# Strong Reversal of Simple Isotope Scaling Laws in Tokamak Edge Turbulence

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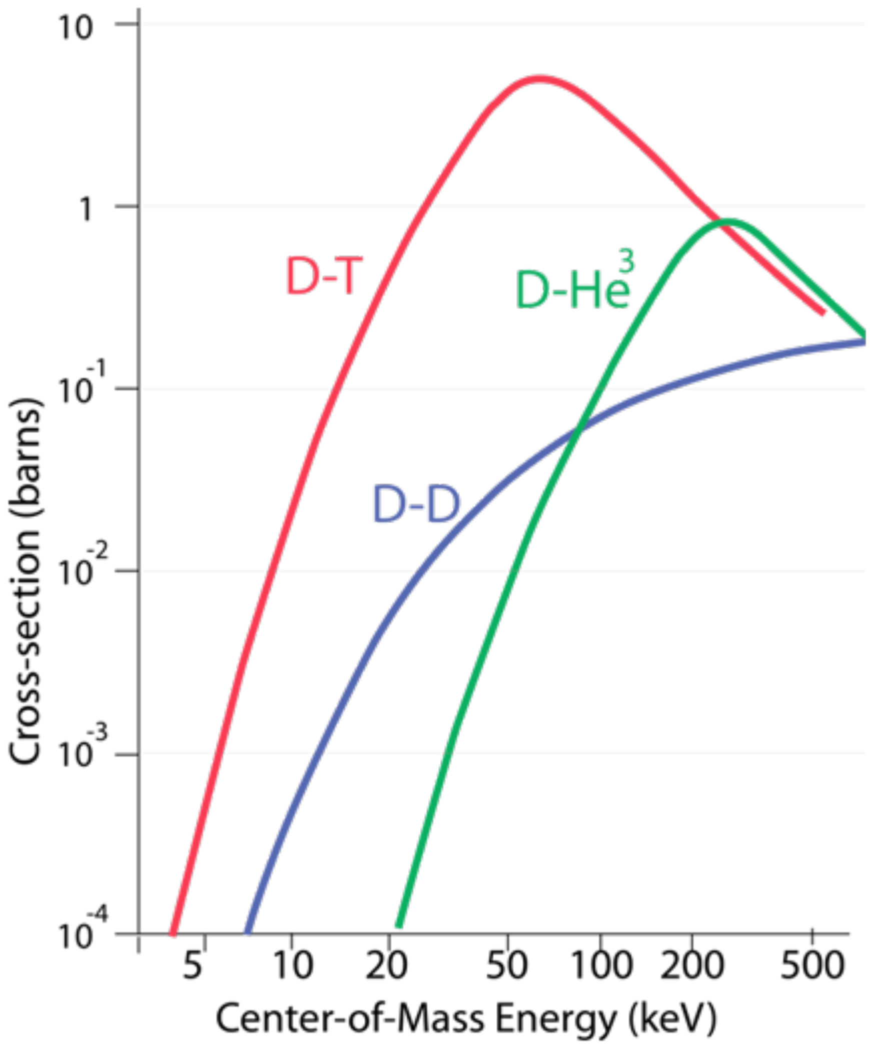
**General Atomics**

Presented at the
IAEA Fusion Energy Conference

May 2021

Supported by U.S. DOE under DE-FG02-95ER54309 and DE-FC02-06ER54873

# Understanding scaling of energy confinement time w/ hydrogenic isotope mass is important in moving toward reactor-relevant DT.




http://hyperphysics.phy-astr.gsu.edu

ITER Operational Phases:

- **H**/He
- **D**
- **50:50 DT**

DT Tokamak Experiments:

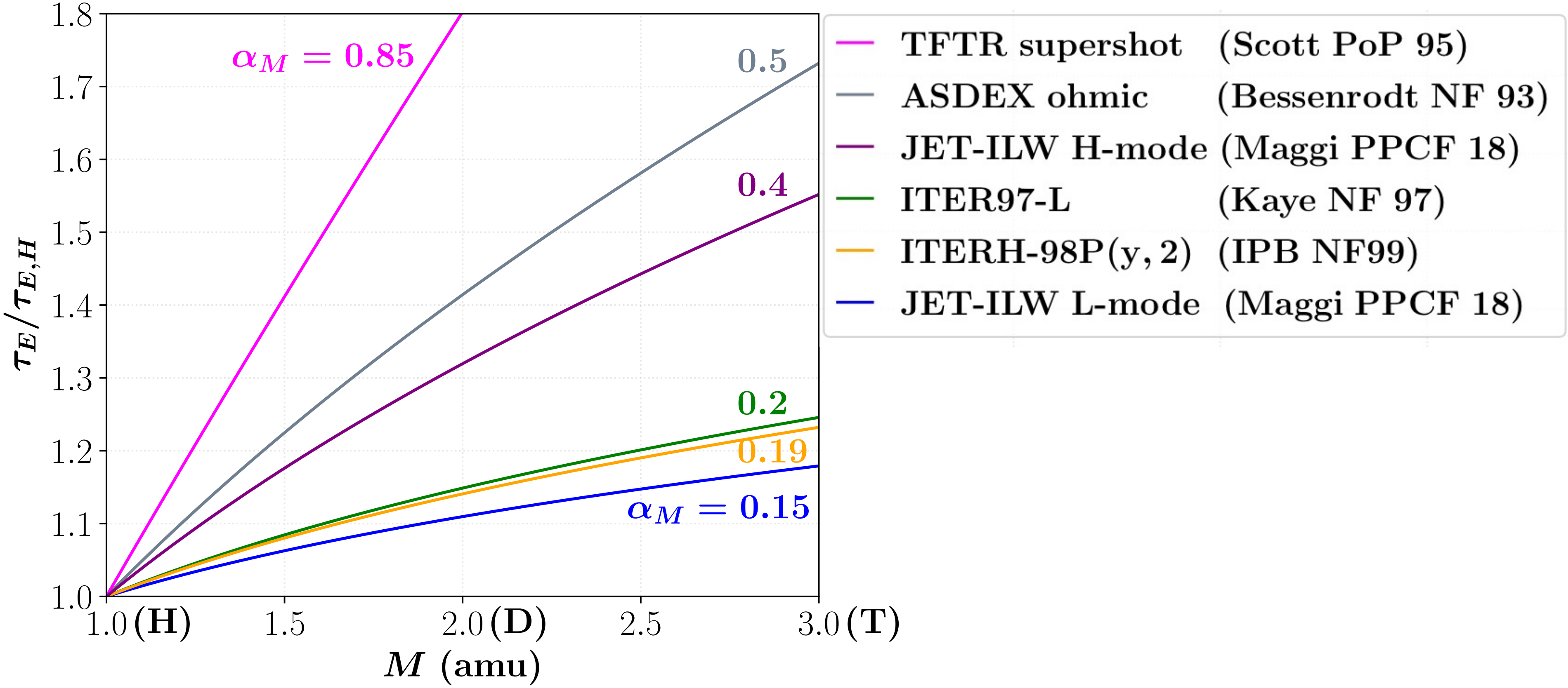
- TFTR 1993-1997
- JET DTE1 1997
- JET DTE2 2021

# We have developed a theoretical framework for understanding the hydrogenic isotope mass dependence of turbulent transport.

- Long-standing problem known as the "**isotope effect**"
- Theoretical basis for gyrokinetic isotope mass scaling of the turbulent energy flux (**naive gyroBohm scaling**)
- Transition of theoretical mass scaling from the ion-dominated core to the electron-dominated edge
- Role of the **nonadiabatic electron drive** in **reversing naive GB mass scaling**; **New scaling law** for **electron-to-ion mass dependence** of flux
- Implications for global confinement and L-H power threshold

# Experiments generally find an increase in global thermal energy confinement time with increasing hydrogenic isotope mass.

$$\tau_E = C I^{\alpha_I} B^{\alpha_B} \bar{n}^{\alpha_n} P^{\alpha_P} R^{\alpha_R} \kappa^{\alpha_\kappa} \epsilon^{\alpha_\epsilon} S_{cr}^{\alpha_S} M^{\alpha_M} \;\rightarrow\; \tau_{E,H} < \tau_{E,D} < \tau_{E,DT}$$



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# Simple gyroBohm-scaling theoretical arguments contradict with experimental observations.

## Naive GyroBohm Scaling

$$\chi_i \sim \frac{\Delta x^2}{\Delta t} \sim \frac{\rho_i^2}{(a/\mathrm{v}_{ti})} \Rightarrow \boxed{\boldsymbol{Q_i = c_0 Q_{GBi}}} \qquad Q_{GBi} = \left(n_0 T_0 \mathrm{v}_{ti} \rho_{*i}^2\right)$$

$$Q_{GBi} = Q_{GBD}\sqrt{\frac{m_i}{m_D}} \qquad Q_{GBD} = \left(n_0 T_0 \mathrm{v}_{tD} \rho_{*D}^2\right)$$

$$\boxed{Q_i = c_0 Q_{GBD}\sqrt{\frac{m_i}{m_D}}}$$

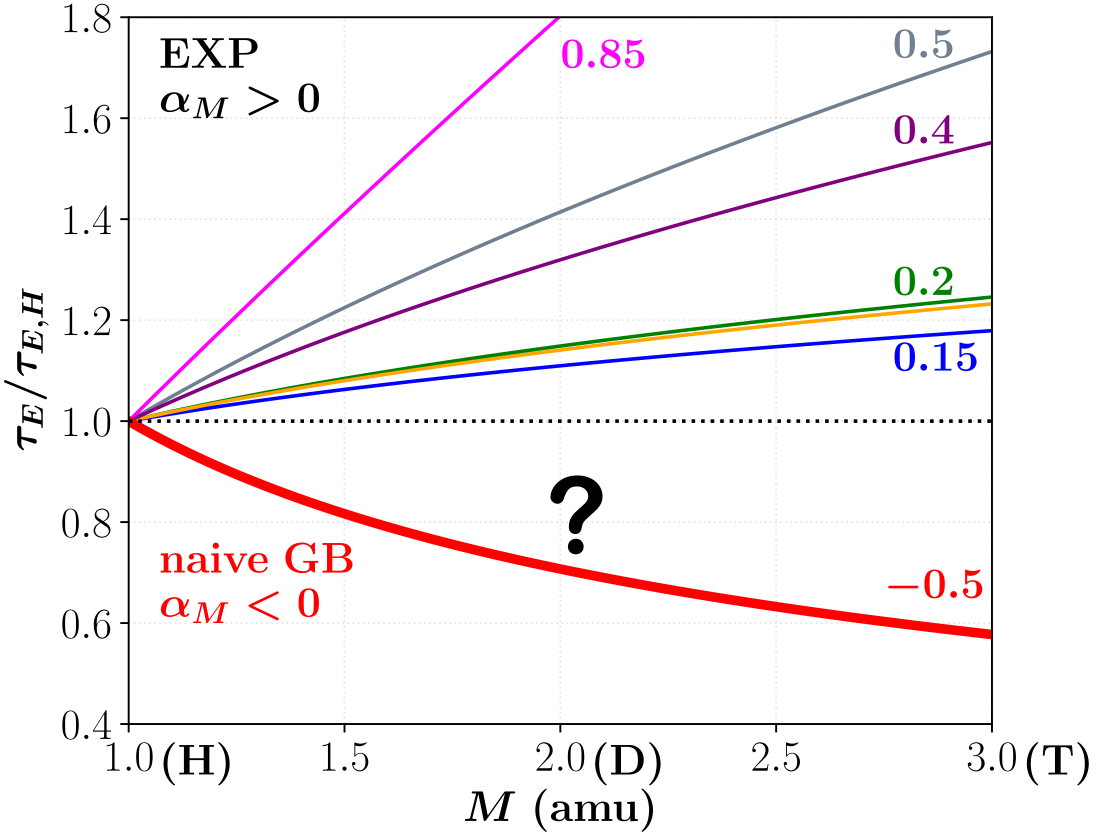
$$\tau_E \sim a^2/\chi_i \qquad \boldsymbol{\tau_E \sim M^{-0.5}}$$

$$\boxed{Q_H < Q_D < Q_{DT}} \Rightarrow \boxed{\tau_{E,H} > \tau_{E,D} > \tau_{E,DT}}$$

# Simple gyroBohm-scaling theoretical arguments contradict with experimental observations.

$$\tau_E = C I^{\alpha_I} B^{\alpha_B} \bar{n}^{\alpha_n} P^{\alpha_P} R^{\alpha_R} \kappa^{\alpha_\kappa} \epsilon^{\alpha_\epsilon} S_{cr}^{\alpha_S} M^{\alpha_M}$$

The "**isotope effect**"




**EXP**

$$\tau_{E,H} < \tau_{E,D} < \tau_{E,DT}$$

**Naive GB**

$$\tau_{E,H} > \tau_{E,D} > \tau_{E,DT}$$

# Proposed mechanisms that can lead to deviation from naive gyroBohm mass scaling of turbulent ion energy flux

- $\vec{E}\times\vec{B}$ flow shear (Garcia NF 17)
- Electromagnetic fluctuations (Garcia NF 17, Manas NF 19)
- Collisions (Nakata PRL 17, Bonanomi NF 19)
- Impurities (Pusztai PoP 11)
- Fast ions (Garcia NF 18, Bonanomi NF 19)
- Kinetic electrons (Estrada PoP 05, Pusztai PoP 11, Bustos PoP 15)

**We present a theoretical framework for the role of the nonadiabatic electron drive in transition of isotopic dependence of turbulent transport from core to edge.**



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# The nonadiabatic electron drive can alter – and even reverse – naive gyroBohm mass scaling.

$$Q_i = c_0 Q_{GBi} = c_0 Q_{GBD}\sqrt{\frac{\boldsymbol{m_i}}{m_D}}$$

$$\Downarrow$$

$$Q_i = \tilde{c}_0 \left(\frac{\boldsymbol{m_e}}{\boldsymbol{m_i}}\right) Q_{GBi}$$

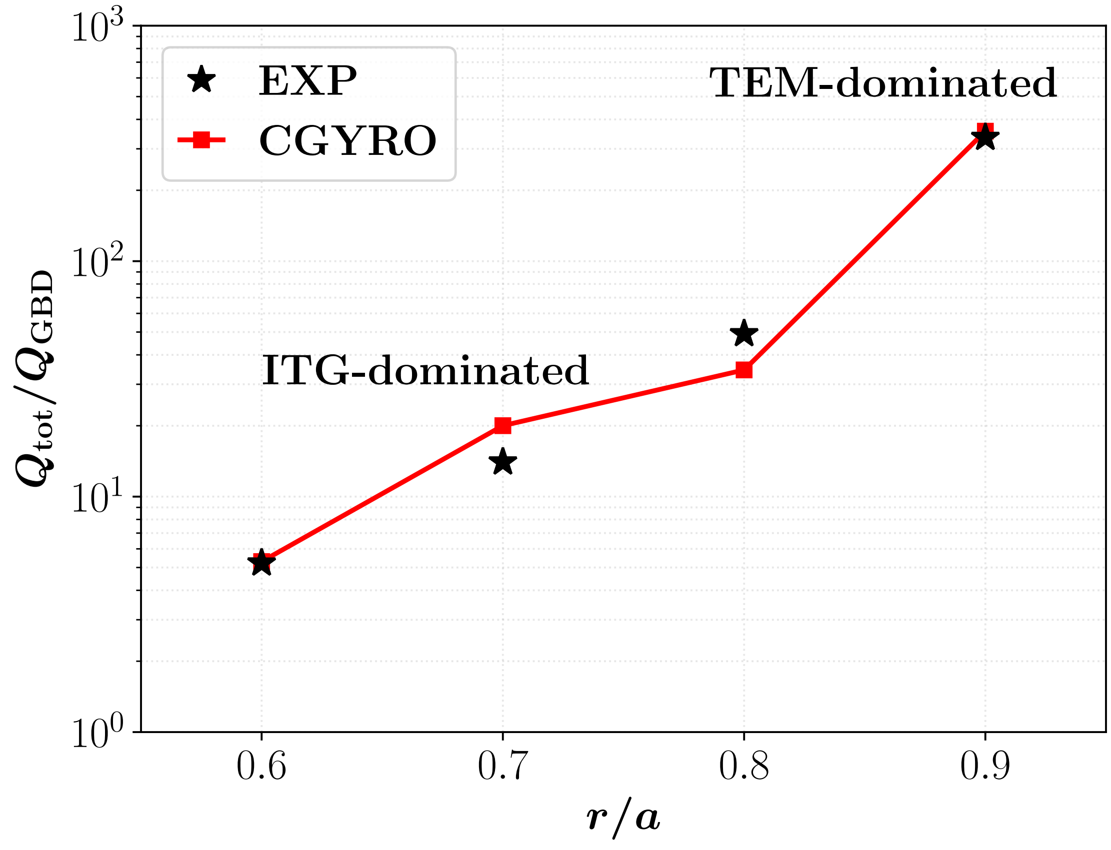
$$\boxed{Q_H < Q_D < Q_{DT}} \quad \tau_{E,H} > \tau_{E,D} > \tau_{E,DT}$$

$$\Downarrow$$

$$\boxed{Q_H > Q_D > Q_{DT}} \quad \tau_{E,H} < \tau_{E,D} < \tau_{E,DT}$$

# CGYRO turbulence simulations match experimental DIII-D power balance (D+e) in the core and the edge.

DIII-D L-mode #173147



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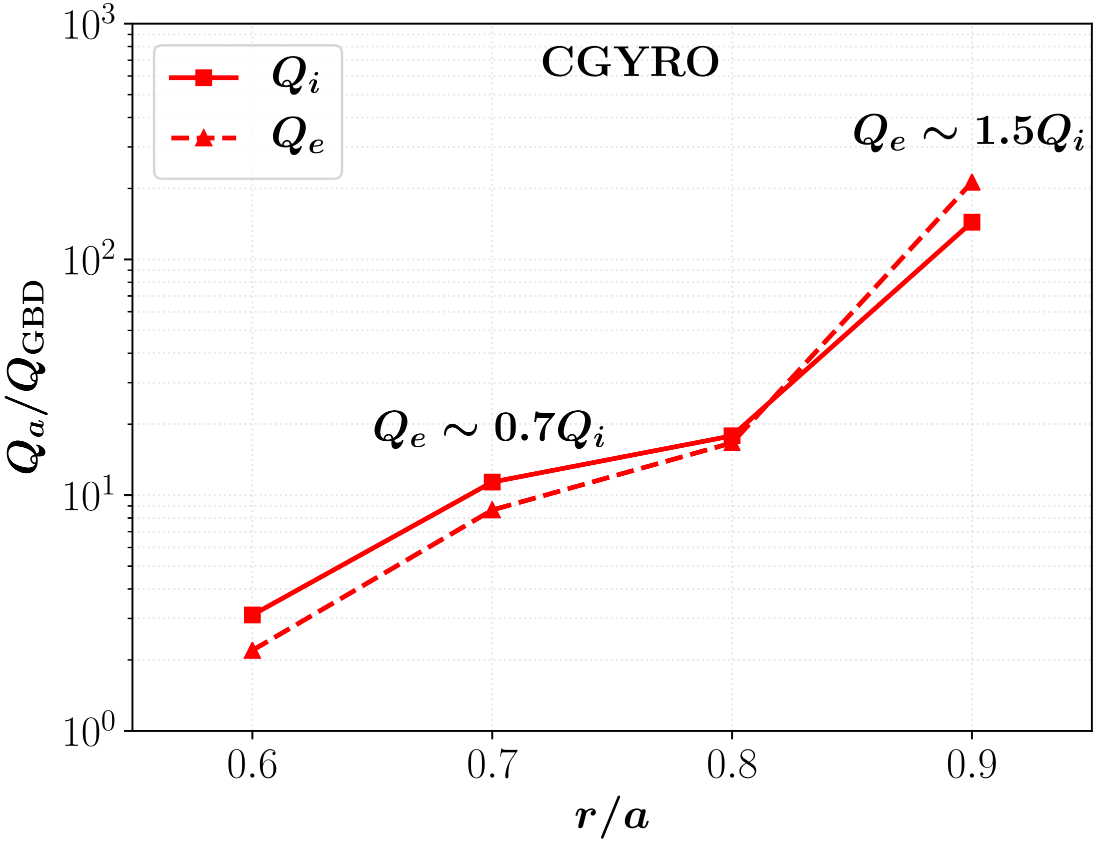
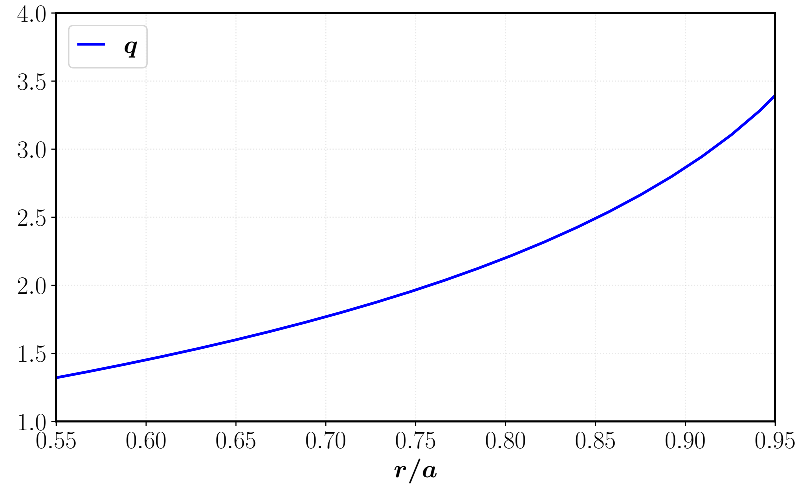
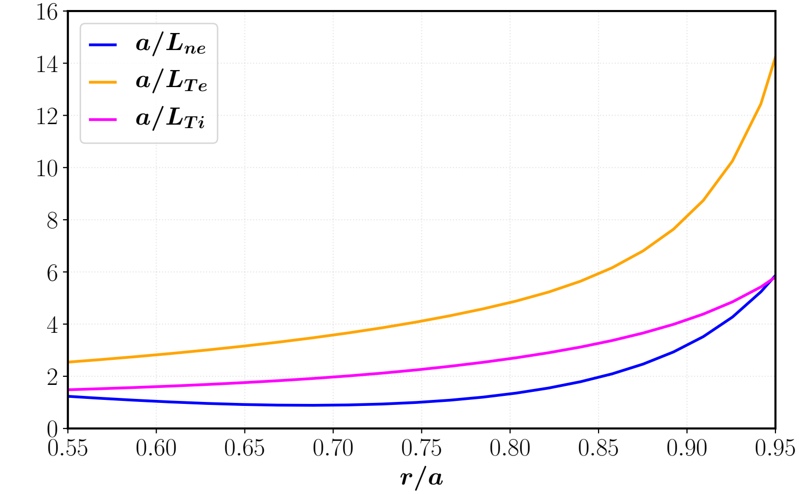
# In the edge, transport becomes explosive due to parameters (large q & gradients) that enhance nonadiabatic electron drive.

## DIII-D L-mode #173147



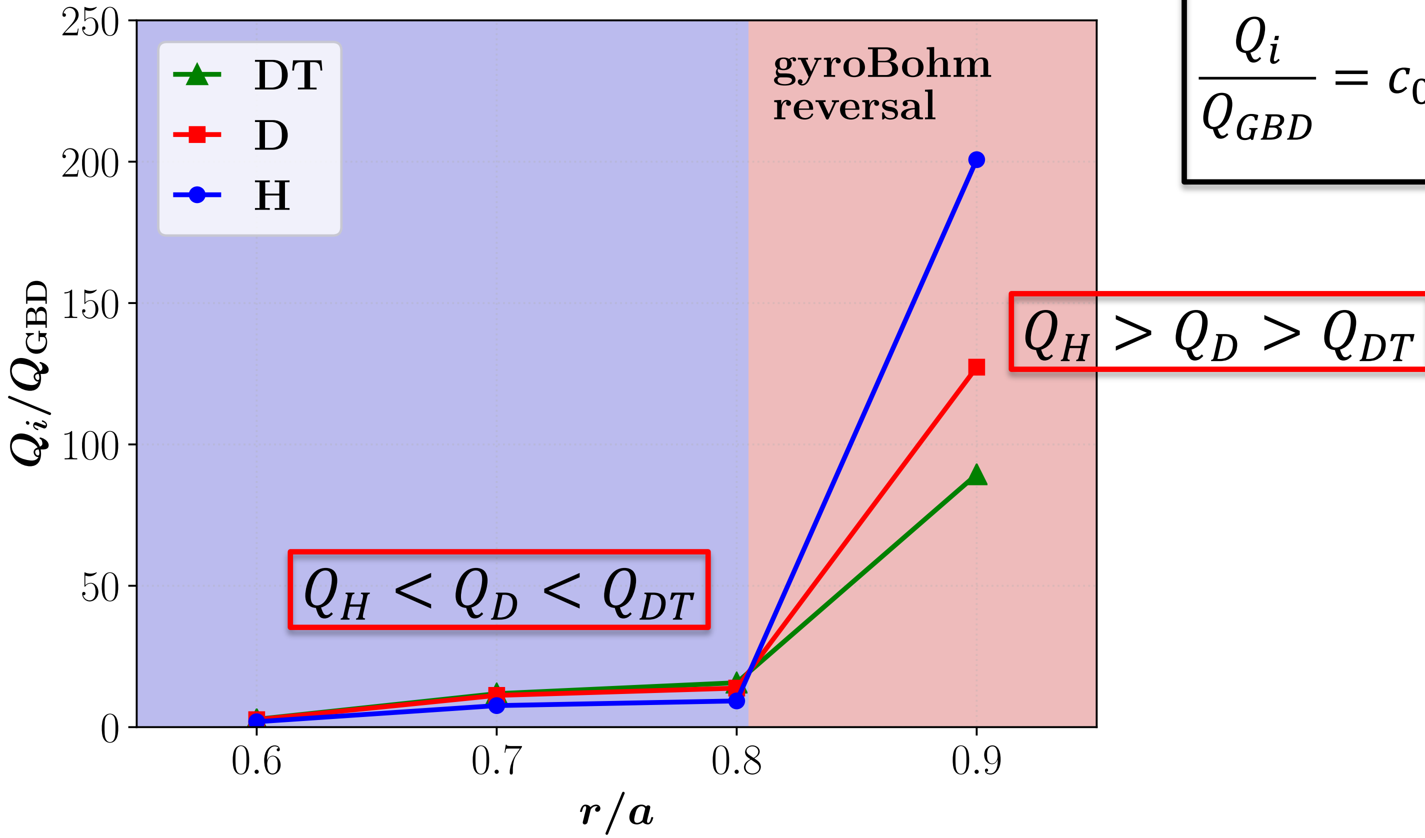



# A favorable reversal of the naive gyroBohm isotope mass scaling is found in the TEM-dominated edge.

naive GB:

$$\frac{Q_i}{Q_{GBD}} = c_0 \sqrt{\frac{m_i}{m_D}}$$




*D corresponds to DIII-D #173147



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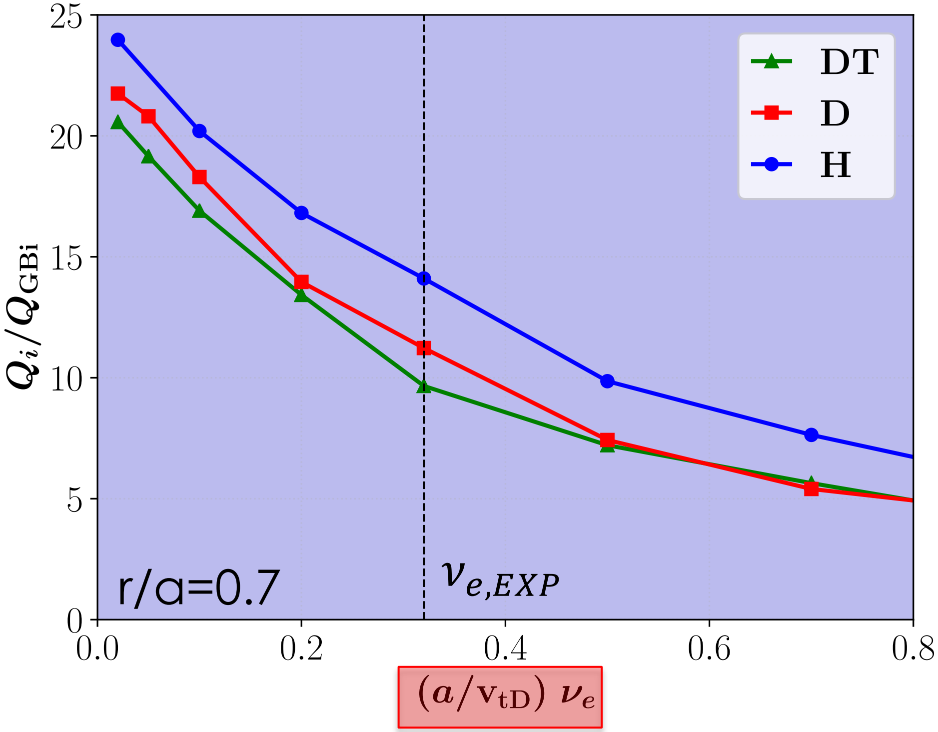
# The finite electron-mass dependence of the turbulent flux that breaks naive gyroBohm scaling enters through several effects.

$$Q_i = \tilde{c}_0 \left(\frac{m_e}{m_i}\right) Q_{GBi}$$

| | | |
|---|---|---|
| Electron-ion collisions | non-negligible | can be rescaled |
| Plasma rotation | non-negligible | can be rescaled |
| Electromagnetic fluctuations | negligible | can be rescaled |
| Finite electron Larmor radius | negligible | irreducible |
| **Electron parallel motion** | dominant | **irreducible** |

# Lighter isotopes are more weakly stabilized by collisions (in absolute units).

## "Fixed Collisions Effect"



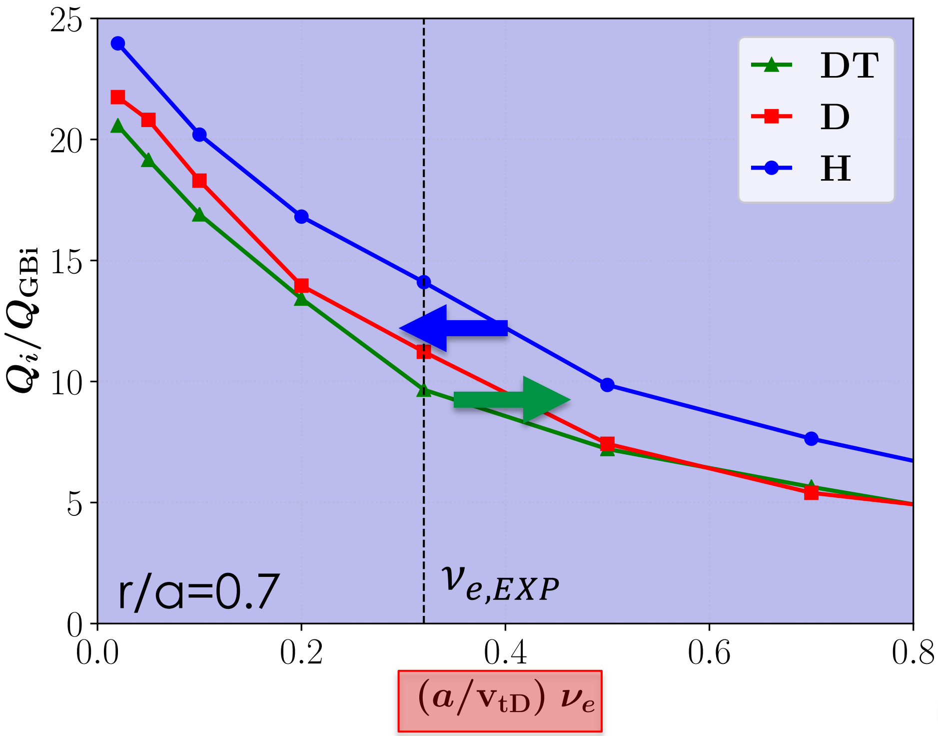

$$\frac{\nu_e}{\omega_{ti}} \sim \sqrt{\frac{m_i}{m_e}}$$

naive GB:

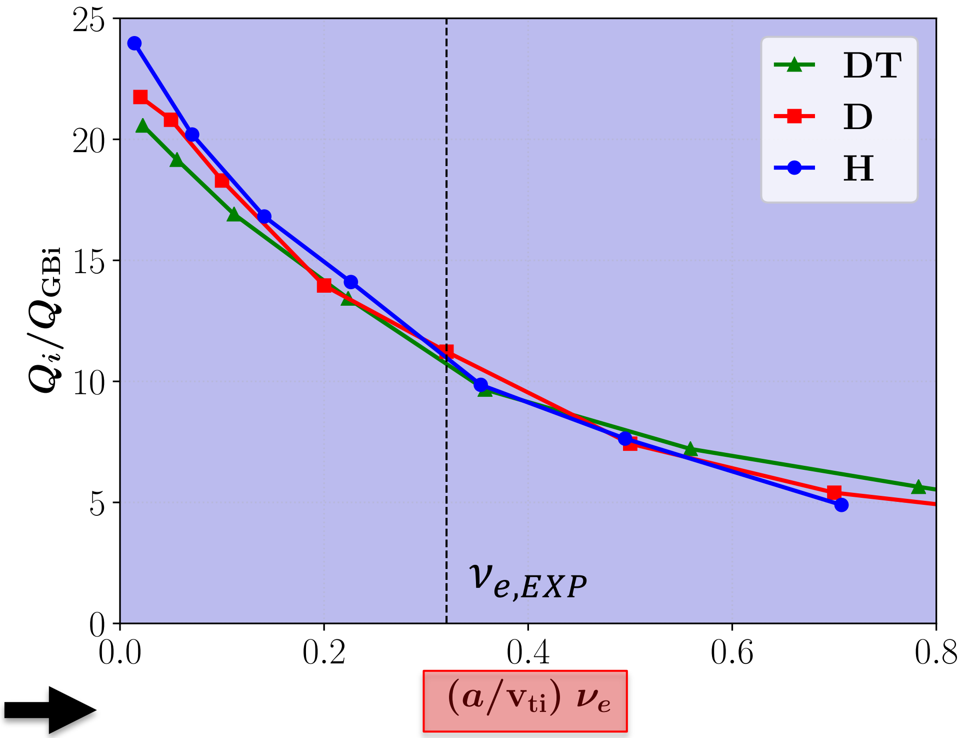
$$\frac{Q_i}{Q_{GBi}} = c_0$$

# Mass-dependence from collisions can be eliminated by rescaling the collision rate with respect to the main ion time scale.

## "Fixed Collisions Effect"




## Rescaled with respect to $a/v_{ti}$



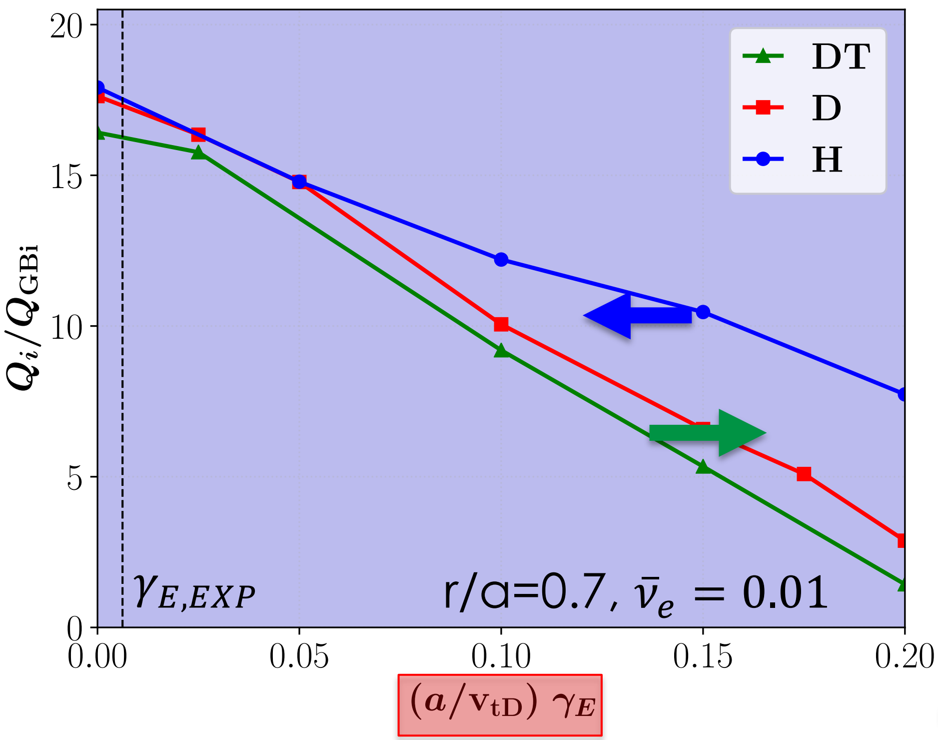

Naive GB mass scaling is recovered

naive GB:

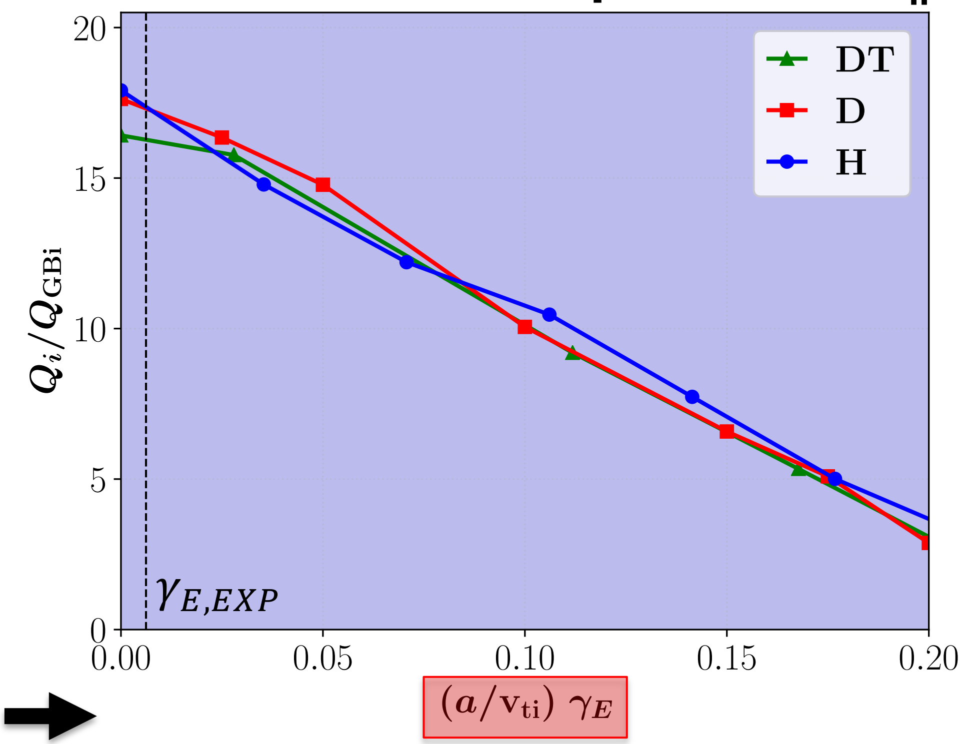
$$\frac{Q_i}{Q_{GBi}} = c_0$$

# Mass-dependence from rotation can be eliminated by rescaling the ExB rotation rate with respect to the main ion time scale.

## "Fixed Rotation Effect"




## Rescaled with respect to $a/v_{ti}$




Naive GB mass scaling is recovered

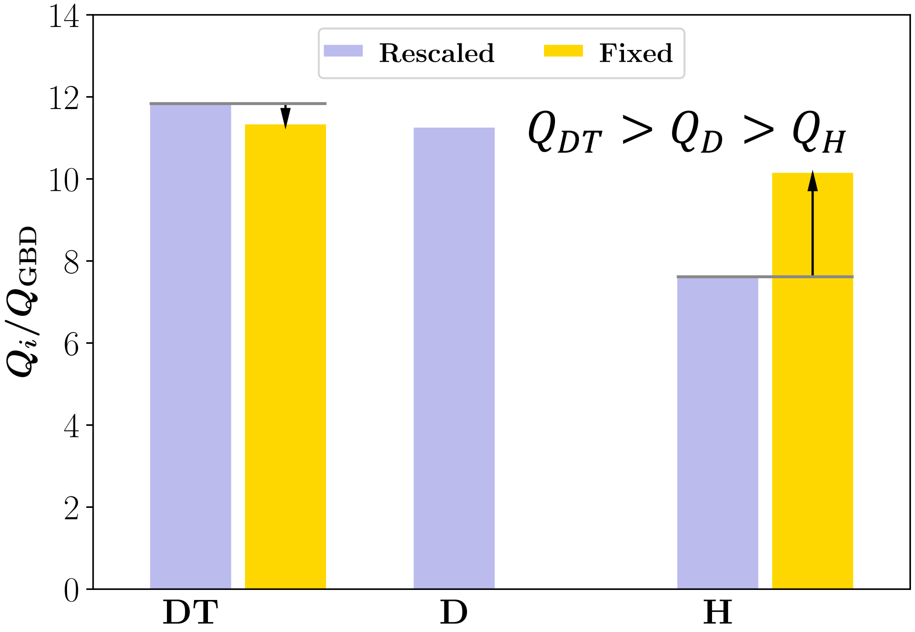
naive GB:

$$\frac{Q_i}{Q_{GBi}} = c_0$$

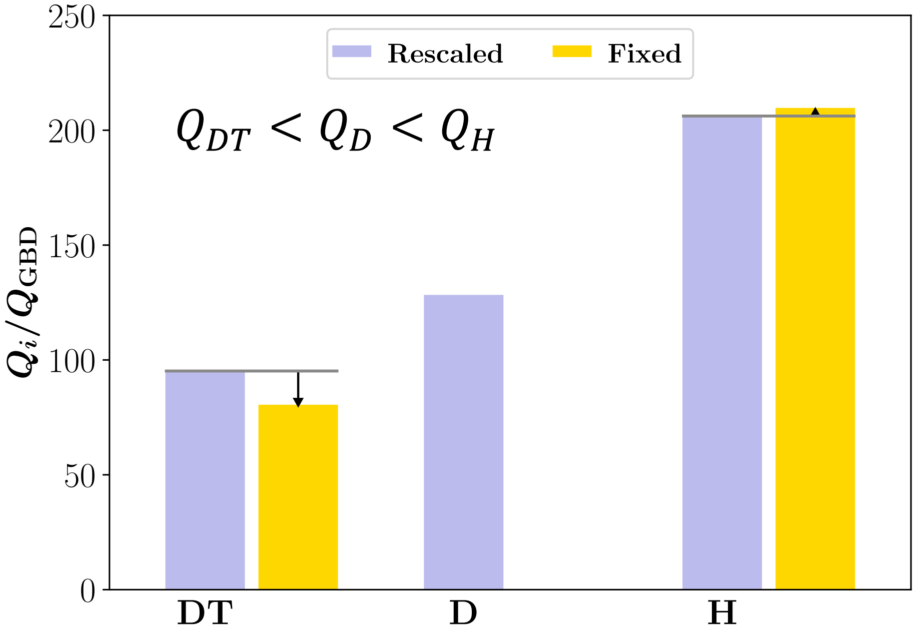
# The "fixed collisions effect" and "fixed rotation effect" enhance the breaking of naive gyroBohm scaling.

**Rescaled:** $\frac{(\nu_e, \gamma_E)}{\mathrm{v}_{ti}/a} = c$

**Fixed collisions & rotation:** $\frac{(\nu_e, \gamma_E)}{\mathrm{v}_{tD}/a} = c$




r/a=0.7




r/a=0.9

DIII-D L-mode #173147



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# The dominant electron-mass dependence of the turbulent flux is through the electron parallel motion.

$$Q_i = \tilde{c}_0 \left(\frac{m_e}{m_i}\right) Q_{GBi}$$

| | | |
|---|---|---|
| Electron-ion collisions | non-negligible | can be rescaled |
| Plasma rotation | non-negligible | can be rescaled |
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| Finite electron Larmor radius | negligible | irreducible |
| **Electron parallel motion** | dominant | **irreducible** |

# A new isotope-mass scaling law is developed to describe the electron-to-ion mass ratio dependence of turbulent energy flux.

Electron mass dependence arises from electron parallel motion:

$$\frac{\partial H_i}{\partial \tau_i} + \frac{u_\parallel(\theta)}{q\mathcal{R}}\frac{\partial H_i}{\partial \theta} = G_i(H_i, \Phi, \boldsymbol{p}) \qquad \frac{\partial H_e}{\partial \tau_i} + \sqrt{\frac{\boldsymbol{m_i}}{\boldsymbol{m_e}}}\frac{u_\parallel(\theta)}{q\mathcal{R}}\frac{\partial H_e}{\partial \theta} = G_e(H_e, \Phi, \boldsymbol{p})$$

$$\boldsymbol{p} = \begin{bmatrix} q, s, \epsilon, \kappa, \ldots & geometry \\ \frac{T_i}{T_e}, \frac{a}{L_{Ti}}, \frac{a}{L_{Te}}, \ldots & profile \\ \frac{a\gamma_E}{\mathrm{v}_{ti}}, \frac{a\gamma_p}{\mathrm{v}_{ti}}, \ldots & rotation \\ \frac{a\nu_{ee}}{\mathrm{v}_{ti}}, \frac{a\nu_{ei}}{\mathrm{v}_{ti}}, \ldots & collisions \end{bmatrix} \qquad \tau_i = \left(\frac{\mathrm{v}_{ti}}{a}\right) t$$

**“fixed”: independent of electron-to-ion mass ratio**

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Corrections to bounce-averaged limit obtained by expanding in $\frac{\boldsymbol{\omega}}{\boldsymbol{\omega_{be}}}$:

$$H_e = \langle H_e \rangle_b + \varepsilon H_e^{(1)} + \varepsilon^2 H_e^{(2)} + \cdots \qquad \boldsymbol{\varepsilon \doteq q\mathcal{R}\sqrt{m_e/m_i}}$$



GENERAL ATOMICS

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$$\Downarrow$$

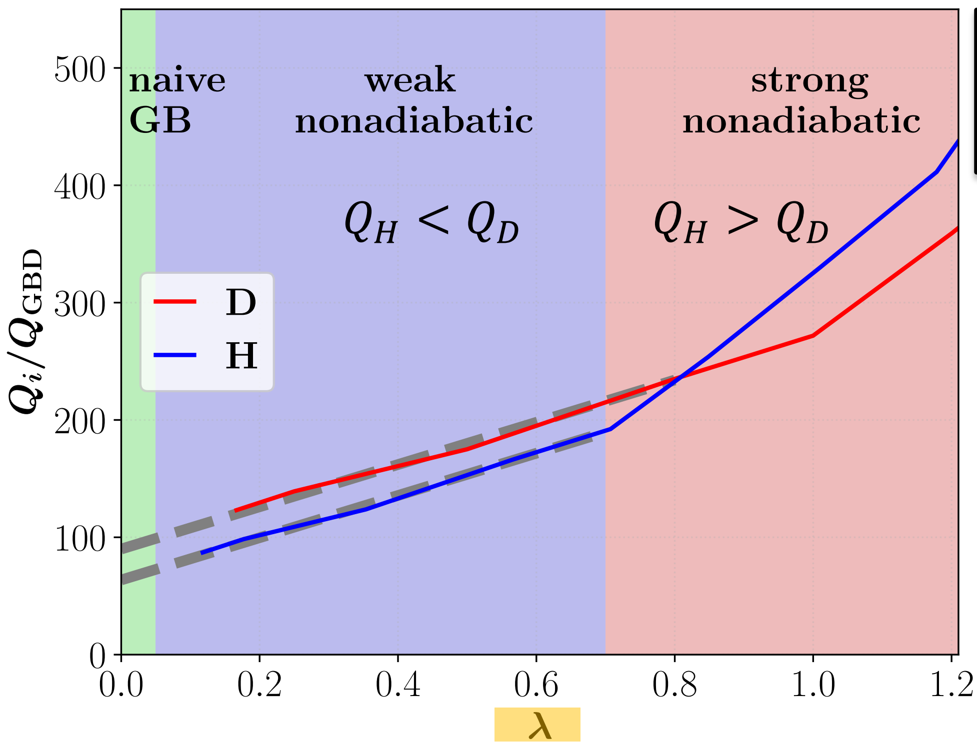
$$Q_i = Q_{GBi}[c_0(\boldsymbol{p}) + \varepsilon c_1(\boldsymbol{p}) + \varepsilon^2 c_2(\boldsymbol{p}) + \ \ldots]$$

$$\frac{Q_i}{Q_{GBD}} = \boldsymbol{c_0(p)}\sqrt{\frac{\boldsymbol{m_i}}{\boldsymbol{m_D}}} + \boldsymbol{c_1(p)q\mathcal{R}}\sqrt{\frac{\boldsymbol{m_e}}{\boldsymbol{m_D}}} + \boldsymbol{c_2(p)(q\mathcal{R})^2}\frac{\boldsymbol{m_e}}{\sqrt{\boldsymbol{m_i m_D}}}$$

| naive gyroBohm | weak nonadiabatic | strong nonadiabatic |
|---|---|---|

# As $\lambda$ increases, the turbulence transitions from a weakly nonadiabatic regime to a strongly nonadiabatic regime (reversal of GB mass scaling).

DIII-D L-mode #173147, r/a=0.9




$$\frac{Q_i}{Q_{GBD}} = c_0\sqrt{\frac{m_i}{m_D}} + c_1 q\mathcal{R}\sqrt{\frac{m_e}{m_D}} + c_2 (q\mathcal{R})^2 \frac{m_e}{\sqrt{m_i m_D}}$$

naive GB — weak nonadiabatic — strong nonadiabatic

$$\frac{\partial H_e}{\partial \tau_i} + \frac{1}{\lambda}\sqrt{\frac{m_i}{m_e}}\frac{u_\parallel}{q\mathcal{R}}\frac{\partial H_e}{\partial \theta} = G_e(H_e, \Phi, p)$$

Electron parallel response time

$$\frac{v_{ti}}{a}\tau_e^\parallel \sim \lambda q\mathcal{R}\frac{v_{ti}}{v_{te}} \sim \lambda q\mathcal{R}\sqrt{\frac{m_e}{m_i}}$$



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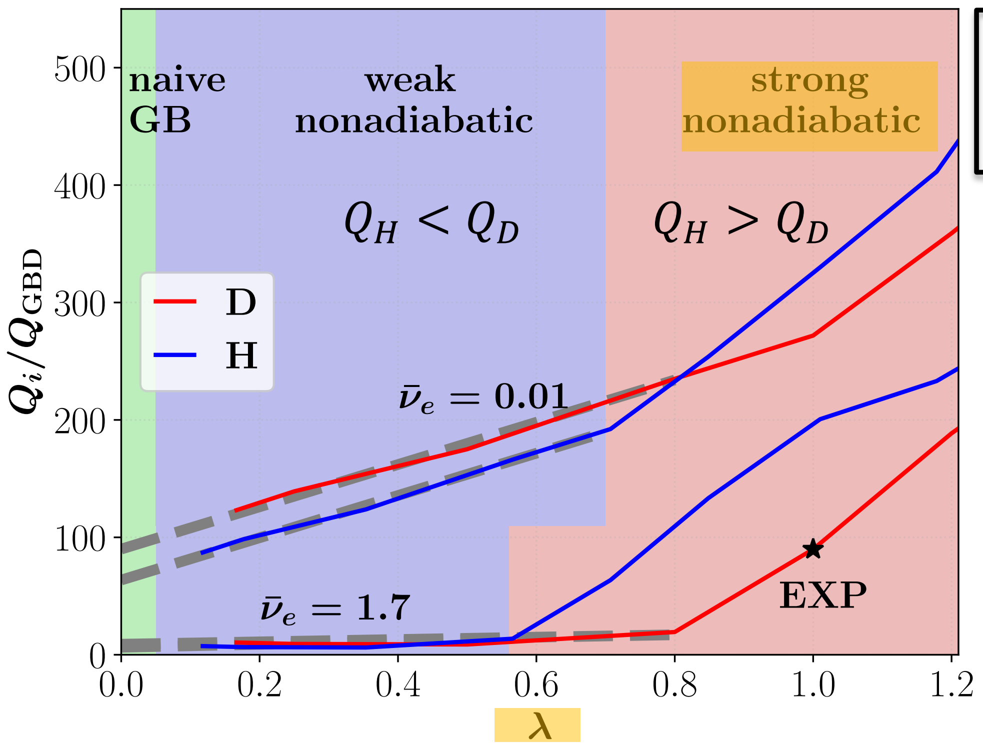
# In the edge, the finite electron mass correction dominates the mass scaling & plays a key role in reversing GB mass scaling.

DIII-D L-mode #173147, r/a=0.9




$$\frac{Q_i}{Q_{GBD}} = c_0\sqrt{\frac{m_i}{m_D}} + c_1 q\mathcal{R}\sqrt{\frac{m_e}{m_D}} + c_2(q\mathcal{R})^2\frac{m_e}{\sqrt{m_i m_D}}$$

naive GB | weak nonadiabatic | strong nonadiabatic

$$\frac{\partial H_e}{\partial \tau_i} + \frac{1}{\lambda}\sqrt{\frac{m_i}{m_e}}\frac{u_\parallel}{q\mathcal{R}}\frac{\partial H_e}{\partial \theta} = G_e(H_e, \Phi, p)$$

Electron parallel response time

$$\frac{v_{ti}}{a}\tau_e^{\parallel} \sim \lambda q\mathcal{R}\frac{v_{ti}}{v_{te}} \sim \lambda q\mathcal{R}\sqrt{\frac{m_e}{m_i}}$$



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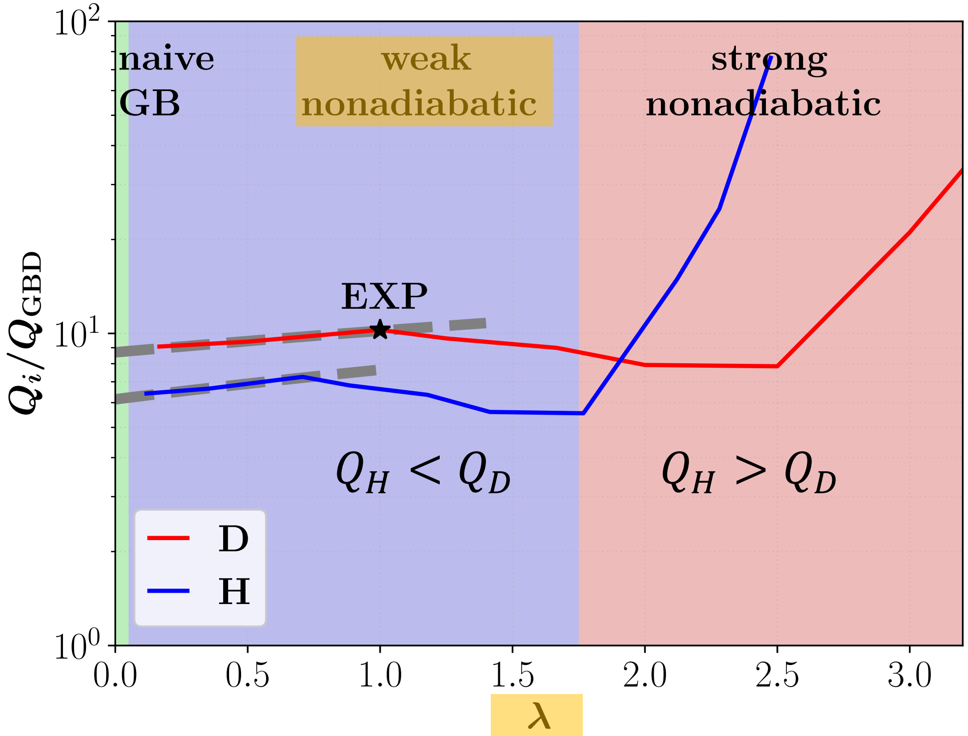
# In the ITG core, the finite electron mass correction is weakly nonadiabatic & a reversal of GB mass scaling is not expected.

DIII-D L-mode #173147, r/a=0.7




$$\frac{Q_i}{Q_{GBD}} = c_0\sqrt{\frac{m_i}{m_D}} + c_1 q\mathcal{R}\sqrt{\frac{m_e}{m_D}} + c_2(q\mathcal{R})^2\frac{m_e}{\sqrt{m_i m_D}}$$

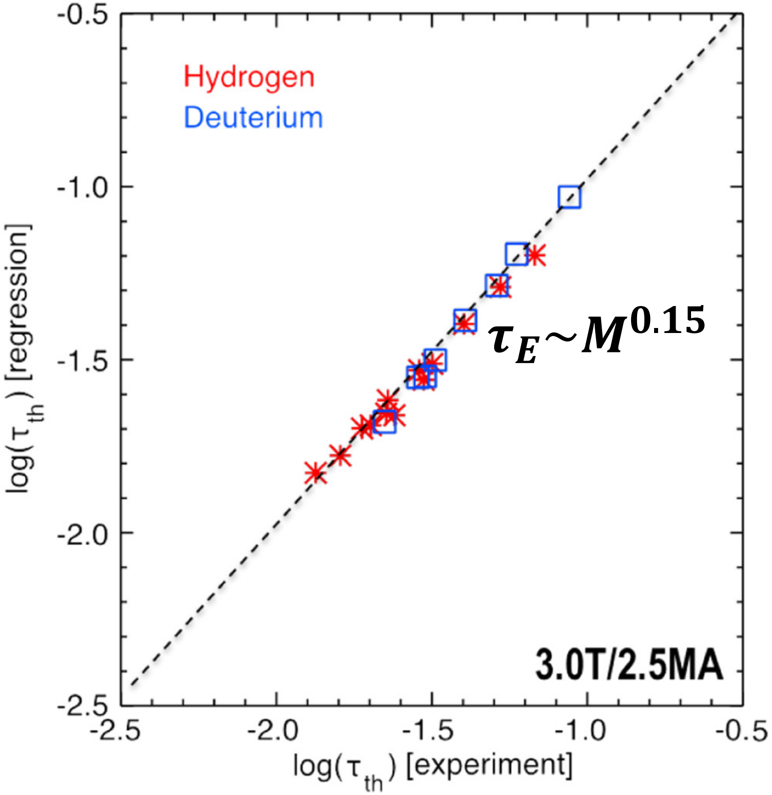
naive GB | weak nonadiabatic | strong nonadiabatic

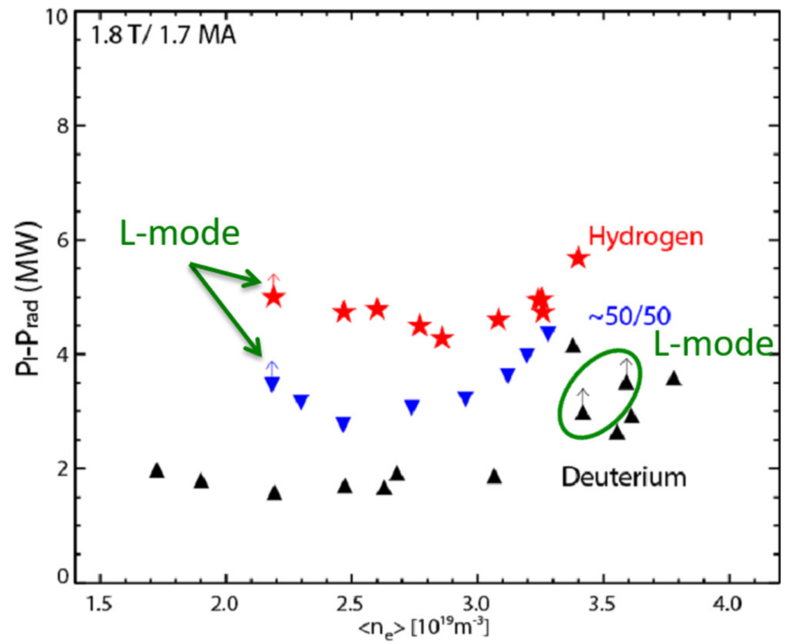
$$\frac{\partial H_e}{\partial \tau_i} + \frac{1}{\lambda}\sqrt{\frac{m_i}{m_e}}\frac{u_\parallel}{q\mathcal{R}}\frac{\partial H_e}{\partial \theta} = G_e(H_e, \Phi, p)$$

# Reversed GB scaling implies favorable increase in global confinement & lowering L-H power threshold, in agreement with experimental trends.

$$Q_H > Q_D > Q_{DT} \Rightarrow \tau_{E,H} < \tau_{E,D} < \tau_{E,DT} \Rightarrow P_{LH,H} > P_{LH,D} > P_{LH,DT}$$

$$\frac{\partial\langle n_i T_i\rangle}{\partial t} + \frac{1}{V'}\frac{\partial}{\partial r}(V' Q_i) = S_{Ti}$$




Results from JET-ILW: Maggi PPCF 2018

# Summary

- A **new isotope-mass scaling law** is proposed to describe **electron-to-ion mass ratio dependence** of turbulent energy fluxes in both ion-dominated **core** & electron-dominated **edge** transport regimes.
- Electron-to-ion mass ratio dependence arises from the nonadiabatic response associated with **fast electron parallel motion**.
- The **nonadiabatic electron drive** strongly regulates the turbulence levels and plays a key role in **altering** – and in the L-mode edge, **reversing** – **naive gyroBohm ion mass scaling**.
- The finite-$m_e$ correction is larger for light ions and higher q such that it is weak in the ITG core but **dominates the mass scaling in the edge**.
- Additional info: E. Belli et al., PRL 125, 015001 (2020)
  E. Belli et al., PoP 26, 082305 (2019)



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