Multi-machine Scalings of Thresholds for n=1 and n=2 Error Field Correction

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Simple error field penetration scalings set the tolerances & correction-coil designs for new machines like ITER

- Nonaxisymmetric fields cause tearing, locking, and disruptions
- A large database has been accumulated for n=1 penetration threshold scaling
- EFC criterion scale a robust coupling metric with basic equilibrium parameters
- Initial data shows n=2 thresholds can have similar magnitude & scaling

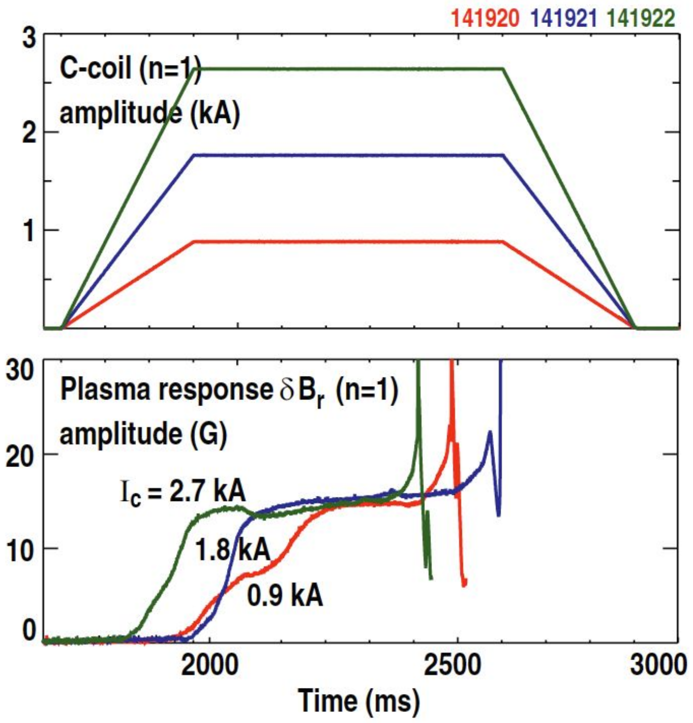
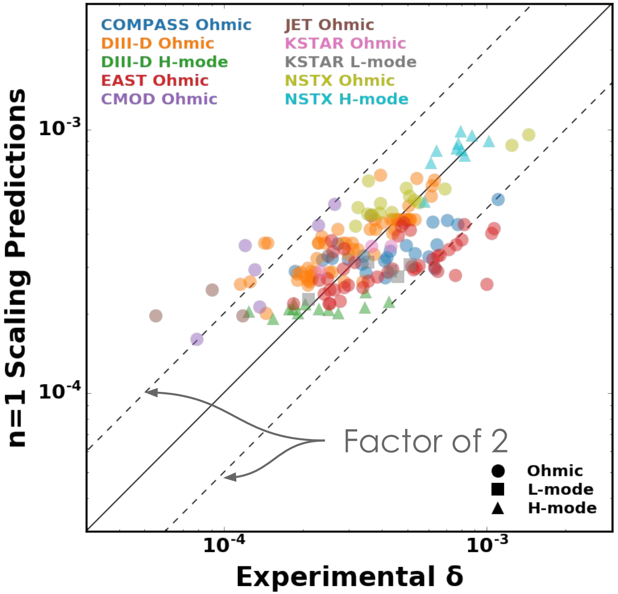
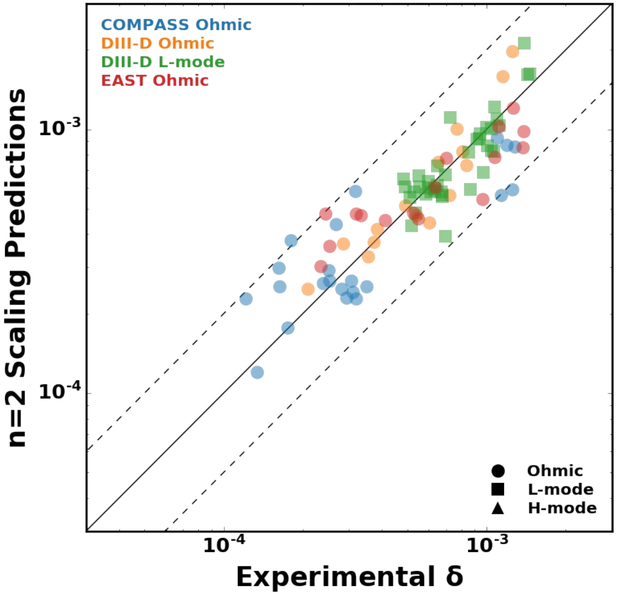


Fig. from Strait, et. al, *Nucl. Fusion*, **54**, 2014

This poster shows how simple scalings have been developed for multi-machine databases for n=1 and n=2 Error Fields





- EF magnitude “δ” determined by ideal MHD resonance
- Broad scaling done using simplest possible 0D parameters: n_e, B_T, R_0, β_N/ℓ_i
- Robustly describes thresholds spanning factor of ~20 within a factor of ~2

Error field magnitude parameterized by ideal MHD resonance

Ideal MHD is used to identify the most dangerous component of the Error Field

- Error Fields (EFs) drive natural modes of the plasma response that amplify or shield the external perturbations and an effective EF metric for avoidance of core tearing modes should incorporate this plasma response
- A broadly validated model of the detailed nonlinear response and penetration across all machines has proven difficult due to the complexity of this physics
  - Modeling requires (and is sensitive to) detailed kinetic profile information not always available
- We use the ideal MHD plasma response to represent the "outer layer" away from the resonant surfaces and empirical scalings to represent the more complex "inner layer" dynamics at the rational surface
  - Use “overlap” metric (next page) rather than 2/1 resonant field to maximize robustness
  - Fundamental 0D plasma parameters are used in this scaling to maximize the utility for design of future tokamaks for which accurate knowledge of detailed profiles may not be available
  - 0D is also necessary when incorporating Ohmic discharges with limited profile measurements

This approach scales a external field metric that includes some knowledge of the plasma response

Experiment $\longrightarrow$ $\delta_{res}(b_{ext}, q_{95}, \kappa, \ldots) \leq \delta_{pen}(n_e, B_T, R_0, \beta_N, \omega, \ldots)$ $\longleftarrow$ Scaling

3D equilibrium on the fast ideal MHD time scale

Experimental evaluation of "dangerous" EF calculated by GPEC[1]

Nonlinear, resistive, resonant layer evolution on slower time scales

Relies on empirical scaling

[1] J.-K. Park, A.H. Boozer, J.E. Menard, and M.J. Schaffer, Nuclear Fusion **48**, 45006 (2008).

This approach scales an external field metric that includes some knowledge of the plasma response

$$\delta_{res}(b_{ext}, q_{95}, \kappa, \ldots) \leq \delta_{pen}(n_e, B_T, R_0, \beta_N, \omega, \ldots)$$

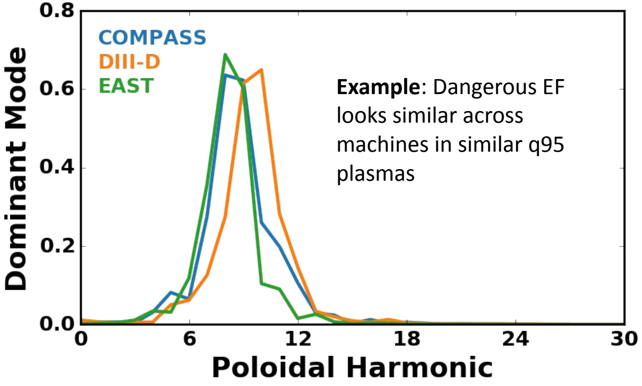
3D equilibrium on the fast ideal MHD time scale

$$b_{res} = C \cdot b_{ext} \sim e_1 v_1 \cdot b_{ext}$$

Linear coupling between external 3D field and total resonant field including the plasma response

v_1 is first right singular vector of C: robust feature of equilibrium with q_{95}, and shaping dependences

Eigenvalue e_1 can be more sensitive to details of equilibrium reconstruction



This approach scales an external field metric that includes some knowledge of the plasma response

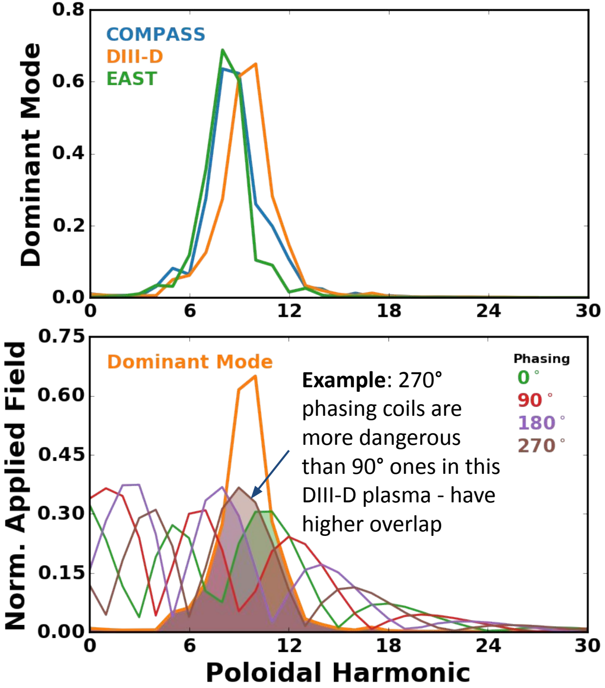
$$\delta_{res}(b_{ext}, q_{95}, \kappa, \ldots) \leq \delta_{pen}(n_e, B_T, R_0, \beta_N, \omega, \ldots)$$

3D equilibrium on the fast ideal MHD time scale

$$\mathbf{I}_{res} = \mathbf{C}\cdot \mathbf{b}_{ext} \sim \mathbf{e}_1\mathbf{v}_1 \cdot \mathbf{b}_{ext}$$ ← **v_1 robust, with common structure** →

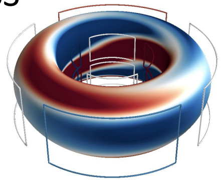
$$\delta = \mathbf{v}_1 \cdot \mathbf{b}_{ext} / B_T$$ ← **"Overlap" metric easily obtained from arbitrary vacuum EF**

Ideal MHD quantifies the dangerous part of the vacuum EF that drives resonances in the plasma

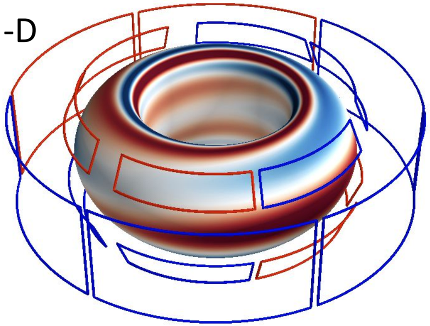


The overlap metric, δ, unifies the many different coils and their different couplings to various plasma scenarios

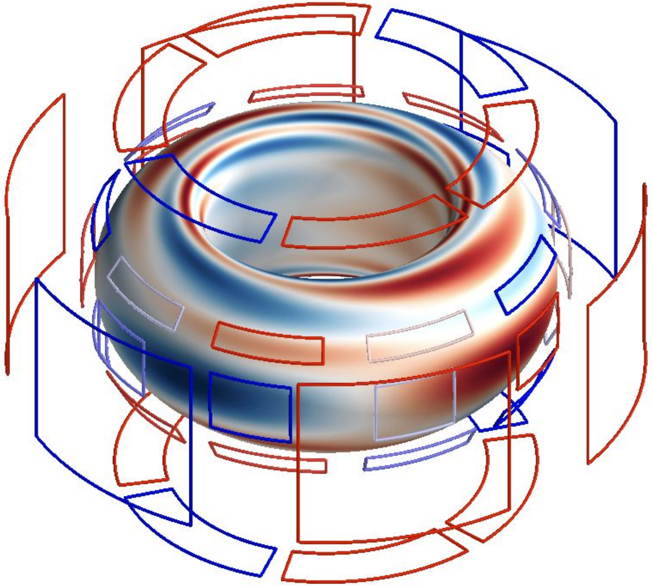
COMPASS



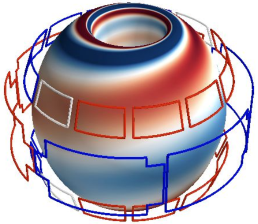
DIII-D



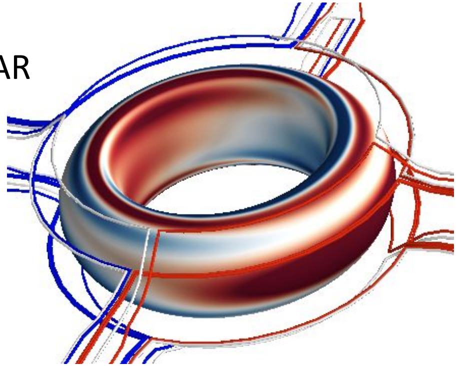
ITER



NSTX*



KSTAR



*Including potential off-mid-plane coil design

Scalings fit using multivariable regression on experimental database of thresholds built by applying intentional error fields

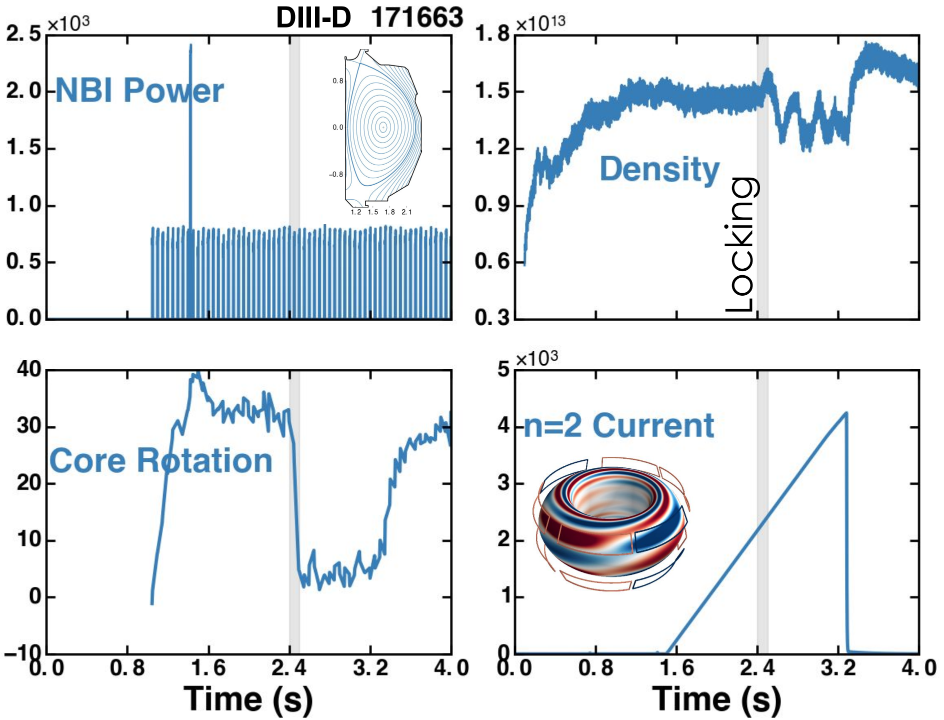
Multivariate regression on large databases requires large amounts of experimental data on many machines

- Decades of individual experimental work has reported a wide range of scaling exponents for common dependencies
  - Density scaling exponents ranging from ~0.5 to ~1.0 [1,2]
  - Toroidal field exponents ranging from negative to positive [3]
- Data from 7 machines has been collected into a common database as part of a ITPA joint experiment effort (MDC-19)
- Experiments performed/published in the last 2 years expanded this database to include toroidal mode number n=2 [4]
  - Fits are separated by n number in this work
  - Combined fits suggest a linear dependence on n due to less amplification of the least stable n=2 and deeper location of the 3/2 surface, but recent KSTAR results suggest details of the unconstrained rotation can alter this [5]
- Multivariate regressions on these databases are used to project thresholds to ITER

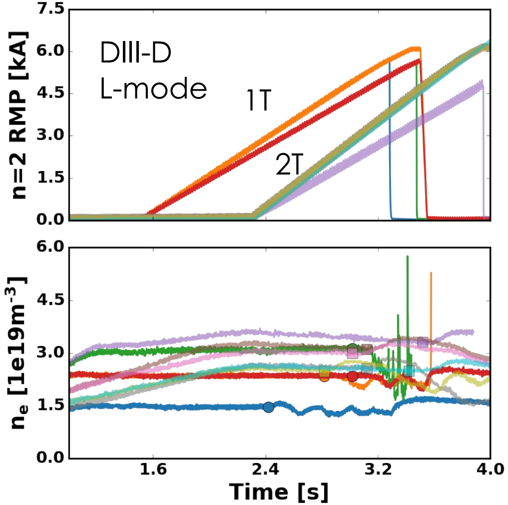
[1] Buttery, Nucl. Fusion 2000 [2] Lazzaro, Phys. Plasmas 2002 [3] Wang, Nucl. Fusion 2020 [4] Logan, Nucl. Fusion 2020 [4] Yang, Nucl. Fusion 2021 (submitted)

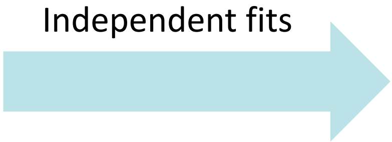
Experiments in each machine explore thresholds in applied error fields using 3D coils

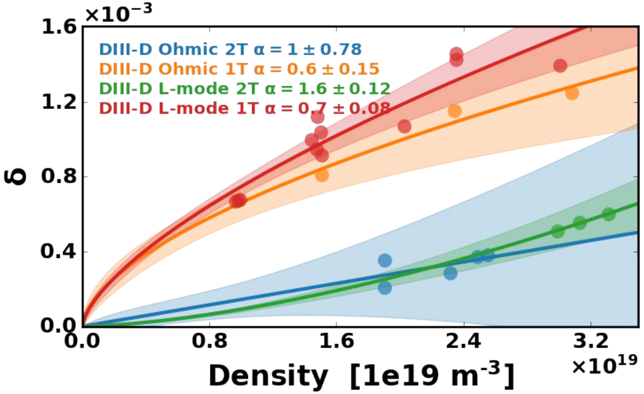
- Fixed 3D coil configuration ramped in amplitude until penetration
  - Time identified by density, rotation & magnetics
  - Expect ~10% error from coil current & penetration time identification
- Devoted scans of n_e, B_T, P_{INJ}, etc. done in various machines
  - Majority used upper single null Ohmic/L-modes



Diverse scalings in different experiments / conditions exemplify dangers of single-experiment trends

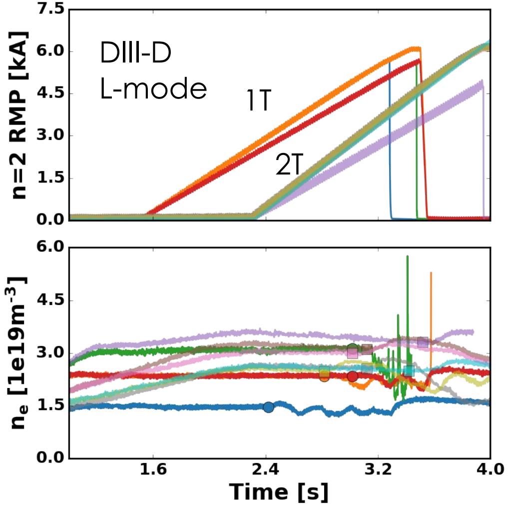




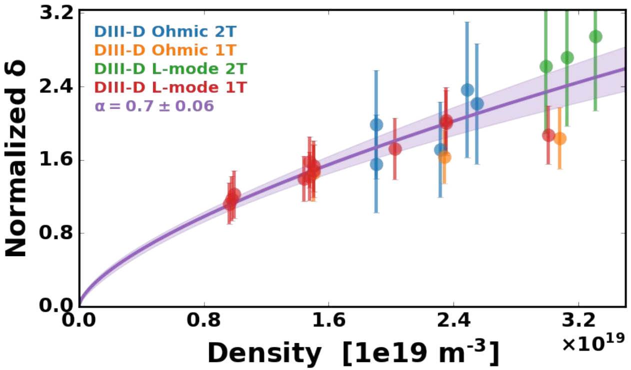


- Single scan is often ~4-8 shots → fits may give varied scalings
  - Access to low range of scan variable is essential
  - Compensating for other variables (BT, R0, βN/ℓi) does not collapse individual scalings
- Resulted in a wide variety of density and toroidal field scalings reported by individual machines in the past

Combining all data into a single, multidimensional regression aligns discrepancies and reduces the uncertainties

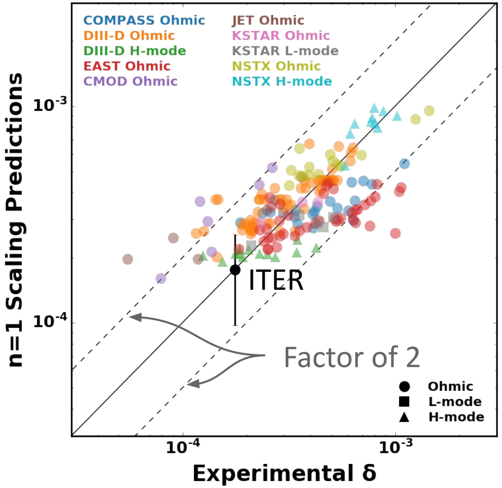


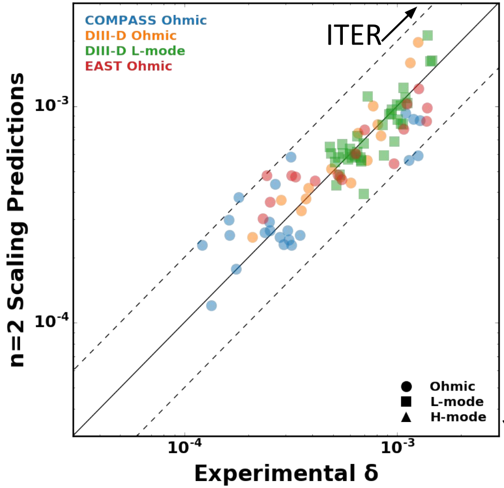
Single fit



- Takeaway: Need large database in variety of conditions for robust scaling
- This is even more true when including multiple machines!
  - Luckily, the ITPA MDC-19 joint experiment has built a large multi-machine database

Performing multivariate regression on the ITPA database, threshold scalings look promising for ITER so far





[1] Logan, Nucl. Fusion 2020

- Projected ITER EF thresholds are comparable or larger than those commonly dealt with on existing devices[1]

$$\delta_{n=1} \leq 10^{-3.7\pm0.03}\ n_e^{0.6\pm0.06}\ B_T^{-1.1\pm0.07}\ R_0^{0.1\pm0.07}\ (\beta_N/\ell_i)^{-0.2\pm0.05} \rightarrow \delta_{ITER,n=1} = 1.37\pm0.36$$

$\times 10^{-4}$

$$\delta_{n=2} \leq 10^{-3.4\pm0.06}\ n_e^{1.1\pm0.1}\ B_T^{-1.5\pm0.2}\ R_0^{1.5\pm0.1}\ (\beta_N/\ell_i)^{0.4\pm0.1} \rightarrow \delta_{ITER,n=2} = 8.8\pm4.4 \times 10^{-3}\ *$$

* n=2 database includes only 2 sizes,and lacks H-mode EF amplification physics. Removing R from the regression results in $\delta_{ITER} = 1.7 \times 10^{-4}$

Large database regressions focus on broad trends, not local dynamics

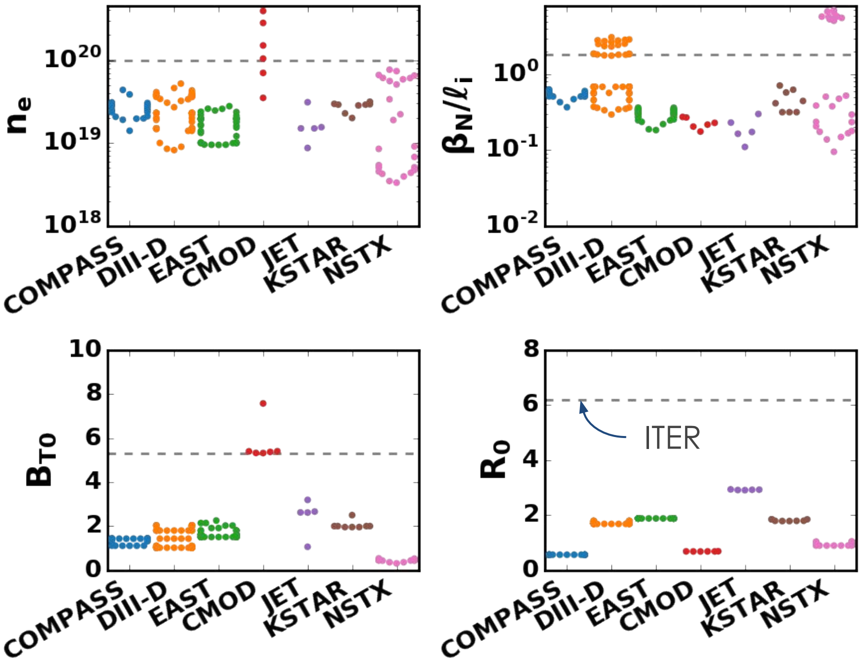
The goal of these database scalings is to provide a robust, trustworthy projection of the allowable EF for future machines

- Distribution of experimental data is not even across the explored parameters
  - Large number of experimental data points available from recent experiments on similarly sized DIII-D, EAST and KSTAR
  - Sparse data available at extremes of B_T and density (from C-MOD)
- Sensitivitivities of the regressions to sampling bias has been studied in detail [1]
  - Any change in particular exponents tends to be countered by other exponents
  - Projections to ITER do not change much
- Monte Carlo downsampled or kernel density weighted regressions help describe boradest trends across the many machines
  - Individual experiments sometimes reveal unique local behavior
  - We do not want these local physics details to dominate the general scaling
- The ultimate goal is, similar to confinement scalings, is to provide a broad trend for projection to new devices at the cost of ignoring (interesting) local phenomena

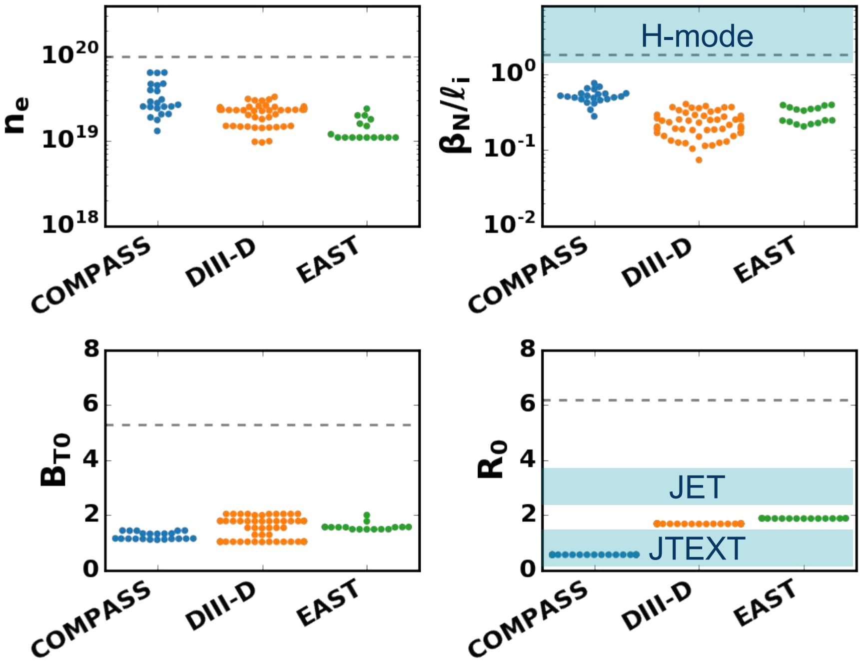
[1] Logan, Plasma Phys. Cont. Fusion 2020

Multi-machine databases already encompass many individual ITER parameters for n=1, but not n=2

n=1

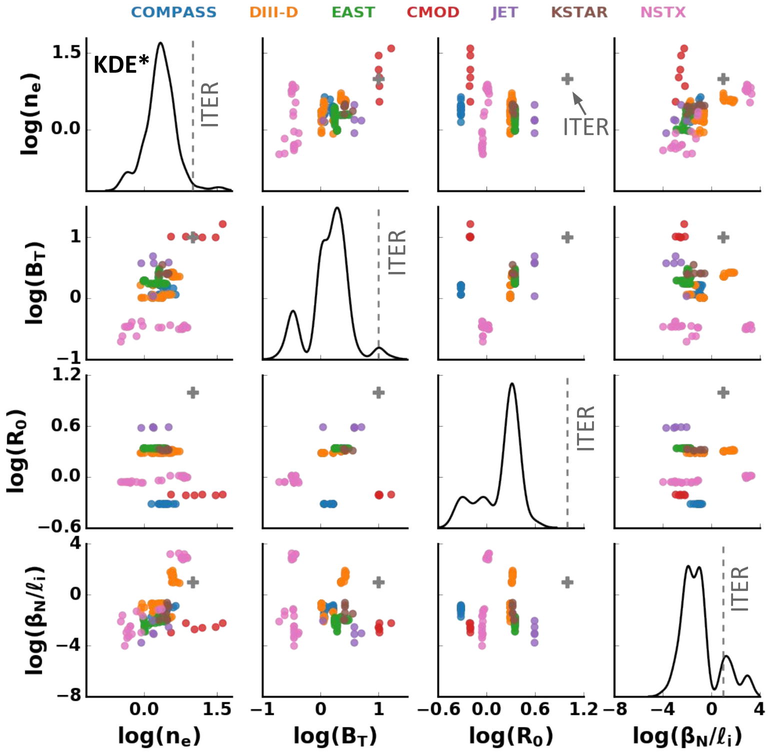


n=2



OMFIT

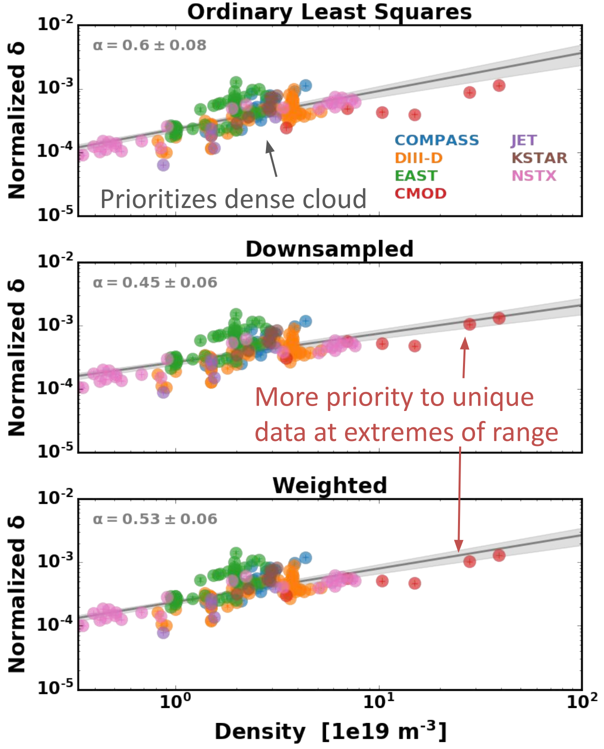
Most impactful additions to data must come from combination of new experiments and data mining from old machines



- Sparse data at high field & high density
  - Renewed effort underway to mine more data from C-MOD (Wolfe, Hughes)
  - COMPASS-U, SPARC data will be valuable
- Gap in toroidal field values between NSTX and DIII-D
  - NSTX-U to fill this gap soon
  - MAST data would help
- Valuable opportunity for all existing devices to contribute meaningful data to this effort in high normalized pressure plasma scenarios

* Kernel Density Estimate

Various regressions have been used to study sensitivities to uneven distribution of data across fit parameters



[1] Logan, Plasma Phys. Cont. Fusion 2020

- Least Squares Regression for exponents uses log-linear regression

$$D \bullet \alpha = \delta_{res}, \quad D_j = [\ \log(10)\ \ \log(n_{e,j})\ \ \log(B_{T,j})\ \ \log(R_{0,j})\ \ \log(\beta_{N,j}/I_{i,j})\ ]$$

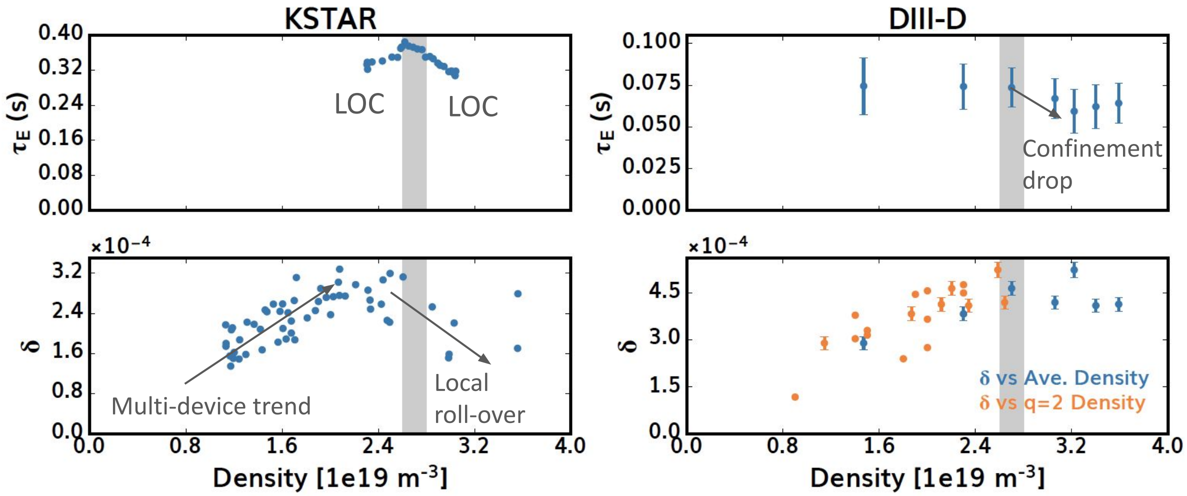
Experimental parameters at each penetration event

Experimental thresholds

- Downsampled regressions sample even numbers of experimental points from each machine
  - Monte Carlo of downsampling choices provides uncertainty estimates
- Weighted regressions weight each point by the inverse kernel density estimate (KDE, previous page)
  - Assigns more weight to sparse C-MOD and JET data
- δ projections to ITER are 1.93, 1.66, and 1.87 x10^{-4} respectively [1], giving confidence in robust result

Local behavior is not always consistent with broad scalings, providing important physics insight

Good reminder that theoretical scalings often assume LOC



Good reminder that simple 0D scalings do not include profile effects

- High density "roll-over" observed in the threshold on both KSTAR and DIII-D
- KSTAR due to Linear Ohmic Confinement (LOC) to saturated (SOC) regime
  - ITPA confinement scaling combined with reduced theory EF scalings predicts $\alpha_n = -4/70$ [1]
- DIII-D transitions from SOC to deteriorated confinement
  - Breaks correlation between line average density and density at q=2

[1] Yang, Nucl. Fusion 2021 (submitted)

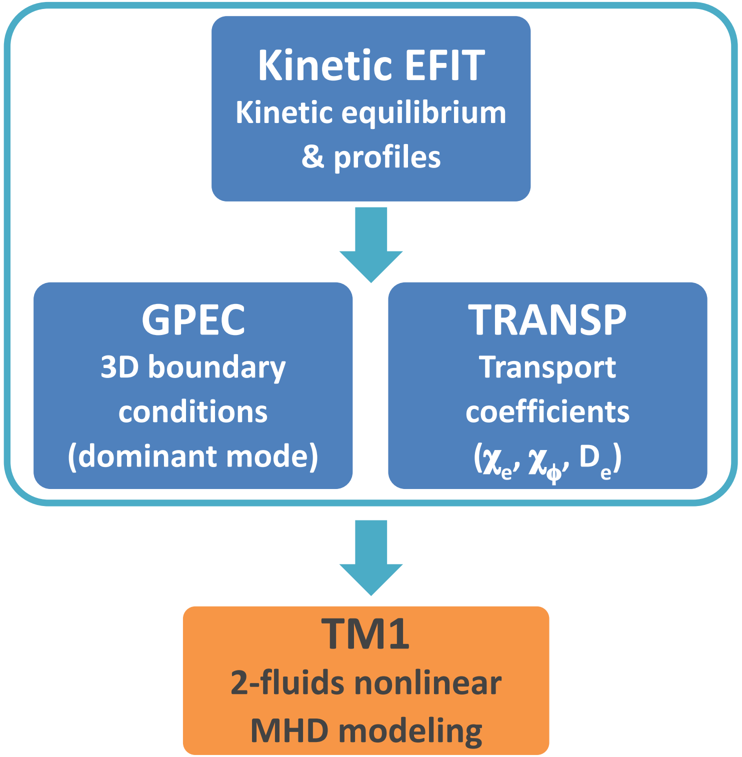
Nonlinear MHD modeling agrees with primary scalings and provides insight into experimental needs

Nonlinear single and two fluid MHD modeling with TM1 has proved an invaluable companion to empirical scalings

- The TM1[1] model is a nonlinear resistive MHD model including the screening or penetration of 3D fields on resonant surfaces
  - Simplified geometry enables nonlinear calculations at real magnetic Reynolds numbers
  - GPEC[2] dominant mode used to set appropriate boundary conditions in shaped plasmas
  - TRANSP[3] used to set bulk transport coefficients (TM1 includes enhanced transport across islands)
- TM1 reproduces the experimentally observed toroidal field and β_N/l_i scaling, and shows the scalings are consistent out to ITER values
- TM1 density scaling exponent falls below the experimental n= 2 fit, closer to the better constrained n= 1 empirical fit
- TM1 predicts n= 2 thresholds in ITER roughly 2-3 times that of the n= 1 thresholds, consistent with the ITPA database
- Two fluid modeling reveals additional dependencies between scaling coefficients and the plasma rotation, motivating modernization of the experimental database to include rotation

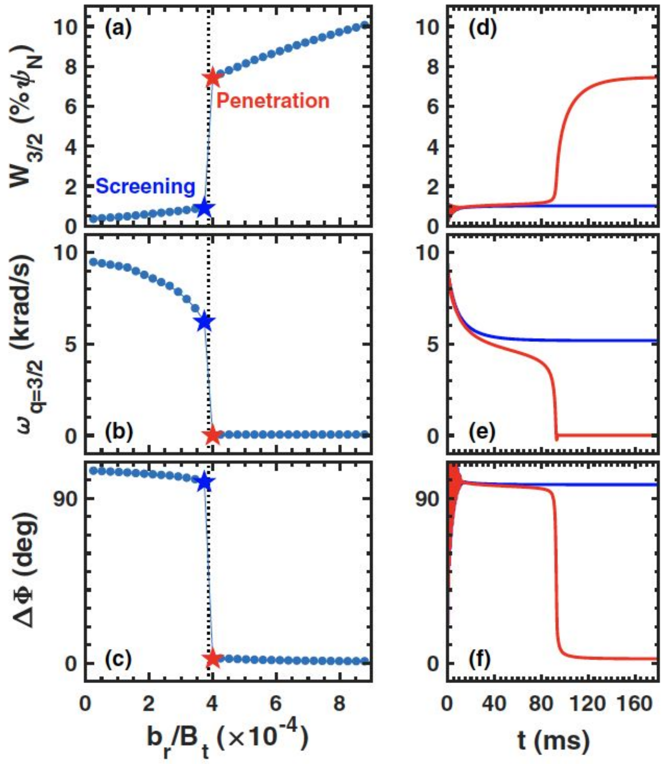
[1] Yu, Phys. Plasmas 10, 2004 [2] Park and Logan, Phys. Plasmas 24, 2017 [3] J. Breslau, TRANSP v18.2 2018

A suite of codes is used to obtain quantitative predictions of island penetration in the core of diii-d and iter plasmas



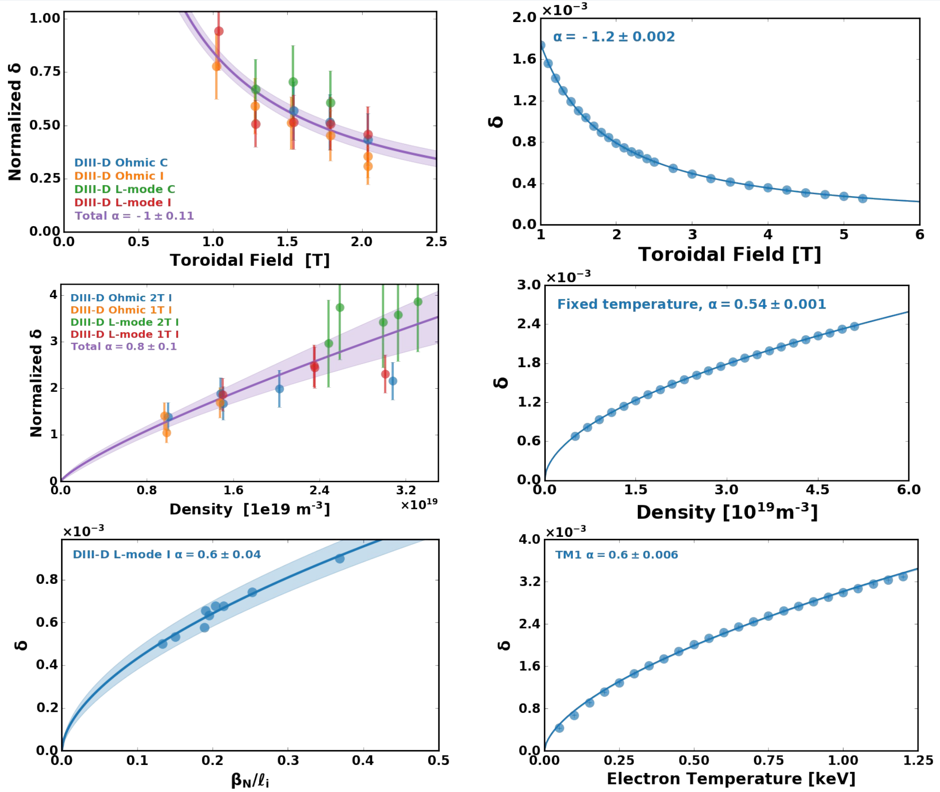
- OMFIT workflow manager used to obtain all necessary inputs for TM1 modeling
- Experimental profiles and transport parameters used from 1 Tesla, L-mode DIII-D EF threshold experiments
- GPEC dominant mode normalization of 2/1 boundary condition in cylindrical model used to connect to experiment

Efficiency of TM1 code enables thorough scans to find the precise penetration threshold



- Experiments ramp error fields and the finite ramp rate introduces some uncertainty
- Modeling applies constant error fields, using many independent runs to scan the amplitude
  - Weak EFs are shielded by the plasma as indicated by the phase difference between the rational surface response and applied field ΔΦ
  - Screening currents drive a finite flux perturbations leading to small effective island widths $W_{3/2}$
  - At a threshold amplitude, the phase jump into alignment with the applied field, the width jumps up, and the rotation locks
- Amplitude scans are repeated for scans of the equilibrium parameters to obtain scalings

Single fluid n=2 scalings support experimental regressions



- Interpolation of experimental equilibria from DIII-D n=2 database used in model scalings for tight connection to experiments
- Modeling agrees well with experiments & remains consistent when projected to ITER parameters ($\delta_{ITER,TM1} = 6.6\text{x}10^{-4}$)[1]
- As shown in last section, local scans do not always match full database regressions:

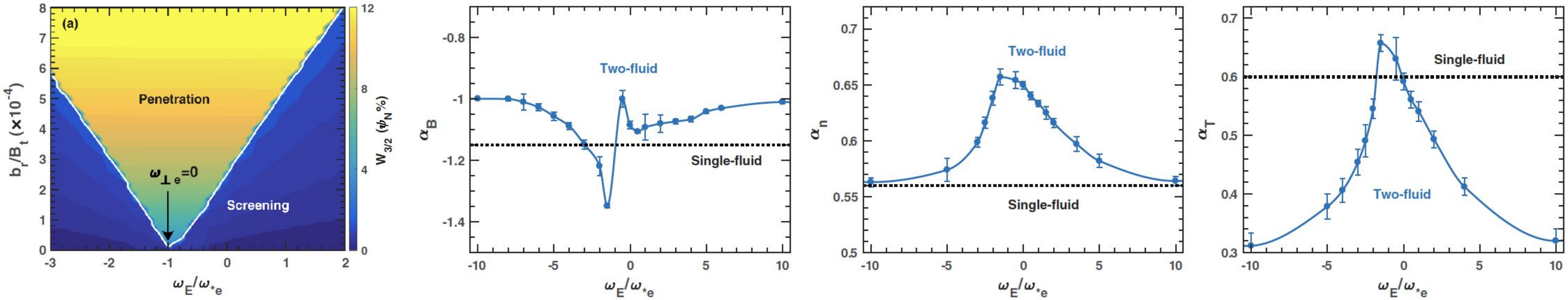
$$\delta_{n=2,\text{ Full DB}} \propto n_e^{1.1\pm0.1} \quad B_T^{-1.5\pm0.2} \quad R_0^{1.5\pm0.1} \quad (\beta_N/\ell_i)^{0.4\pm0.1}$$

$$\delta_{n=2,TM1} \propto n_e^{0.5} \quad B_T^{-1.2} \quad R_0^{0.82} \quad T_e^{0.6} \quad \omega$$

~ visco-resistive

[1] Logan, Nucl. Fusion 2020

Two fluid scalings reveal additional rotation dependencies



- Two fluid penetration threshold is linearly proportional to the perpendicular flow frequency, with an offset minimum in ω_E/ω_{*e} corresponding to $\omega_{\perp e}$=0 (left plot)
  - Very small initial islands near the offset explain the finite experimental thresholds here
- In addition to this macro rotation scaling, the choice of fixed rotation also impacts the individual scaling exponents of toroidal field, density and temperature [1]
  - Steep gradients near the diamagnetic frequency explain some of the variation in single-machine experimental scans and indicate a floor for full database uncertainties

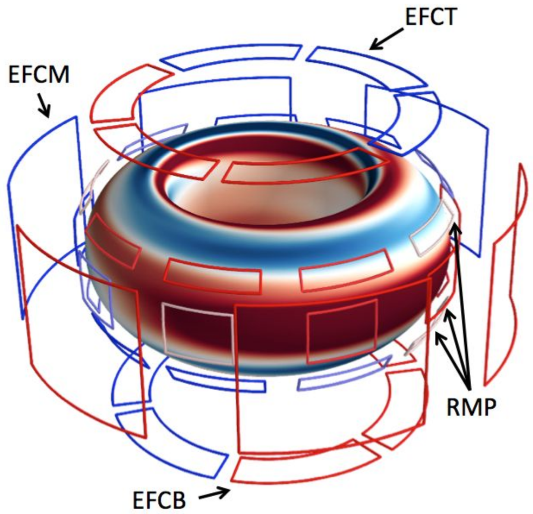
[1] Hu, Nucl. Fusion 60 2020

Conclusions

Simple error field penetration scalings describe large multi-machine databases for projection to new machines

- n=1 scaling describes a database for of 7 machines, spanning many of the ITER parameters
- Initial data shows n=2 thresholds can have a similar magnitude & scaling
- Investigation of sampling bias confirms robustness of regressions and identifies opportunities for new data
- TM1 nonlinear MHD modeling supports empirical scalings and corroborates the associated projections to ITER
- Scalings set construction tolerances and correction coil requirements in new machines like ITER, COMPASS-U & SPARC

ITER 3D Coils



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