# Predator-Prey model interpretation of nonlinear dynamics of Alfvénic instabilities

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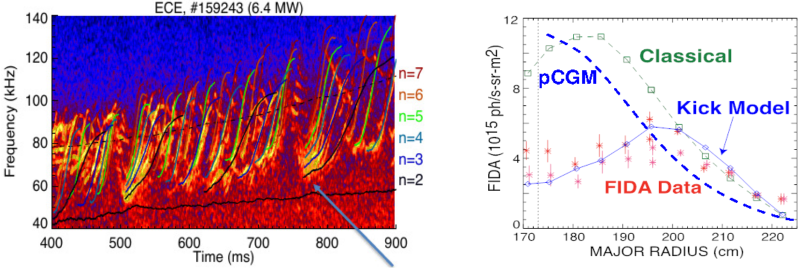
# *How Alfvén mode induced EP fluxes evolve? Can they oscillate within QL approach?*

Consider critical gradient DIII-D experiment, established resilience of EP profiles to injection geometry, shot #159243 *(Collins et al, PRL'16, Heidbrink et al., PoP'17)*



*805msec is chosen near rational $q_{min}$ for detailed study*

- *Well diagnosed and studied DIII-D plasma can be used for deeper understanding of EP losses.*
- *AE modes are localised from near axis region to near the edge.*
- *Can Predator-Prey model be used to understand EP relaxation in experiments?*
- *What is Predator and what is Prey?*

*Is Resonance-Broadened-Quasi-Linear (RBQ) approach compatible with oscillatory behaviour of EP fluxes in experiments? (see oscillations in Ghantous et al., PoP'14)*

# Outline

1. RBQ simulates AEs with oscillations
   - QL equations
   - Rigorous verifications are undertaken
2. Predator-Prey Model can explain EP flux oscillations
   - PPM with one mode
   - Two mode PPM
   - Multiple mode simulation by RBQ
3. Summary and Plans

# *Resonance-Broadened Quasi-Linear code (RBQ) is used*

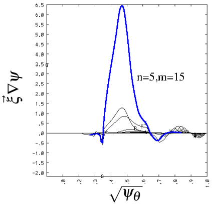
RBQ is a post NOVA/NOVA-K processor to compute EP dynamics in the presence of Alfvénic modes.

- RBQ is in its 1D version and includes:
  - Eigenmode solver (NOVA or others).
  - AE evolution and EP distribution function.
  - Resonances are broadened by resonance islands and effective $\chi$ scattering.
  - QL dynamics allows diffusion in Constants of Motion space ⇒ oscillations!!
  - Postprocessing using probability density function for EP diffusion in the velocity space.
- Connected with TRANSP to compute long time simulations.
- Extensively verified against analytic theory *(Gorelenkov et al., APS'18, Duarte et al. NF'19,'17).*
- Being validated against DIIID steady state critical gradient experiments *(Gorelenkov et al., NF'18).*

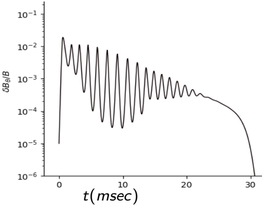
RBQ aims at complex multiple AE instabilities expected in BP conditions given its efficient calculations.

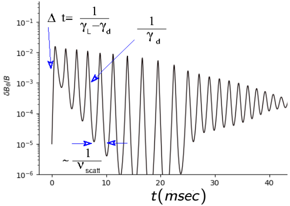
# *RBQ model captures interplay of three time scales for one mode*

*RSAE structure*



*constant* $\nu_{scatt} = 5msec^{-1}$ & $2.5msec^{-1}$





- Interplay between 3 time periods explains oscillations:
  - Linear growing phase: $\gamma_L + \gamma_d$.
  - Damped phase: $\gamma_d$.
  - Recovering phase: $\nu_{scatt}$.
- Periodicity (oscillatory time evolution) is due to Coulomb scattering - effective source of resonance ions.

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# *Quasi-Linear equations include 3 time scales: $\gamma_L$, $\gamma_d$, and $\nu_{eff}$*

Action-angle formalism through flux variables results in a set of equations for fast ion DF: *(Kaufman,PhFl'72, Berk, Breizman, NF'95)* and adapted for RBQ1D *(Duarte, PhD'17, Gorelenkov, NF'18)*

$$\frac{\partial}{\partial t} f = \pi \sum_{\mathbf{l},k} \frac{\partial}{\partial P_\varphi} C_k^2 \mathscr{E}^2 \frac{G^*_{m'p} G_{mp}}{\left|\partial \Omega_{\mathbf{l}} / \partial P_\varphi\right|_{res}} \mathscr{F}_{\mathbf{l}} \frac{\partial}{\partial P_\varphi} f + \nu_{eff}^3 \left|\frac{\partial \Omega_{\mathbf{l}}}{\partial \bar{P}_\varphi}\right|^{-2} \frac{\partial^2}{\partial P_\varphi^2}\left(f - f_0\right),$$

where EP distribution is evolved due to scattering terms on RHS amended by the scattering "source" operator.

AE amplitudes satisfy

$$C_k\left(t\right) \sim e^{\left(\gamma_L+\gamma_d\right)t} \Rightarrow \frac{dC_k^2}{dt} = 2\left(\gamma_L+\gamma_d\right) C_k^2.$$

*AE growth rates $\gamma_L$ are evolved, $\gamma_d$ are fixed.

Critical for RBQ *multiple mode cases (Dupree'66, Berk'95, White'18) is*

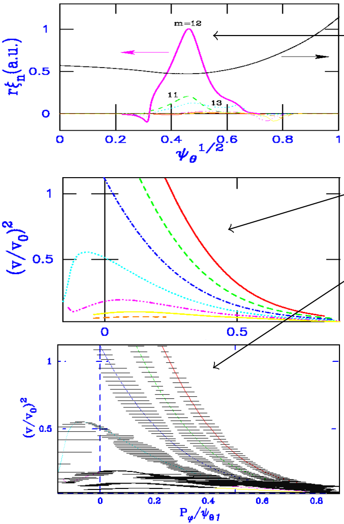
resonant frequency and its broadening by nonlinear bounce $\omega_{bNL}$ and effective scattering $\nu_{eff}$: *(Duarte et al., poster this meeting)*

$$\delta\left(\Omega = \omega + n\dot{\varphi} - m\dot{\theta} - l\omega_b\right) \to window\ \ function\ \mathscr{F}\left[\Delta P_\varphi = \left(c_\omega \omega_{bWPI} + c_\nu \nu_{scatt}\right)/\Omega'_{P_\varphi}\right].$$

RBQ(1D) benefits are:

- Time efficient.
- Realistic computations of current drive, loss distribution over the first wall, intermittency.

# *RBQ workflow illustration for $n = 4$ Reversed Shear Alfvén Eigenmode (at $q_{min}$)*



- Ideal MHD NOVA finds RSAE structure $f = 84kHz$ *(Collins, PRL'16)*.
  - This mode provides a channel for ion diffusion and hollow fast ion pressure profiles: resonant particles are close to the injected pitch angle.
- NOVA-K code computes resonances for particle interactions with the mode and $\langle \boldsymbol{v} \cdot \boldsymbol{E} \rangle$ matrices.
- RBQ1D broadens those resonances along $P_\varphi$ direction using QL prescriptions for each mode. Shown is the broadening at measured amplitude $\delta B_\theta / B = 7 \times 10^{-3}$.
- Monte-Carlo TRANSP package post-processes RBQ diffusion to compute the fast ion distribution function evolution.
  - The Probability Density Function for ion diffusion in the velocity space for further processing within TRANSP is evaluated.
  - Two versions of RBQ1D are developed:
    - Interpretive and predictive.
- RBQ1D is the solver to find the diffusion in the constant of motion space.
  - Employ kick model probability density function technique to describe QL diffusion.

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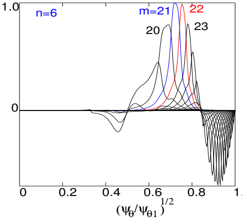
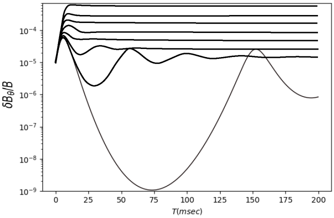
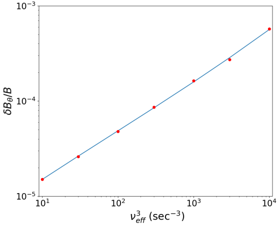
# *RBQ verification via Coulomb collisions*

Global $n = 6$ TAE saturates over $\sim$ *msec*





RBQ shows $\delta B_\theta/B \propto \nu_{eff}^{1.65}$



- TAE amplitude scales with fast ion Coulomb scattering frequency, $\delta B_\theta/B \sim \nu_{eff}^{\mathbf{2}} \sim \nu_\perp^{\mathbf{2/3}}$, where $\nu_{eff}^{\mathbf{3}} = \nu_\perp \left|\frac{\partial\Omega}{\partial\chi}\right|^{\mathbf{2}}$ *(Berk et al.,Phys. Fluids B'90).*
- Dirichlet boundary conditions, $f_h(\bar{\Psi}_\theta \to 0) = const$ and $f_h(\bar{\Psi}_\theta \to 1) = 0$, are required to account for Coulomb scattering.
- At higher $\nu_{eff}^{\mathbf{3}}$ the effect of the resonant island is weakening, less oscillatory evolution.

- Intermittency (fluctuations in losses) is expected in predictive RBQ simulations!!

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# A heuristic Predator-Prey Model (PPM) for 1 mode

*Three elements are essential:*
*(motivated by Borba et al., Theor Fus. Plasm.'92; PPM for fishbones)*

- *Constant background damping, $\gamma_d$.*
- *Growth rate oscillating in time together with particle density, $\gamma_L \sim f_{part}$.*
- *Sources of energetic driving particles, cyclically recovering with $\nu_{scatt}$.*

*"Prey" equation:*

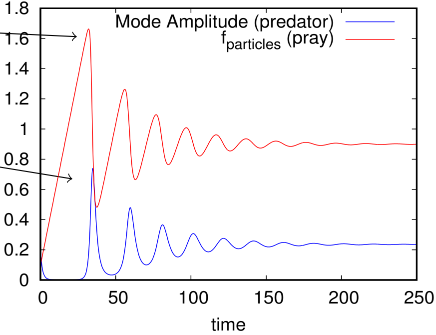
$$\frac{\partial f_{part}}{\partial t} = -A^2 f_{part} + \nu_{scatt} f_0$$

*"Predator" is feeding on prey:*

$$\frac{\partial A}{\partial t} = \gamma_d A + \gamma_L \frac{f_{part}}{f_0} A$$

$\Rightarrow$ *saturated state:*

$$f_{part} = -f_0\gamma_d/\gamma_L \text{ and } A^2 = \nu_{scatt} f_0/f_{part}$$
$$\text{or } A^2 = -\nu_{scatt}(\gamma_d+\gamma_L)/\gamma_d.$$



*Parameters are $\gamma_d = -0.9$, $\gamma_L = 1$, $\nu_{scatt} = 0.05$.*
*Predator-Prey system converges to a saturated state.*
*Not cyclical due to loss rate proportional to $f_{part}$, not constant.*

# Outline

# *Multimode PP Model exhibits complex intermittent evolution*

*Two modes: two ensembles of particles + two dampings/growth rates*

- *Sources/sinks of EP driving particles are interchangeable, cyclically recovering at $\nu_{scatt}$ rate.*

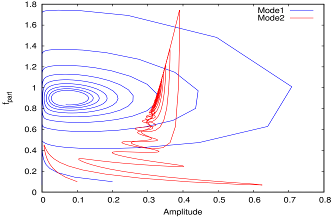
*2 PPM systems:*

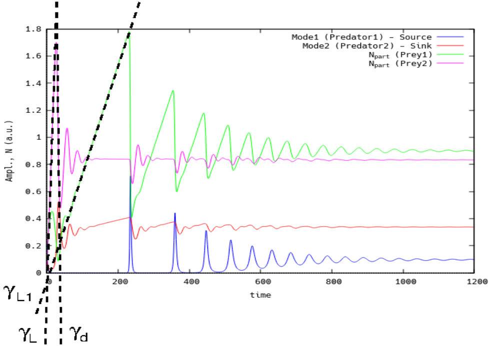
$$\text{Dominant}: \quad \frac{\partial f_{part}}{\partial t} = -A^2 f_{part} + \nu_{scatt}\left(f_0 - f_{1part}\right)$$

$$\frac{\partial A}{\partial t} = \gamma_d A + \gamma_L \frac{f_{part}}{f_0} A$$

$$\text{Subdominant}: \quad \frac{\partial f_{1part}}{\partial t} = -A_1^2 f_{1part} + \nu_{scatt}\left(f_0 + f_{part}\right)$$

$$\frac{\partial A_1}{\partial t} = \gamma_{d1} A_1 + \gamma_{L1} \frac{f_{1part}}{f_0} A$$





*Parameters are:*
$\gamma_d/\gamma_{d1} = -0.9/-0.25$, $\gamma_L/\gamma_{L1} = 1/0.3$,
$\nu_{scatt} = 0.05$.
*Not cyclical due to loss rate proportional to $f_{part}$, not constant.*

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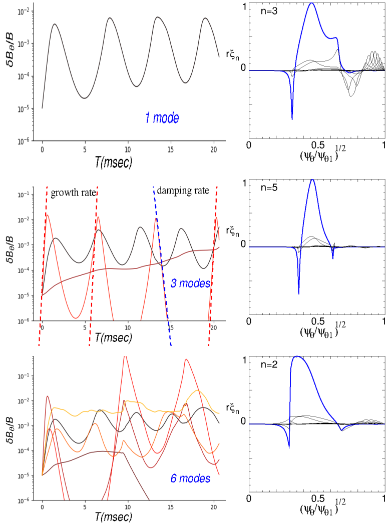
# *RBQ with multimode runs exhibits oscillations at fixed Coulomb scattering rate*

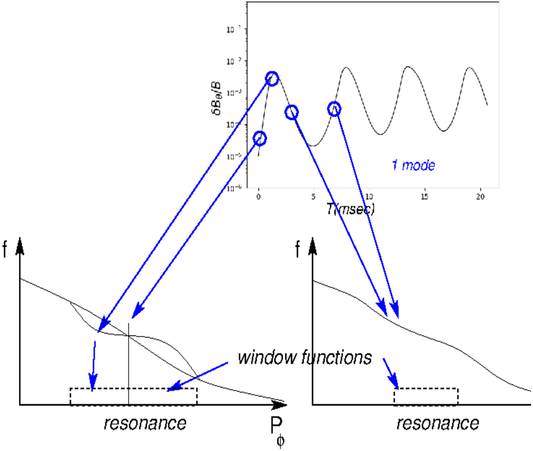
## *Dominant RSAE $\delta B_\theta/B(t)$ & structures*



- RBQ exhibits interplay for 1, few modes.
  - With 1-3 modes growth/damping rates can be measured.
  - Similarity with DIII-D observed oscillatins, Van Zelland et al., I-1
- AE evolutions may capture growth/damping rates.
- If AE amplitude is cyclical growth phase repeats at $\gamma_L(t_0) \sim \gamma_L(t_0 + nT)$. The depth of the cycles should be near zero.

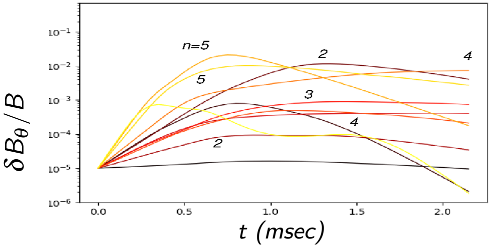
RBQ needs sources and sinks to be consistent with measured amplitude evolution.

## Sketch of resonant particle dynamics during oscillations



- QL methodology allows to represent resonant dynamics.
- Measurable interplay of growth/damping rate scales is important for experiments.

# *Selfconsistent evolutions do not show oscillations with Coulomb collisions*



- RBQ1D computes Alfvén Eigenmode amplitudes consistent with measured values $O\left(10^{-4}-10^{-2}\right)$ *(Collins et al., PRL'16)*.
- Amplitudes (diffusion coefficients) at saturation are sensitive to growth rate values:

  ⇒ need to be robustly computed!
  Amplitudes are sensitive to the model of QL broadening: nonlinear resonant island & scattering effects *(Ghantous et al., PoP'14)*.
- With Coulomb scattering rate no time oscillations are observed in multimode simulations.
  - $10-100$ times stronger pitch angle scattering (turbulence?) is required to model intermittencies consistent with experiments *(Van Zeeland et al., NF'19)*.

# Summary

- AE instability induced fluxes can oscillate in the presence strong pitch angle scattering.
    - If scattering is strong and only few modes are present AE amplitudes oscillate in time with observable characteristic damping, growth rates and scattering time.
- PPM helps to identify growth/damping rates in RBQ QL simulations (experiment?).
- For consistency with experiments an additional scattering in pitch angle needs to be provided, turbulence??
- At the moment 2D version of RBQ is being developed within ISEP SciDAC for realistic simulations.

# *Analytic solution for amplitude evolution near threshold*

Near marginal stability, the amplitude governed by

$$\frac{dA(t)}{dt} = A(t) - \frac{1}{2}\int d\Gamma \mathscr{H} \left\{ \int_0^{t/2} dz z^2 A(t-z) \times \right.$$
$$\left. \times \int_0^{t-2z} dy e^{-\hat{\nu}_{eff}^3 z^2 (2z/3+y)} A(t-z-y) A^*(t-2z-y) \right\}.$$

*(Berk et al., PRL'96)*

At large $\nu_{eff}$ ($>$ net growth rate) only recent time history dictates the WPI dynamics, i.e. when $y, z \to 0$:

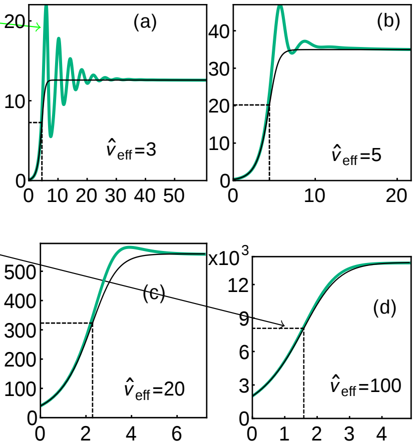
$$A(t) = \frac{A(0)e^t}{\sqrt{1 - bA^2(0)(1-e^{2t})}}$$

where $A(0)$ is the initial amplitude and

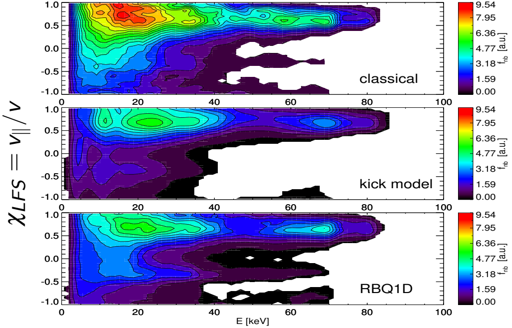
$b \equiv \left[ \int d\Gamma \mathscr{H} \frac{\Gamma(1/3)}{6\hat{\nu}_{eff}^4} \left(\frac{3}{2}\right)^{1/3} \right]$

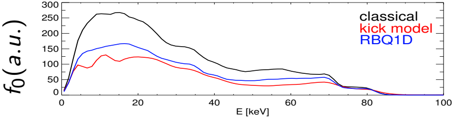
*(V.Duarte et al., NF'19).*

Amplitude $A$ vs time $t$ for full cubic equation (green) and its analytical solution (black)



# *Backups: Distribution function has similar properties with the kick model distribution*

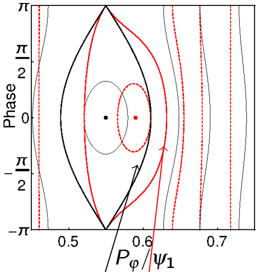




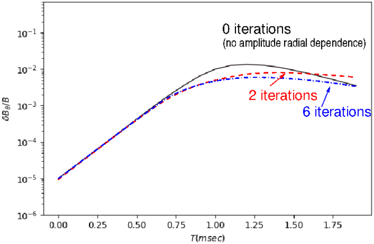
- Co-going passing ions are strongly redistributed.
- Amplitudes are kept constant throughout observed times.
- Neutron rate includes radial and energy dependence within TRANSP simulations.
- (Near) hollow EP density is due COM location sensitive diffusion.
- Rotation is ignored!!
  It can be significant and could lead to EP energy shift $\sim E_0/2$ in DIII-D.

# *Resonant ion island dynamics is accounted for using Hamiltonian technique*

EP islands for “Gaussian” mode



RBQ needs ˜2 iterations to converge well. Lowers saturation ampl.

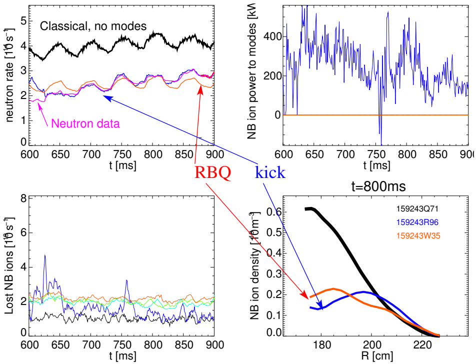


$0^{th}$ iteration
(Berk-Breizman approach)

$6^{th}$ iteration ("new" island) accounting for RSAE radial structure

- Low amplitude $\Delta P_\varphi \sim \Delta\Omega = 4\omega_b$ at $\delta B_\theta/B \lesssim (1\div 5)\times 10^{-4}$ *(via ORBIT modeling, G.Meng, NF'18).* Supports resonant frequency approach for nonlinear wave particle interaction.
- Radial amplitude structure limits NL resonance frequency *(R.White et al., PoP'18).*

# *Compare RBQ1D, kick simulations with neutron deficit using TRANSP*



- Distributions are evolved by TRANSP Monte-Carlo package.
- Kick model agrees with FIDA data over the velocity space region.
- RBQ1D and kick model simulations are consistent.