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Validation of GAE simulation and theory for NSTX(-U) and DIII-D

Elena Belova[1],
J. Lestz[3], N. Crocker[2], E. Fredrickson[1]

1) Princeton Plasma Physics Laboratory, Princeton NJ, USA
2) University of California, Los Angeles, California 90095, USA
3) UCI, Irvine CA

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Motivation

- Correlation between GAE/CAE observations and flattening of T_e profile at increased NBI power in NSTX suggests that GAE/CAE can reduce the efficiency of NBI heating.
  - Important to correctly identify and predict these instabilities
- Simulations were needed to conclusively identify nature of high-frequency modes (ω/ω_{ci}~ 0.6) in DIII-D.
  - Previously identified as compressional Alfven eigenmodes (CAEs) in DIII-D [Heidbrink, NF2006], however both theory and NSTX simulations predict stronger GAE instabilities [Belova PoP2019, Lestz, NF2021].
  - In NSTX(-U), GAEs were more common especially for lower v_{beam}/v_A.
  - New dedicated experiments in DIII-D suggest SAW dispersion for these modes [S.Tang, PRL 2021]
- Validate HYM code against DIII-D observations.
- DIII-D results indicate that GAEs can be excited for larger aspect-ratio devices and smaller injection velocity $v_{beam}/v_A \lesssim 1$, therefore they can be unstable in ITER.

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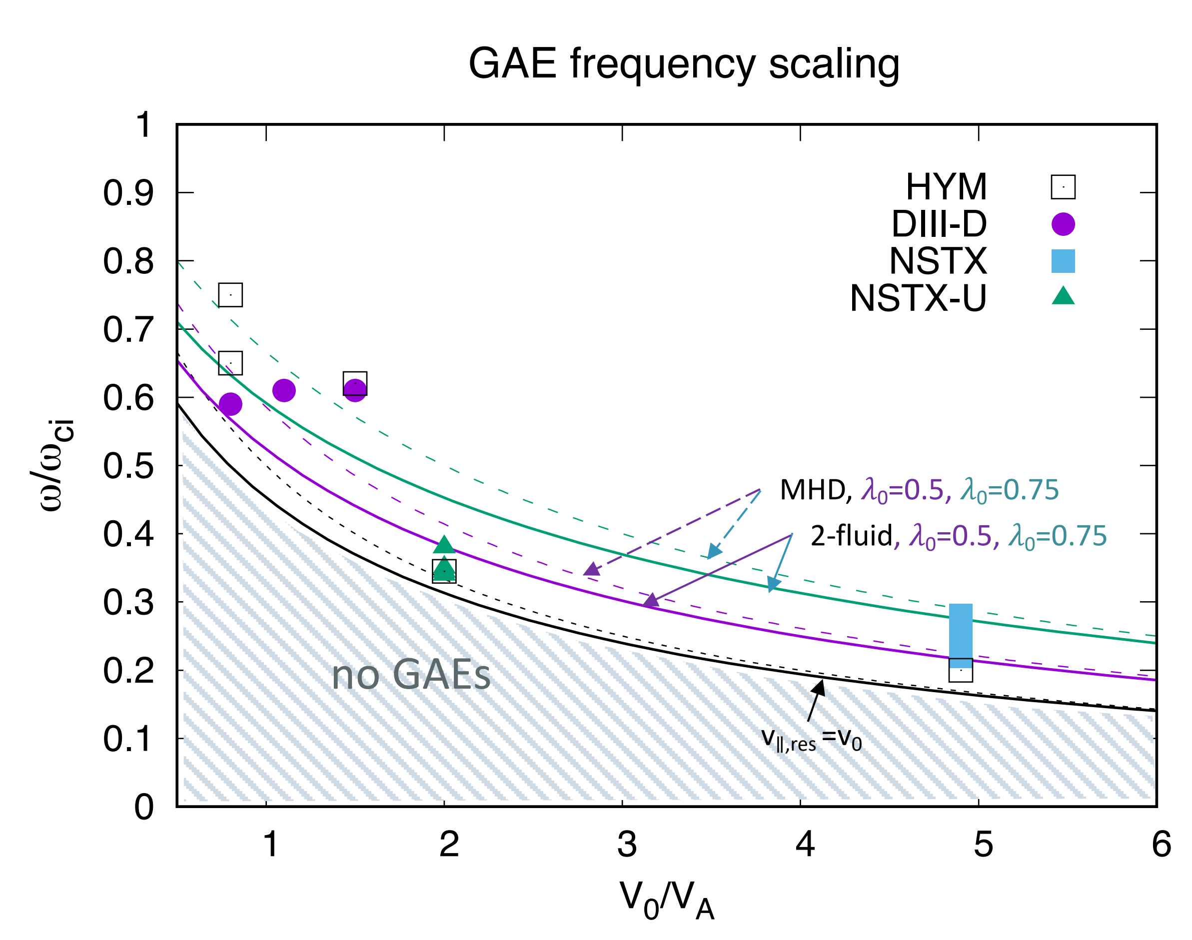
Theory and simulations explain GAE frequency scaling with NBI parameters in NSTX, NSTX-U and DIII-D

Numerical model and theory for sub-cyclotron frequency modes developed for NSTX(-U) have been successfully applied to explain DIII-D observations.

- HYM simulations for DIII-D demonstrate that modes with $\omega/\omega_{ci} \sim 0.6$, previously mis-identified as compressional Alfven eigenmodes (CAEs), have shear polarization $\delta B_\perp >> \delta B_{||}$ (GAEs).
- Simulation results match the observed frequencies in DIII-D for high- and low- B_{tor} experiments [S.Tang, PRL 2021, Heidbrink NF06].



Scaling of experimentally observed GAE frequency with injection velocity vs predicted by theory (lines) and most unstable modes in simulations (black squares). Color lines – 2-fluid (solid) or MHD (dashed) condition for peak instability calculated for $k_\perp/k_{||}$=1. Black lines show $v_{\|,res} \leq v_0$ boundary.

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Theory predict scaling of most unstable GAE with B_{tor}, n_e, λ_0

Predicted range of most unstable counter-GAEs:

$$\frac{1}{(1+v_0/v_A)} < \frac{\omega}{\omega_{ci}} \le \frac{1}{(1+\frac{v_0}{v_A}\sqrt{1-\lambda_0})},$$

[Belova, PoP 2019]
[Lestz, PoP 2020]

- $\omega \sim \omega_{ci} \rightarrow$ nearly linear scaling with B_{tor}
- Weaker than $1/\sqrt{n_e}$ scaling with density.
- Larger v_0/v_A results in smaller values of ω/ω_{ci}:

  $\omega/\omega_{ci} \approx 0.6$ for $v_0/v_A \sim 1$ and $\lambda_0 \sim 0.6$ (DIII-D),

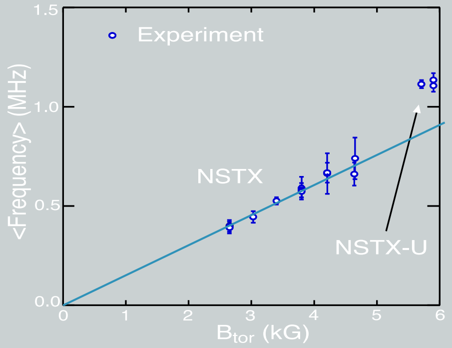
  $\omega/\omega_{ci} \approx 0.4$ for $v_0/v_A \sim 2$ (NSTX-U),

  $\omega/\omega_{ci} \lesssim 0.2$ for $v_0/v_A \gtrsim 4$ (NSTX),

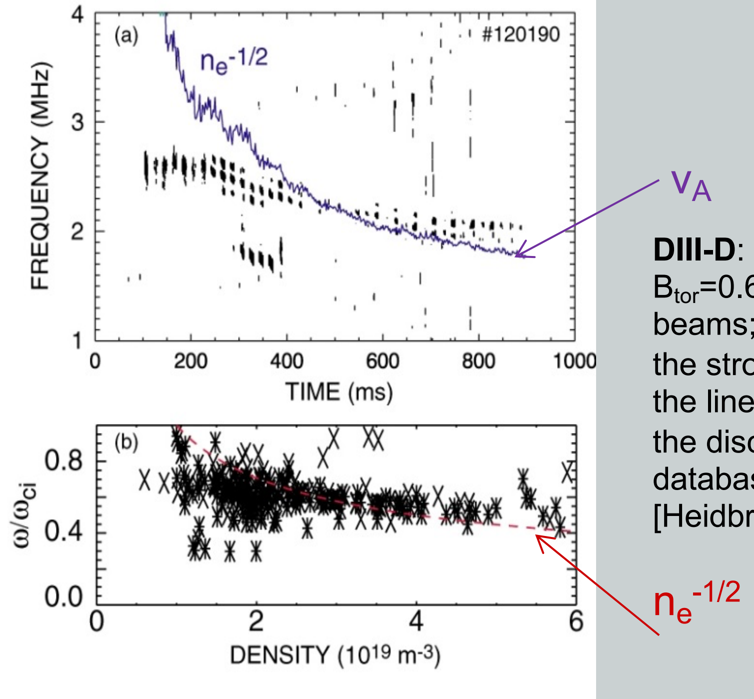
- Scaling with λ_0: larger $\lambda_0 \rightarrow$ larger ω/ω_{ci}

  $\omega/\omega_{ci} \approx 0.6$ for $\lambda_0=0.5$,
  $\omega/\omega_{ci} \approx 0.7$ for $\lambda_0=0.8$ ($v_0/v_A \sim 1$)

– consistent with DIII-D ‘left’ and ‘right’ beam sources ($\omega/\omega_{ci}=0.56$ and $\omega/\omega_{ci}=0.69$ [Heidbrink,NF06]) .



A roughly linear scaling with B_{tor} is seen for GAE frequency in **NSTX-U** [Fredrickson,NF18]



DIII-D: (a) Spectra for B_{tor}=0.6 T with 80 keV left beams; (*b*) Frequency of the strongest mode vs the line-average n_e for all the discharges in the database [Heidbrink,NF06].

HYM simulations for DIII-D

Two basic cases are considered:

1. NSTX-similarity experiments on DIII-D from [W. Heidbrink et al, NF 2006]

   $\mathbf{B_{tor}}$**= 0.6T**, R_0= 1.63m, a= 0.56m, I= 0.6MA, q_0=1.2, q_{max}=4.5, β_{tot} ~9%

   Beam parameters: E= 80keV, $\mathbf{V_0/V_A}$**= 1.5**, n_b/n_e ~4%, β_{beam} ~3%

   Observed mode parameters: f= 2.5MHz, f_{ci}= 4.5MHz

2. More recent dedicated GAE/CAE experiments [S.Tang PRL 2021, N.Crocker 2021]:

   $\mathbf{B_{tor}}$**= 1.24T**, R_0= 1.72m, I= 0.62MA, β_{tot} ~2%

   Beam parameters: E= 78keV, $\mathbf{V_0/V_A}$**= 0.8**, n_b/n_e ~3%, β_{beam}~0.5%

   Observed mode parameters: f= 5.5MHz, f_{ci}= 9.5MHz

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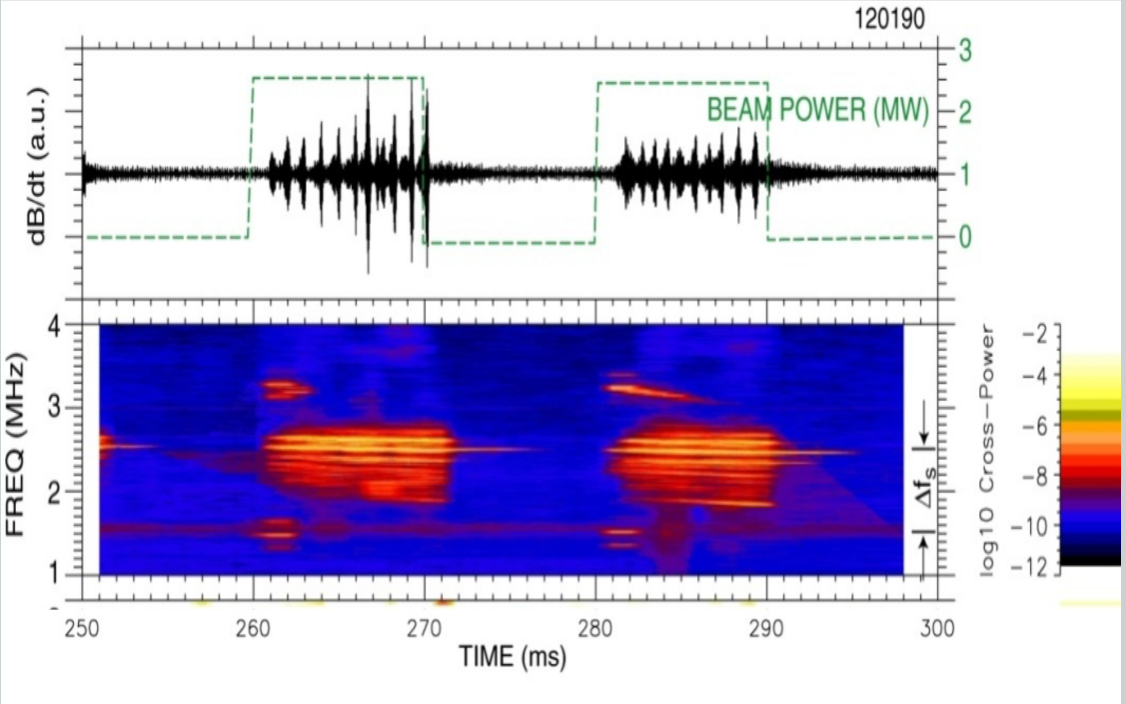
Sub-cyclotron frequency Alfven Eigenmodes were observed in low toroidal field experiments in DIII-D



Time evolution of the magnetic signal and spectra in a discharge with periodic injection of 80 keV left beams; B_T = 0.6 T; f_{ci}=4.5Mhz. [Heidbrink, NF2006]

- High-frequency AEs were observed in DIII-D in low toroidal field discharges (NSTX similarity experiments) mostly when $v_b \gtrsim v_A$.
- These modes are counter-propagating and driven unstable by Doppler shifted cyclotron resonance with beam ions.
- Mode polarization was not measured directly, but large dB_φ was observed near the edge.
- They were identified as compressional Alfven eigenmodes (CAE) based on high frequency ($f \sim 0.6 f_{ci}$) and comparison with previous theoretical instability conditions.
- For comparison:
  NSTX → $f_{GAE} \sim$ 0.1-0.3 f_{ci} ; $f_{CAE} \sim$0.3-0.5 f_{ci},
  NSTX-U → $f_{GAE} \sim$ 0.4 f_{ci}

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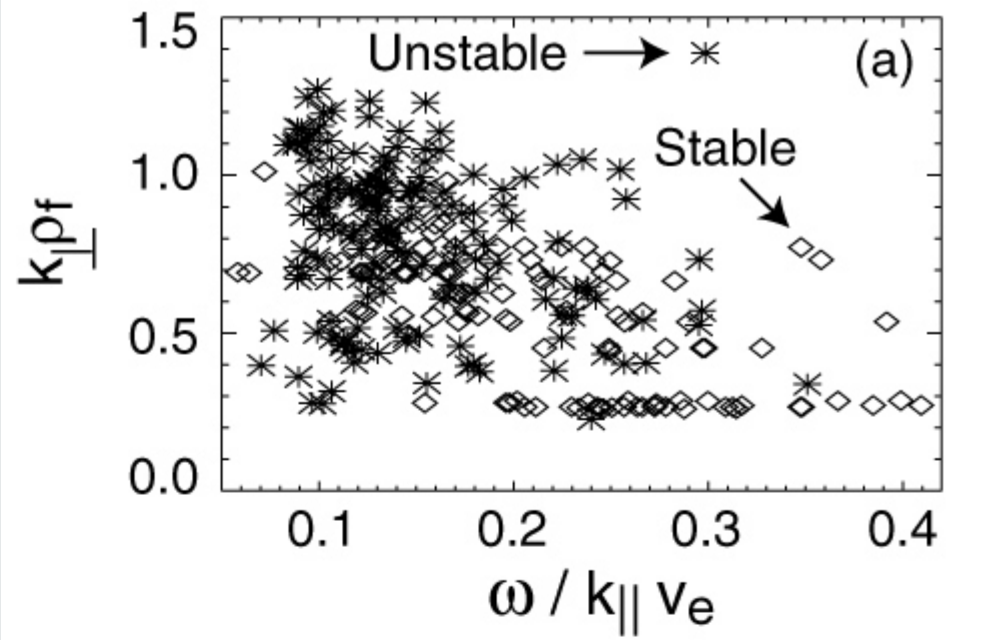
Early CAE/GAE theory predicted instability for large values of $k_\perp\rho_b$



[Heidbrink, NF2006]

- Experimental estimates got $k_\perp\rho_b \lesssim 1$ (based on CAE dispersion ie from observed ω/v_A) and local B value.
- Re-scaling for B on axis gives even lower values: $k_\perp\rho_b \lesssim 0.6\ k_\perp v_A/\omega$.
- Previously theory predicted the CAE instability for:

  $1 < k_\perp\rho_b < 2$
- For Global Alfven eigenmode (GAE) instability:

  $2 < k_\perp\rho_b < 4$
- New GAE/CAE theory [Belova,2019, Lestz, 2020] predicts stronger instability for small $k_\perp/k_{||}$ ($k_\perp\rho_b$<<1), and GAEs more unstable than CAEs.

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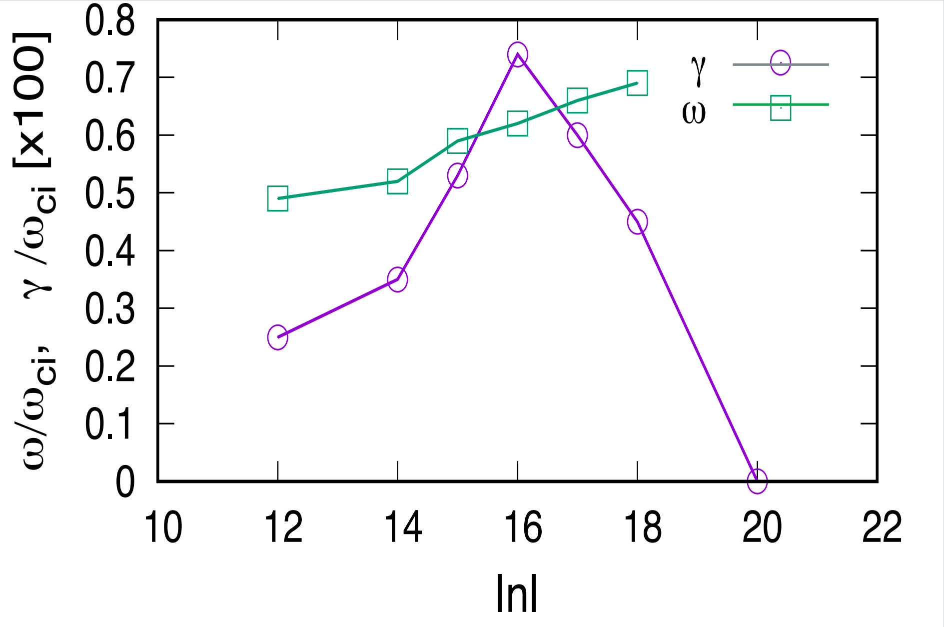
Case 1: NSTX-similarity experiments on DIII-D

B_{tor}= 0.6T, R_0= 1.63m, a= 0.56m, I= 0.6MA, q_0=1.2, q_{max}=4.5, β_{tot} ~9%
Beam parameters: E= 80keV, **V_0/V_A= 1.5**, n_b/n_e ~4%, β_{beam} ~3%
Observed mode parameters: f= 2.5MHz, f_{ci}= 4.5MHz

HYM parameters: **V_0/V_A= 1.5**, n_b/n_e =4%, λ_0= 0.75, $\Delta\lambda$= 0.2

Simulation results:

- Unstable modes have shear Alfven polarization with $\delta B_{||} << \delta B_\perp$ and are counter-rotating GAEs
- |n|=12-18 with m~1-5,
- frequencies ω/ω_{ci}=0.5-0.7, and growth rates γ/ω_{ci}=0.003 – 0.0075.
- Can estimate unstable |n| from n≈ $R_0 k_{||}$ and SAW dispersion to get: n≈ $R_0\, \omega/v_A$ ~ 15.



Growth rates and frequencies of unstable counter-GAEs from HYM simulations for v_0/v_A=1.5 and n_b=4%.

Toroidal mode numbers were not measured in experiments but estimated as n= -O(10). For #120196 shot n=-16+/-5 was inferred from correlation with changes in Mirnov signal [W. Heidbrink et al, NF 2006]

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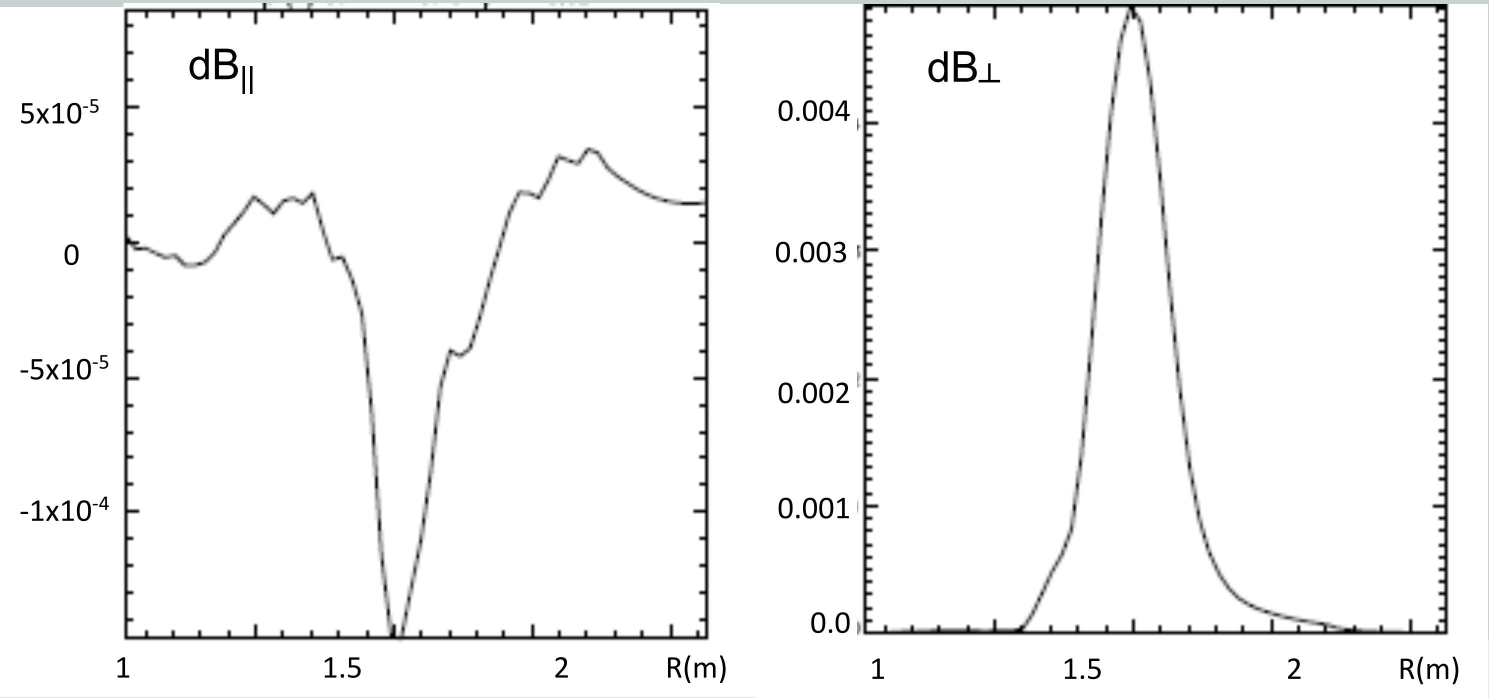
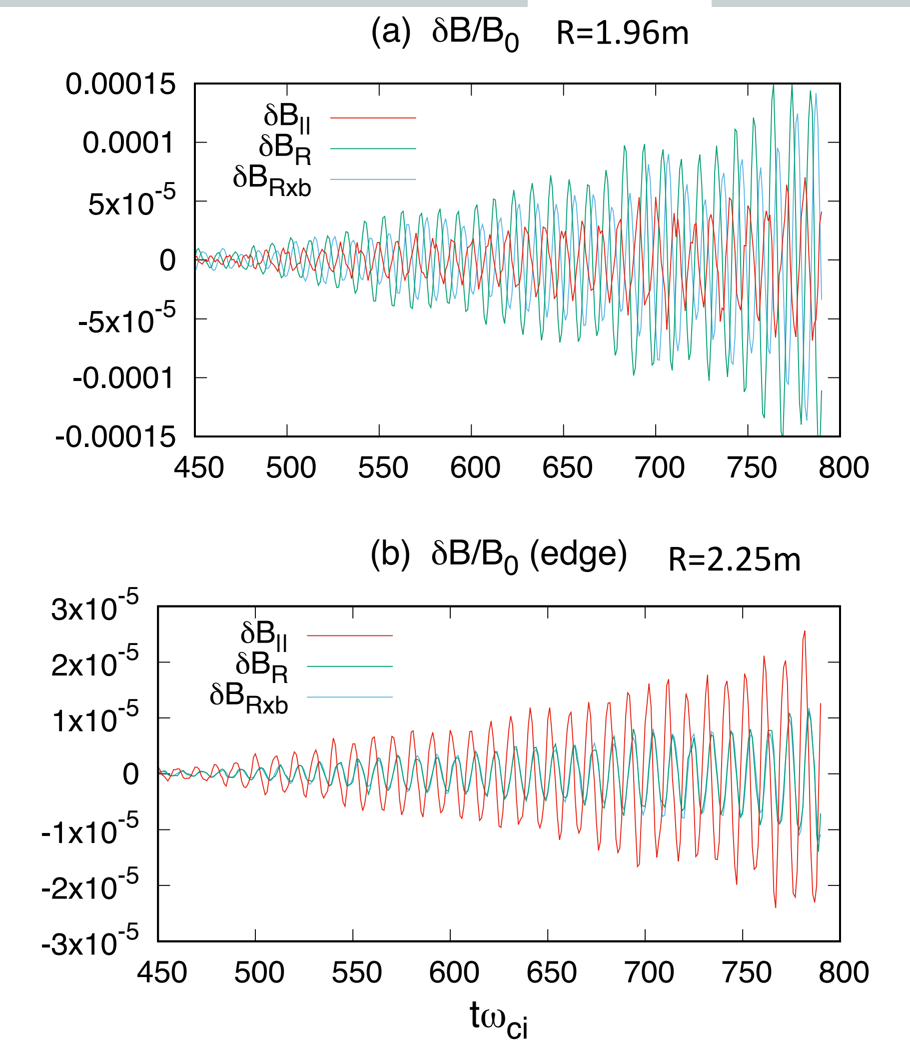
Modes have shear polarization in the core (GAEs)



Radial profiles of **δB** for *n= -16* counter-GAE

- Simulations show unstable counter-propagating GAEs with peak values of $\delta B_{||} \sim 0.05\ \delta B_{\perp}$
- Mode is located near the magnetic axis R=1.6m; $\delta B_{||}$ - radial profile is wider.
- $k_{\perp}\rho_b \sim 1$ for m= 2-3, $v_{||}$= 0.7v, and v~ $0.8v_0$

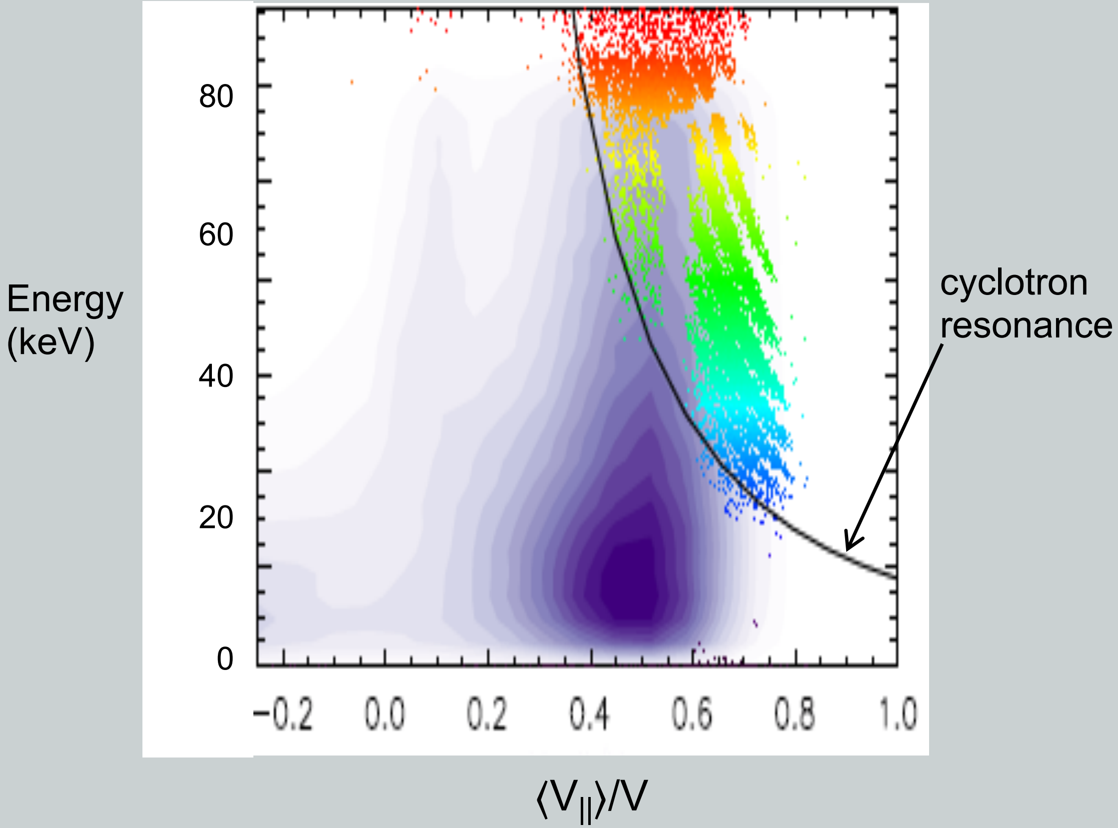
Compressional perturbations dominate at the edge



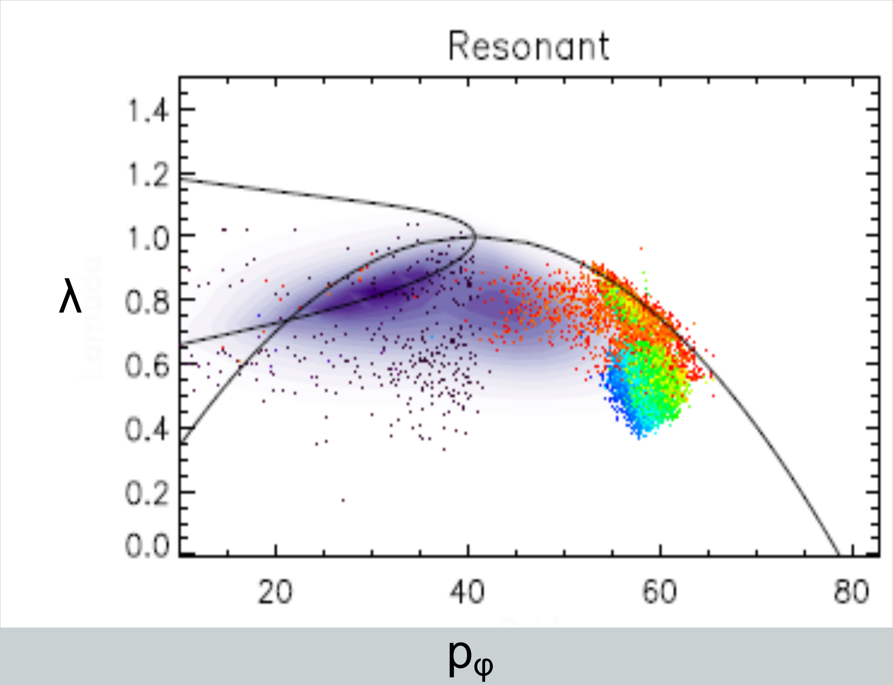
Time evolution of different **δB** components for *n= -15* counter-GAE at two radial locations away from the axis (R_0=1.61m): R=1.96m and R=2.25m.

$\delta B_{||}$ has much wider radial profile compared to $\delta B_{\perp}$ => compressional perturbations dominate at the edge where: $\delta B_{||} \sim 2|\delta B_{\perp}|$.

Large number of sideband resonances can be seen for each unstable GAE

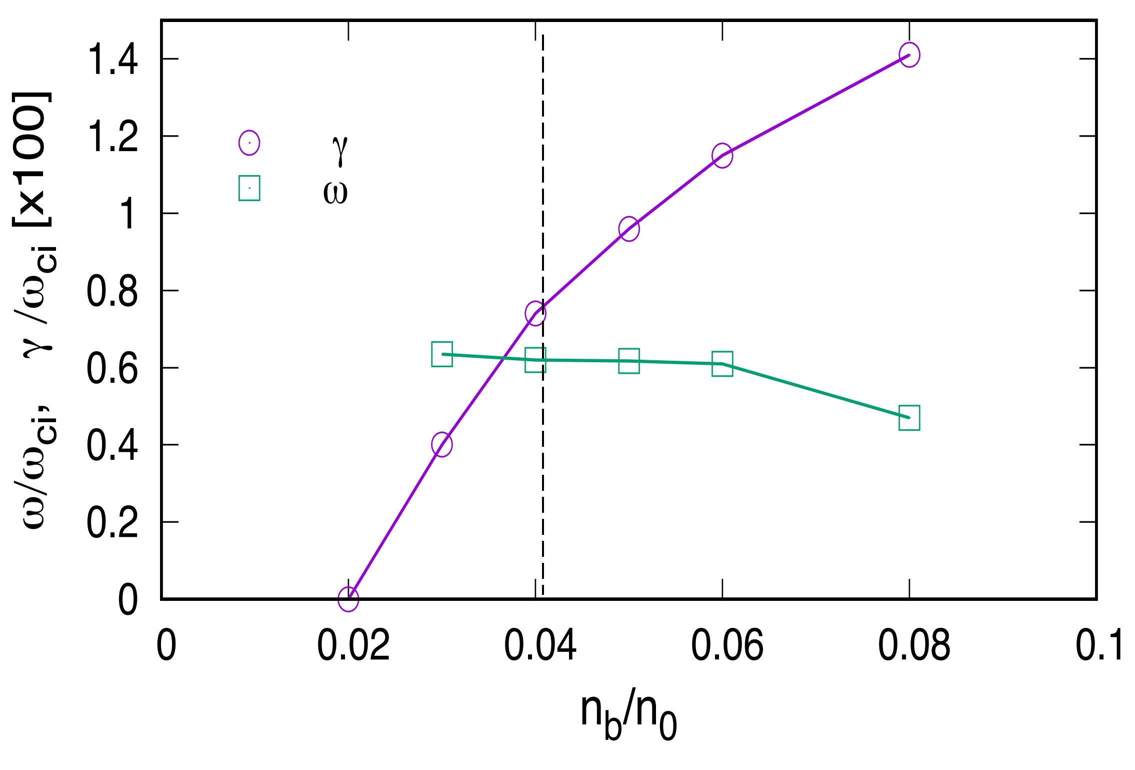


HYM: fast-ion energy vs pitch distribution from n=-16 GAE simulations; resonant line is shown for $v_{||}=0.8v_A$; colour dots show resonant particles.



Location of resonant particles in phase space: $\lambda=\mu B_0/\varepsilon$ vs p_φ. Particle color corresponds to different energies: from E=0 (purple) to E=80 keV (red).

Density scan allows to estimate damping for GAE



Growth rate and frequency of n= -16 GAE vs beam density.

- Estimated stability threshold is $n_b/n_0 \approx 0.02$.
- Dependence is not linear in contrast with NSTX and NSTX-U simulations.
- Assuming that $\gamma = Cn_b - \gamma_d$, the damping rate (continuum) can be calculated.
- For experimental parameters ($n_b/n_0 \approx 0.04$): $\gamma_d \approx 0.5\ \gamma_{drive}$, where $\gamma_{drive}/\omega_{ci} \approx 0.016$.

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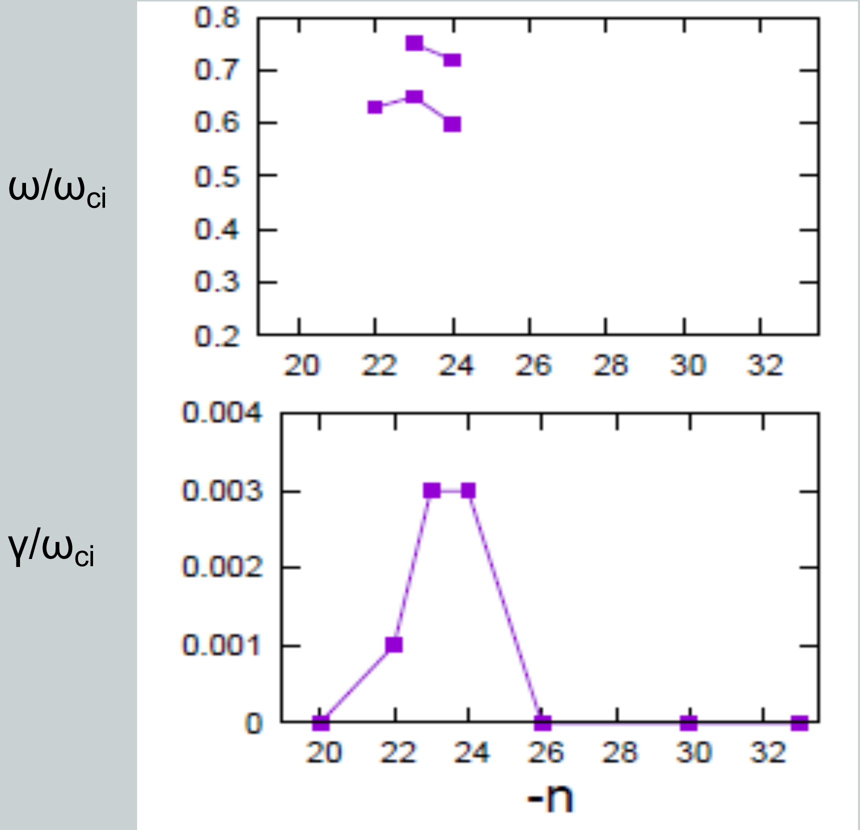
Case 2: Higher B_{tor} experiments on DIII-D

B_{tor}= 1.24T, R_0= 1.72m, I= 0.62MA, q_0=0.94, q_{max}=6, β_{tot} ~2%
Beam parameters: E= 75-80keV, **V_0/V_A= 0.8**, n_b/n_e =3%, β_{beam} ~0.5%
Observed mode parameters: f= 5.5 MHz, f_{ci}= 9.5 MHz

HYM parameters: **V_0/V_A= 0.8-0.9**, n_b/n_e =3-6%, λ_0=0.65-0.75, $\Delta\lambda$= 0.2

- Unstable modes have shear Alfven polarization with $\delta B_{||} << \delta B_{\perp}$ and are counter-rotating – GAEs
- |n|= 22-24 with m~ 3-4,
- frequencies ω/ω_{ci}= 0.6-0.75, and growth rates γ/ω_{ci}=0.001 – 0.003.
- SAW estimate for n: $|n| = R_0\omega/v_A$~ 19 - 22,

DIII-D estimated toroidal mode numbers was n≈ - 28, from 2-fluid SAW dispersion relation [S.Tang, PRL 2021].



Growth rates and frequencies of unstable counter-GAEs from HYM simulations for v_0/v_A=0.8, n_b=6%, λ_0=0.75.

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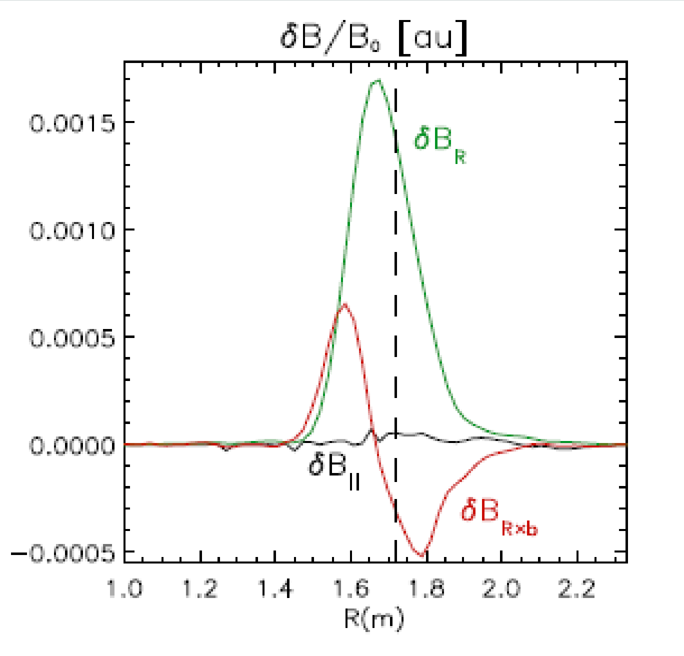
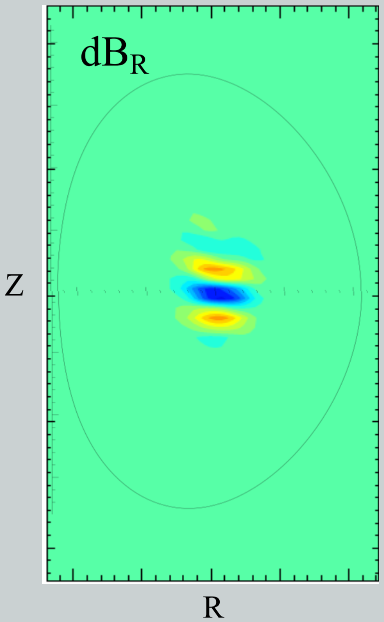
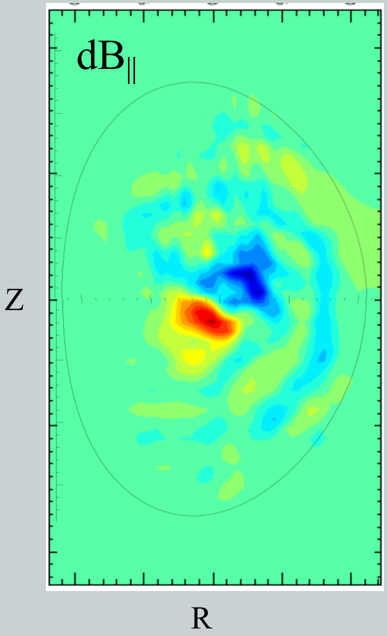
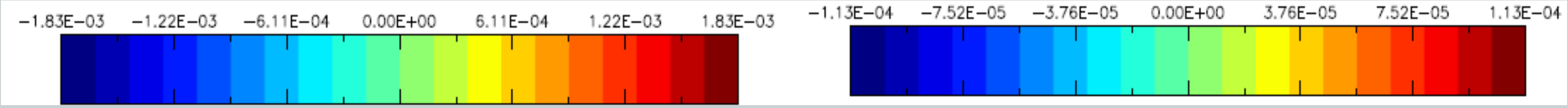
HYM simulations demonstrate that unstable modes in DIII-D have SAW polarization (GAEs)

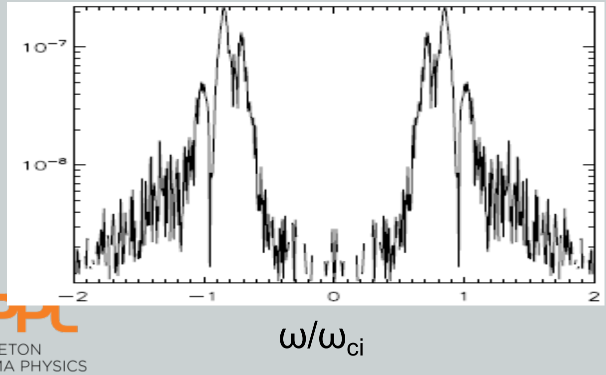


Radial profile of **δB** for *n= -22* counter-GAE









- Simulations show unstable counter-propagating GAEs with $\delta B_\perp > 10\,\delta B_{||}$, but $\delta B_{||}$ has wider radial profile.
- High toroidal mode numbers |n| > 20; $\omega/\omega_{ci} \sim 0.6\text{-}0.7$; $k_\perp \rho_b \sim 0.5$
- Located near the magnetic axis.

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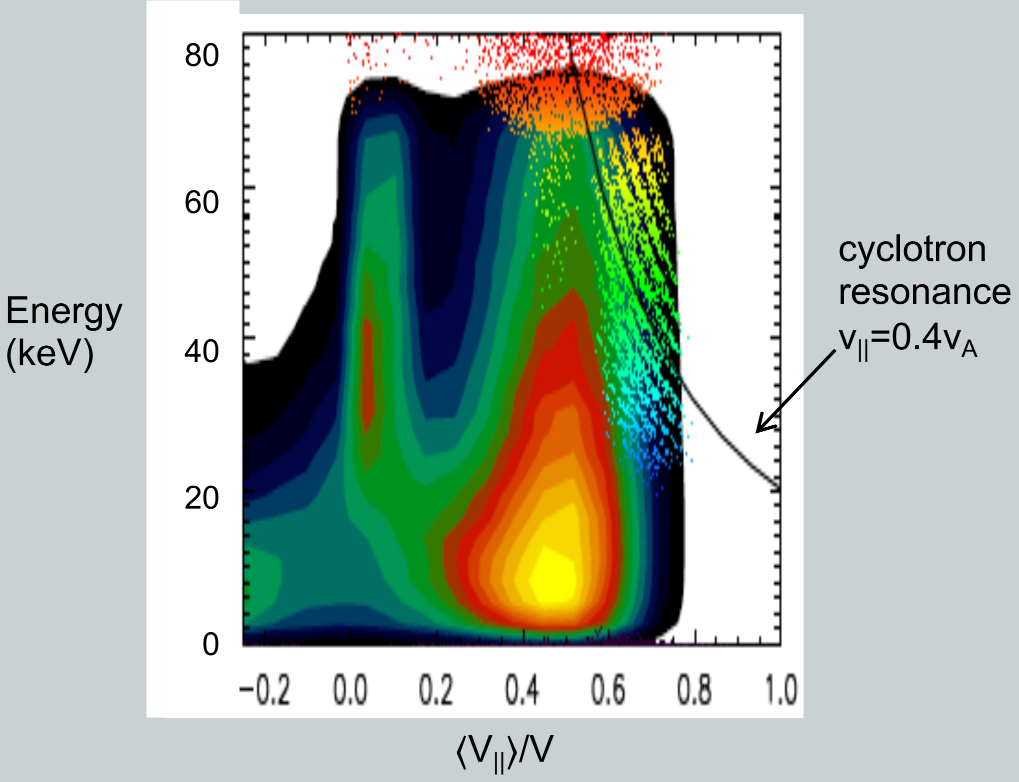
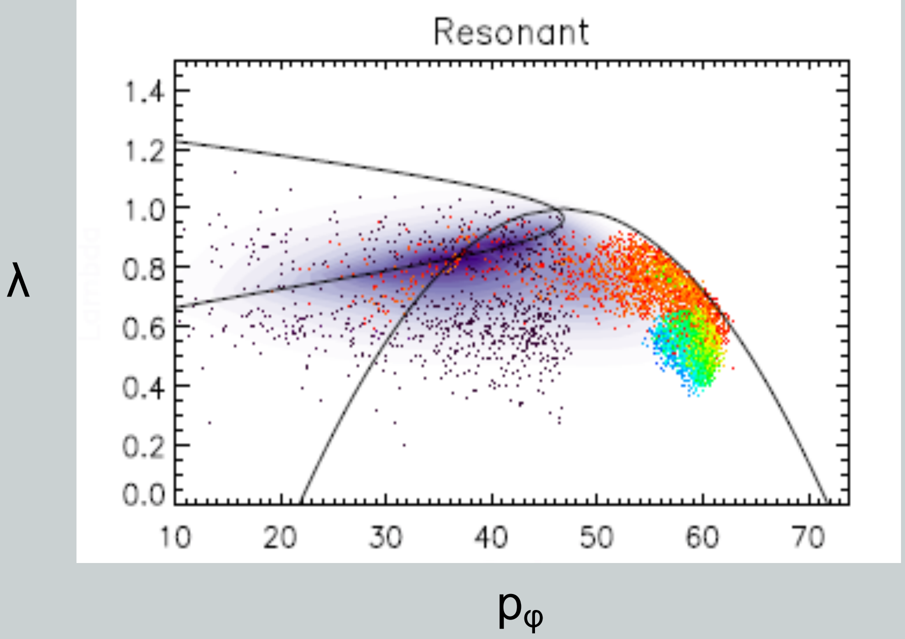
Large number of sideband resonances can be seen for each unstable GAE



HYM fast-ion energy vs pitch distribution from n=-22 GAE simulations; resonant line is shown for $v_{||}=0.4v_A$; colour dots show resonant particles.



Location of resonant particles in phase space: $\lambda=\mu B_0/\varepsilon$ vs p_φ. Particle color corresponds to different energies: from E=0 (purple) to E=80 keV (red).

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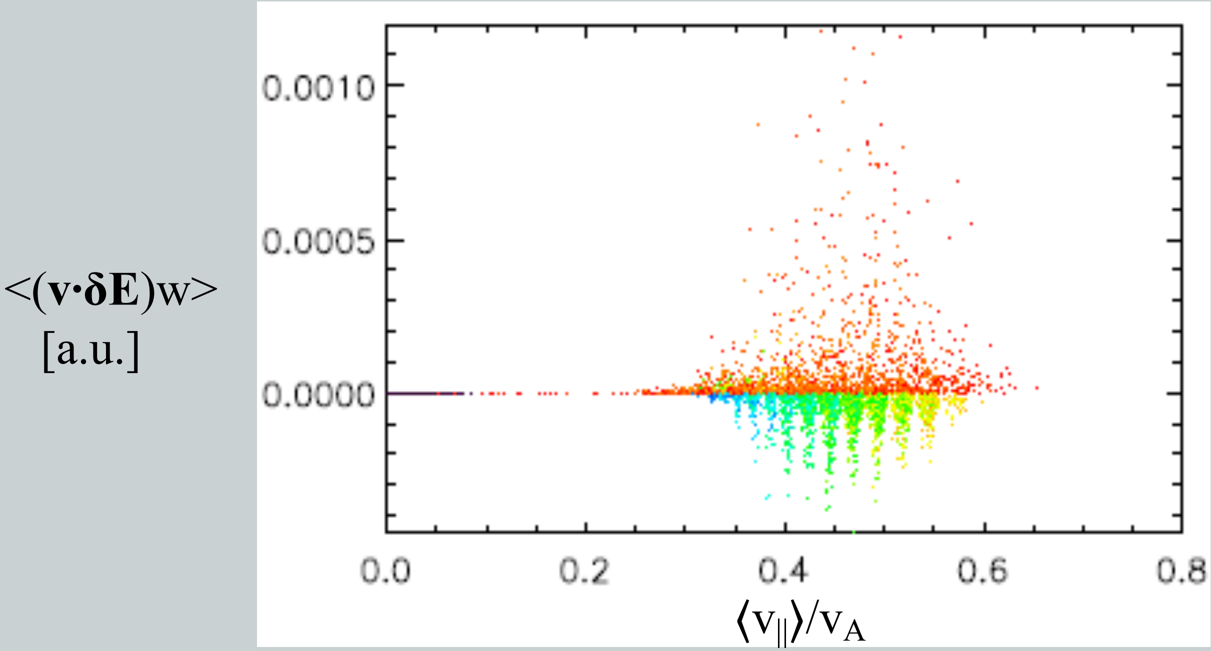
Two groups of resonant particles: driving ($\lambda<\lambda_0$) or damping ($\lambda>\lambda_0$)

Energy exchange rate between the beam ions and the mode $\Delta K=\int(\delta \mathbf{j}_b\cdot\delta \mathbf{E})d^3x=\sum(\mathbf{v}_m\cdot\delta \mathbf{E})w_m$.



Lower energy particles are driving, and particles from the tail are stabilizing GAE.

Scatter plot of resonant particles from linear phase of n=-22 GAE simulations. Time-averaged values of *(**v·δE**)w* [a.u.] of resonant particles. Particle color corresponds to different energies: from E=35keV (green) to E=80keV (red).

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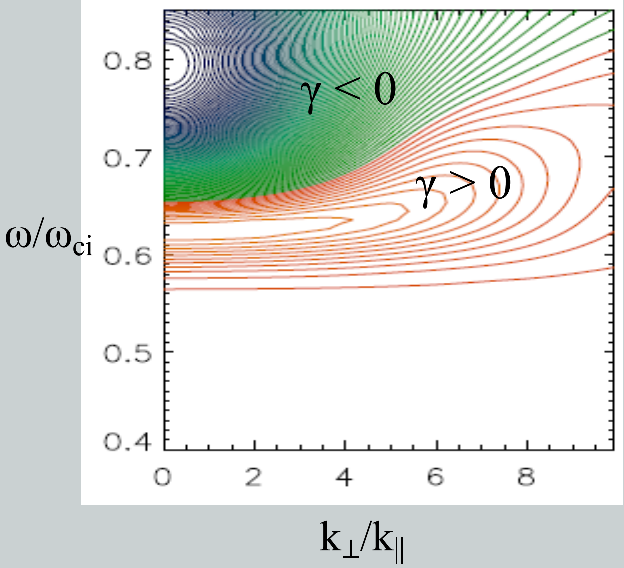
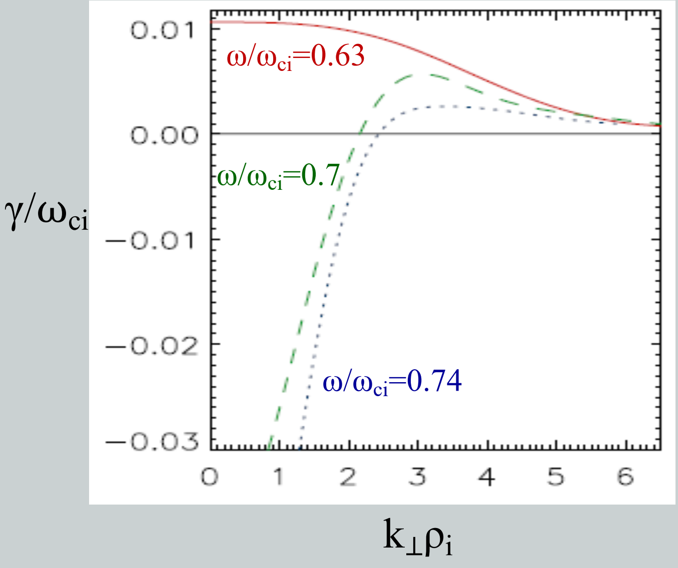
HYM simulations for DIII-D agree with analytic predictions





(a) contour plot of GAE growth rate and (b) plot of $\gamma(k_\perp\rho_i)$ for fixed frequencies; blue-green contours correspond to negative values, and orange-red to positive; γ/ω_{ci} values are between -0.08:0.011. DIII-D beam parameters: V_0/V_A=0.9, n_b/n_e=0.05, λ_0=0.7, $\Delta\lambda$=0.2.

- Frequency of most unstable modes: $\omega/\omega_{ci} \sim 0.6$,
- GAE linear growth rate is largest for small values of $k_\perp$

$$\gamma \approx \pi \frac{n_b}{n_i}\frac{\omega_{ci}^2}{2\omega} v_{||res}^3 A \int_0^{\lambda_m} \frac{d\lambda}{(1-\lambda)^2}\lambda \frac{\partial f}{\partial \lambda}\frac{J_1^2}{\xi^2}\Big|_{v_{||}=v_{res}},$$

$v_{||res} = (\omega_{ci} - \omega)/|k_{||}|$, $\lambda_m = 1 - v_{||res}^2/v_0^2$, v_0 – injection velocity,
$f = A \exp[-(\lambda-\lambda_0)^2/\Delta\lambda^2] / (v^3+v_*^3)$.

- Resonant beam ions drive instability provided: $\partial f/\partial\lambda > 0$, i.e. when $\lambda < \lambda_0$ and stabilizing otherwise.
- Most unstable modes have $k_\perp\rho_b < 1$, and are in the range [1,2]:

$$(1 + v_0/v_A)^{-1} < \omega/\omega_{ci} \leq (1 + v_0/v_A\sqrt{[1-\lambda_0]})^{-1}$$

  - follows from cyclotron resonance condition and the instability condition: $\lambda_m < \lambda_0$
- Higher frequency modes with $\omega/\omega_{ci} > (1 + v_0/v_A\sqrt{[1-\lambda_0]})^{-1}$ have smaller growth rates and unstable if: $2 < k_\perp\rho_b < 4$ (Bessel regime [3]).

[1] Belova, Phys. Plasmas 2019
[2] Lestz, Phys. Plasmas 2020
[3] Gorelenkov, NF03.

Dependence on frequency and $k_\perp\rho_i$ for counter-GAEs

$$\gamma \approx \pi \frac{n_b}{n_i}\frac{\omega_{ci}^2}{2\omega} v_{\|res}^3 A \underbrace{\int_0^{\lambda_m} \frac{d\lambda}{(1-\lambda)^2}\lambda \frac{\partial f}{\partial \lambda}\frac{J_1^2}{\xi^2}}_{G(\lambda)}\Bigg|_{v_\|=v_{res}},$$

$\lambda = v_\perp^2/v^2$, $\lambda_m = 1 - v_{\|res}^2/v_0^2$, $v_{\|res} = (\omega_{ci} - \omega)/|k_\||$

$\xi = k_\perp v_\perp/\omega_{ci}$

$f = A \exp[-(\lambda-\lambda_0)^2/\Delta\lambda^2] / (v^3+v_*^3)$

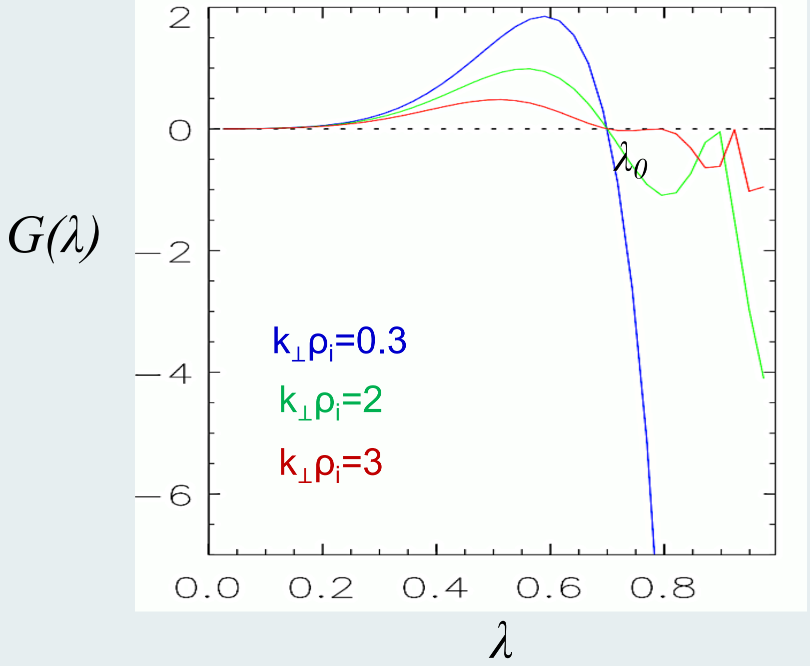
The sign of the integrand is determined by sign of $\partial f/\partial\lambda = 2(\lambda_0 - \lambda) f/\Delta\lambda^2$

→ particles with $\lambda < \lambda_0$ (small $v_\perp$) are destabilizing,

→ particles with $\lambda > \lambda_0$ (large $v_\perp$) are stabilizing.

1. For $\omega < \omega_{ci}$, $v_{\|res} \sim v_0$ and $\lambda_m < 1$
   - $\lambda_m \le \lambda_0$ – sufficient condition for instability (any $k_\perp$) and gives an approximate range of unstable frequencies: $\omega/\omega_{ci} \le (1 + v_0/v_A\sqrt{[1-\lambda_0]})^{-1}$
   - most unstable modes have $k_\perp\rho_i \ll 1$ with $(J_1/\xi)^2 \approx 1/4$

2. High-frequency limit $\omega \approx \omega_{ci}$, $v_{\|res} \ll v_0$ and $\lambda_m \approx 1$
   - for small $k_\perp\rho_i \rightarrow (J_1/\xi)^2 = 1/4$ and γ is negative
   - for $k_\perp\rho_i \gtrsim 2$, Bessel factor reduces stabilizing effect of large $v_\perp$ ($\lambda > \lambda_0$) particles.



This clarifies the role of FLR effects: large $k_\perp\rho_i$ reduces the stabilizing effect of particles with $\lambda > \lambda_0$

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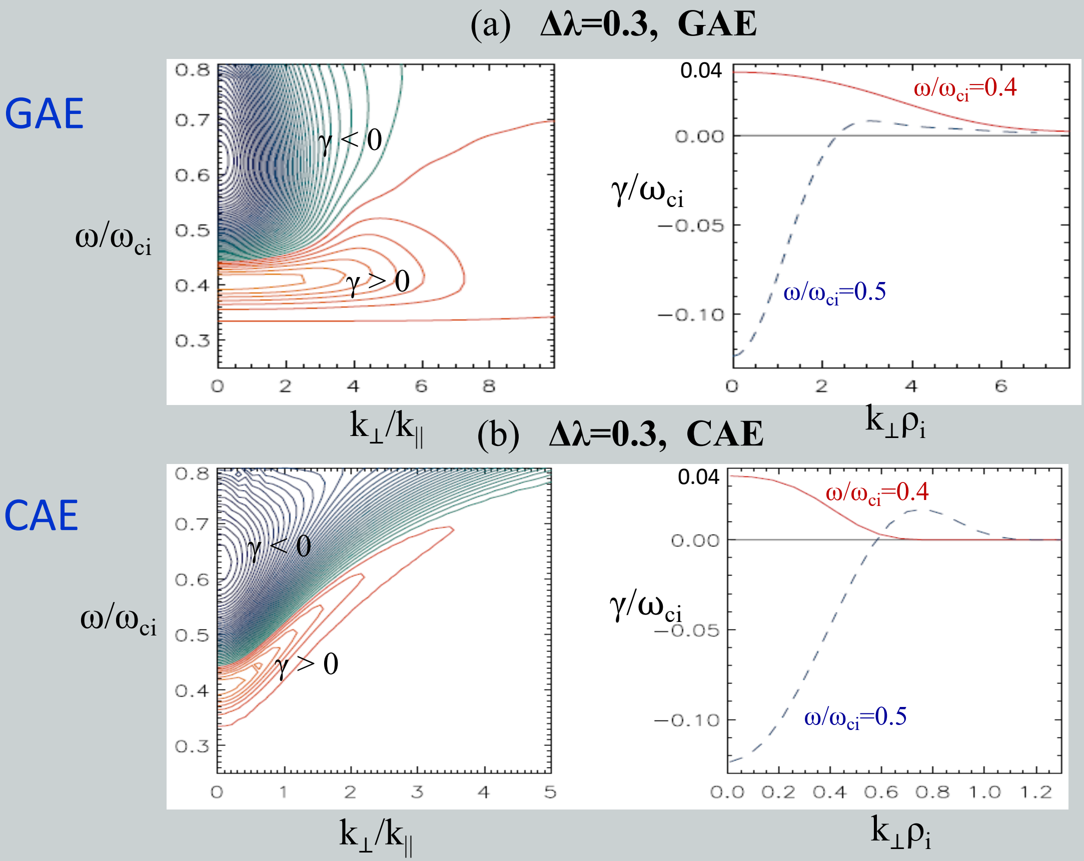
Counter-CAE are predicted to be less unstable than GAEs



Contour plots of growth rate and plots of γ vs $k_\perp\rho_i$ for wide pitch parameter distributions; blue-green contours correspond to negative values, and orange-red to positive.

(a) GAE: $\Delta\lambda_0=0.3$, γ/ω_{ci} values are between -0.22÷0.036;

(b) CAE: $\Delta\lambda_0=0.3$, same range of γ/ω_{ci} values as in (a).

$$\gamma \approx \pi \frac{n_b}{n_i}\frac{\omega_{ci}^2}{2\omega} v_{\|res}^3 A \int_0^{\lambda_m} \frac{d\lambda}{(1-\lambda)^2}\lambda \frac{\partial f}{\partial \lambda}\frac{(J_0^2 - J_2^2)}{4}\bigg|_{v_{\|}=v_{res}},$$

- Same (sufficient) instability condition: $\lambda_m < \lambda_0$ as GAEs, and same γ for $k_\perp=0$ [1,2], but $\omega = kv_A$.
- Most unstable modes have $k_\perp\rho_b<1$, and are in the range:

$$(1 + \alpha v_0/v_A)^{-1} < \omega/\omega_{ci} < (1 + \alpha v_0/v_A\sqrt{[1-\lambda_0]})^{-1}$$

where $\alpha = |k_{\|}/k|$.

- Counter-CAEs have much smaller growth rates than GAEs for $k_\perp/k_{\|} \gtrsim 1$.
- Two-fluid effects / coupling to SAW reduce growth rate of CAEs [2].

[1] Belova et al, Phys. Plasmas 2019

[2] Lestz et al., Phys. Plasmas 2020.

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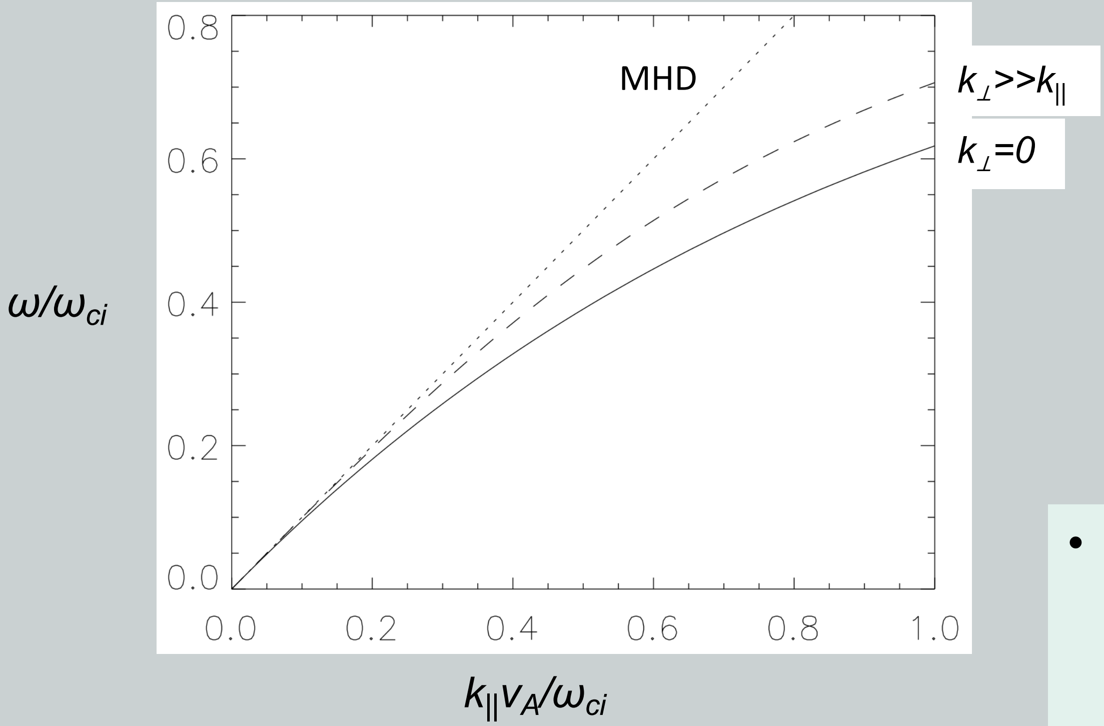
NSTX-U

Finite frequency corrections for GAEs ω/ω_{ci} ~1

$$\omega^2 = k_{||}^2 v_A^2 f^2, \quad \text{where } f^2 = \frac{1}{2}\left[\left(\frac{k^2}{k_{||}^2}(1+\varkappa_{||}^2)+1\right) - \sqrt{\left(\frac{k^2}{k_{||}^2}(1+\varkappa_{||}^2)+1\right)^2 - 4\frac{k^2}{k_{||}^2}}\right], \quad \text{and } \varkappa_{||} = k_{||}v_A/\omega_{ci}$$



Limit $k_\perp = 0$:

$$\omega = k_{||}v_A\left(\sqrt{1+\left(\frac{k_{||}v_A}{2\omega_{ci}}\right)^2} - \frac{k_{||}v_A}{2\omega_{ci}}\right)$$

Limit $k_\perp \gg k_{||}$:

$$\omega = k_{||}v_A \Big/ \sqrt{1+\left(\frac{k_{||}v_A}{\omega_{ci}}\right)^2}$$

- $\lambda_m = 1 - v_{||res}^2/v_0^2 \le \lambda_0$ – sufficient condition for instability (≈ peak growth rate) valid for large frequencies (ω/ω_{ci} ~1) but correct 2-fluid GAE dispersion should be used.
- One-fluid MHD dispersion leads to overestimated most unstable frequency and underestimated $k_{||}$ (n).

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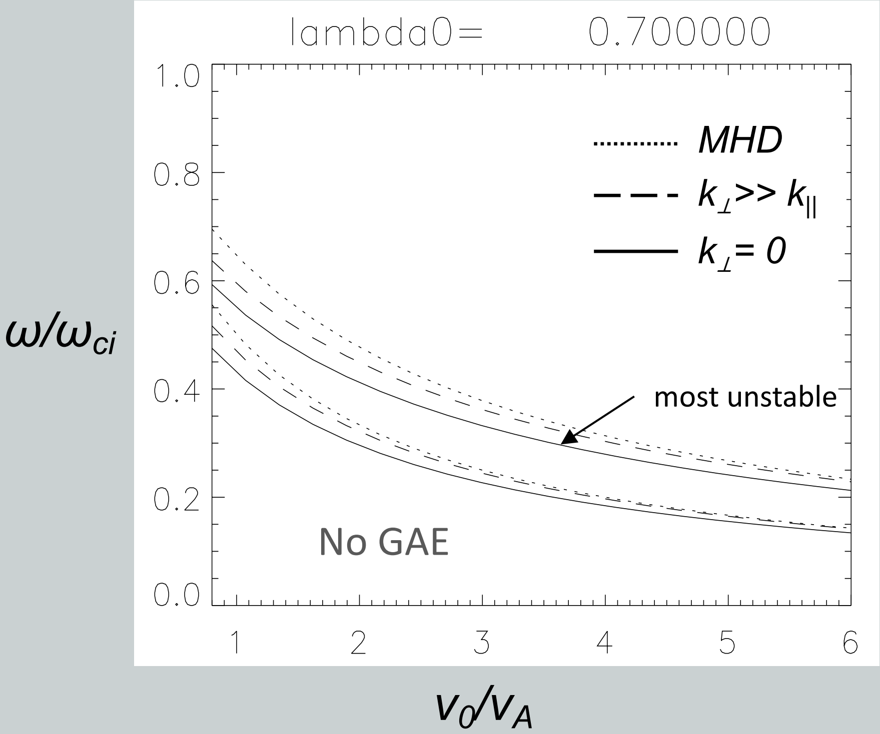
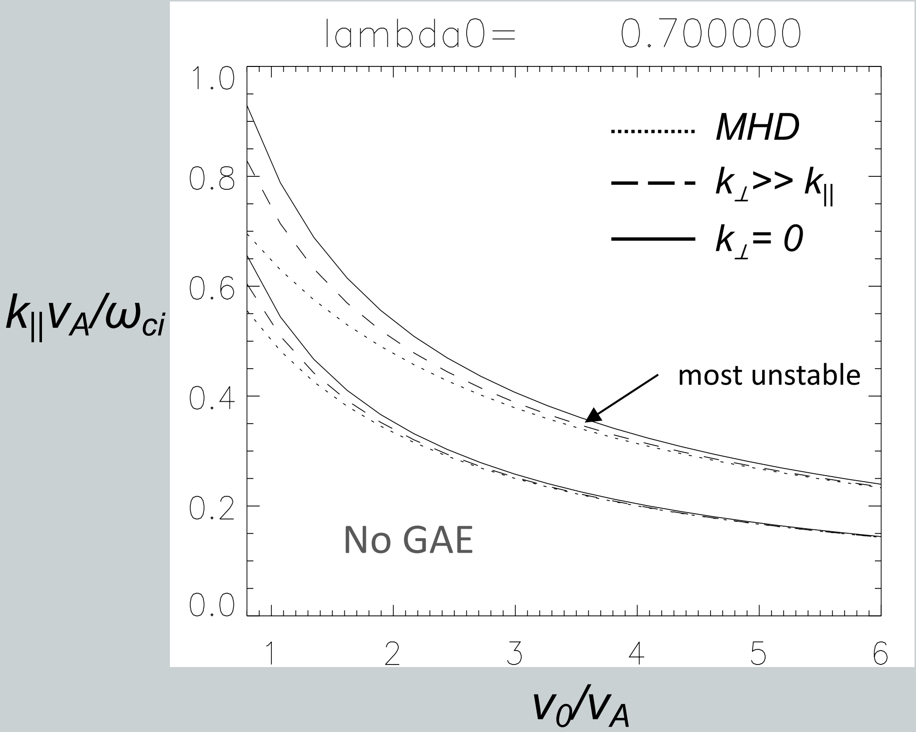
Most unstable frequency and $k_{||}$ range for ω/ω_{ci} ~1





Numerical solution for two conditions using 2-fluid SAW dispersion :

1. $v_{\parallel,res} \leq v_0$ (existence of resonance)
2. $\lambda_m = 1 - v_{\parallel res}^2/v_0^2 \leq \lambda_0$ (max γ)

MHD conditions: $\left(1+\frac{v_0}{v_A}\right)^{-1} < \frac{\omega}{\omega_{ci}} \leq \left(1+\frac{v_0}{v_A}\sqrt{1-\lambda_0}\right)^{-1}$, corresponding to $v_{\parallel,res} < v_0$ and $\lambda_m \leq \lambda_0$ respectively, modified for ω/ω_{ci} ~1 . Due to MHD description of thermal plasma, the HYM code overestimates most unstable frequencies and underestimates most unstable |n|.

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Summary and Future Work

- HYM simulations demonstrate that high-frequency modes (ω/ω_{ci}~ 0.6) previously misidentified in DIII-D as compressional (CAEs), have shear polarization $\delta B \approx \delta B_\perp$ (GAEs).
- Simulations reproduce experimentally observed frequencies and estimated toroidal mode numbers for DIII-D experiments.
- A simple analytical theory based on local dispersion relation is very successful in predicting the counter-GAE instabilities.
- New analytic theory explains range of most unstable modes, and GAE frequency scaling across different devices (NSTX, NSTX-U, DIII-D).
- Counter-GAEs can be unstable in ITER ($v_{beam}/v_A \lesssim 1$) with ω/ω_{ci} ~0.5-0.7.

Future work:

- Need to include 2-fluid (Hall) effects in thermal plasma description to account for finite frequency effects ~$O(\omega/\omega_{ci})$. At present, HYM overestimates unstable frequencies, and underestimates toroidal mode numbers.