

Nonlinear Trapping in Wave-Particle Interactions in Tokamaks

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ABSTRACT

- Wave-particle interactions are ubiquitous in toroidal plasmas, such as tokamaks. Waves or static perturbations can be in resonance with characteristic particle frequencies in tokamaks.
- For example, the resonance condition for circulating particles is

$$\omega + \sigma[l-nq(p_\zeta)]\omega_i + n\omega_d = 0,$$
 and the resonance condition for the trapped particles is

$$\omega + l\omega_b + n\omega_d = 0.$$
- Linear resonance can be decorrelated either by the collisions or by the nonlinear particle dynamics.
- When the (effective) collision frequency ν_{eff} decreases so that it becomes smaller than the bounce frequency of the nonlinearly trapped particles, the physics of nonlinearly trapping becomes important.
- The radial excursion of the particles has to be less than the radial separation of the modes for nonlinear trapping to be observable.
- The distribution in the nonlinearly trapped region in the phase space is flattened, and the corresponding energy loss rate can be comparable with that of the neoclassical even for a small magnitude of the perturbed magnetic field.

BACKGROUND

- At the resonance, the phase of the wave is stagnant.
- The radial excursion moves particles out of the resonance.
- The ‘poloidal’ motion moves particles back to the resonance.
- A nonlinear pendulum-like motion results.
- The results in the following papers are summarized and updated here.
 1. K. Shaing, M. Garcia-Munoz, and E. Viezzer, NF **60**, 056002 (2020)
 2. K. Shaing, M. Garcia-Munoz, E. Viezzer, and M. S. Chu, NF **61**, 046009 (2021)

PHYSICS CONSEQUENCES OF NONLINEAR TRAPPING

ENERGY TRANSFER RATE AND POWER-FLUX RELATION

- Wave-particle interactions usually involve energy transfer between them.
- From the change of the single particle energy $E = v^2/2 + e\phi/M$, the energy loss rate of the particle energy is

$$dW_p/dt = - \int dv M \dot{E} f$$
- If Poynting energy flow S can be ignored, the wave energy gain is, from the energy conservation law,

$$dW/dt = dW_p/dt$$
 where W is the wave energy.
- Of course, the rate of the change of the wave energy also follows the Poynting theorem

$$dW/dt + \text{div}S = - \mathbf{J} \cdot \mathbf{E},$$
 in which the wave-particle energy exchange rate is the $-\mathbf{J} \cdot \mathbf{E}$ term. It is not difficult to show that the energy transfer rate derived from the single particle picture is the same as that from the Poynting theorem when Poynting flux is neglected.

POWER-FLUX RELATION

- Because there is a relation between the rate of the change of the single particle energy E and that of the p_ζ , i.e.,

$$cnM dE/dt = -e\omega dp_\zeta/dt$$

the energy transfer rate is proportional to a part of the particle flux driven by the perturbed fields that break the toroidal symmetry.

NONLINEAR DAMPING OR GROWTH RATE

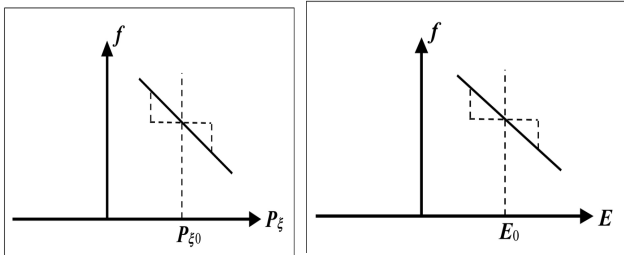
- A nonlinear damping or growth rate γ_{NL} can be calculated to obtain

$$\gamma_{\text{NL}} = d \ln W / dt$$

PHYSICS CONSEQUENCES OF NONLINEAR TRAPPING

FLATTENING OF THE DISTRIBUTION FUNCTION

- It is shown *analytically* that the particle distribution function is flattened in the nonlinearly trapped region in the phase space.



ENHANCED TRANSPORT LOSSES: NEW SCALINGS

- The transport coefficient for the static magnetic perturbations scale as

$$\chi_{na} \sim \nu r^2 \text{Sqrt}(\delta B/B) \text{Sqrt}(Q_p/r)$$
- For the electromagnetic waves,

$$\chi_B \sim \nu r^2 \text{Sqrt}(B_p/B)$$
- The energy loss rates are either weakly dependent on or independent of the equilibrium magnetic field strength; increasing it will not reduce the loss meaningfully
- Even for a small magnitude of the normalized perturbed magnetic field strength of the order of 10^{-4} or smaller, the energy loss rates can be comparable with that of the neoclassical.

CONCLUSIONS

The following physics consequences resulting from the nonlinear trapping in wave-particle interactions in tokamaks are demonstrated:

1. Distribution function is flattened in the the nonlinearly trapped region in the phase space.
2. The energy loss rate for the energetic alpha particles can be comparable to that of the neoclassical theory for normalized perturbed magnetic field $\delta B/B \sim 10^{-4}$ or smaller.
3. The energetic alpha particle energy loss rate is either weakly dependent on or independent of the equilibrium magnetic field strength. Thus, increasing equilibrium magnetic field strength does not improve the loss rate.
4. The energy transfer rate can be derived either from the single particle picture, i.e., dE/dt , or equivalently, from Poynting theorem.
5. Because dE/dt is proportional to dp_ζ/dt , energy transfer rate is proportional to the part of the particle flux that is driven by the perturbed fields that break the toroidal symmetry. This shall be called power-flux relation.
6. The nonlinear damping or growth rate γ_{NL} can also be calculated.
7. Because there is a universal collision frequency landscape for the particle flux, there is such a universal landscape for the energy transfer rate and γ_{NL} as well.

ACKNOWLEDGEMENTS / REFERENCES