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# Analytical and numerical gyrokinetic studies of low-frequency drift Alfvén waves in toroidal fusion plasmas

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## ABSTRACT

- We present an overview of theory and simulation of low-frequency drift Alfvén waves (DAW) in toroidal fusion plasmas based on the framework of the general fishbone like dispersion relation (GFLDR) and gyrokinetic theory.
- The GFLDR is a unified theoretical framework that can help understanding experimental observations as well as numerical simulation and analytic results with different levels of approximation.
- We analyze EP excitations of low-frequency Alfvén eigenmodes in toroidal plasmas.
- We also discuss the important role of core plasma for low-frequency Alfvén and acoustic fluctuations.
- We illustrate the relevant aspects of the Drift Alfvén Energetic Particle Stability (DAEPS) code taking the kinetic structures of the low-frequency continuous spectrum as example.

## THE GENERAL FISHBONE LIKE DISPERSION RELATION

$$2\pi^2\int_0^a dr\left[\frac{k_\vartheta^2 c^2 (d\psi/dr)}{\mathcal{J}B_0^2}\right]_{\vartheta=0}|A(r)|^2\left[\delta\bar{W}_f(r)+\delta\bar{W}_k(r)-i\Lambda(r)\right]=0\,. \qquad (1)$$

- $\Lambda$ represents a generalized inertia, while $\delta\bar{W}_f$ and $\delta\bar{W}_k$ describe the potential energy of the fluctuations accounting, respectively, for the fluid and kinetic plasma response [1-3].
- For radially localized fluctuations, the GFLDR can then be written as

$$i\Lambda(r)=\delta\bar{W}_f(r)+\delta\bar{W}_k(r)\,, \qquad (2)$$

- For radially localized fluctuations The present approach allows extracting spatial and temporal scales of the considered fluctuations as well as the underlying physics: e.g, radial singular structures of the shear Alfvén wave (SAW) continuous spectrum are shown in FIG. 1 using the analytic as well as the numerical solution for the generalized inertia, $\Lambda$, in the MHD fluid limit [4-6].

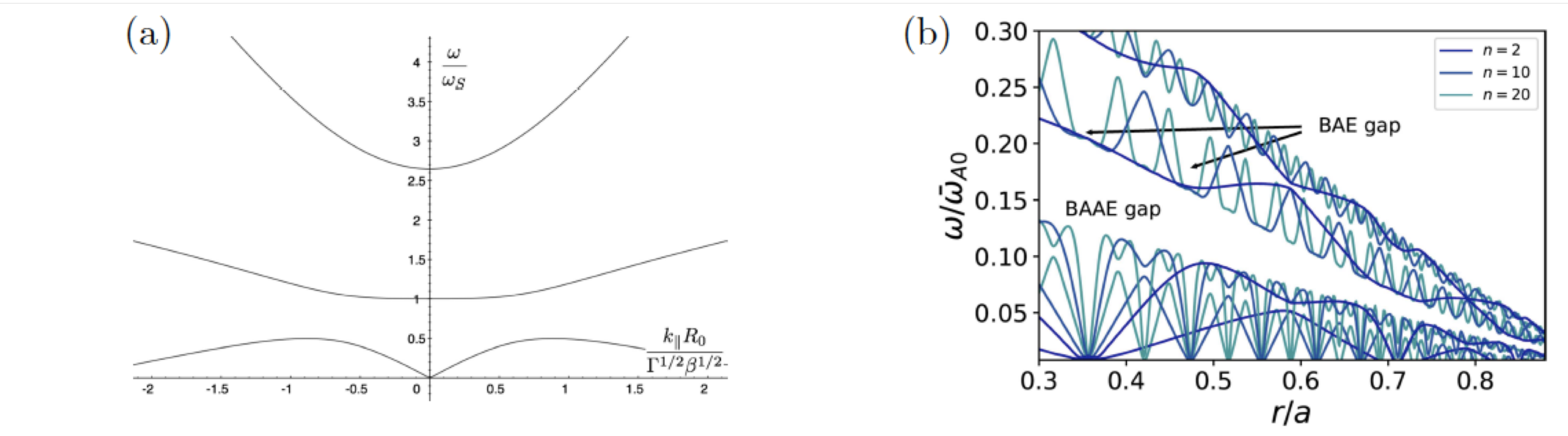


*FIG. 1. (a) Analytic solution of the low frequency SAW continuous spectrum (Adapted from original figure in Ref. [4]). (b) Numerical solution for the low frequency SAW continuous spectrum from a realistic Divertor Tokamak Test (DTT) facility equilibrium (Adapted from original figure in Ref. [5]).*

- Analytic results can be verified with very good agreement for both SAW continuum accumulation points [7, 8] as well as for geodesic acoustic mode oscillations [9].
- Thus, the GFLDR as a unified theoretical framework can help understanding experimental observations as well as numerical simulation and analytic results with different levels .

## GYROKINETIC ANALYSIS OF THE LOW FREQUENCY ALFVÉN ACOUSTIC SPECTRUM

### WEAK EXCITATION OF BAAE BY EPs

- The causality constraint applied to the GFLDR, Eq. (2), dictates that, for discrete eigenmodes,

$$Re(i\Lambda)=\delta\bar{W}_f+Re(\delta\bar{W}_k)<0. \qquad (3)$$

- $\Lambda(\omega)$ generally has multiple-$\omega$ branches and, thus, the GFLDR can also be expressed as

$$\omega_{Fj}(\Lambda)\simeq\omega_{fj}(0)-(1/2)\omega''_{Fj}\left(\delta\bar{W}_f+\delta\bar{W}_{kr}+i\delta\bar{W}_{ki}\right)^2, \qquad (4)$$

  with j=1,2,3 representing the different roots in the MHD fluid limit with accumulation point: 1. $\omega_{BAE}=(\Gamma\beta)^{1/2}v_A/R_0$; 2. $\omega_s=\omega_{BAE}/\sqrt{2}q$; and 3. $\omega=0$ for normal shear
- It is found that

$$|\omega''_{F2}/\omega''_{F1}|\sim|\omega''_{F2}/\omega''_{F3}|\sim O(\beta)\sim O(10^{-2})\ll 1\,. \qquad (5)$$

- Equation (4), indicates that EP coupling to BAAE, $\omega_{F2}$ ,is much weaker, i.e., $O(\beta)$ smaller, than that to BAE, $\omega_{F1}$, or the low-frequency SAW eigenmode, $\omega_{F3}$, which includes the energetic particle mode (EPM) [16].
- The same result holds for the kinetic result, which is needed for collisionless fusion plasmas where that $\Lambda_k(\omega)$ depends on the thermal ion dynamics and BAAE is damped due to thermal ion Landau damping [2,13-15].

$$\omega_{kj}(\Lambda)\simeq\omega_{kj}(0)-(1/2)\omega''_{kj}\left(\delta\bar{W}_f+\delta\bar{W}_{kr}+i\delta\bar{W}_{ki}\right)^2\,. \qquad (6)$$

## GYROKINETIC EXPRESSION OF LOW FREQUENCY GENERALIZED INERTIA

- For $|\omega|<\ \omega_{bi}\simeq\sqrt{\epsilon}v_{ti}/(qR_0)$ (thermal ion bounce frequency), we derive $\omega_{k3}$ as the kinetic-ballooning mode (KBM) with the gap between $\omega=0$ and the thermal-ion diamagnetic drift frequency including enhanced neo-classical inertia

$$\Lambda^2=\left[\frac{1}{2\pi}\int_{-\pi}^{\pi}\frac{\mathcal{J}^2B_0^2}{v_A^2}\omega\left(\omega-\omega_{*pi}\right)d\theta\right]\left(1+\delta\hat{\Phi}_{\parallel 0}/\delta\hat{\Psi}_0\right)$$

$$-2\int_{-\pi}^{\pi}\mathcal{J}^2B_0^2\frac{F^2(\psi)}{|\nabla\psi|^2}\left\langle m_i\widetilde{\left(\frac{v_\parallel}{B_0}\right)}^2\left(\omega-\bar{\omega}_{di}\right)Q\bar{F}_{0i}\right\rangle d\theta$$

- This expression shows that the kinetic generalized inertia at low frequency has the same structure as the MHD limit [4], with : i) the d.c. parallel electric field due to bounce averaged trapped particle response [13-15]; and ii) the anisotropic thermal ion compression response, modified by toroidal precession rate.

## ANALYTICAL RESULTS FOR BETA INDUCED ALFVÉN ACOUSTIC EIGENMODES

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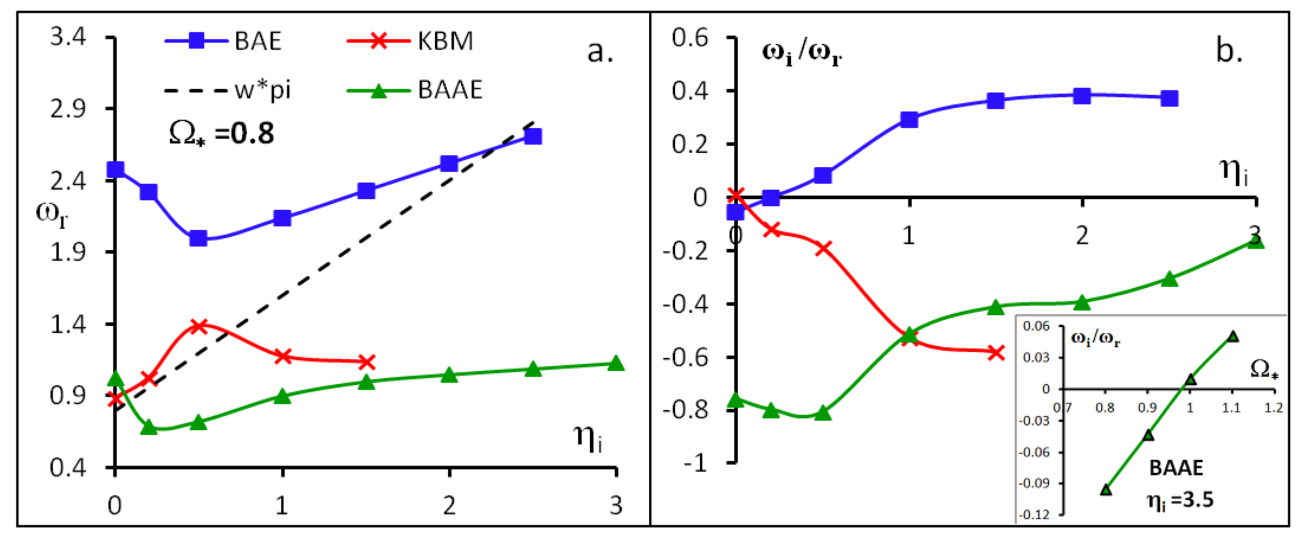
*FIG.2. Coupling of the low frequency modes throw the diamagnetic frequency.*
*a. Real frequency vs. $\eta_i$;*
*b. Normalized growth rate*
$\Omega_*=\omega_{*ni}/\omega_{ti}=0.8$




- For $\eta_i=0$ the BAE is slightly damped do to finite Landau damping, while the KBM is marginally stable. The BAAE on the other hand is heavily damped [14,19]. For increasing $\eta_i$ the BAE accumulation point becomes unstable, and the mode can be excited by the thermal ion temperature gradient at large $\eta_i$ (Alfvén ITG mechanism [20]) (FIG.2a).
- For small values of $\omega_{*pi}$, the KBMs frequency is close to the diamagnetic frequency, but this changes for higher $\eta_i$ values due to the coupling of the KBM to the BAE [18] and BAAE [14] branches: the BAAE damping rate is decreasing (FIG.2b).

## DAEPS CALCULATION OF LOW FREQUENCY KINETIC CONTINUOUS SPECTRUM

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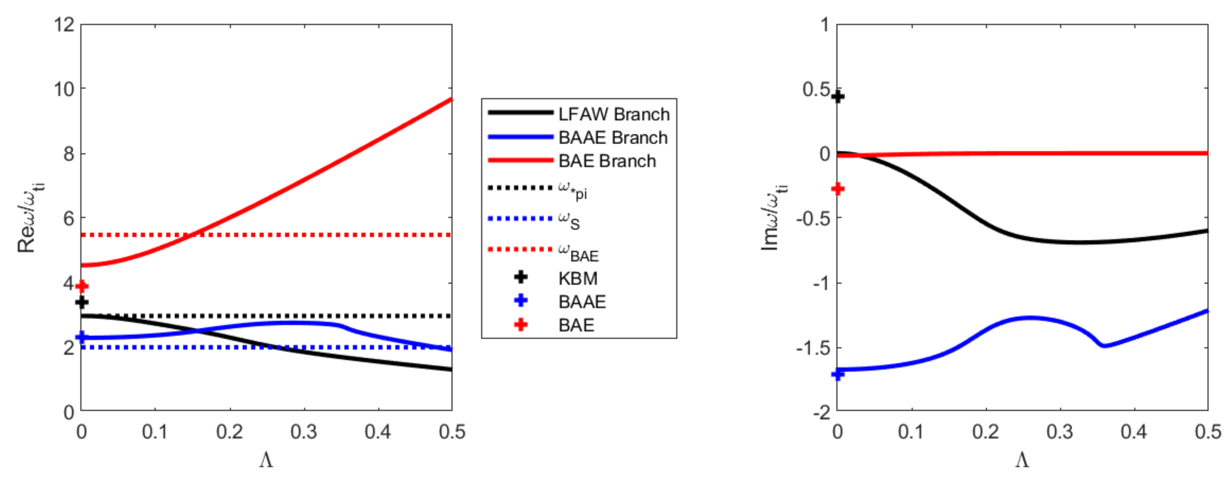
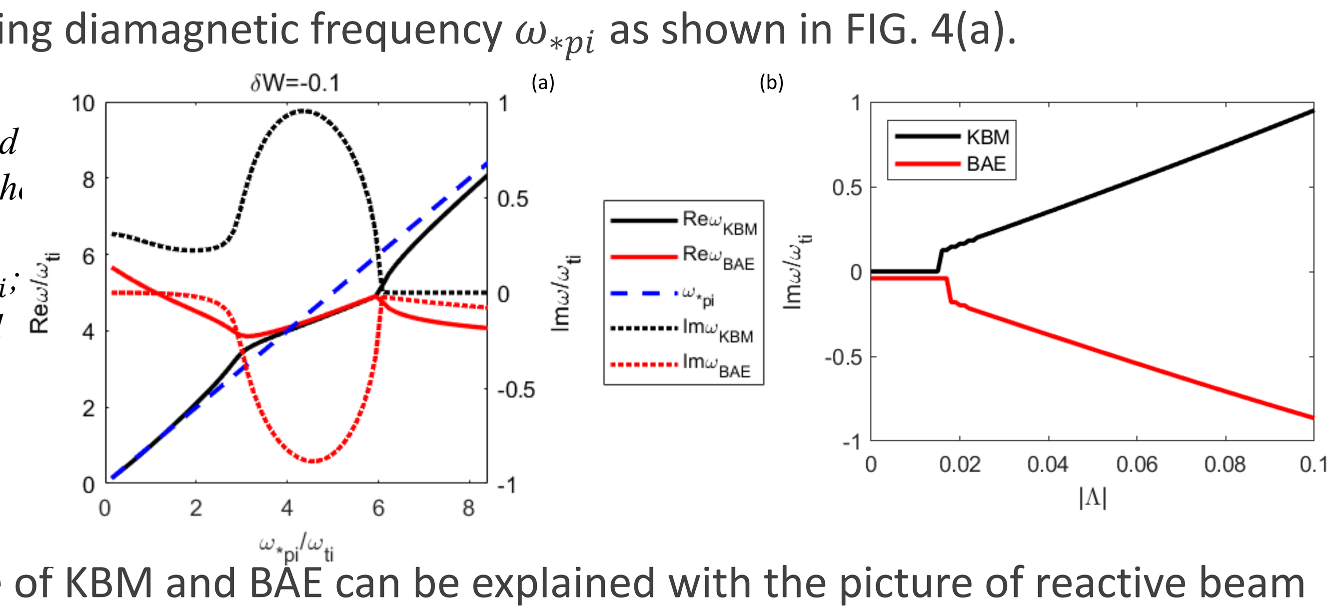
*FIG. 3. Frequency (a) and growth rate (b) of LFAW branch, BAAE branch, and BAE branch and corresponding gap modes with $i\Lambda=-0.1$.*




- It is observed that, when LFAW and BAE branches are close, the frequencies of the gap modes, i.e., KBM and BAE, are close as well, while the growth rates undergo a drastic change for increasing diamagnetic frequency $\omega_{*pi}$ as shown in FIG. 4(a).

*FIG. 4. (a) Frequency (solid lines) and growth rate (dashed lines) of BAE and KBM vs. diamagnetic frequency $\omega_{*pi}$; (b) growth rate of BAE and KBM vs. $|\Lambda|$, with $\delta W=i\Lambda=-0.1$.*




- The drastic change of KBM and BAE can be explained with the picture of reactive beam instability. Defining $\omega\approx\omega_{*pi}+\delta\omega\approx\omega_{BAE}+\delta\omega$, with $\omega_{2i}\ll\mathrm{Im}\delta\omega\ll\omega_{*pi}$, [18]:

$$\Lambda^2=-\left|\delta\bar{W}_f\right|^2=(1-\omega_1/\omega)(\omega-\omega_2)(\omega+\omega_3)/\omega_A^2\quad;\quad\delta\omega^2/\omega\ \approx-\omega_A^2\left|\delta\bar{W}_f\right|^2/2\omega_{*pi}\,.$$

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