Development and Implementation of Integrated $q$-profile+$\beta_N$ Feedback Control Strategies for Advanced Scenarios in EAST


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“Advanced Tokamak” (AT) operational goals for EAST include:
- Steady-state operation
- High-performance operation (high $\beta$, high $q_{\text{min}}$, etc.)
- MHD-stable operation

Active, feedback control of the current density profile, as well as other plasma kinetic profiles and scalars, can play a critical role in achieving these AT operational goals.

High dimensionality
- Nonlinearity
- Magnetic/kinetic coupling

Model-based Control Design

First-principles-driven (FPD) PDE model: Mix of widely accepted first-principles laws and control-oriented models for transport/sources by exploiting both empirical (from physical observations) and analytical scalings as well as neural-network accelerated models.
Modeling Poloidal-Flux+Energy Evolution for Control Design

- Magnetic Flux ($\psi$) Dynamics Modeled by 1D Diffusion Equation
  \[
  \frac{\partial \psi}{\partial t} = \eta(T_e) \left[ \frac{1}{\mu_0 \rho_b^2} \tilde{F} \frac{1}{\tilde{\rho}} \frac{\partial}{\partial \tilde{\rho}} \left( \tilde{\rho} \tilde{F} \hat{G} \hat{H} \frac{\partial \psi}{\partial \tilde{\rho}} \right) + R_0 \hat{H} \frac{\tilde{j}_{NI} \cdot \tilde{B}}{B_{\phi,0}} \right], \quad \frac{\partial \psi}{\partial \tilde{\rho}} \bigg|_{\tilde{\rho}=0} = 0, \quad \frac{\partial \psi}{\partial \tilde{\rho}} \bigg|_{\tilde{\rho}=1} = -\frac{\mu_0 R_0}{2\pi \hat{G} \hat{H}} I_p(t) \tag{1}
  \]

  - Resistivity
  - Geometric Parameters
  - Non-inductive CD

- Fast Evolving Kinetic Profiles Modeled by Singular Perturbation
  \[
  T_e(\hat{\rho}, t) = T_{e, prof}(\hat{\rho}) \frac{I_p(t)^{\alpha} P_{tot}(t)^{\beta}}{n_e(t)^{\gamma}}, \quad n_e(\hat{\rho}, t) = n_{e, prof}(\hat{\rho}) \bar{n}_e(t) \tag{2}
  \]

- Profiles Consistent with Stored Energy ($W$) Dynamics Modeled by 0D Power Balance
  \[
  \frac{dW}{dt} = -\frac{W}{\tau_W} + P_{tot}(P_{tot} = P_{aux} + P_{ohm} + P_{rad}) \Rightarrow \beta_N = \frac{a(2W/3)}{I_p B_{\phi,0}/(2\mu_0)}, \quad \tau_W \propto I_p^\alpha P_{tot}^{-\beta_s} n_e^{-\gamma_s} \tag{3}
  \]
**Electron Temperature Profile Modeled by Heat Transport Equation**

Assuming diffusion is dominant transport mechanism, the $T_e$ dynamics is given by

$$\frac{3}{2} \frac{\partial}{\partial t} \left[ n_e T_e \right] = \frac{1}{\rho_b^2 \hat{H}} \frac{1}{\hat{\rho}} \frac{\partial}{\partial \hat{\rho}} \left( \hat{\rho} \frac{\hat{G} \hat{H}^2}{\hat{F}} \left( \chi_e(\cdot)n_e \frac{\partial T_e}{\partial \hat{\rho}} \right) \right) + Q_e^{ohm} + Q_e^{rad} + \sum_i Q_e^{aux} \tag{4}$$

with boundary conditions $\frac{\partial T_e}{\partial \hat{\rho}}(0, t) = 0$, $T_e(1, t) = T_{e, bdry}$, and where $Q_e^{aux} = Q_{e_i}^{dep}(\hat{\rho})P_{aux}(t)$

1. Thermal conductivity $\chi_e$ can be modeled as an analytical scaling law.
2. Thermal conductivity $\chi_e$ can be modeled as an empirical scaling law, e.g. $\chi_e = k_{\chi_e} T_e^\gamma n_e^{\nu} q^{\mu} s^{\pi}$
   + Multi-linear regression from $\chi_e$ computed by physics models (TRANSP) to determine structure.
   + Nonlinear optimization to determine constants:
     $$\min_{\theta} J, \quad J = \int_{t_0}^{t_f} \left\{ \sum_{i=1}^{N} \alpha \left[ q_{e_i}^{\exp}(\hat{\rho}_i, t) - q(\hat{\rho}_i, t) \right]^2 + \beta \left[ T_{e_i}^{\exp}(\hat{\rho}_i, t) - T_e(\hat{\rho}_i, t) \right]^2 \right\} dt, \quad \theta = [k_{\chi_e} \gamma \nu \mu \pi].$$
3. Thermal conductivity $\chi_e$ can be modeled as state model, e.g. $\chi_e = f(T_e, n_e, q, s)$
   + Machine Learning techniques → Neural Network training (NEO, TGLF, MMM, ...)

**NOTE:** Sources $\frac{\langle \hat{j} \cdot \hat{B} \rangle}{B_{\phi,0}}$ and $Q_e^{aux}$ can also be modeled using Machine Learning.
Several plasma-response characterization experiments were conducted before the $q$-profile+$\beta_N$ feedback-control experiments. Plasma-response data was generated by exciting the plasma through different available actuators. Figure shows typical response of $q$ profile at two spatial locations ($\hat{\rho} \in [0.05, 0.3]$) in response to open-loop excitation of $P_{\text{LH2}}$ (4.60 GHz LHW source power) during flattop in shot #77643. This data was used to tailor the control-oriented model (1)-(3) to the EAST scenario of interest.

This tailored control-oriented model was used in this work to optimize the gains of the employed fixed-structure controller and to test the PCS implementation of the control algorithm in closed-loop Simserver simulations before experiments.
TRANSP simulations are run in both interpretative and predictive modes to produce plasma response data for the development of lower-complexity, faster, control-oriented models.

Equilibrium reconstruction constrained by POlarimeter-INTerferometer (POINT) plays critical role in comparing model-predicted $q$-profile+$\beta_N$ evolutions with experimental data.
First-principles-driven Models are Engine of COTSIM

LU Control-Oriented Transport SIMulator (COTSIM)

- 1D transport code
- Matlab/Simulink-based
- Control-design friendly
- Modular configuration
- Variable physics complexity
- Closed-loop capable
- Optimizer wrappable
- Equilibrium: Prescribed → 2D Solver
- Fast (offline simulations)
- Very fast (real-time control)

NN models: NUBEAM, MMM
NN model for LHCD in EAST (MIT)
Fixed-structure PID-type Feedback Control Algorithm

- The feedback (FB) control algorithms use a proportional-integral-derivative (PID) structure, i.e.

\[
u^{FB}(t) = K_P e(t) + K_I \int_0^t e(t) + K_D \frac{de(t)}{dt}
\]  \hspace{1cm} (5)

where the input/output vectors are defined as

\[
u^{FB} = [I_p^{FB} P_{LH1}^{FB} P_{LH2}^{FB} P_{NBI1}^{FB} P_{NBI2}^{FB} P_{NBI3}^{FB} P_{NBI4}^{FB}]^T, \quad e = \begin{bmatrix} q(0.1) - q^{tgt}(0.1) \\ q(0.5) - q^{tgt}(0.5) \\ q(0.9) - q^{tgt}(0.9) \\ \beta_N - \beta^{tgt}_N \end{bmatrix}.
\]  \hspace{1cm} (6)

- Actuators considered in this work: total plasma current \( I_p \), 2.45 \( \text{GHz} \) LWH source power \( P_{LH1} \), 4.6 \( \text{GHz} \) LHW source power \( P_{LH2} \), individual co-current NBI powers \( P_{NBI1} \) (NBI1L, NBI1R)), and individual counter-current NBI powers \( P_{NBI3} \) (NBI2L, NBI2R)).

- \( K_P, K_I, K_D \) are gain matrices optimized in simulations based on control-oriented model (1)-(3).

- The superscript \( tgt \) denotes target values for the to-be-controlled plasma properties.
Model-based PID Gain Optimization Before Experimental Testing

\[ J = \int_{t_i}^{t_f} (q^{tgt} - q)^2 + K (\beta_N^{tgt} - \beta_N)^2 \, dt \]

\[ u^{FB} = K_P e + K_I \int e \, dt + K_D \frac{de}{dt} \]

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Model-Based Scenario Control in EAST

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Profile control algorithm has been coded by LU Plasma Control Group: DIII-D → EAST

- Interface with real-time pEFIT + (POINT)
- Interface with actuators. Actuators must be under PCS.
- Interface with user data.
Overall input for plant $G$ (EAST):

$$ u = u^{FF} + u^{FB} + u_d + (s), \quad (7) $$

Overall input for Controller $K$:

$$ y^{FB} = y + y_d - y_r. \quad (8) $$

To-be-tracked target:

$$ y^{FB}_t = y_t - y_r. \quad (9) $$

- $u^{FF}$: feedforward control, $u^{FB}$: feedback control (output of controller $K$), $u_d$: input disturbance.
- $u^{FF} = u_r + u_c$, $u_r$: input reference, $u_c$: output of feedforward compensator.
- $s$: output of an optional anti-windup (AW) compensator (signal added only when AW is on).
- $y$: overall plant output, $y_d$: output disturbance, $y_r$: output reference (associated with $u_r$).
- $y^{FB}_t$: reference-modified output target (linearized-model-based controllers), $y_t$: output target.
One controller implemented in Profile Control category has linear state-space representation:

\[
\begin{align*}
    x_{k+1} &= Ax_k + B \begin{bmatrix} y_t - y_r \\ y + y_d - y_r \end{bmatrix}_k, \\
    u_k^{FB} &= Cx_k + D \begin{bmatrix} y_t - y_r \\ y + y_d - y_r \end{bmatrix}_k,
\end{align*}
\]

(10)

**IMPORTANT:** After time discretization, proposed controller (5) can be implemented in the Profile Category by using this linear discrete-time state-space representation.

Controller (10) is complemented by an anti-windup compensator in discrete-time state-space form:

\[
\begin{align*}
    x_{aw}^{k+1} &= A_{aw}x_{aw}^k + B_{aw} [sat(u) - u]_k, \\
    s_{k+1} &= C_{aw}x_{aw}^k + D_{aw} [sat(u) - u]_k,
\end{align*}
\]

(11)

The saturation function is defined as

\[
sat(\cdot) = \begin{cases} 
    (\cdot)^{min} & \text{if } (\cdot) < (\cdot)^{min} \\
    (\cdot) & \text{if } (\cdot)^{min} \leq (\cdot) \leq (\cdot)^{max} \\
    (\cdot)^{max} & \text{if } (\cdot) > (\cdot)^{max}
\end{cases}
\]
Pulse Width Modulation for the Command of NBI Power

A pulse width request $t_{PW}^{\text{request}}$ is first defined based on a chosen averaging time interval $t_{av}$ and a given duty cycle $D_c$ defined by the requested/maximum NBI power ratio, i.e.

$$t_{PW}^{\text{request}} = D_c t_{av}, \quad D_c = \frac{P_{\text{NBI}}}{P_{\text{max NBI}}}.$$  \hfill (12)

Algorithm below guarantees fulfillment of minimum on/off times:

1. If $t_{PW}^{\text{request}} < \frac{t_{\text{on}}}{2}$
   - Yes: $t_{PW}^{\text{request}} = 0$
2. If $\frac{t_{\text{on}}}{2} \leq t_{PW}^{\text{request}} < \frac{t_{\text{on}}}{2}$
   - Yes: $t_{PW}^{\text{request}} = t_{\text{on}}$
3. If $t_{\text{on}} \leq t_{PW}^{\text{request}} < t_{av} - t_{\text{off}}$
   - Yes: $t_{PW}^{\text{request}} = t_{\text{request}}$
4. If $t_{av} - t_{\text{off}} \leq t_{PW}^{\text{request}} < t_{av} - t_{\text{off}}$
   - Yes: $t_{PW}^{\text{request}} = t_{\text{off}}$
5. If $t_{av} - t_{\text{off}} \leq t_{PW}^{\text{request}} < t_{av} - \frac{t_{\text{off}}}{2}$
   - Yes: $t_{PW}^{\text{request}} = t_{av} - t_{\text{off}}$
6. If $t_{av} - \frac{t_{\text{off}}}{2} \leq t_{PW}^{\text{request}}$
   - Yes: $t_{PW}^{\text{request}} = t_{av}$
Connection is built between response model (1)-(3) ($\psi, W \rightarrow q, \beta_N$ dynamics) and PCS

- Enables debugging of the algorithm implementation in the Profile Control category
- Validates real-time computations carried out by the implemented control algorithm
  - Uses model-based predicted diagnostic data before experimental testing
Simultaneous Feedback $q$-profile Regulation at Edge & Core Was Demonstrated for the First Time by Using 4.60 GHz LHW Source

- Tracking of desired $q$ profile at $\hat{\rho} = 0.1$ and $\hat{\rho} = 0.9$ is achieved by using $I_p$ and $P_{LH2}$ actuation.

- Feedback control (FB) is turned on for $2s < t < 8s$ (indicated by light-gray background in figures).

- Feedforward-control components are modified by feedback controller so that actual evolutions (solid-blue) track targets (dashed-red).

- Target evolutions for the $q$ profile at these 2 points were obtained from actual shot to ensure feasibility.
Simultaneous Feedback $q$-profile Regulation at Edge & Core Was Demonstrated for the First Time by Using 4.60 GHz LHW Source

- Feedforward (FF) control (dashed-orange lines) is corrected by feedback (FB) controller to produce requested actuation (dashed-green lines).
- There is a bias between requested (dashed-green lines) and delivered (solid-blue lines) LHW power due to the way this actuator is controlled.
- In spite of bias, the FB controller is capable of tracking targets due to presence of integral action.
- The requested actuation (dashed-green lines) is the result of constraining the actuation computed by the FB controller (solid-yellow lines) by the physical limits associated to the different actuators.
- These saturation limits (dashed-black lines) were not active in this discharge.
New Beam Power Modulation Algorithm Implemented in PCS for Simultaneous $q$-profile $+\beta_N$ Control Showed Good Average Tracking
New Beam Power Modulation Algorithm Implemented in 2018 for Simultaneous $q$-profile + $\beta_N$ Control Showed Good Average Tracking

- Tracking of desired $q$ profile at $\hat{\rho} = 0.1$, $\hat{\rho} = 0.9$ and $\beta_N$ is achieved by using $I_p$, $P_{LH2}$ and $P_{NBI1}$ actuation
- PWM algorithm (12) was used with mixed results to command the NBI1L source ($P_{NBI1} = P_{NBI1L}$)
- The targets are tracked in average but the PWM algorithm introduces significant perturbations due to:
  + Minimum on/off time constraints significantly impacting this relatively low-$\beta_N$ plasma
  + Detected implementation issues: i- FF control set to zero, ii- time delay introduced by PWM algorithm
Simultaneous Feedback $q$-profile Regulation at Three Points Was Demonstrated for the First Time by Using two LHW Sources
Simultaneous Feedback $q$-profile Regulation at Three Points Was Demonstrated for the First Time by Using two LHW Sources

- Tracking of desired $q$ profile at $\dot{\rho} = 0.1$, $\dot{\rho} = 0.5$, $\dot{\rho} = 0.9$ is achieved by using $I_p$, $P_{LH1}$, and $P_{LH2}$ actuation.
- Solid-magenta lines show $q$-profile evolutions at these points for feedforward-only EAST shot #95176.
- FF control needs to be modified by FB control for actual (solid-blue) profile to track target (dashed-red).
- Saturation in the 4.60 GHz LWH power ($P_{LH2}$) is briefly observed at the beginning of FB-on window.
- Around 1MW of ECRF H&CD power was used in this and subsequent shots to keep plasma in H-mode.
Simultaneous Feedback Regulation of Two Points of the $q$ Profile and $\beta_N$ Was Experimentally Tested by Using two LHW Sources
Simultaneous Feedback Regulation of Two Points of the $q$ Profile and $\beta_N$ Was Experimentally Tested by Using two LHW Sources

- Tracking of desired $q$ profile at $\dot{\rho} = 0.1$, $\dot{\rho} = 0.9$ and $\beta_N$ is achieved by using $I_p$, $P_{LH1}$, and $P_{LH2}$ actuation
- Solid-magenta lines show $q$-profile evolutions at these points for feedforward-only EAST shot #95176.
- FF control needs to be modified by FB control for actual (solid-blue) profile to track target (dashed-red)
- Saturation in the 2.45 GHz LWH power ($P_{LH1}$) is observed after around 5 sec. as the combined $q$-profile+$\beta_N$ controllers tries to track the $\beta_N$ target more closely while controlling $q$ at $\dot{\rho} = 0.1$, $\dot{\rho} = 0.9$. 
Simultaneous Feedback $q$-profile Regulation at Three Points Was Demonstrated Even Under the Presence of Input Disturbances

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Model-Based Scenario Control in EAST

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Simultaneous Feedback $q$-profile Regulation at Three Points Was Demonstrated Even Under the Presence of Input Disturbances

- Tracking of desired $q$ profile at $\hat{\rho} = 0.1$, $\hat{\rho} = 0.5$, $\hat{\rho} = 0.9$ is achieved by using $I_p$, $P_{LH1}$, and $P_{LH2}$ actuation.

- Solid-magenta lines show $q$-profile evolutions at these points for feedforward-only EAST shot #95176.

- FF control needs to be modified by FB control for actual (solid-blue) profile to track target (dashed-red).

- Shot similar to #95183 but introducing 0.3 MW perturbation in the 4.60 GHz LWH power ($P_{LH2}$) for $t \in [4, 6]$.

- FB controller starts reducing request of LHW power after actual (solid-blue line) $q$ values at $\hat{\rho} = 0.1$ and $\hat{\rho} = 0.5$ exceed targets. Tracking improvement is limited by lower-limit saturation of $P_{LH2}$ after 6 sec.
Successful $q$-profile+$\beta_N$ control was demonstrated for the first time in EAST

Task 1: Number of actuators under the Profile Category in the PCS should be increased by:
- Enhancing the NBI PWM algorithm and testing it in H-mode plasmas
- Incorporating the command of ECRF and ICRF H&CDs

Task 2: The quality of the real-time reconstruction of the $q$ profile needs to be improved by constraining pEFIT with POINT measurements

Task 3: The accuracy of the control-level models used for control design should be enhanced by further developing control-physics understanding and continuing validation efforts

Completion of these tasks will further augment capability of tightly regulating $q$-profile and $\beta_N$ to routinely enable access to long-pulse, disruption-free, high-performance operation in EAST

It is anticipated that this augmented control capability will be achieved by employing more sophisticated, model-based, optimal, control algorithms.