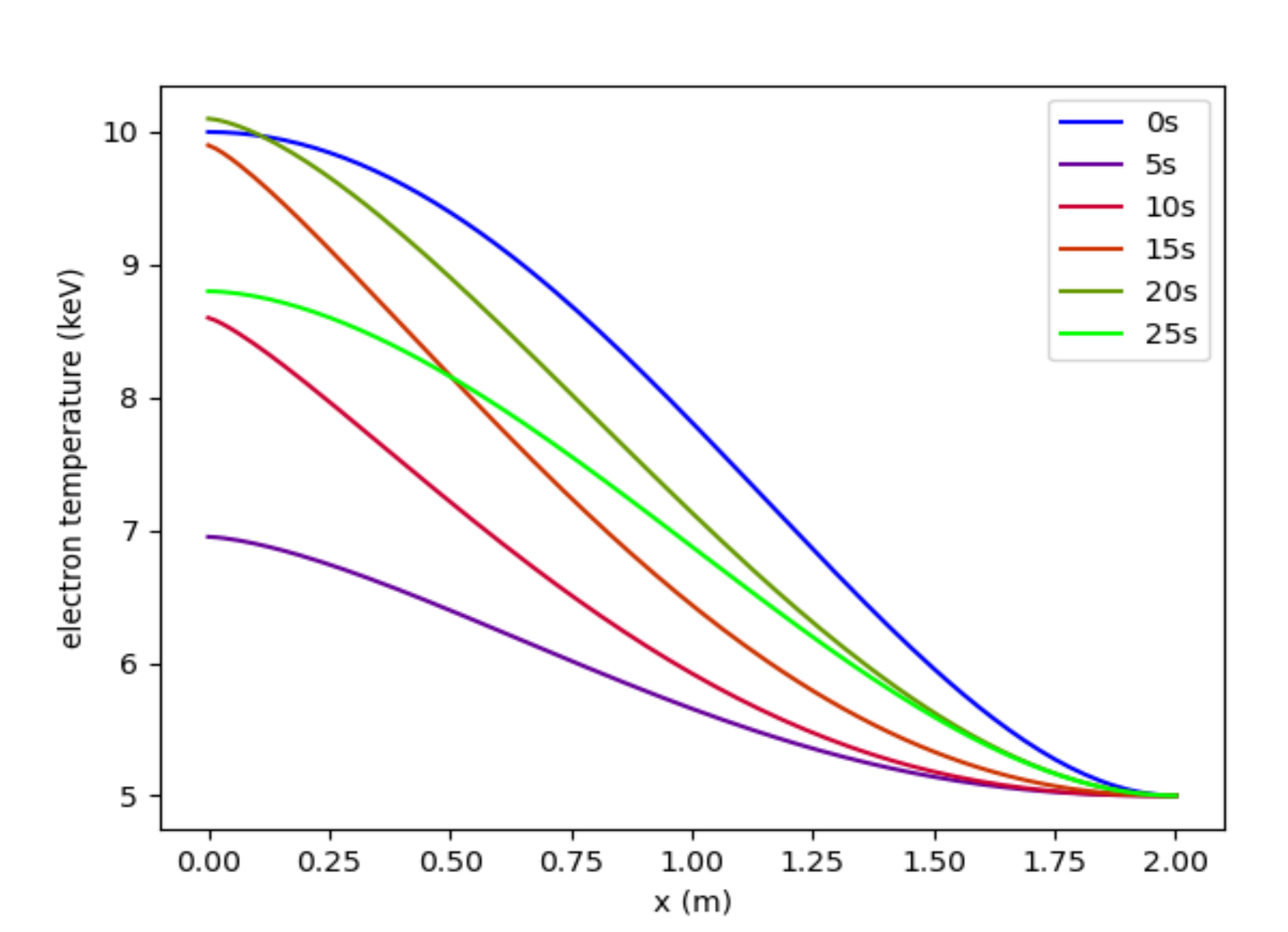


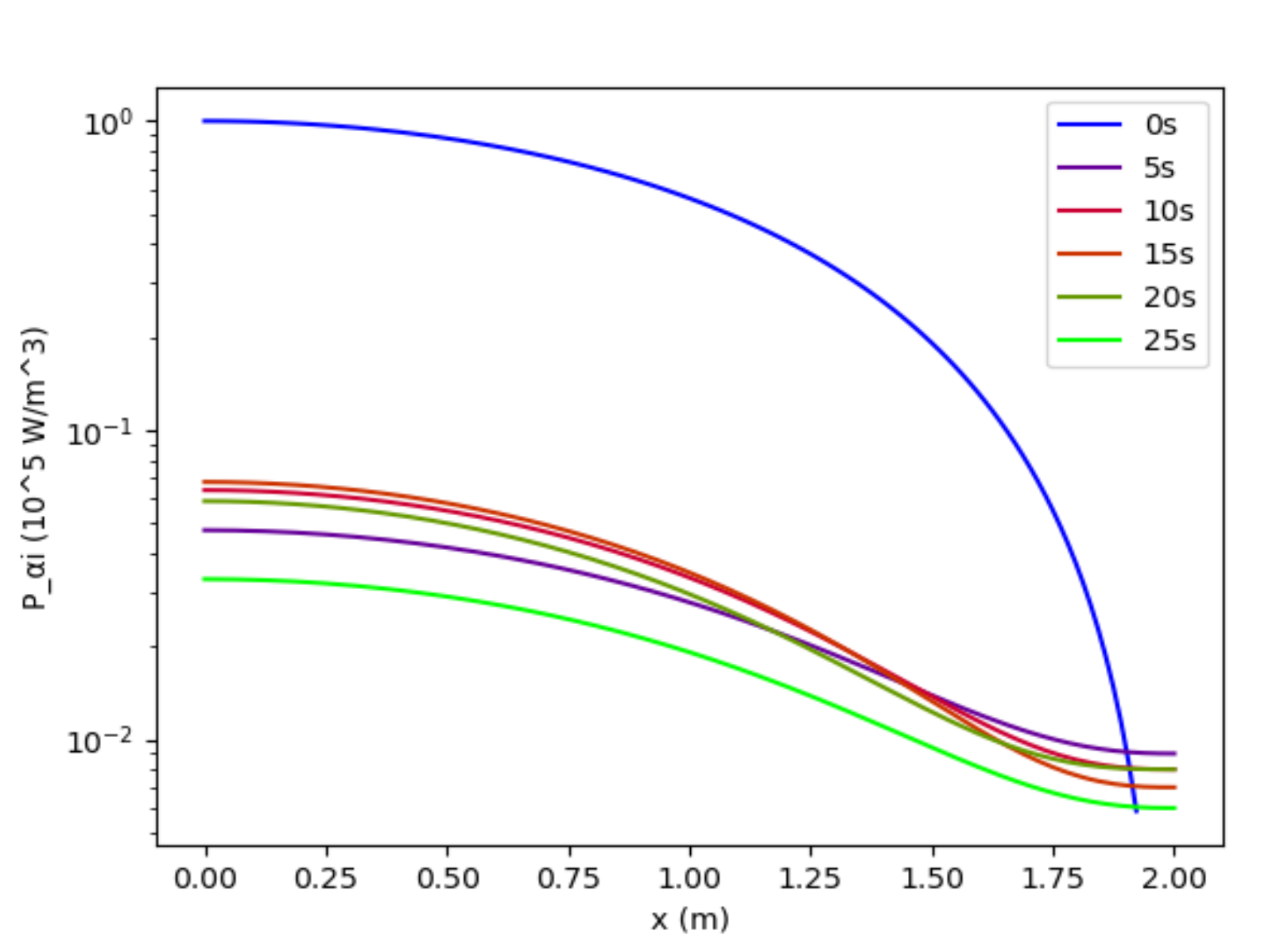
Toroidal Beta

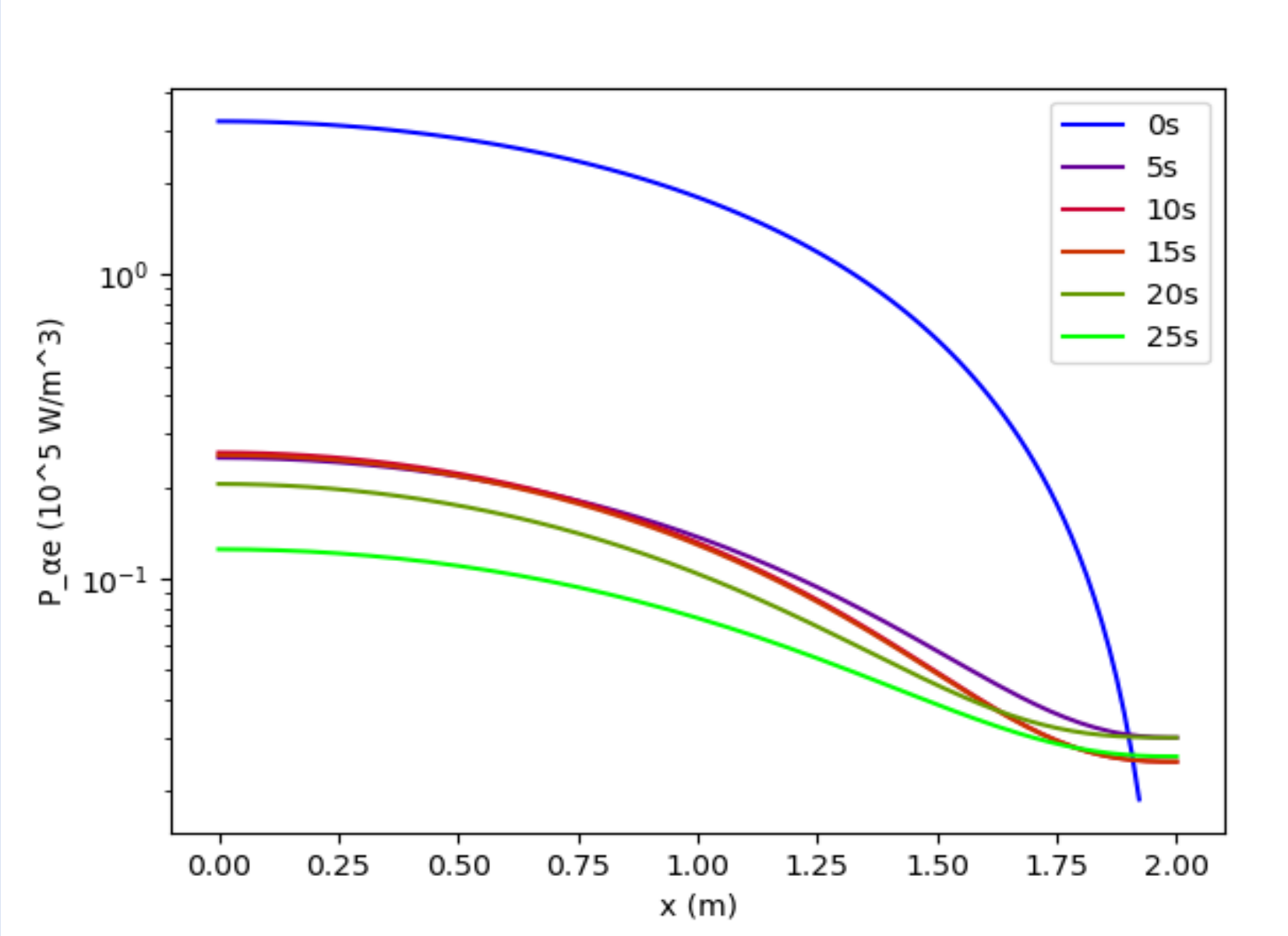












REFERENCES

[1] F. L. Hinton, R. D. Hazeltine; *Theory of plasma transport in toroidal confinement systems*; Reviews of Modern Physics, 48, 2, Part I, 1978
[2] S. Wang, L. Qiu, *Alpha Particle Classical Transport in Tokamaks*; Nuclear Fusion 36, 5, 1996
[3] L. L. Lao, S. P. Hirshman, R. M. Wieland; *Variational Moment Solutions to the Grad-Shafranov Equation*; Phys. Fluids, 24, 8, 1981
[4] J. P. Boris et al; *LCPFCT - Flux-Corrected Transport Algorithm for Solving Generalized Continuity Equations*; NRL/MR/6410-93-7192, 1993
[5] M. Erba et al; *Development of a non-local model for Tokamak heat transport in L-mode, H-mode and transient regimes*; Plasma Physics and Controlled Fusion, 39, 2, 1997
[6] O. Sauter, C. Angioni, Y.R. Lin-Liu; *Neoclassical conductivity and bootstrap current formulas for general axisymmetric equilibria and arbitrary collisionality regime*; Physics of Plasmas, 6, 7, 1999