Global theory of BAE excited by trapped energetic¹¹⁵⁴ electrons

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ABSTRACT

- The global dispersion relation of high-n beta-induced Alfvén eigenmodes (BAEs) excited by trapped energetic electrons (EEs) in tokamaks are investigated using gyrokinetic theory.
- •The contribution of the trapped EEs to the global e-BAE dispersion relation is limited to the ideal MHD structure of the BAE.

•The different radial eigenstates can be unstable simultaneously; the radial mode structure is twisted by EEs and shows opposite deformation directions compared with that in the presence of EIs; the radial symmetry breaking mode structure w.r.t. k_{||} has a potential impact on toroidal momentum transport.

Keeping the leading-order contributions, Eq. (1) becomes [R. Ma et al 2021NF]

 $\begin{bmatrix} \frac{\partial^2}{\partial z^2} + Q(\omega; z) \end{bmatrix} A(z) = 0 \quad (2)$ $z \equiv (nq - m_0) \simeq |nq'(r_0)|(r - r_0) \text{ and } q'(r_0) = dq/dr \text{ at } r = r_0;$ $Q(\omega; z) = a + bz^2 \text{ with } a \equiv 2(i\Lambda - \delta W_{f0} - \delta W_{k0})/(\delta W_{f\eta_k}^{(2)} + \delta W_{k\eta_k}^{(2)})$ and $b \equiv 2\delta W_{k0}/[L_{PE}^2 k_{\theta}^2 s^2 \left(\delta W_{f\eta_k}^{(2)} + \delta W_{k\eta_k}^{(2)}\right)].$ The two turning points $z_{T\pm}$ are given by $Q(\omega; z) = 0$. The matching of solutions yields the global dispersion relation for e-BAE, i.e.,

BACKGROUND

•BAE, arising from the combined effects of the ion compressibility and the geodesic curvature of the magnetic field [M. Chu 1992PFB], can be affected by both EPs and thermal ions [F. Zonca et al 1999POP, X. Wang et al 2010PPCF] and has received considerable interest recently.

- •BAEs excited by EEs (e-BAEs), have been observed on the low magnetic field side in HL-2A during the Ohmic and ECRH heating [W. Chen et al 2011NF] experiments.
- •Gyrokinetic simulations [J. Cheng et al 2016POP, Y. Chen et al 2019POP] show that the EE precessional resonance is responsible for the e-BAE linear excitation and the opposite direction of the triangle mode structure w.r.t. that in the energetic ion (EI) driven BAE case.
- •This work is to offer some ideas for multi-mode coexistence of e-BAE in the HL-2A [X.T. Ding et al 2013, 2018NF]; to explain the twisted mode structures;

$$\int_{zT-}^{zT+} \sqrt{Q(\omega;z)} dz = (L+1/2)\pi, L = 0, 1, 2, ...$$
 (3)

The integer L: the radial eigen-mode number labeling the different

resonance states.

NUMERICAL RESULTS



Fig.2 The dependence of the mode frequencies & the growth rates on ϵ_{nE} . Figure 2 shows that the ground (L=0) and excited radial eigenstates (L=1,2)

can he unstable simultaneously

Z/a

Z/a

Zla



to illustrate the effect of radial symmetry breaking of mode structure on toroidal momentum transport.

GLOBAL DISPERSION RELATION OF e-BAE

THEORETICS MODEL AND DISPERSION RELATION

- $\succ \text{ The wavelength and frequency orderings: } k_{\theta}\rho_{Li} \approx \delta, k_{\theta}\rho_{Le} \approx \delta^{5/2}, k_{\theta}\rho_{Li} \approx \delta^{2}, \\ \text{ and } \varpi_{dEe} \approx \omega_{*i} \approx \omega_{ti} \approx \omega \approx O(\delta^{1/2})\omega_{A};$
- > The scalar potential $\delta \phi$ and the parallel (to **b** = **B**/B) vector potential $\delta A_{||}$
- $\equiv -(c/\omega)\mathbf{b} \cdot \nabla \delta \psi$: to describe the electromagnetic fluctuations;
- Vorticity equation + gyrokinetic equation + quasi-neutrality condition: to investigate e-BAEs.

Methods: two spatial scales + asymptotic matching [L. Chen et al 1984PRL, L. Chen 1994POP].

 $\longrightarrow D(\omega; r, \eta_k) \equiv i\Lambda(\omega) - \delta W_f(r, \eta_k) - \delta W_k(\omega, r, \eta_k) = 0$ (1)

The deeply trapped EEs are dominant in the ideal MHD region of e-BAE dispersion relation [R. Ma et al 2020NF].

> δW_f and δW_k correspond to the generalized potential energy due to fluid-like plasma response and the kinetic EE behavior. $\Lambda(\omega)$ accounts for the generalized inertia response, including both thermal ion transit resonances and diamagnetic effects [F. Zonca et al 1996PPCF].

Fig.3 The Stokes structures in the complex-z plane.

Figure 3 shows that the most unstable mode is determined by the pressure

gradient of EEs and the width of the mode itself.



Figure 4 shows that the 2D poloidal mode structures are twisted by EEs. The L affects the parity of mode structures. The magnetic shear and

EE density affect the mode structures.

Solution Assumption: the radial dependence of $D(\omega; r, \eta_k)$ is mainly due to the profile of the energetic electrons via α_{Ee} , i.e., $\alpha_{Ee} = \alpha_{Ee0} \exp\left[-\frac{(r-r_0)^2}{L_{PE}^2}\right]$. **THE GLOBAL EIGENMODE ANALYSIS** The solution of Eq. (1) yields the condition of the most unstable e-BAE. The local equilibrium parameters: $r_0 = 0.5$, q = 2.0, s = 1, $\epsilon = 0.2$, $\tau = 1.0$, $\theta = 0.04$, $\frac{\omega_* ni}{\omega_* ni} = 4.000$

local equilibrium parameters: $r_0 = 0.5$, q = 2.0, s = 1, $\epsilon = 0.2$, τ 1.0, $\beta_i = 0.01$, $\frac{\omega_{*ni}}{\omega} = 1$, $\eta_i = 0.25$, $\frac{L_{ni}}{R_0} = 0.171$, $\eta_E = 0$, $k_{\theta}\rho_{LE} = 0.01$, $T_{Ei} \equiv \frac{T_{Ee}}{T_i} = 9$, and n = 4.

Figure 1 shows that the mode growth rate is maximum at $\eta_k = 0$ and $r = r_0$ and decreases as $|\eta_k|$ and $|r - r_0|$ increase. Taylor expanding $D(\omega; r, \eta_k)$ for (r, η_k) around $(r_0, 0)$, respectively.



Fig.4 Poloidal contour plots of electrostatic potential $\delta \phi_n(r, \theta)$ in (R, Z) plane.



ACKNOWLEDGEMENTS

• The work is supported in part by the ITER-CN under Grant No. 2018YFE0304103 and NNSF of China under Grant Nos. 11705050, and the ITER-CN under Grant No. 2017YFE0301900.