



Analysis of TAEs and FBs induced fast ions redistribution and losses in MAST using a reduced fast ion transport model

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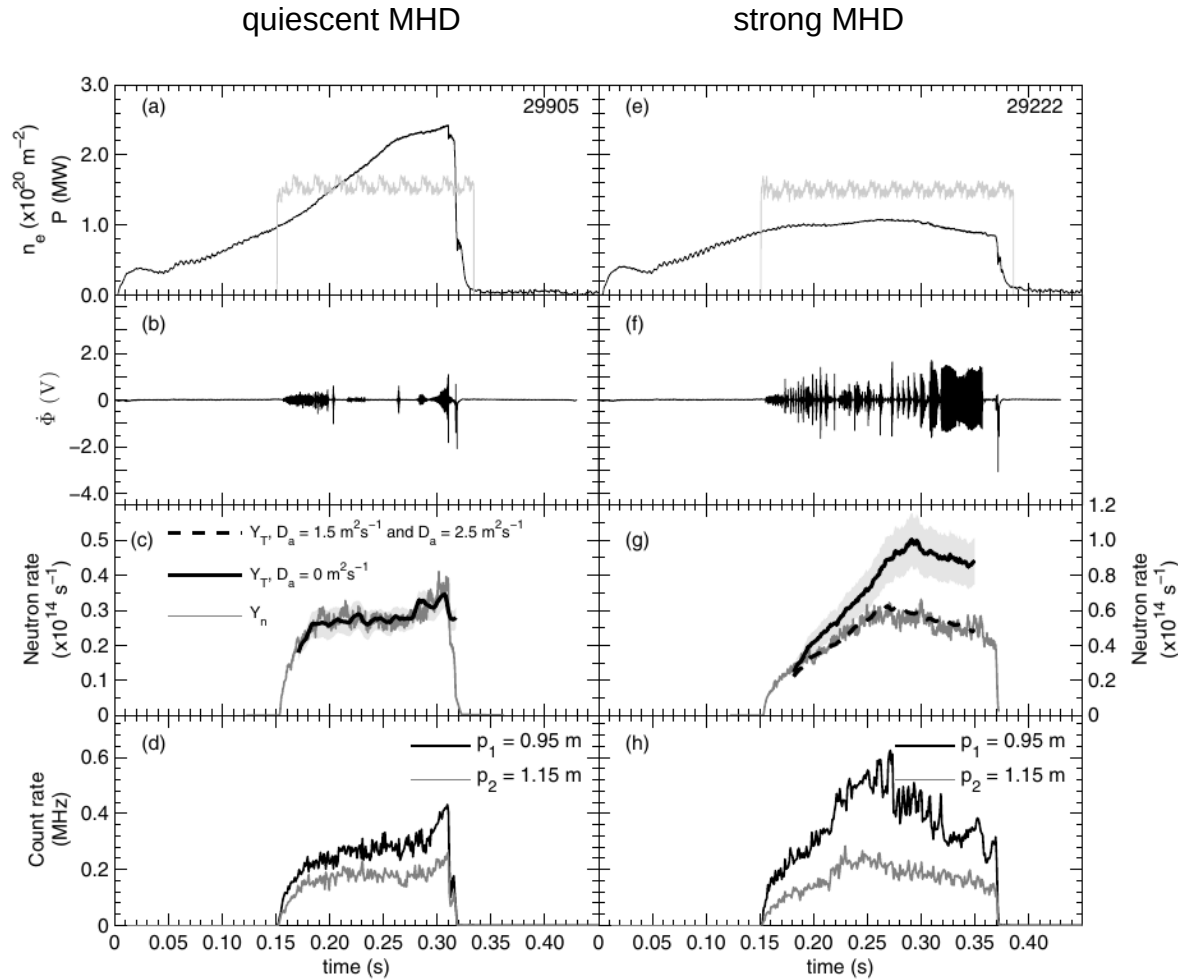
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Some background ...

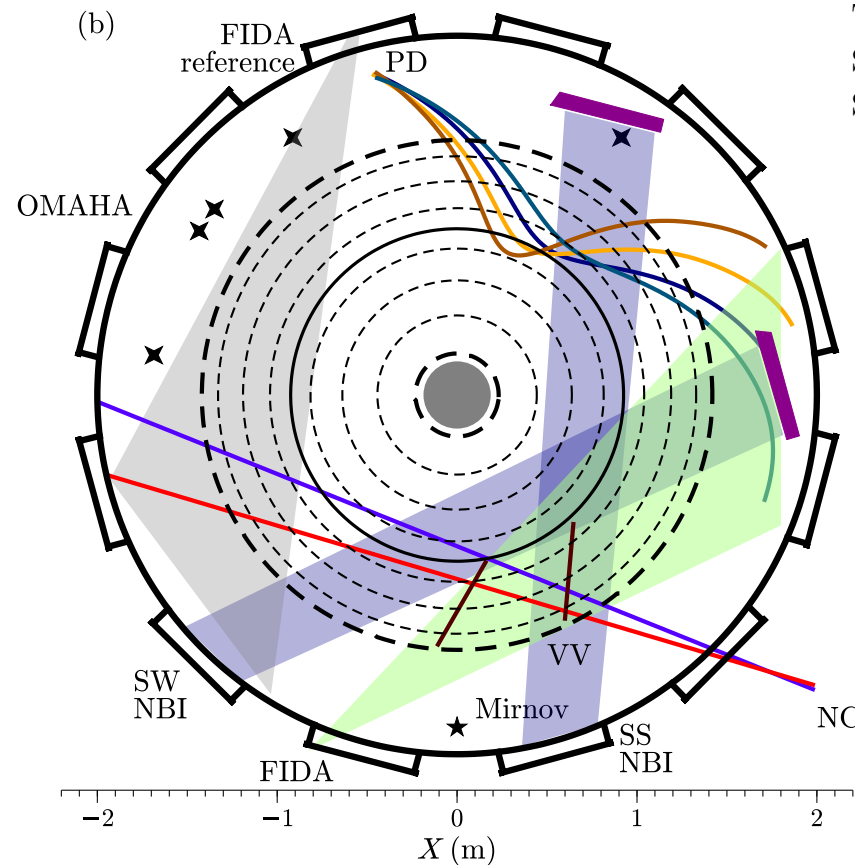
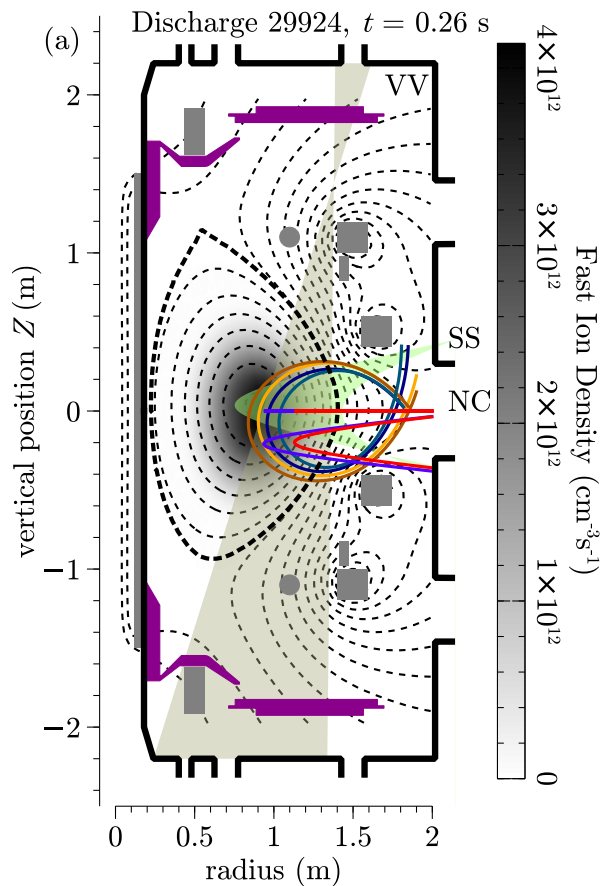


- the beam-thermal neutron emission is strongly suppressed in presence of resonant MHD instabilities such as TAEs and FBs;
- this is due to the strong redistribution and losses of FIs caused by these MHD instabilities;
- so far, at MAST, the effect has been modelled in TRANSP/NUBEAM by introducing a time dependent anomalous fast ion diffusion coefficient whose amplitude has been adjusted “ad hoc” to match the neutron rate,
- however no physics model nor justification for the “ad hoc” adjustments;
- reduced transport kick model: modelling and comparison with global and local measurements of the FI population.

Klimek I, Cecconello M, Gorelenkova M, Keeling D, Meakins A, Jones O, et al. TRANSP modelling of total and local neutron emission on MAST. Nuclear Fusion. 2015 Feb 1;55(2):023003.

Fast Ions diagnostics on MAST

In addition to a fission chamber MAST was also equipped with:



Neutron Camera (2 mid-plane channels)

Time resolution: 1 ms
 Spatial coverage: 0.2 - 1.3 m
 Sensitive to: FI closest to injection energy

FIDA (tangential)

Time resolution: 1 ms
 Spatial coverage: 0.8 - 1.4 m
 Fast ion range: 30 - 70 keV
 Sensitive mainly to passing FI

CFPD (4 channels)

Time resolution: 1 ms
 Spatial coverage: 0.8 - 1.4 m
 Sensitive to: FI closest to injection energy

Cecconello M, et al. Energetic ion behaviour in MAST. Plasma Physics and Controlled Fusion. 2015 Jan 1;57(1):014006.

A series of experiments was carried out in 2013 to assess the effect of different instabilities on the fast ion populations taking advantage of the combined information from all these diagnostics.

Comparison between TRANSP simulations and experimental measurements.

Energetic fast ions in MAST: TAE phase of #29976

L-mode scenario used to explore TAEs, fishbones and LLMs characterized by:

- $P_{\text{NBI}} = 3 \text{ MW}$, $E_{\text{NBI}} = 70 \text{ keV}$ (44 keV)
- $n_e(0) = 3 \times 10^{19} \text{ m}^{-3}$
- $I_p = 0.8 \text{ MA}$
- $B_0 = 0.52 \text{ T}$
- $v_{\text{NBI}} = 2 - 2.6 \times 10^6 \text{ m/s}$
- $v_A(0) \approx 1.5 \times 10^6 \text{ m/s}$

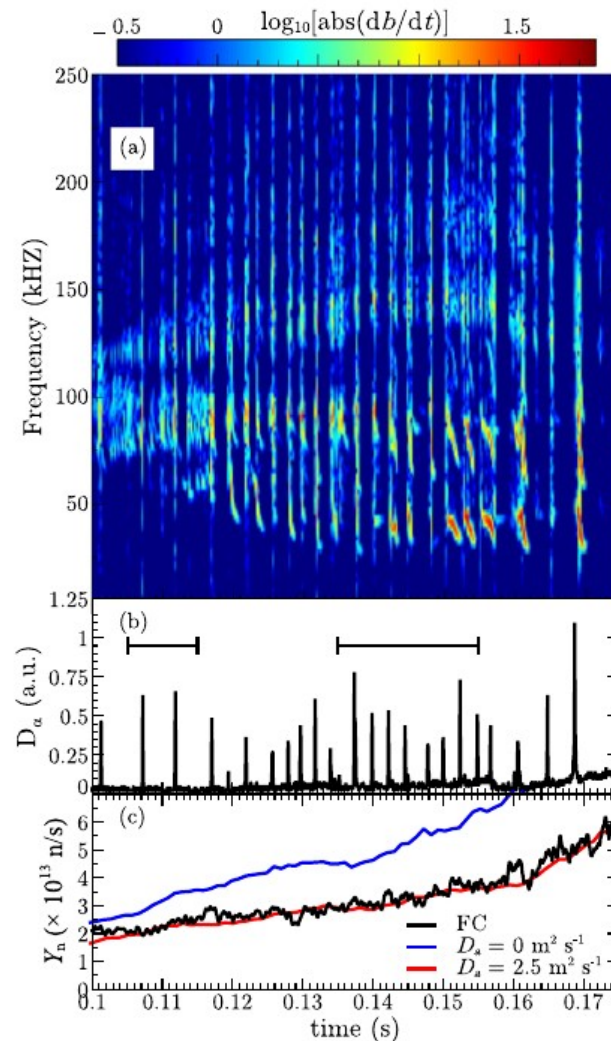
with a safety factor profile $q(r)$ initially reversed, with $q(0) \geq 1$, evolving into a monotonic profile with $q \approx 1$ in the plasma core towards the end of the pulse.

Observations:

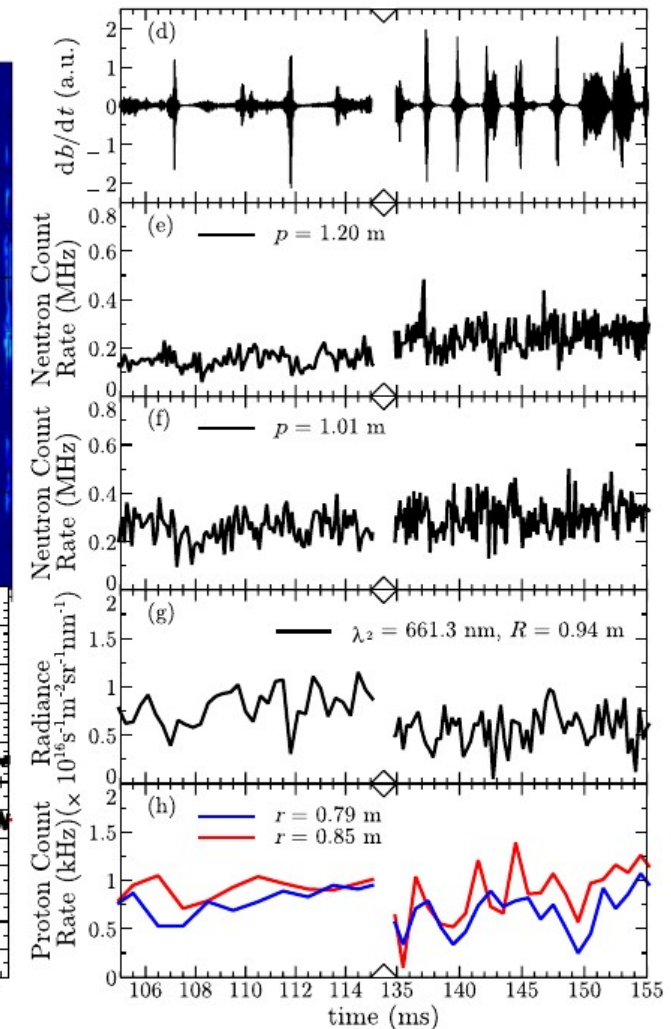
- FI ejection due to bursting TAEs is small however
- strong redistribution from the core to the edge is inferred from the need to impose an ad-hoc **anomalous fast ion diffusion coefficient** $D_a = 2.5 \text{ m}^2 \text{ s}^{-1}$

$$\Gamma_{\text{FI}} = -D_a \nabla_r n_{\text{FI}}$$

Spectrogram and neutron rates

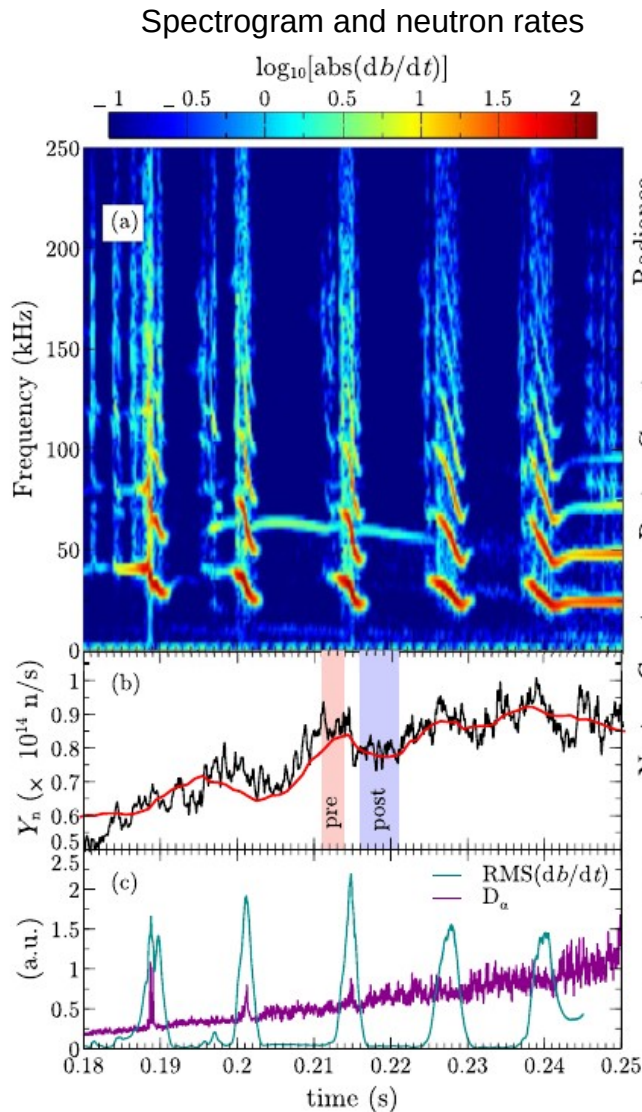


FI diagnostics time traces (@ 1ms)

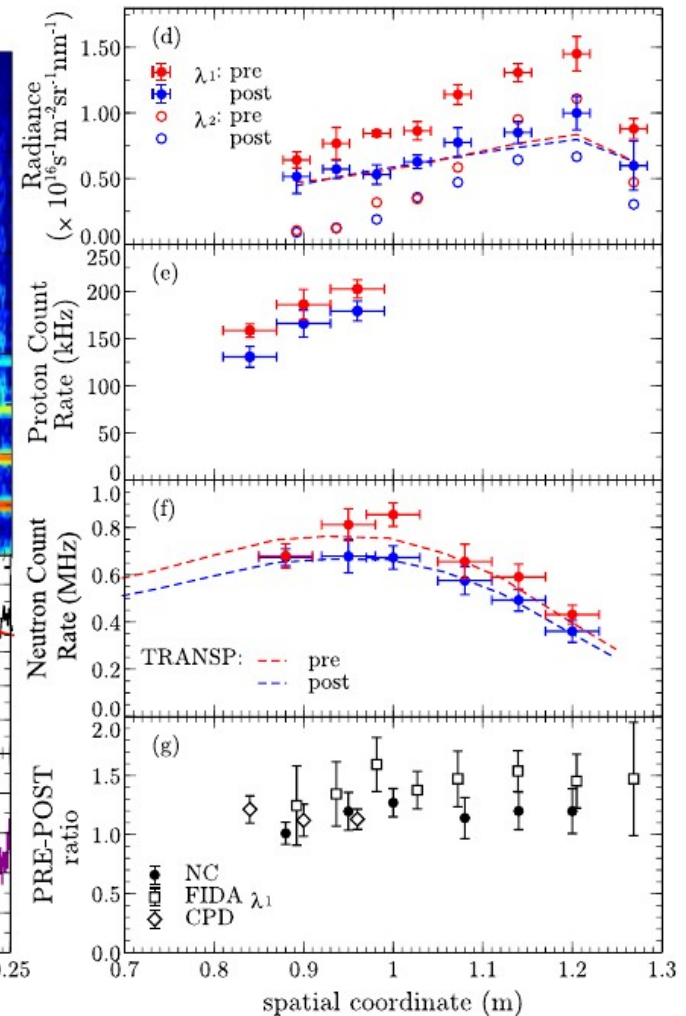


Energetic fast ions in MAST: bursting FBs phase #29976

- An overall decrease in the fast ion population is observed by all FIDs across the measured radial positions;
- The trend of the FC neutron rate is matched by setting $D_a = 0.5 \text{ m}^2 \text{ s}^{-1}$
- Drops at the fishbone bursts have been more difficult to reproduce in a consistent manner:
 - Selectively removing trapped/barely passing ions at high energies reproduced the behaviour of the global neutron rate and NC signal but failed to account for the observed reduction in FIDA signal
 - while removing passing fast ions in a narrow region of pitch ($|\lambda| \in [0.88 - 0.91]$) at all energies up to the injection energy was required to match the relative size of the drops in FIDA signal, but failed to reproduce the observed drops in NC signal.



FI diagnostics spatial profiles (@ 2-3 ms)



Modelling the effect of TAEs and FBs using the reduced fast ion transport (“kick”) model

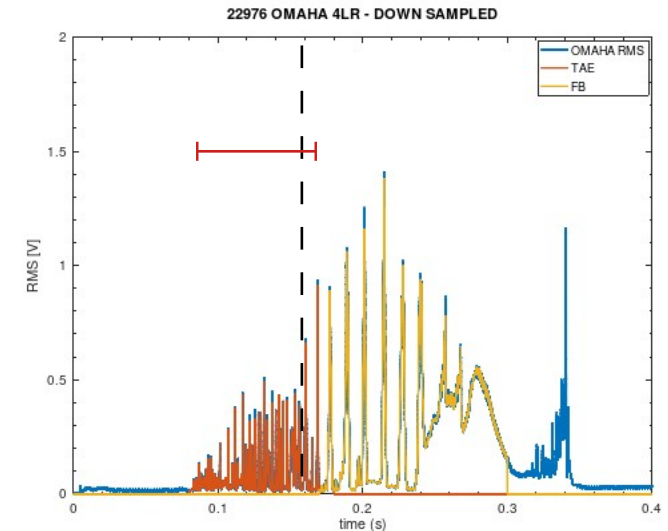
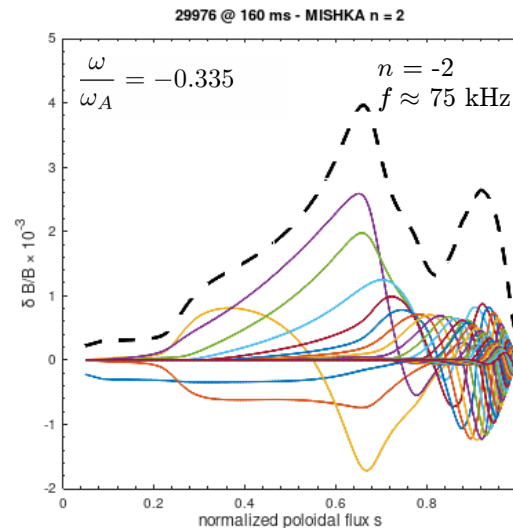
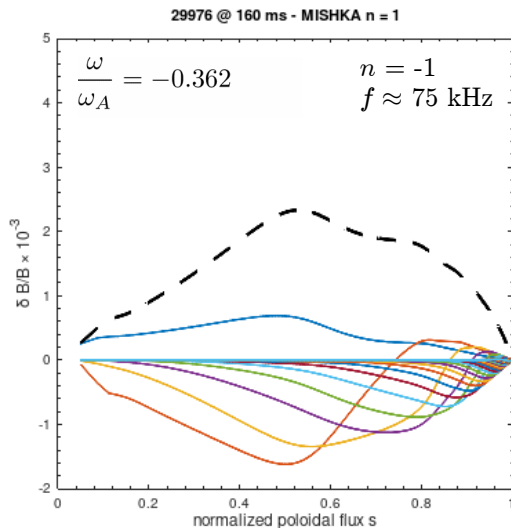
Estimate of the “kicks” in energy and toroidal canonical momentum on the fast ions due to perturbations requires:

(1) Modelling the perturbation spatial structure
(for example at $t = 160$ ms)

(2) its amplitude in time

MISHKA: all m but ≤ 0 used in the calculation of $p(\Delta E, \Delta P_\zeta)$:

Time evolution of the perturbation based on the RMS of a Mirnov coil signal.



NB: the eigenfunctions are zero at the LCFS while the perturbation is measured outside the LCFS! Difficult to constrain the eigenfunction amplitude with external measurements.

RMS window width = 512 samples; time intervals for

- TAE: [80, 170] ms
- Set to zero outside and downsampled (0.128 ms time step).
- TAE mode amplitude x 3 used in ORBIT

TAE kick probability matrix pdf at $t = 160$ ms

$$p(\Delta E, \Delta P_\zeta | E, P_\zeta, \mu)$$

The effect of the modelled perturbation(s) on the FI motion, i.e. the amplitude of the kicks, is calculated using the guiding-centre code ORBIT.

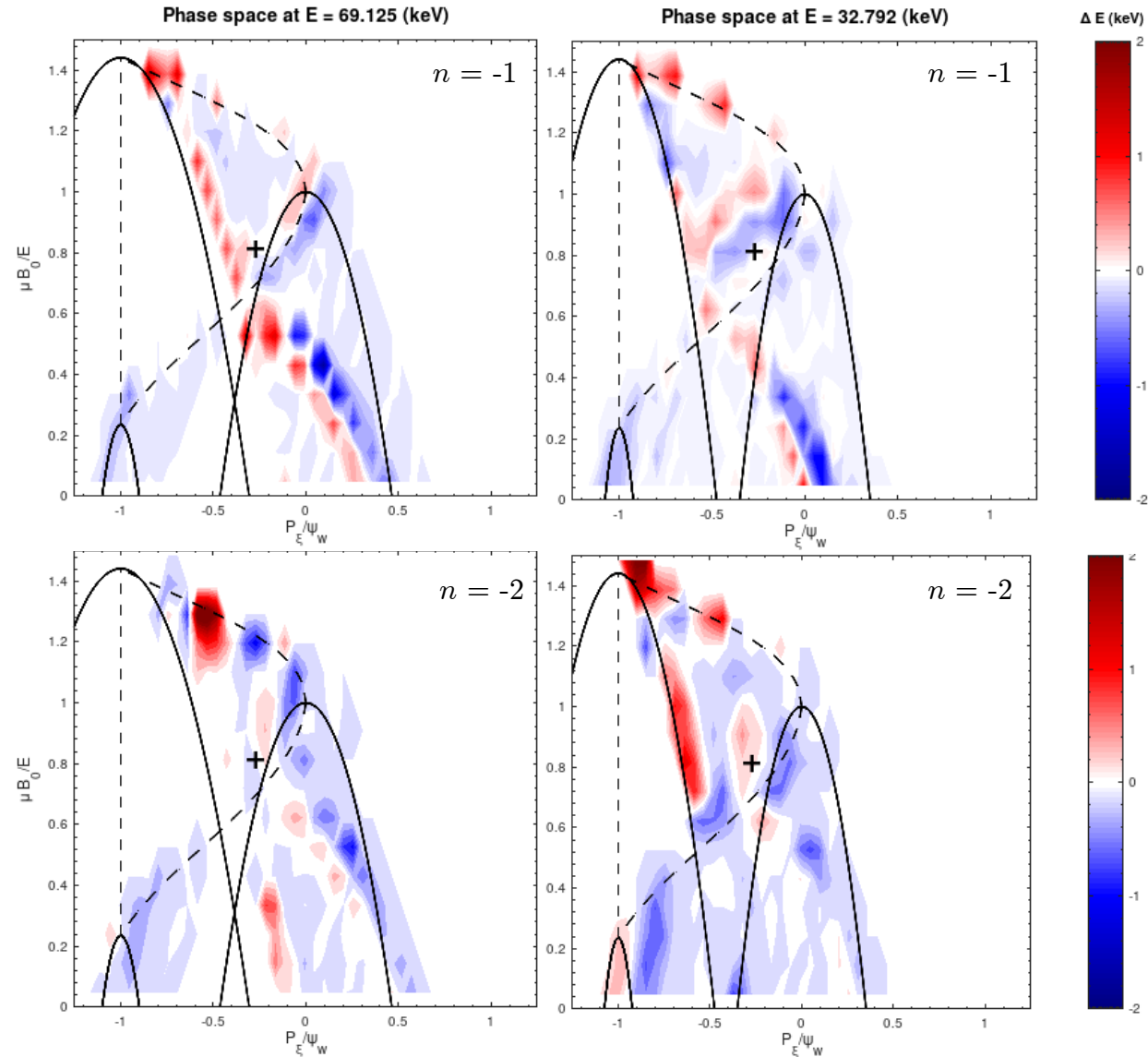
Typical settings:

- 29×29 bins $\Delta E \times \Delta P_\zeta$, 1 ms simulation time
- $12 E \times 40 P_\zeta \times 16 \mu$
- 2000 particles per run uniformly sampled in CoM phase space (30 runs).

Average amplitude of the kicks that a FI with a given energy will experience depending on its P_ζ and μ :

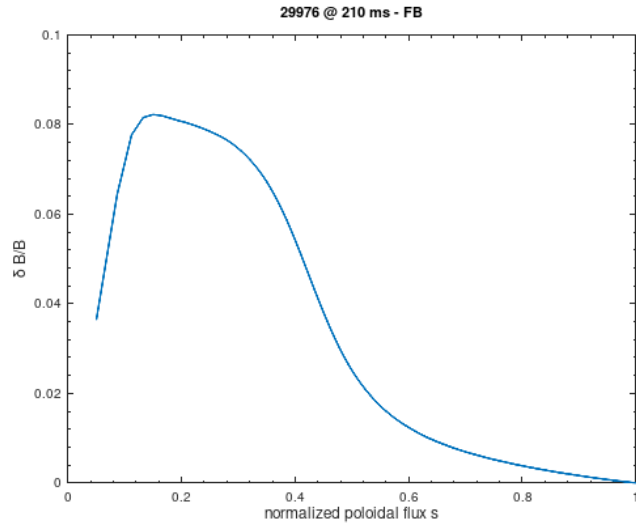
$$\Delta E_{\text{RMS}} = \frac{\sum_i \Delta E_i S(\Delta E_i | E, P_\xi, \mu)}{\sum_i S(\Delta E_i | E, P_\xi, \mu)}$$

$$S(\Delta E_i | E, P_\xi, \mu) = \sum_j S(\Delta E_i, \Delta P_{\xi,j} | E, P_\xi, \mu)$$



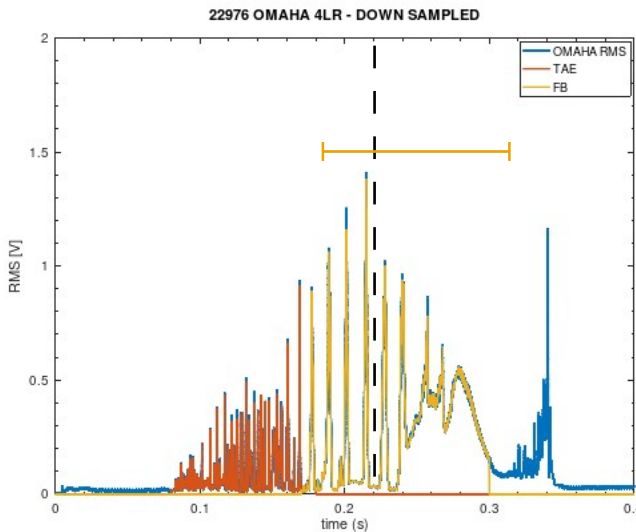
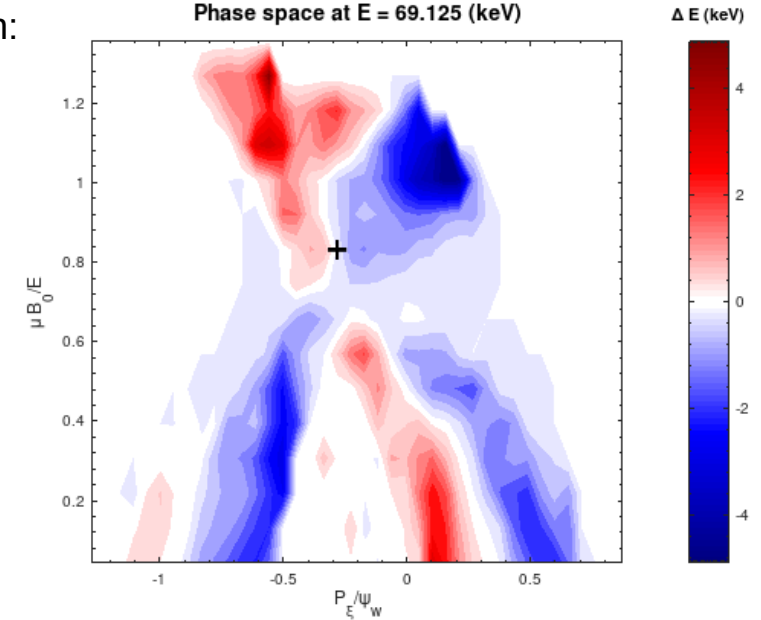
Modelling the perturbation for the calculation of the kicks pdf at $t = 210$ ms

Analytical expression for $(m,n) = (1,1)$ displacement of the plasma column:



$$p(\Delta E, \Delta P_\zeta | E, P_\zeta, \mu)$$

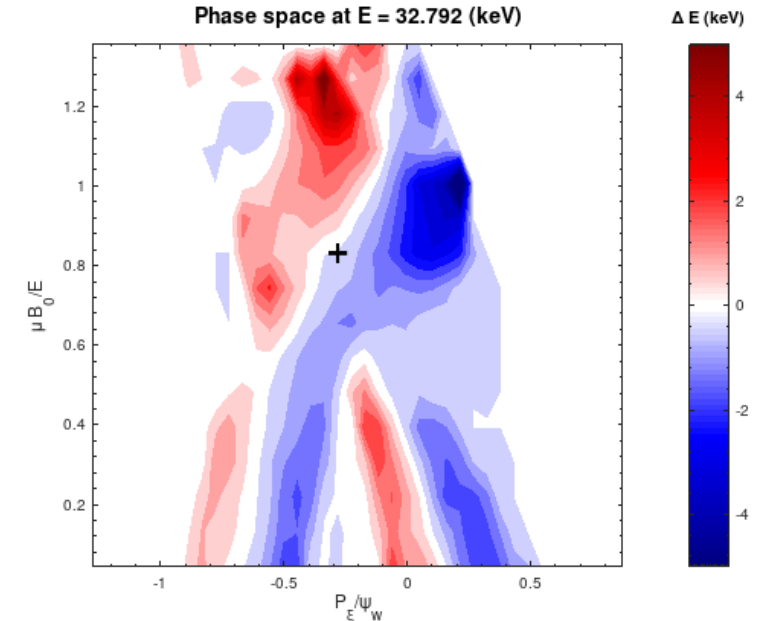
Phase space at $E = 69.125$ (keV)



Drops in FIDA rates correlated with the RMS of the Mirnov coils signal rather than the amplitude of the perturbation.

Drops in FC rates correlated to amplitude of the perturbation.

Phase space at $E = 32.792$ (keV)



Time intervals for

- FB: [170, 300] ms
- Set to zero outside and downsampled (0.128 ms time step)

TRANSP simulation of the neutron rates using the kick matrices

- A “free” parameter of the model is the amplitude of the perturbation A_M (i.e. of the kicks)
- In this study, adjusted iteratively to match the fission chamber neutron rates

First results:

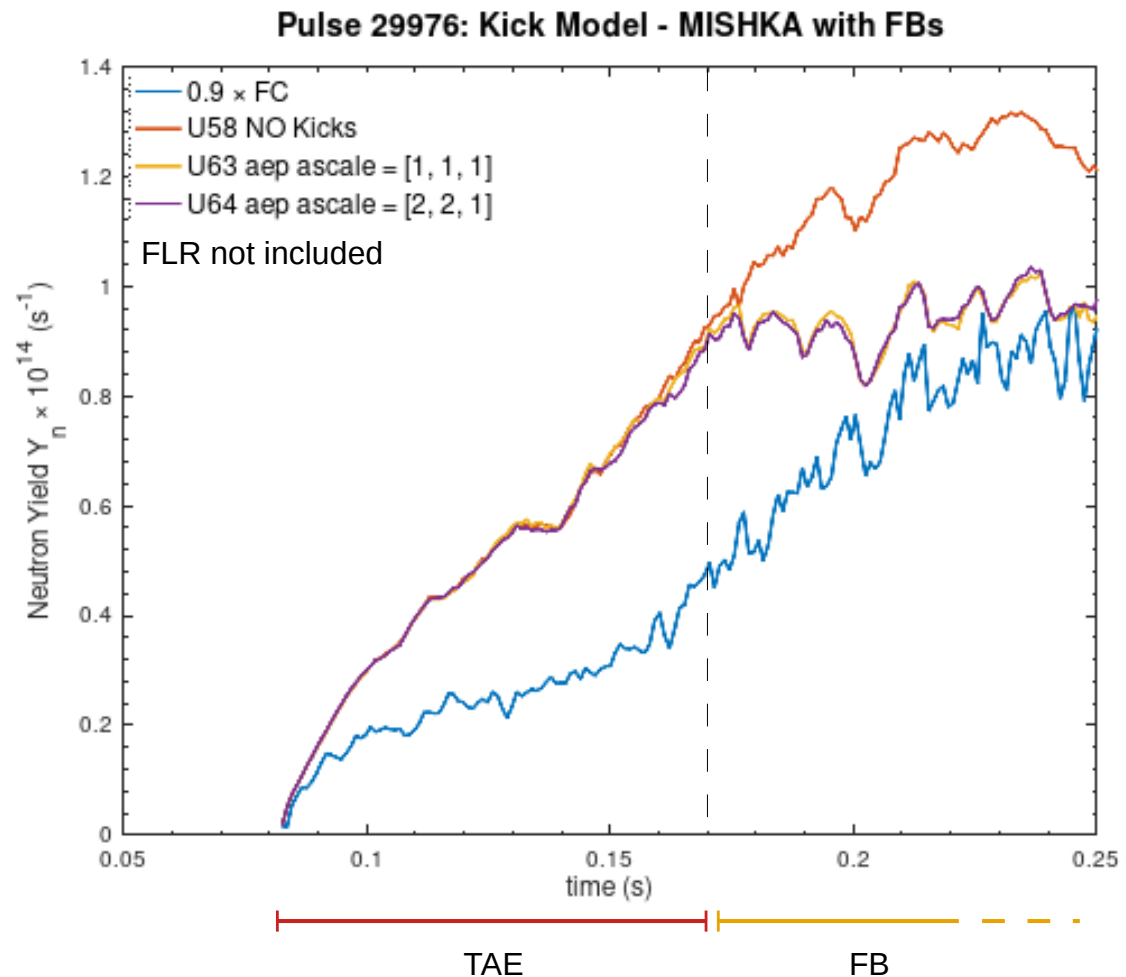
1. TAEs with approximately 75 kHz frequency have little effect on the neutron rate;
2. increasing the amplitude further causes the TRANSP equilibrium solver to fail due to a too large radial displacement of the plasma;
3. amplitude scaled at constant A_M :

$$k \times p(\Delta E, \Delta P | E, P_\zeta, \mu, A_M)$$

but a better choice is:

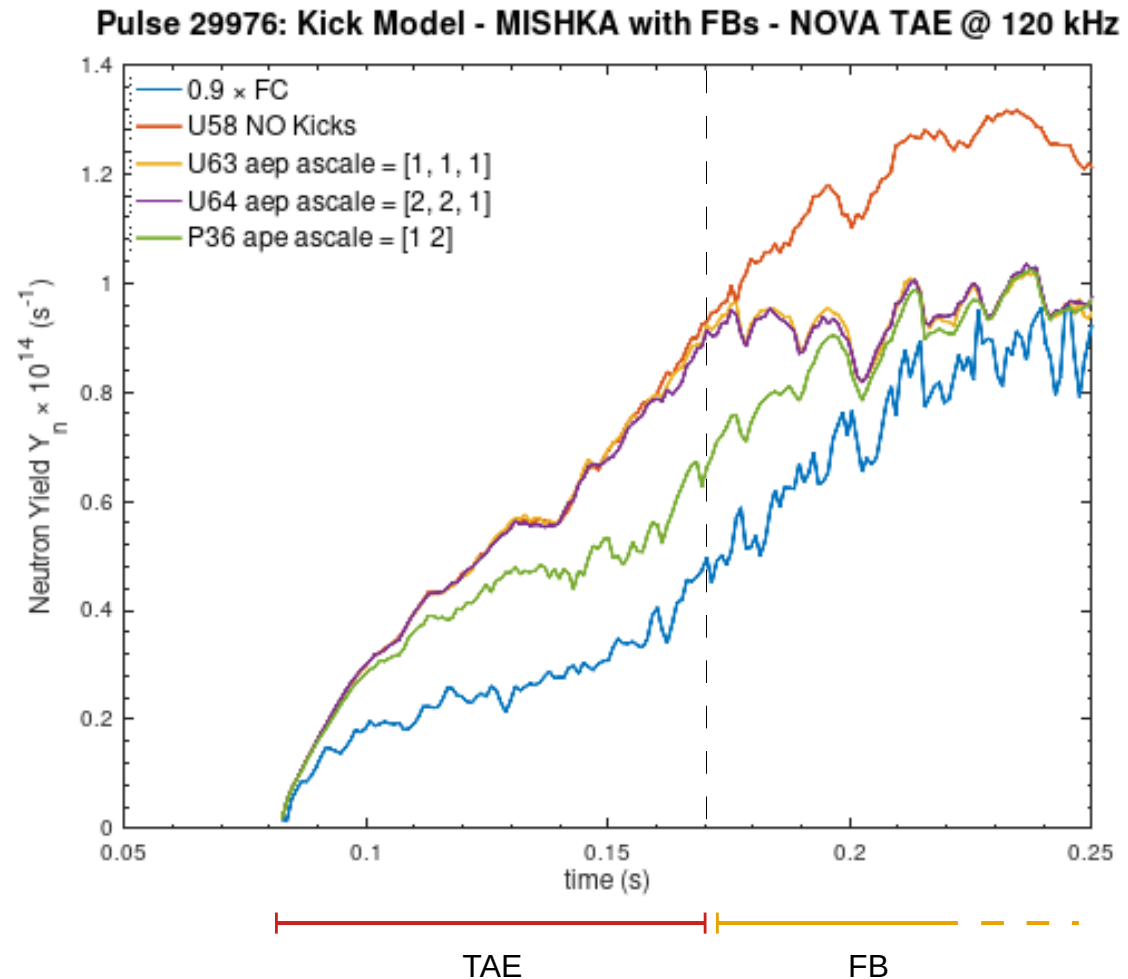
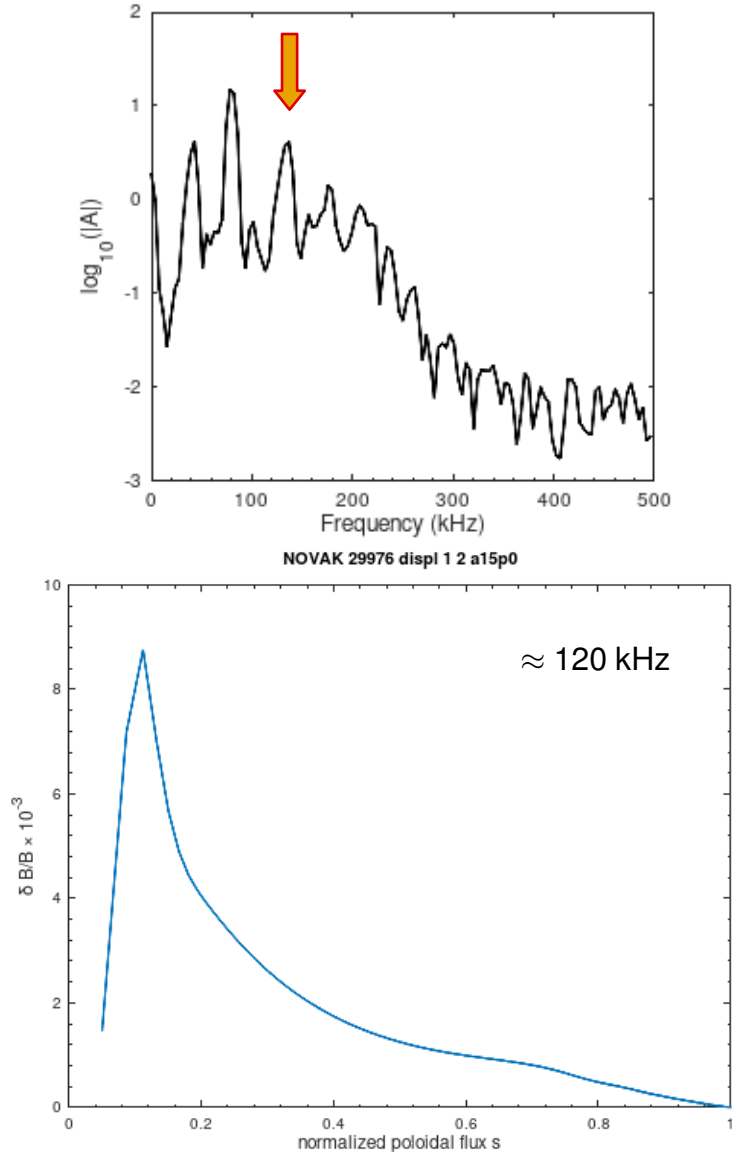
$$p(\Delta E, \Delta P | E, P_\zeta, \mu, k \times A_M)$$

4. FB recipe has a clear impact in suppressing the neutron rates.



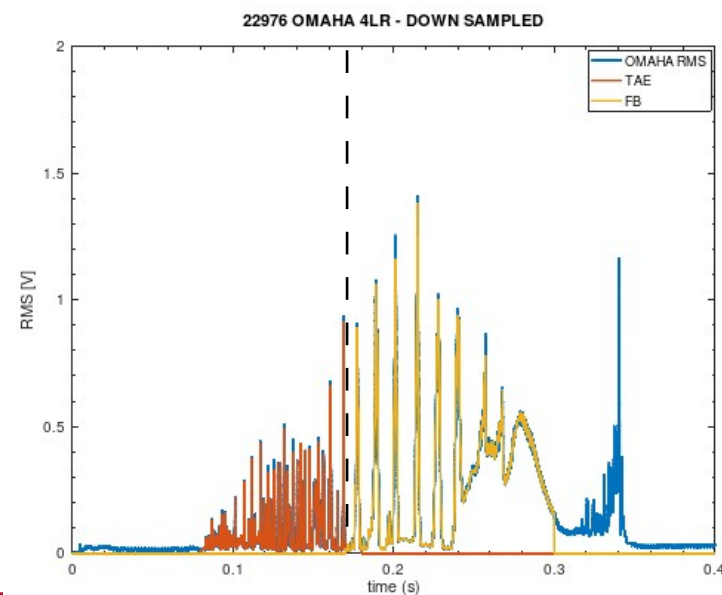
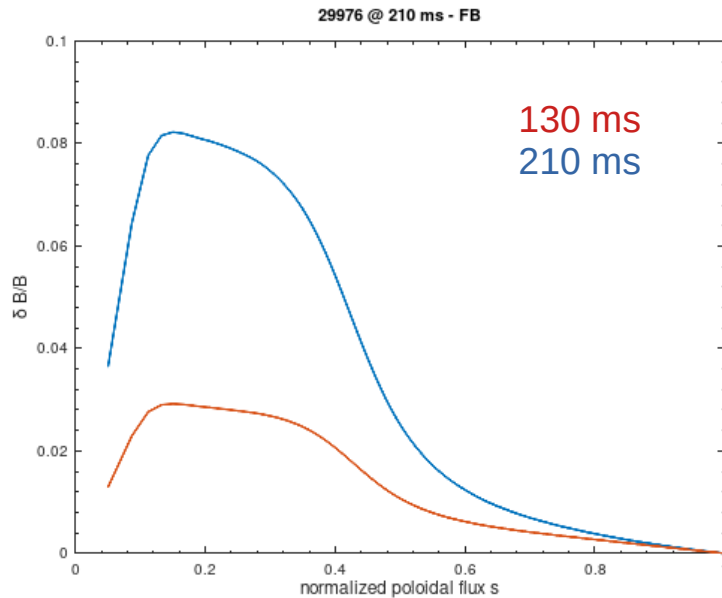
Kick-model with TAEs and FBs perturbations (NOVA ≈ 75 and 120 kHz)

Inclusion of higher frequency TAEs eigenfunctions calculated by NOVA-K on TRANSP equilibrium (**different than the one used in MISHKA**) has a somewhat larger effect on the neutron rates:

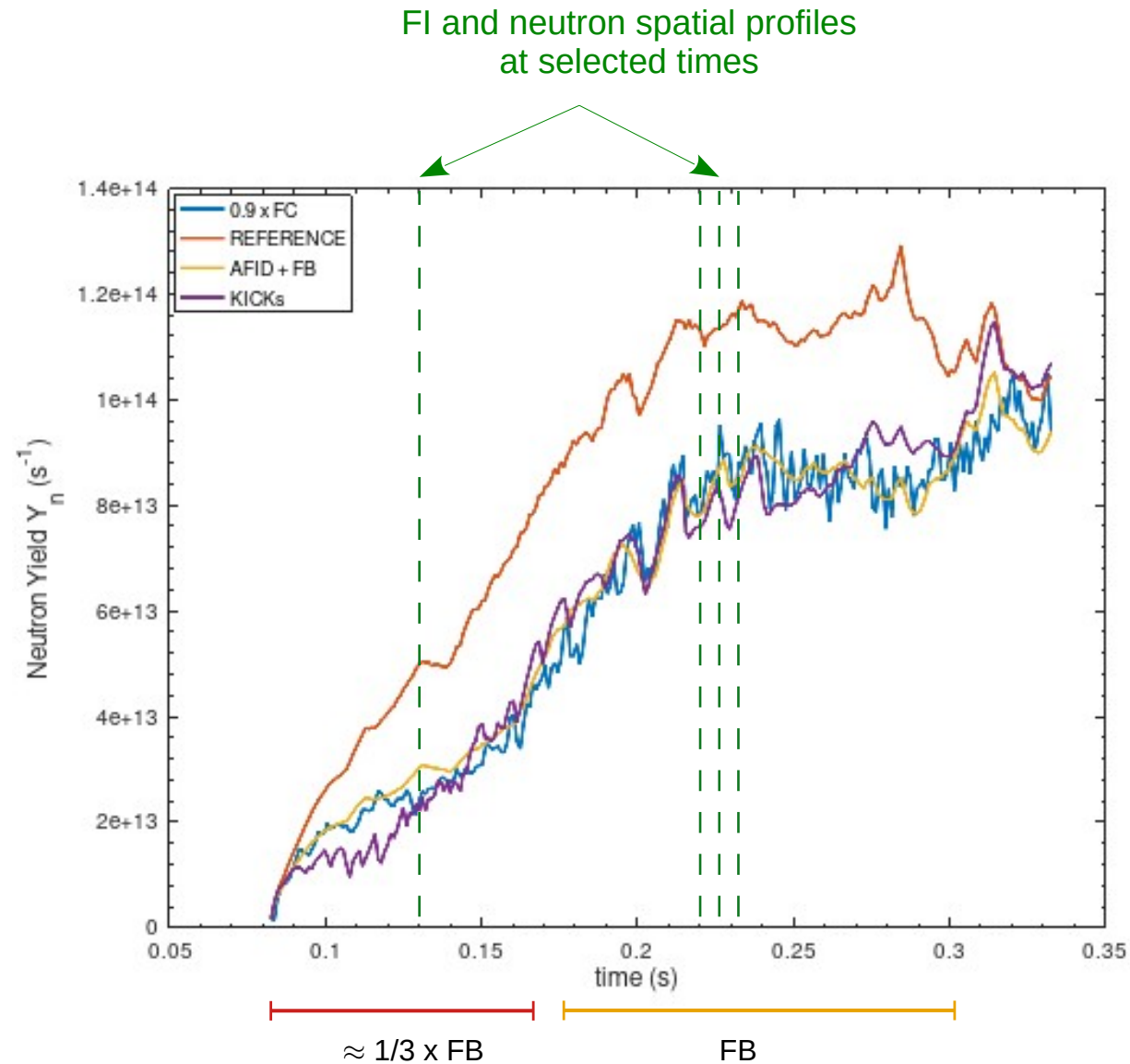


Perturbations used for profile analysis

Analytical expression for $(m,n) = (1,1)$ of the plasma displacement at 130 and 210 ms but with different amplitude:



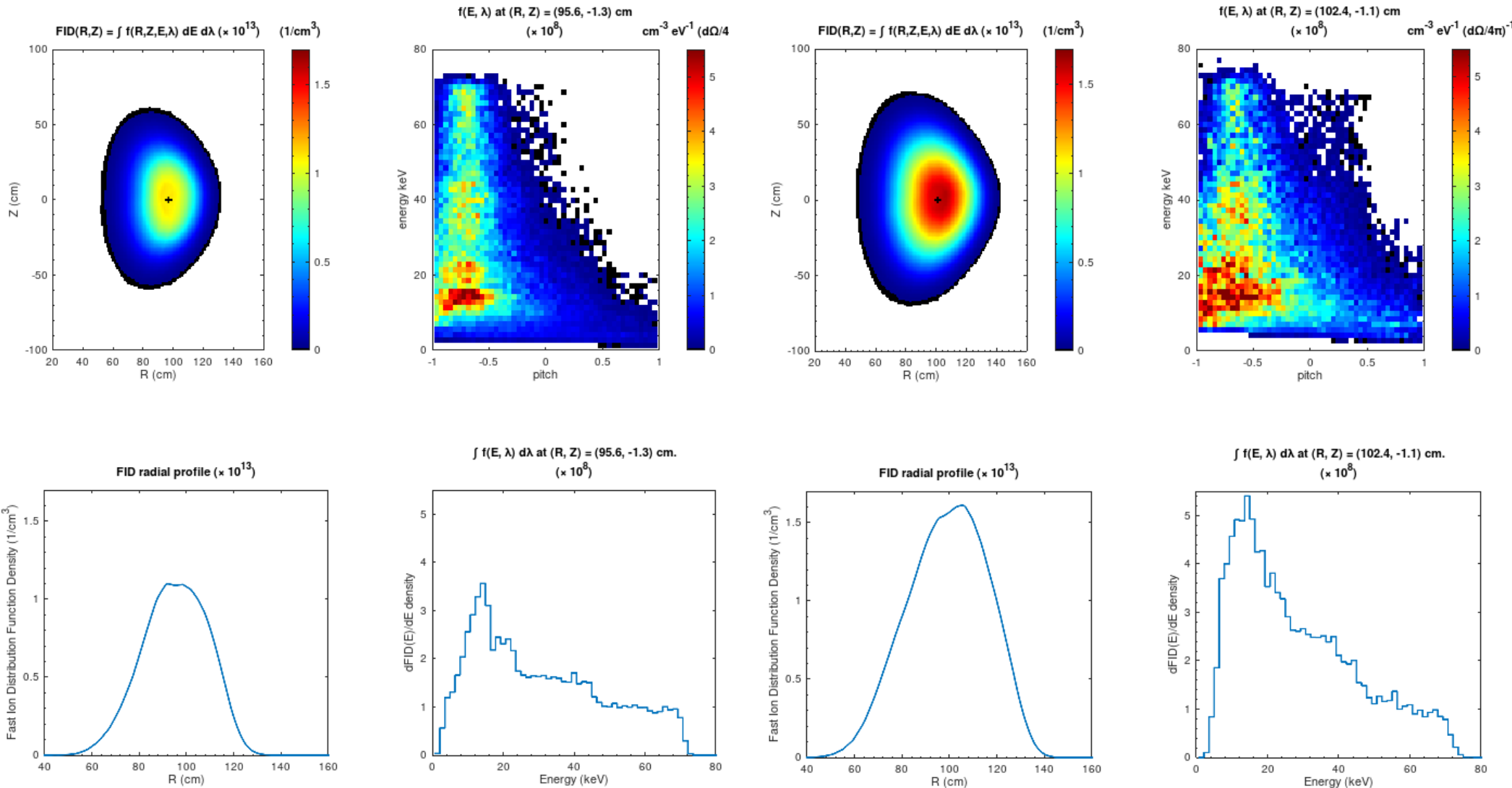
The “kick” and “AFID” models provide similar agreement between predicted and measured neutron rates:



Fast ion spatial distribution for reference TRANSP run (no AFID/Kicks)

$t = 130$ ms

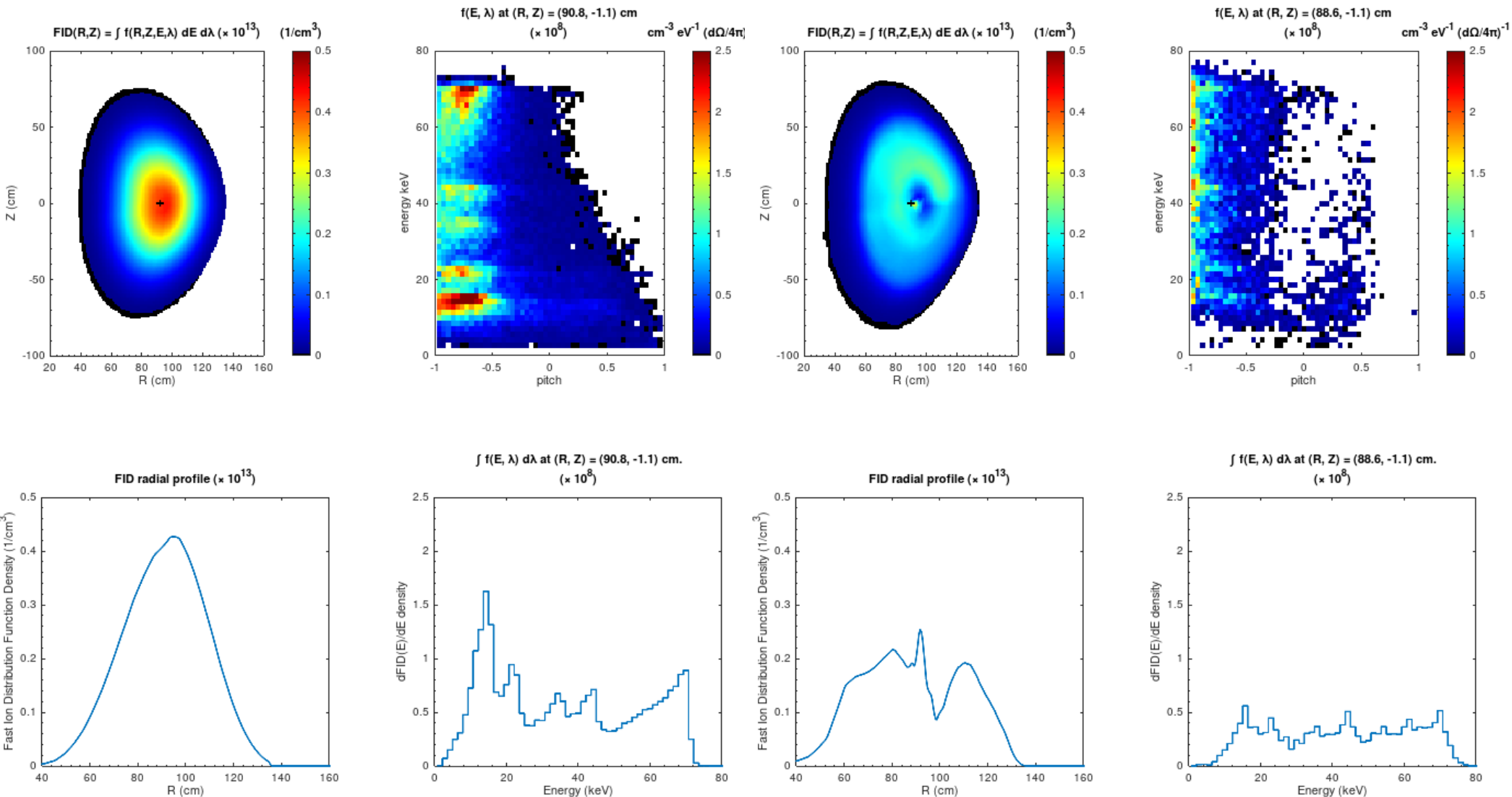
$t = 212$ ms



Fast ion spatial distribution: AFID vs Kick model at 130 ms

Anomalous Fast Ion Diffusion

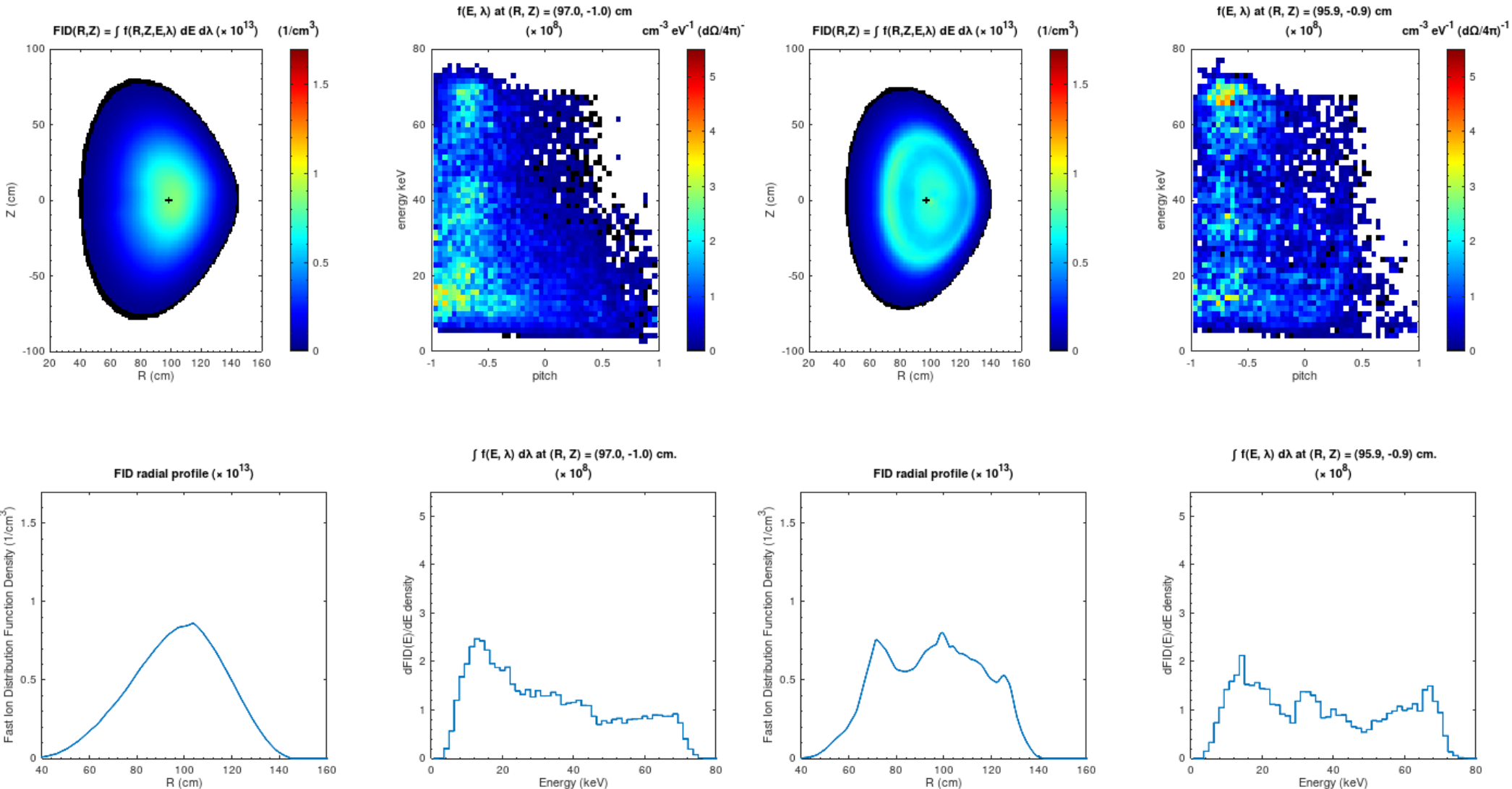
Kick-model



Fast ion spatial distribution: AFID vs Kick model at 212 ms (pre-FB burst)

Anomalous Fast Ion Diffusion

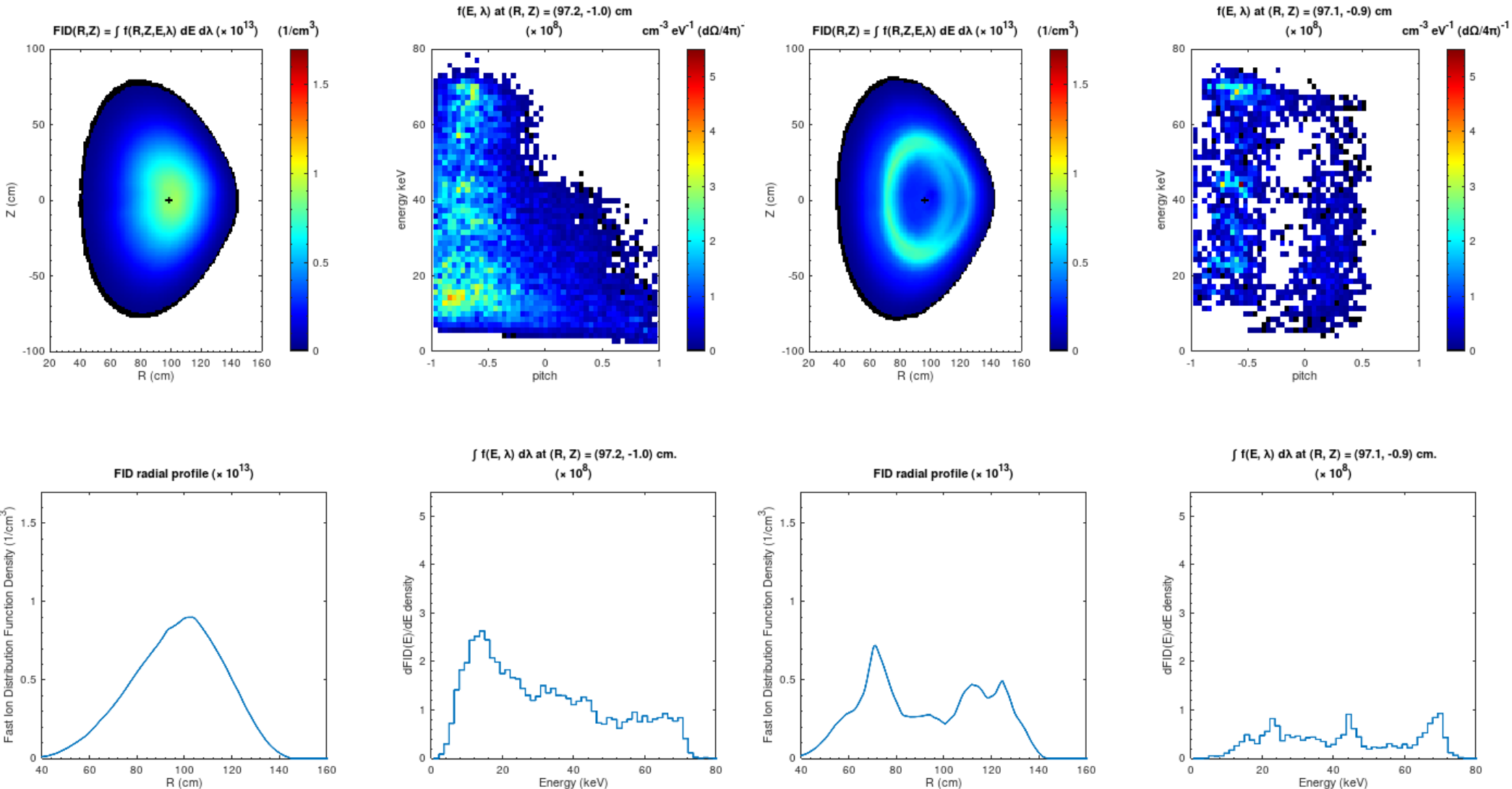
Kick-model



Fast ion spatial distribution: AFID vs Kick model at 217 ms (post-FB burst)

Anomalous Fast Ion Diffusion

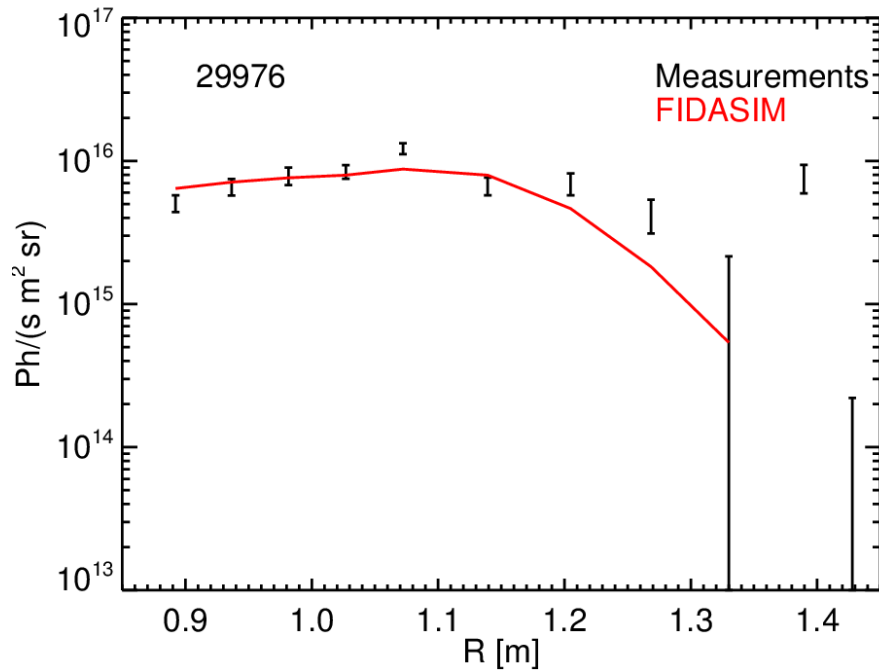
Kick-model



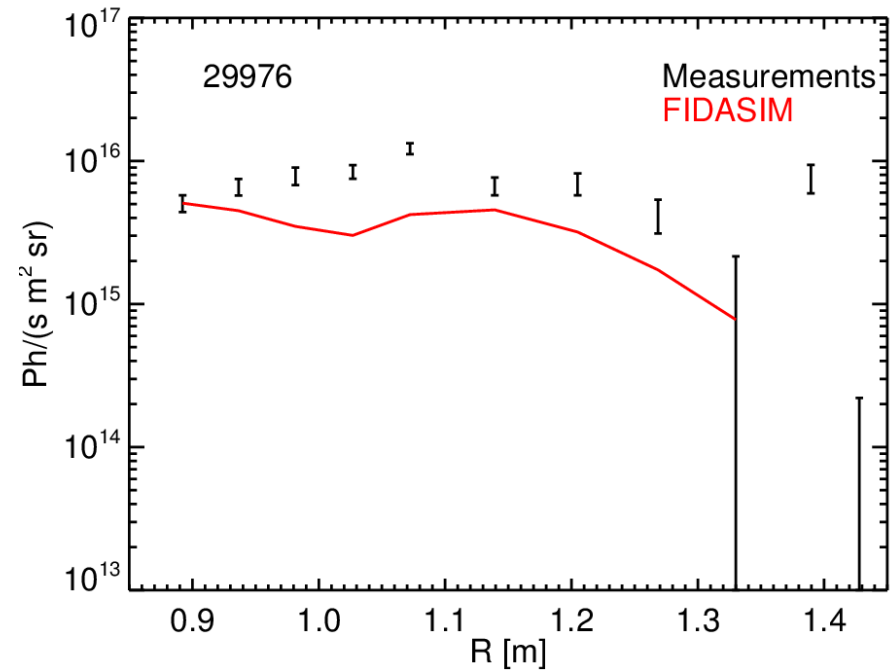
Fast ion spatial distribution: comparison with experimental profiles (130 ms)

TRANSP/FIDASIM compared with tangential FIDA measurements:

Anomalous Fast Ion Diffusion



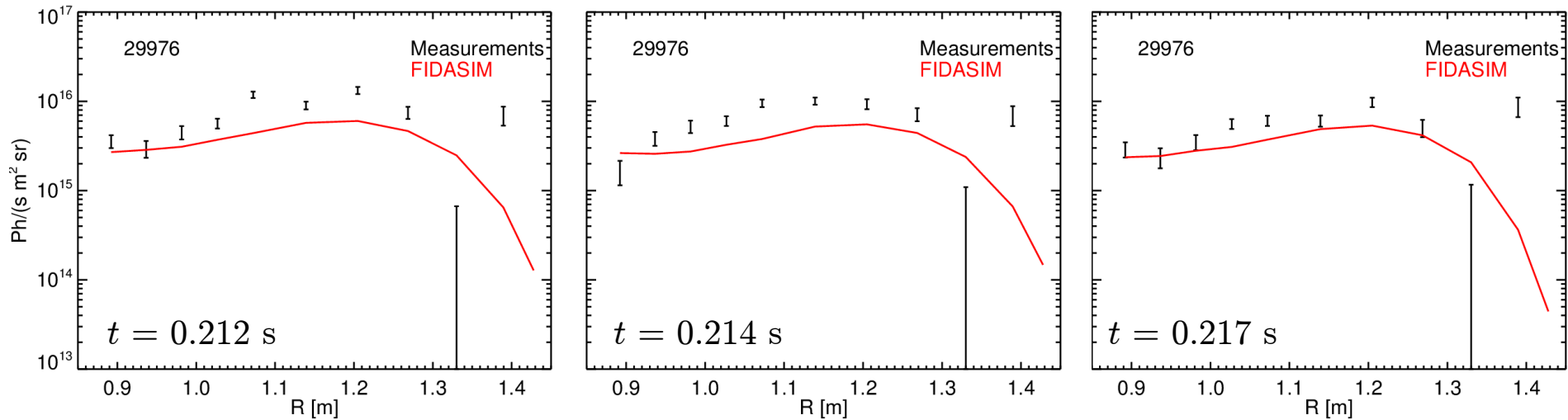
Kick-model



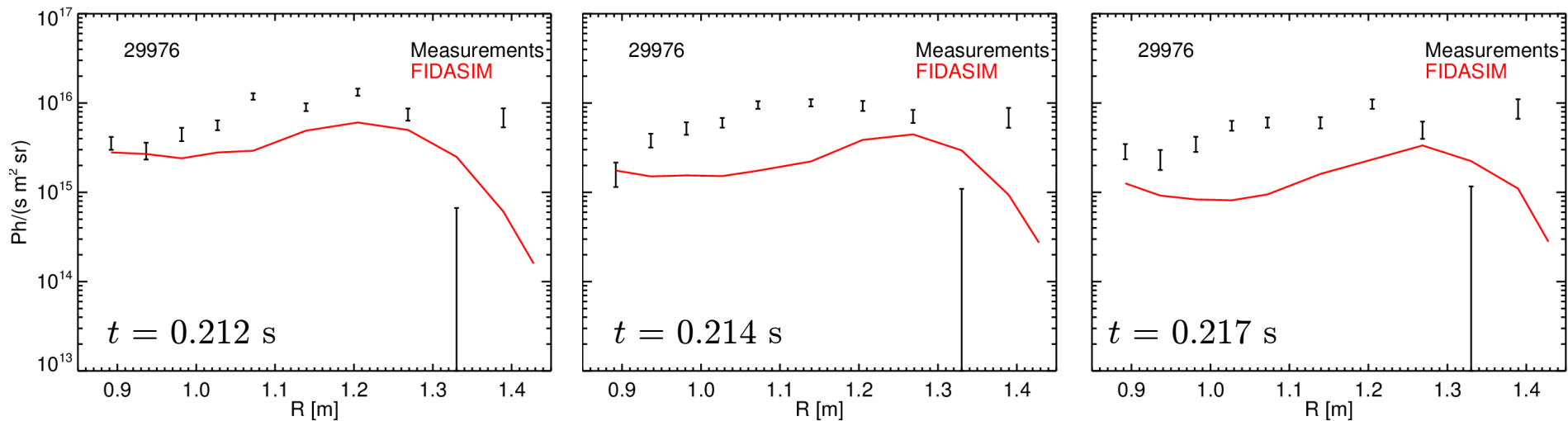
FIDA measurements integration time: 1 ms

Fast ion spatial distribution: **TRANSP/FIDASIM** comparison with experimental profiles (FB)

Anomalous Fast Ion Diffusion

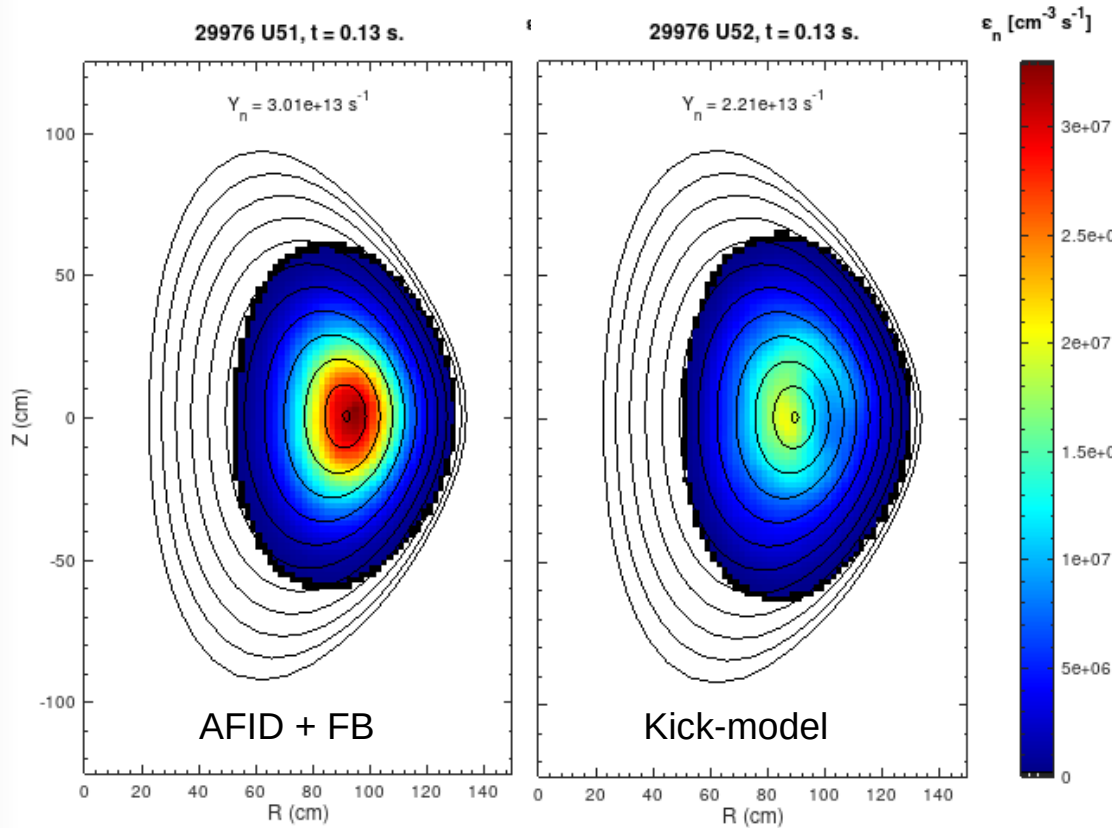


Kick-model

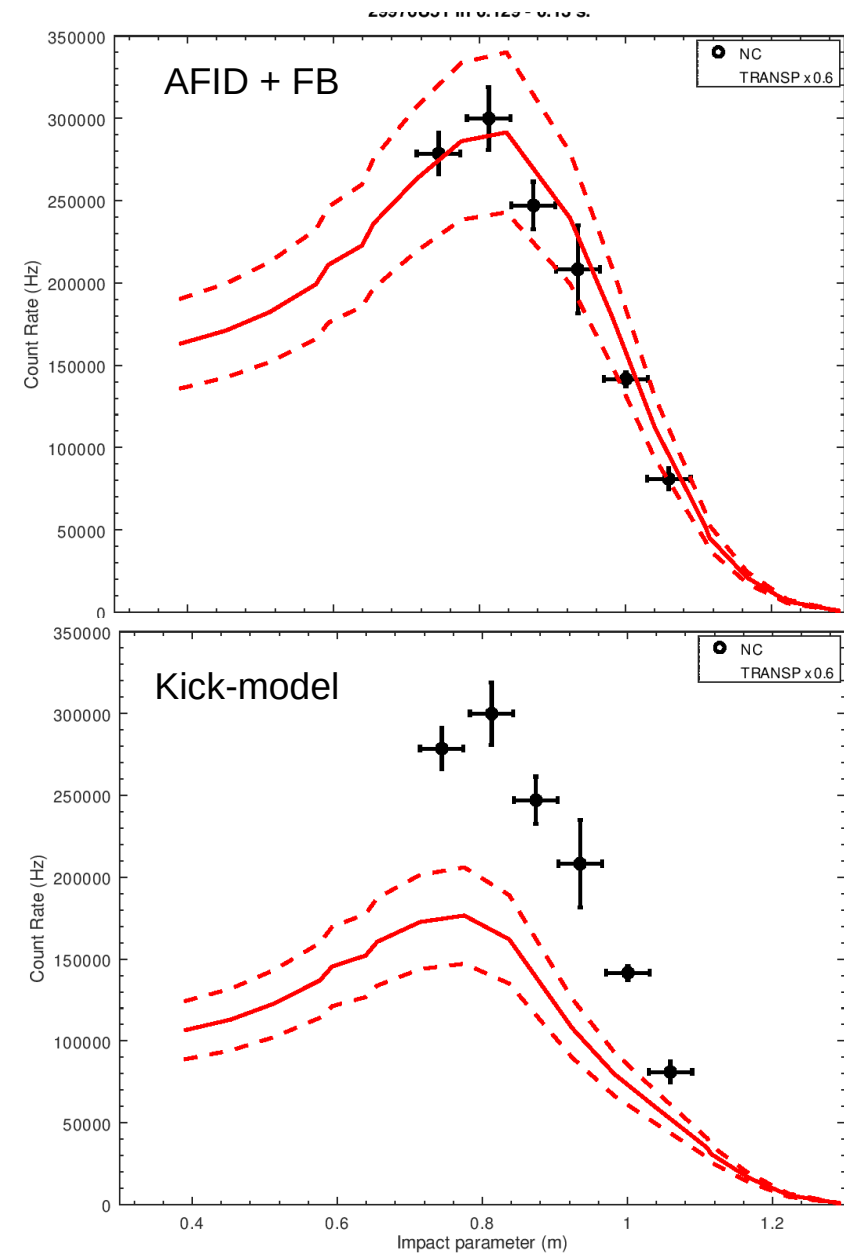


Neutron emissivity spatial distribution: comparison with experimental profiles (130 ms)

Neutron emissivity from TRANSP:

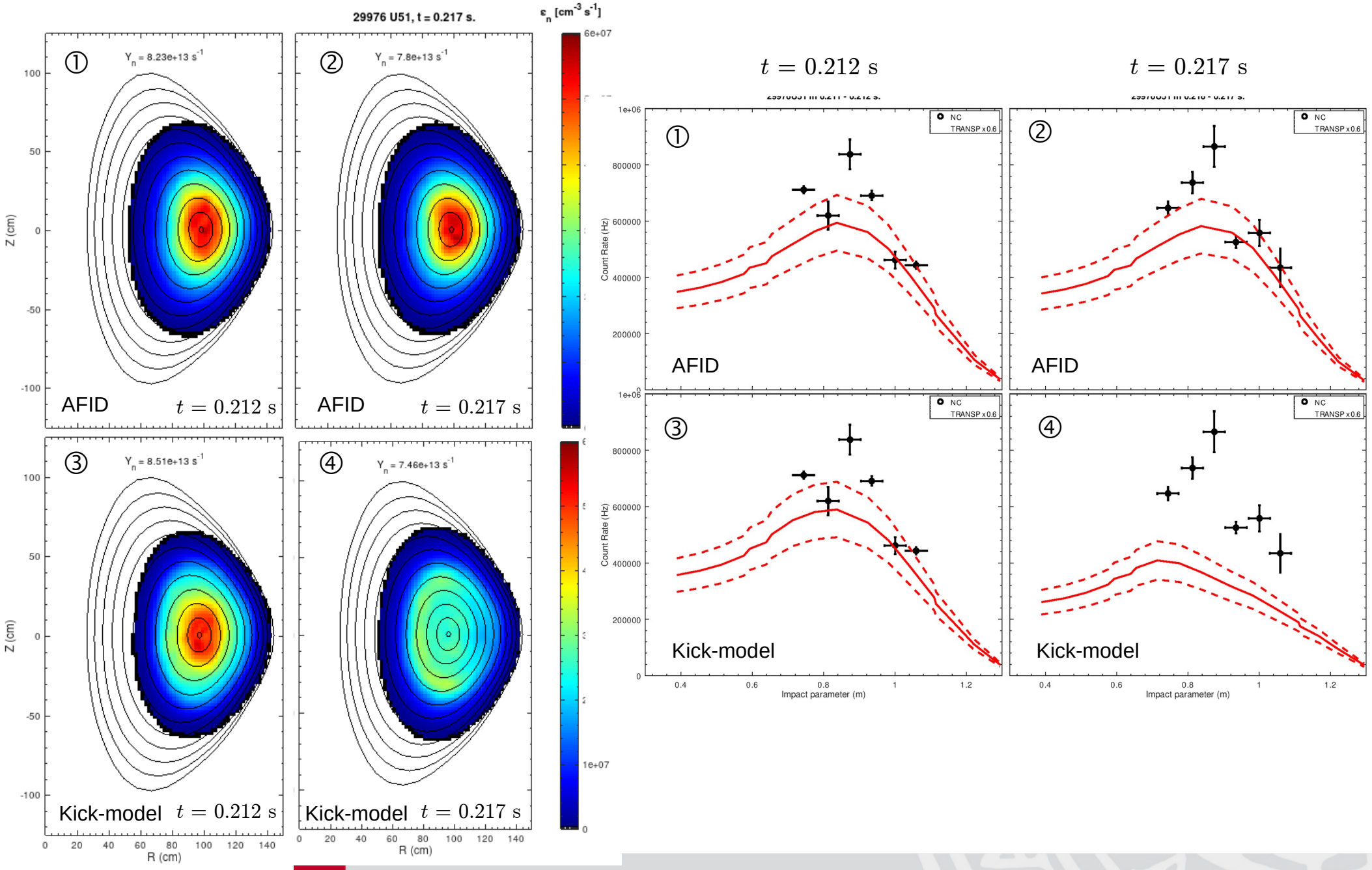


- Neutron camera with integration time of 1 ms
- Profiles from multiple, similar plasma discharges
- TRANSP rates scaled by a constant factor of 0.6^(*)



^(*) Cecconello M, et al. Discrepancy between estimated and measured fusion product rates on MAST using TRANSP/NUBEAM. Nucl Fusion. 2019 Jan 1;59(1):016006.

Neutron emissivity spatial distribution: comparison with experimental profiles (FBs)



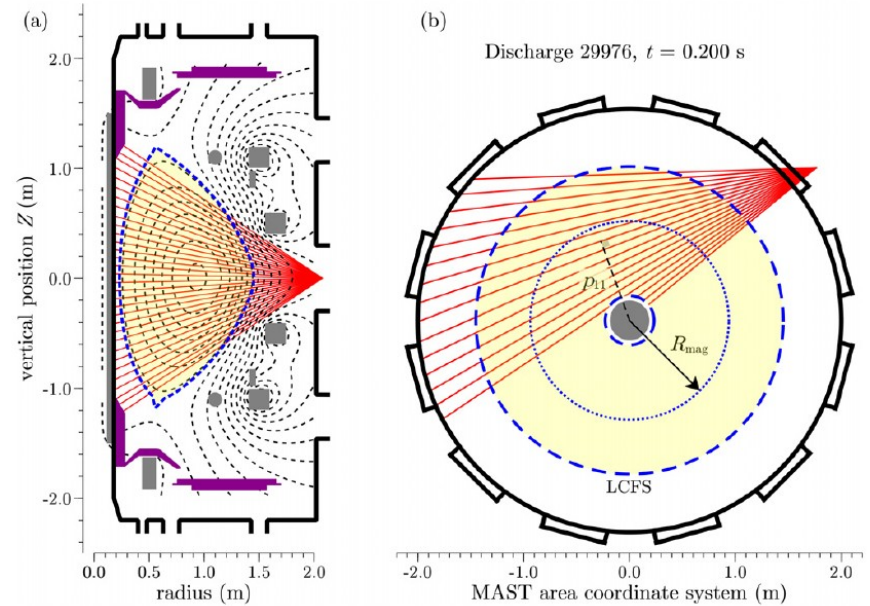
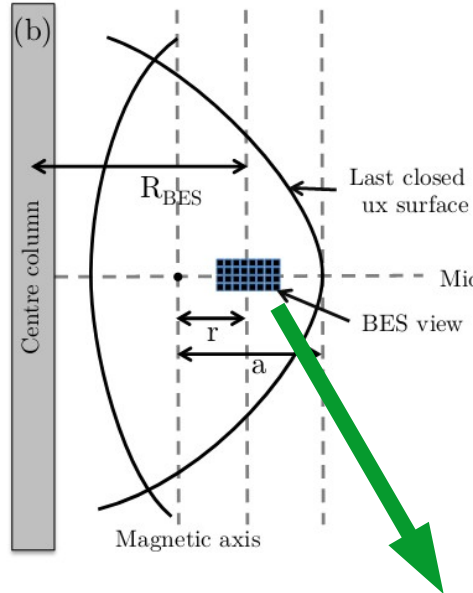
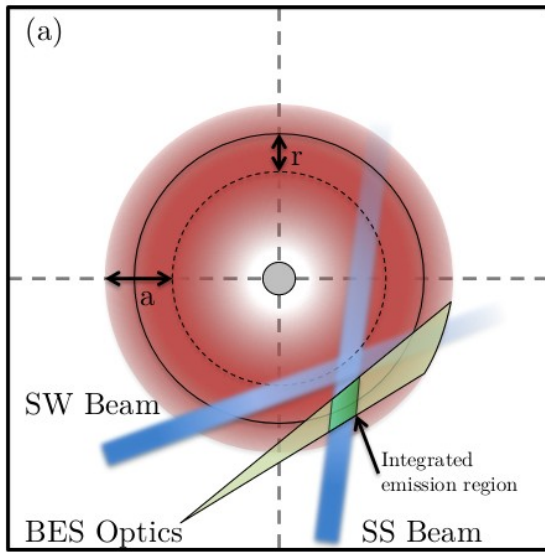
First conclusions from this preliminary study (work in progress)

- AFID provides a better qualitative match between the predicted radial profiles and FIDA and NC measurements;
- The kick-model, with a simple description of the FB displacement, is able to reproduce the global neutron rates but does not agree with FIDA and NC experimental profiles;
- Spatial **structure of the perturbation** clearly impact the redistribution of fast ions and the neutron rates:
 - At 75 kHz, the TAEs calculated by NOVA-K and MISHKA are localized at around $s \gtrsim 0.5$ with (almost) no resonances near the magnetic axis;
 - At 120 kHz, the TAEs calculated by NOVA-K have eigen functions with large amplitude near the magnetic axis resulting in some redistribution;
 - rigid plasma displacement is significantly larger than zero at radial positions;
- **Matching global measurements** (such as neutron rates) might not be sufficient to produce fast ion distribution close to experimental ones;
- Constraining the amplitude and spatial structure of the perturbation **using profile measurements** is necessary as some **measurements of core fluctuation amplitude and localization**
- Ideal modes are assumed to be responsible for the FI redistribution during the TAE phase: maybe we are in presence of an **EPM instead** with completely different frequency and eigenfunction?
- More work is clearly required...

Fluctuation diagnostics on MAST (and on MAST Upgrade)

External and internal:

- BES (8 x 4 chs, 14 x 6 cm, 2 MHz SF, 1 ms time resolution)
- SXR (multichannel, MHz SF)
- OMAHA and Mirnov pick-up coils (up to 1 MHz SF)

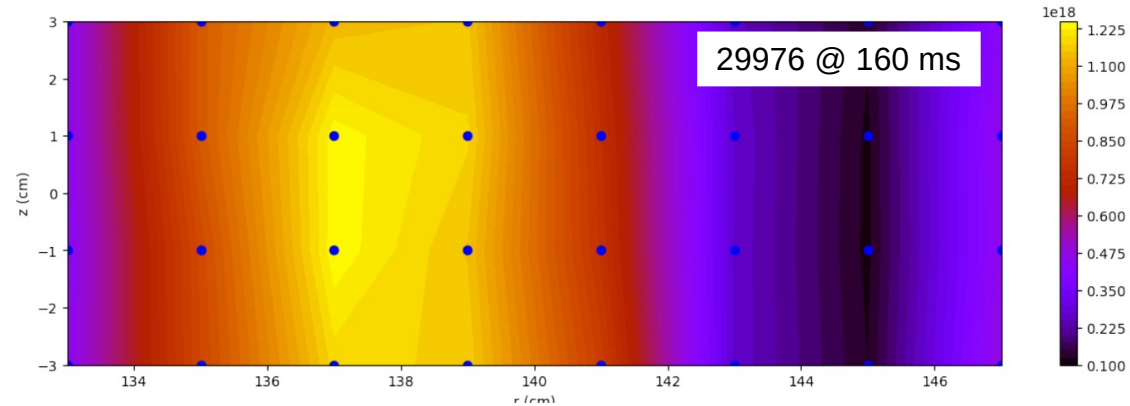


M. Cecconello et al. Impurity transport driven by fishbones in MAST, Nucl. Fusion 55 (2015) 032002

$$s \approx 0.97$$

M. Fox, Statistical structure of plasma turbulence from BES measurements in MAST and the effect of flow shear, PhD Thesis, Oxford Univ. (2016)

$$\tilde{n} = |(\tilde{I}_f(t)/I_o)_{TAE}| * n(r)$$



What next?

- More detailed work on:
 - TAEs modes eigenfunctions and of their amplitude using BES and SXR measurements
 - modelling of EPMS
 - increase time “resolution” with more than 2 temporal intervals and modes, especially during the current ramp-up phase;
- Kick-model applied to TRANSP runs with EFIT++ constrained equilibrium;
- Determination of resonance maps;
- In addition to FIDA and NC, comparison with CFPD;
- HALO/ORBIT kick-matrices comparison for FLR studies.

On MAST Upgrade;

- tangential FIDA and BES will be available on MAST Upgrade
- improved diagnostics: 6 channels NC and 12 channels CFPD
- new FI diagnostics: FILD and SSNPA
- several experiments dedicated to FI physics studies.

Additional material

- Kick model overview and work flow
- MHD spectroscopy and EFIT++ equilibrium for 29976 (MSE and TS constrained)
- MISHKA and NOVA-K eigenfunctions, FB and mode amplitude
- Kick matrices for MISHKA and NOVA-K modes
- Examples of kick modelling in NSTX/DIII-D
- 29976 Global parameters and spectrogram
- AFID on MAST: some examples and 29976 in particular
-
- MISHKA and NOVA-K eigenfunctions, FB and mode amplitude
- Kick matrices for MISHKA and NOVA-K modes
- FIDA and NC details
- More FIDA and NC profiles
-
-

The kick-model: an overview

The kick-model estimates the fast ions diffusion coefficients in the (λ, E) phase space by a combination of:

- an estimation of the time dependent perturbation's amplitude based on experimental measurements (Mirnov coils, BES, soft X-rays, reflectometry)
- an estimation of the perturbation(s) eigenfunction (spatial structure and mode numbers)
- a Monte Carlo simulation of the effect of such perturbation(s) on the motion of fast ions

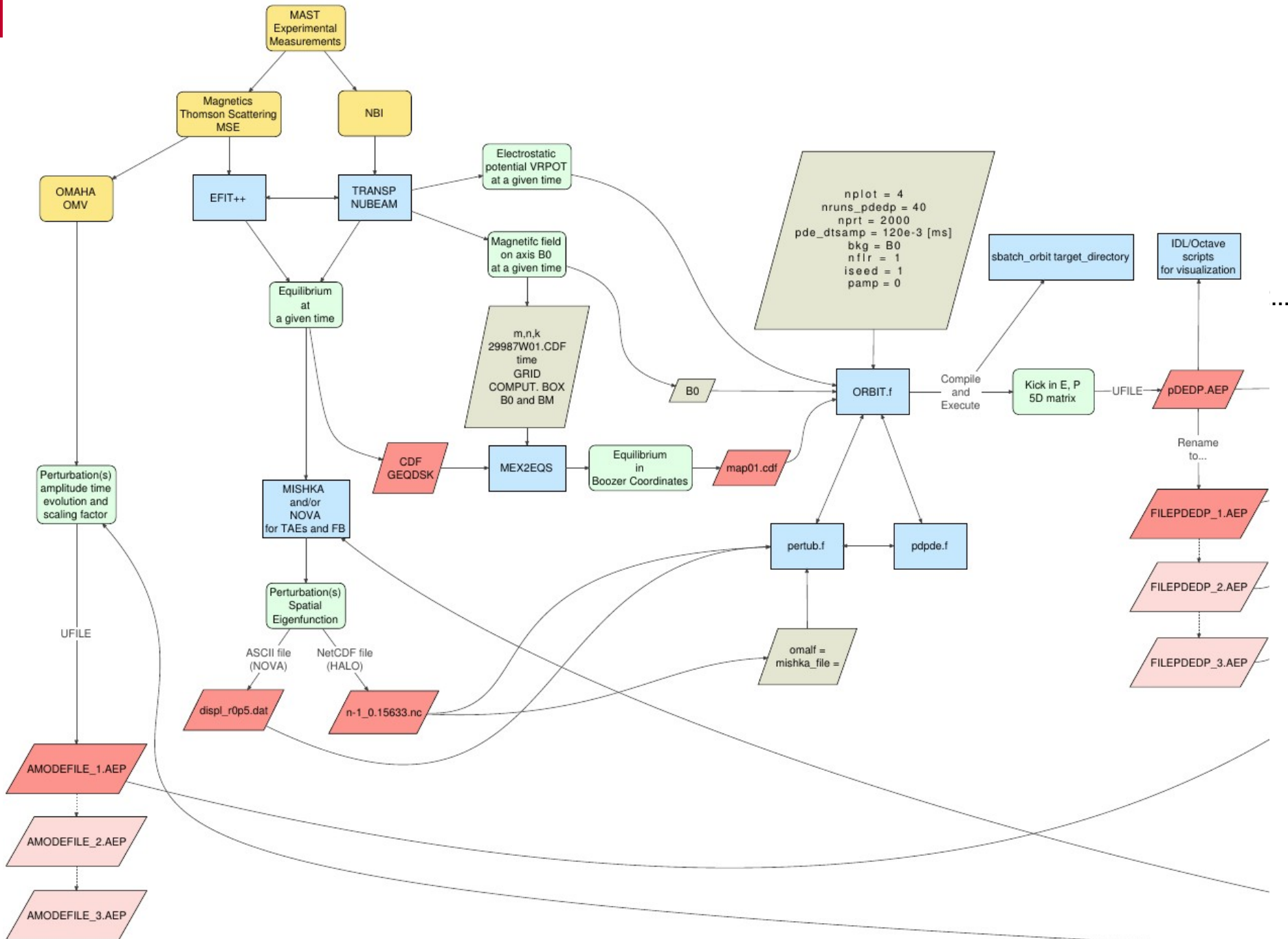
from which spatial diffusion emerges naturally from the diffusion in (λ, E) phase space due to changes in the fast ion orbit topologies.

Features and present limitations of the kick model:

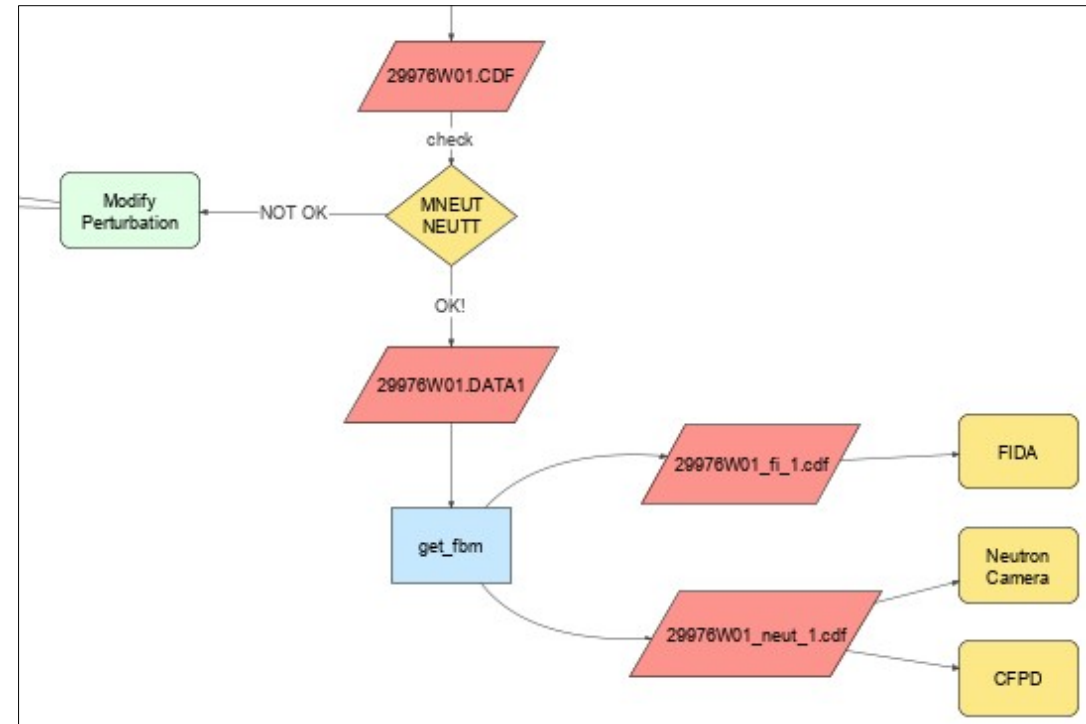
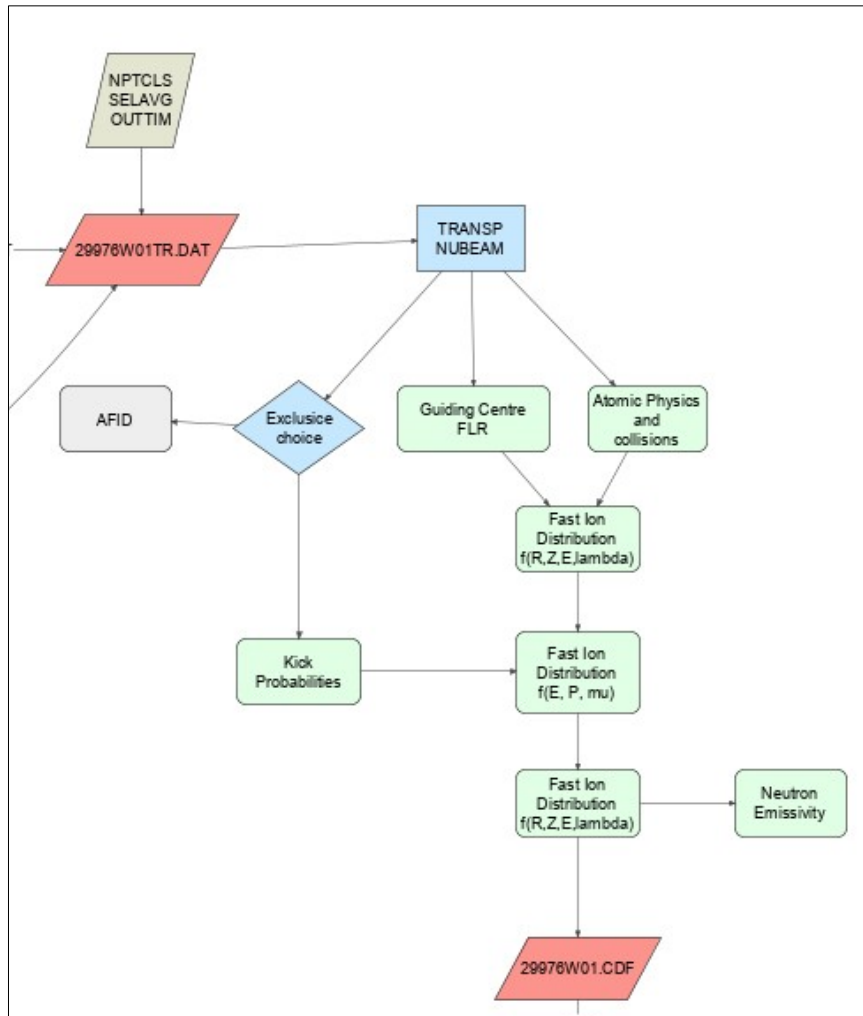
- requires a good equilibrium (possibly with MSE and kinetic profiles)
- it is equilibrium dependent, that is each phase in a discharge (ramp-up, steady-state) need separate modelling
- assumes conservation of the magnetic moment μ
- guiding-centre based (with FLR corrections)
- uses NUBEAM guiding-centre (with FLR corrections)
- degree of freedom (arbitrariness) in the scaling factor of the perturbation amplitude



Kick-model workflow: step 1



Kick-model workflow: step 2



The kick-model: the method

The kick-model integrates the following codes:

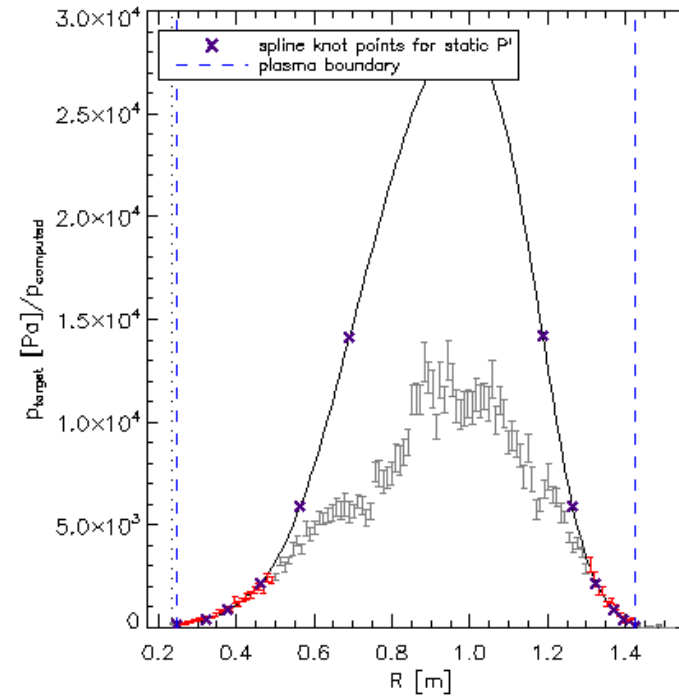
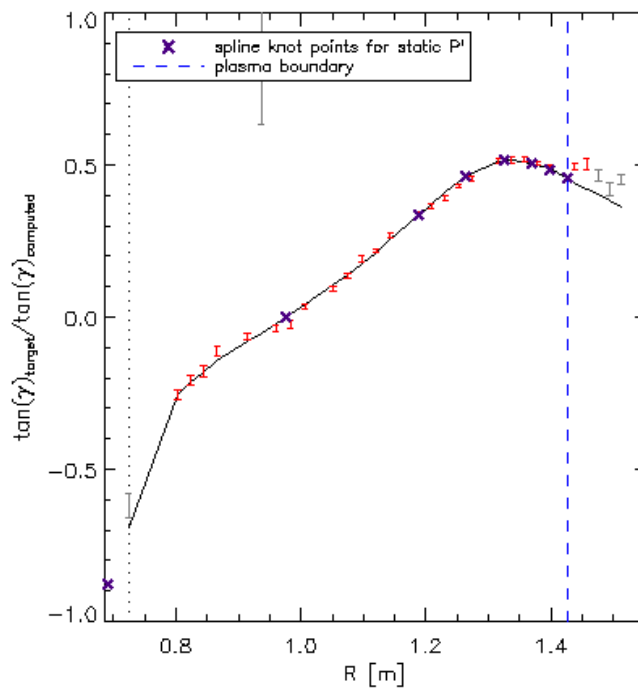
- EFIT++ for the calculation of the equilibrium
- TRANSP/NUBEAM for the calculation of the equilibrium and the evolution of the fast ion distribution (including the fast ion GC motion)
- ORBIT (adapted by M. Podesta) for the calculation of fast ion in an given equilibrium in presence of perturbations calculated by
- NOVA-K (MISHKA) for the calculation of the perturbation(s) spatial eigenfunction(s)

A two-steps approach:

- 1) estimate the diffusion in phase space due to the perturbation(s) in terms of a probability distribution function of kicks in energy and toroidal canonical momentum
- 2) evolve the fast ion distribution in NUBEAM including the random sampling of this distribution as one of physical processes in addition to classical collisions and atomic physics

EFIT++ Equilibrium reconstruction

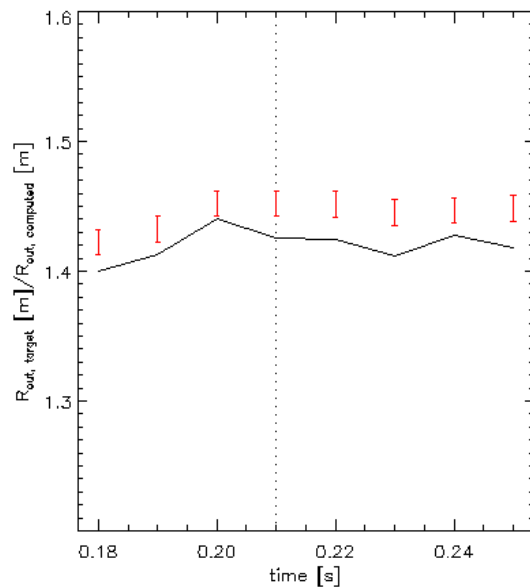
Fits to MSE and pressure profile edges look “reasonable”.



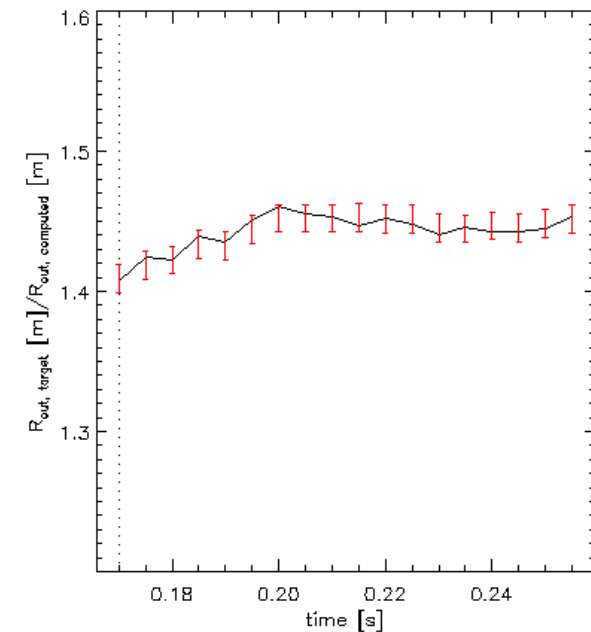


Equilibrium: boundary and strike Point

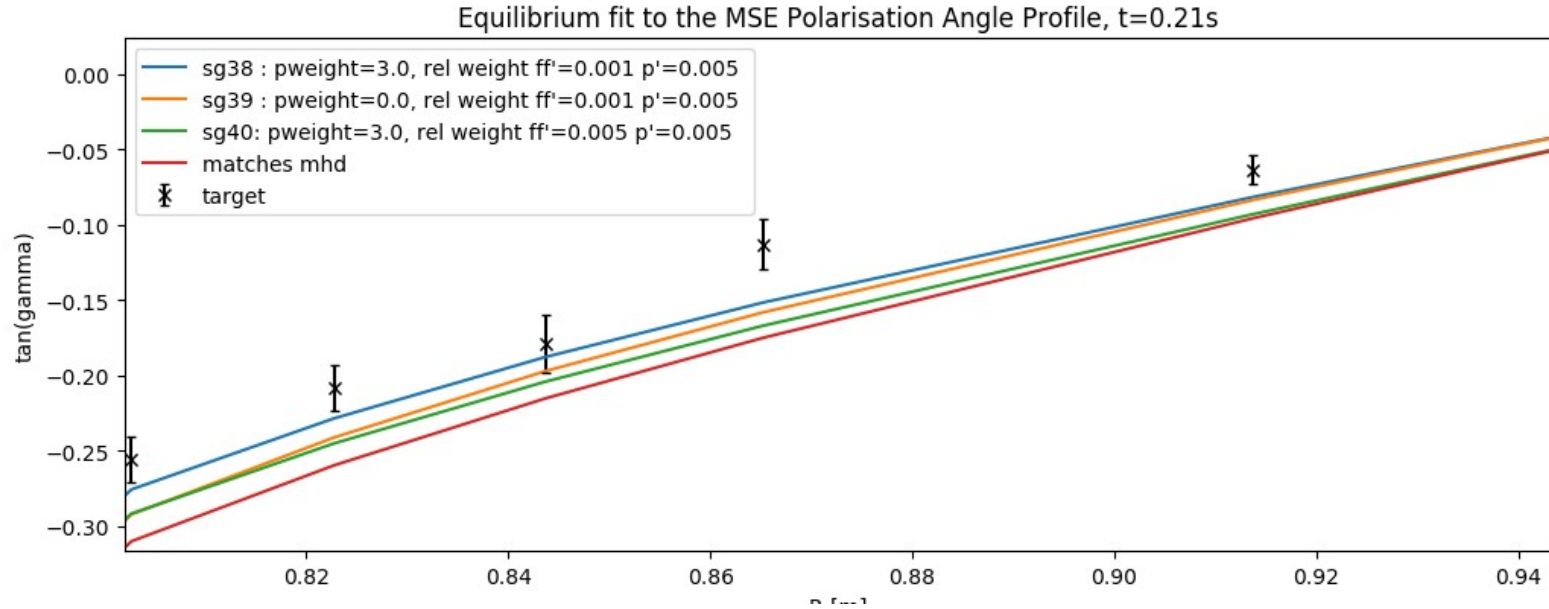
- Fit to the boundary position from D alpha camera is poor, resulting in misplaced strike point
- Need to strike a balance between number of pressure points used on the outer edge/weight on the D alpha camera to ensure strike points are in the correct place.
- Changed pressure range to be from 1.27-2.0m, lowered the boundary weight to 3
- Fit to the boundary measurement improved
- Strike point position fixed - $\sim 0.98\text{m}$ for $t = 0.21\text{s}$



Measured $\sim R=1.43-1.45\text{m}$
Computed $\sim R=1.40-1.44\text{m}$



MSE data analysis

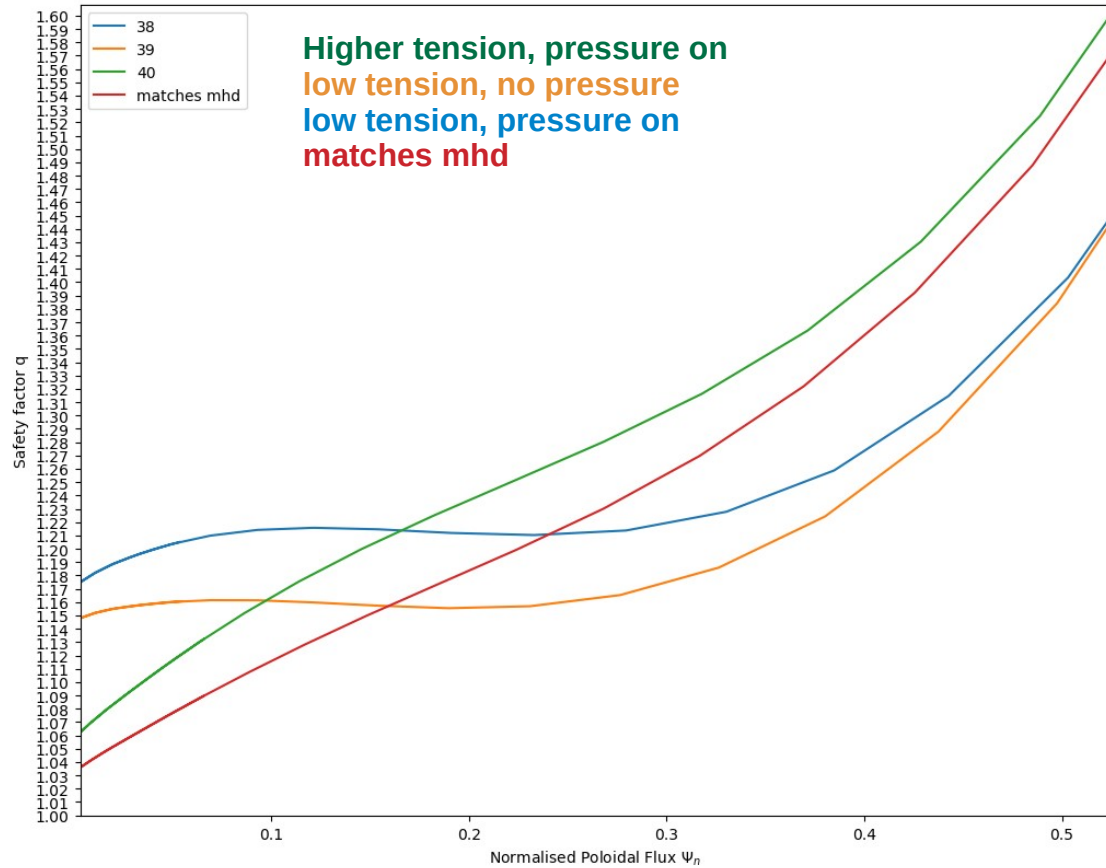




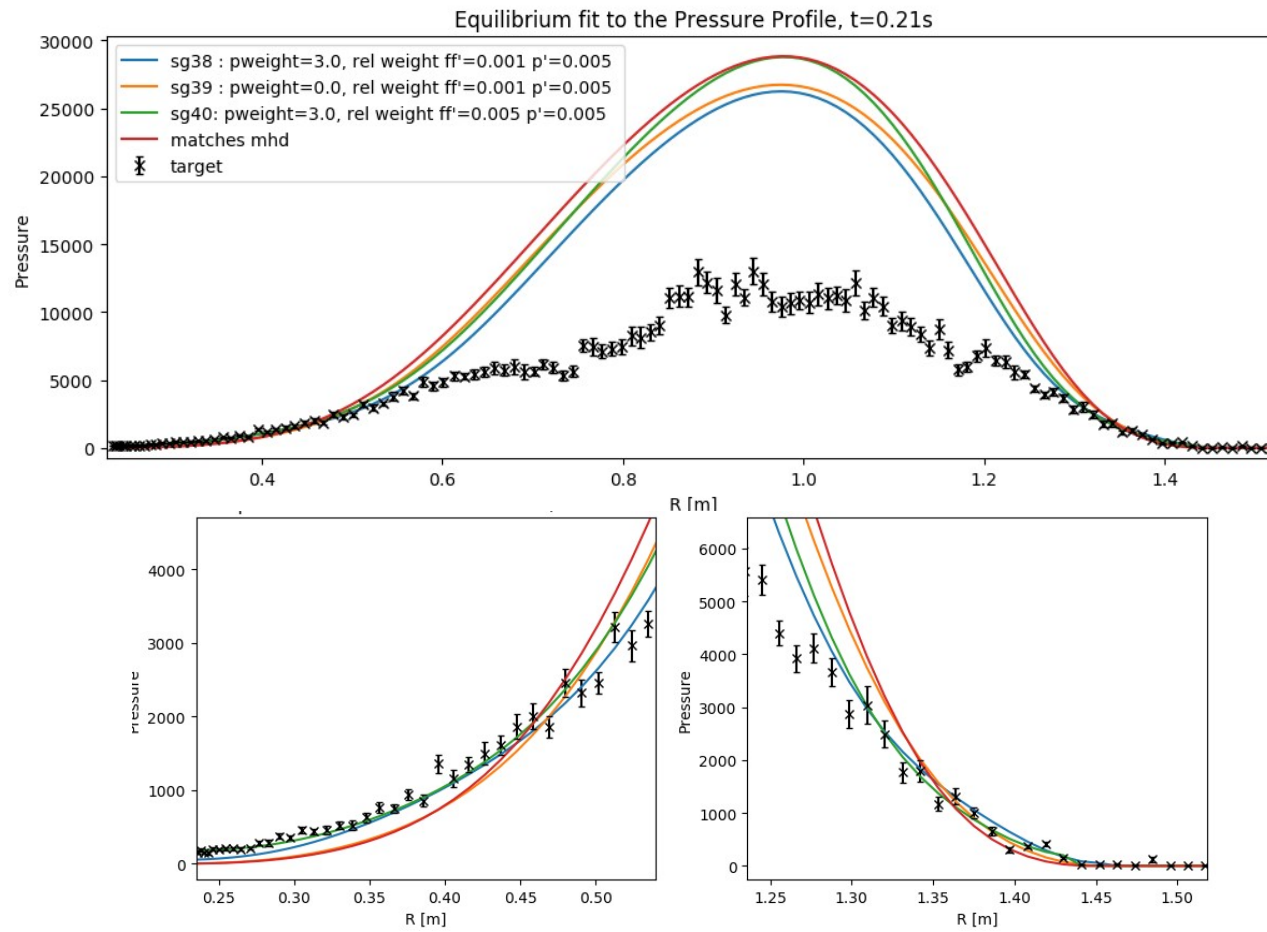
q profile analysis

Normalised chi squared values for pressure/MSE fits

Run	Pressure chi squared	MSE chi squared
38	113	5.03
39	142	5.13
40	141	6.83
match_mhd	175	6.86



Pressure profile analysis





Step 1: calculating the kick PDF

The kick-model: resonance between FI and MHD modes

The kick-model:

- characterizes the orbit topologies in terms of energy E , magnetic momentum μ and canonical toroidal momentum P_ζ
- calculates variations of E and P_ζ consistently

For a single mode:

$$\omega P_\zeta - nE = k$$

mode frequency \rightarrow ω n \rightarrow toroidal mode number

k \rightarrow constant

from which:

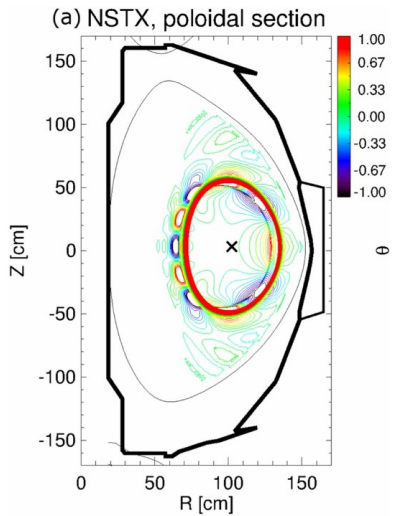
$$\frac{\Delta P_\zeta}{\Delta E} = \frac{n}{\omega}$$

The main ingredient of the new model is the probability that a particle, whose orbit is characterized by (P_ζ, E, μ) experiences a change over a time δt in energy and canonical angular momentum of magnitude ΔE and ΔP_ζ in the presence of a mode with amplitude A_m .

The kick-model: perturbation eigenfunctions using NOVA-K and MISHKA-1

Podestà M, et al. Computation of Alfvén eigenmode stability and saturation through a reduced fast ion transport model in the TRANSP tokamak transport code. Plasma Physics and Controlled Fusion. 2017 Sep 1;59(9):095008.

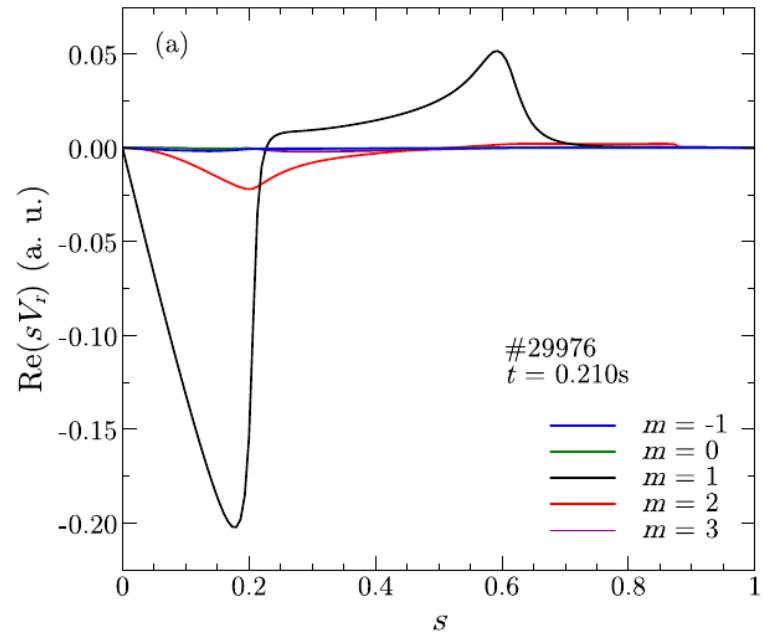
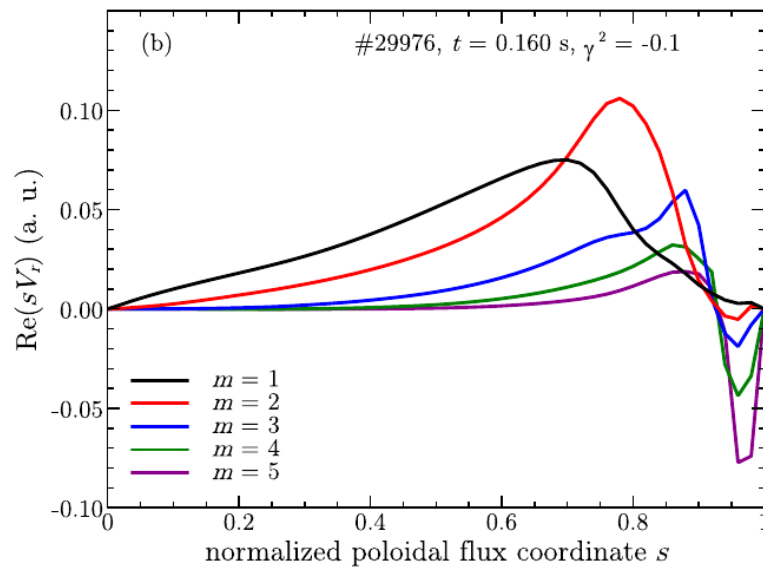
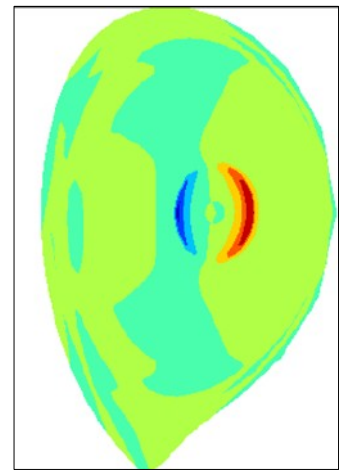
Toroidal Alfvén eigenfunctions



- more spatially localized
- multiple m modes

Fishbones

- broader spatial profile
- one dominant m mode



Cecconello M et al. Energetic ion behaviour in MAST. Plasma Physics and Controlled Fusion. 2015 Jan 1;57(1):014006.

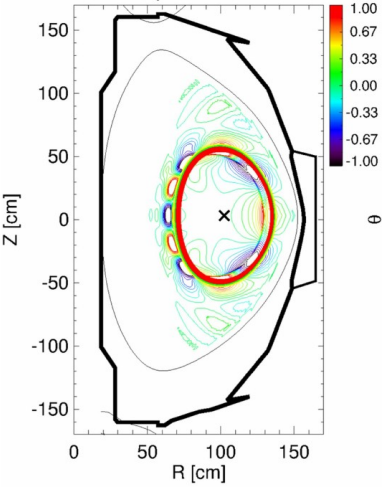
Fitzgerald M et al. Full-orbit and drift calculations of fusion product losses due to explosive fishbones on JET. Nuclear Fusion. 2019 Jan 1;59(1):016004.

Perturbations' characteristic frequencies

Podestà M, et al. Computation of Alfvén eigenmode stability and saturation through a reduced fast ion transport model in the TRANSP tokamak transport code. Plasma Physics and Controlled Fusion. 2017 Sep 1;59(9):095008.

Toroidal Alfvén eigenfunctions

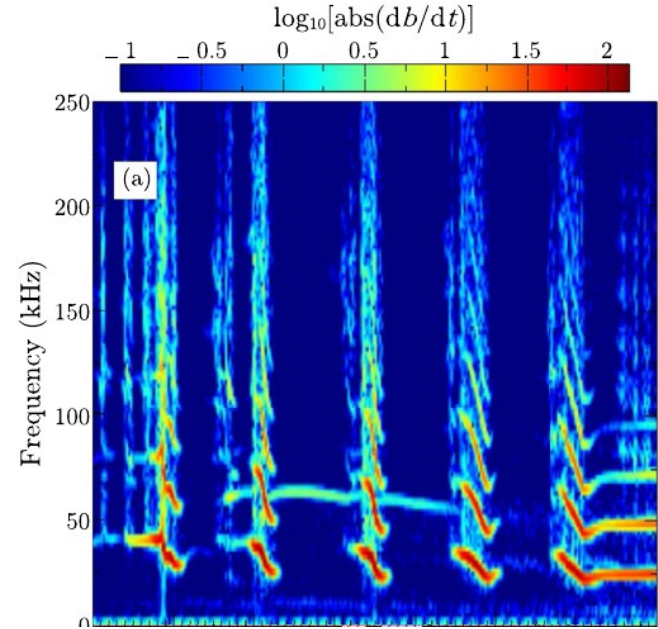
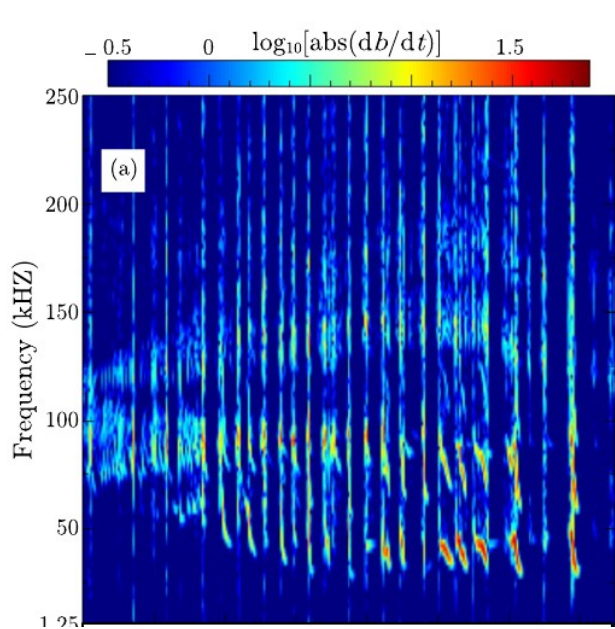
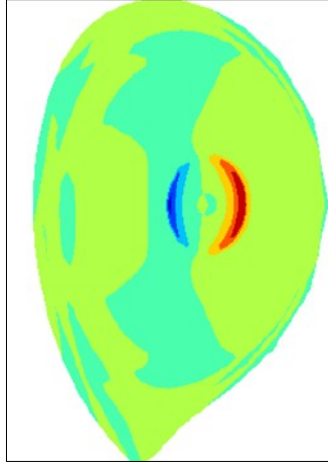
(a) NSTX, poloidal section



- more spatially localized
- multiple m modes

Fishbones

- broader spatial profile
- one dominant m mode



Cecconello M et al. Energetic ion behaviour in MAST. Plasma Physics and Controlled Fusion. 2015 Jan 1;57(1):014006.

Fitzgerald M et al. Full-orbit and drift calculations of fusion product losses due to explosive fishbones on JET. Nuclear Fusion. 2019 Jan 1;59(1):016004.

The kick-model: from the ΔE and ΔP_ζ kicks to the kick probability

The main ingredient of the new model is the **probability** that a particle, whose orbit is characterized by (P_ζ, E, μ) experiences a change over a time δt in energy and canonical angular momentum of magnitude ΔE and ΔP_ζ in the presence of a mode with amplitude A_m :

$$p(\Delta E, \Delta P_\zeta | P_\zeta, E, \mu, A_m) \quad \text{which can be reduced to} \quad p(\Delta E, \Delta P_\zeta | P_\zeta, E, \mu)$$

by setting $A_m = 1$ resulting in a 5D matrix and the mode amplitude is just a scaling factor provided as an external input since:

$$\sigma_{E, P_\zeta}(A_m) \propto A_m \sigma_{E, P_\zeta} |_{A_m=1}$$

The ensemble of kicks:

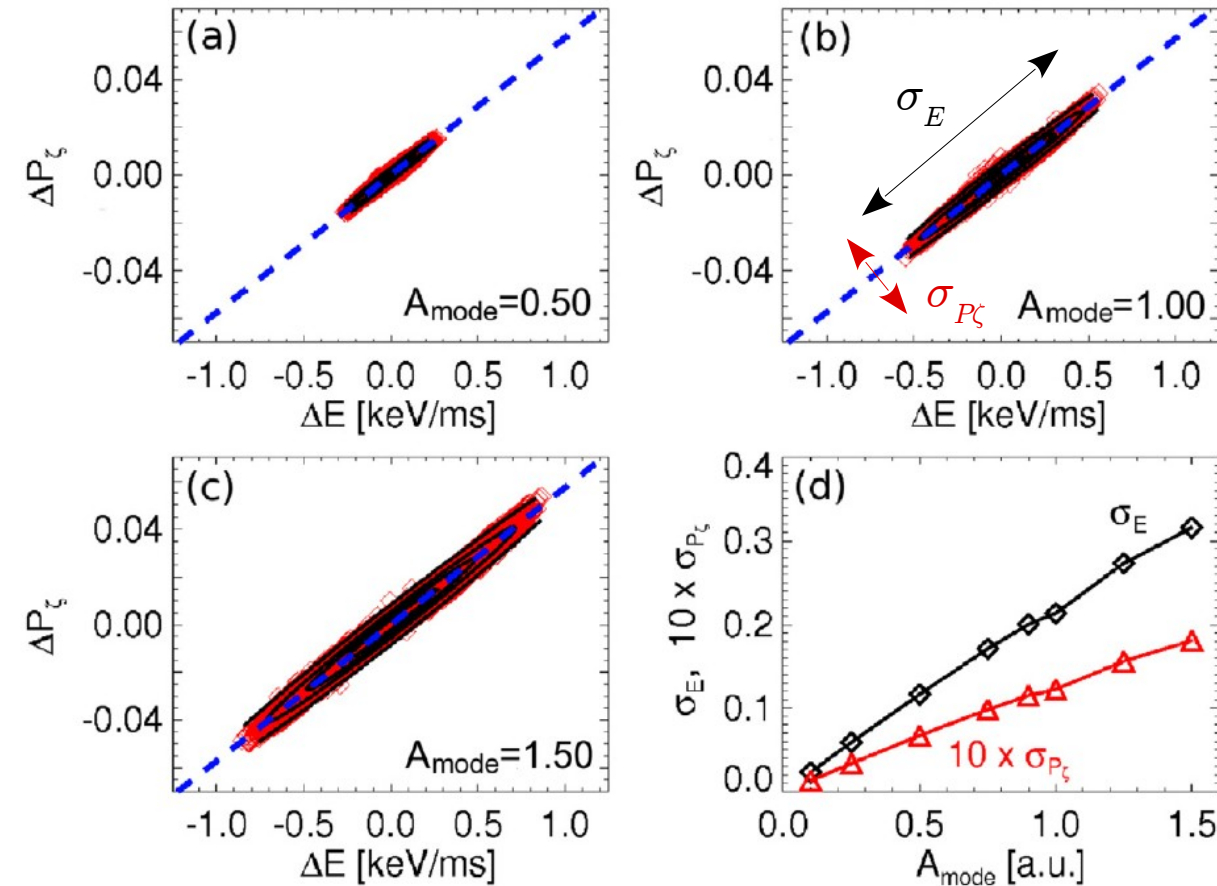
$$\Delta E(E, P_\zeta, \mu)$$

$$\Delta P_\zeta(E, P_\zeta, \mu)$$

is re-sampled over a grid in (P_ζ, E, μ) space and in each voxel of this 3D grid a 2D histogram is built of the corresponding ΔE and ΔP_ζ giving the probability distribution function with the normalization condition:

$$\sum_{\Delta E, \Delta P_\zeta} p(\Delta E, \Delta P_\zeta) = 1$$

The kick-model: example of variations of energy and toroidal canonical momentum



The initial particle distribution consists of **5000 particles**, all **within the same** (P_{ζ}, E, μ) voxel but with different toroidal locations and hence different phases with respect to a single mode TAE.

The effect of three different mode's amplitudes are shown.

Approximate linear scaling between the variations in P_{ζ} , E and the mode amplitude

Podesta M. A reduced fast ion transport model for the tokamak transport code TRANSP. Plasma Phys Control Fusion. 2014;56:055003.

Calculating the E and P_ζ kicks

A single ORBIT run with a sufficiently large number of test particles is used to calculate:

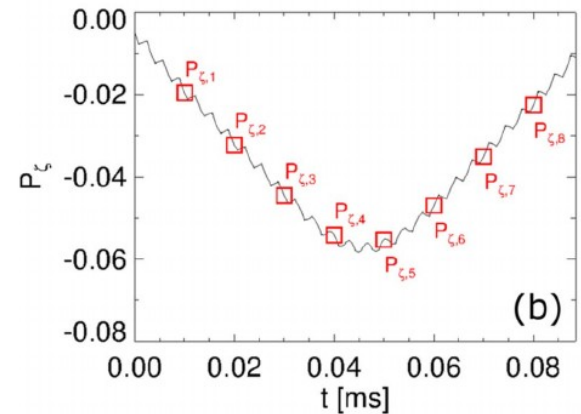
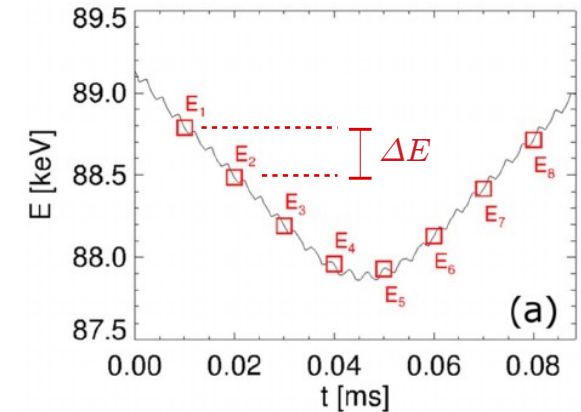
$$p(\Delta E, \Delta P_\zeta | P_\zeta, E, \mu)$$

in which initial particle orbits are randomly sampled from a uniform distribution in the (P_ζ, E, μ) space and the variations in energy and canonical momentum are tracked at fixed time intervals $\delta t_\Delta = \min(1/f) = 10 \mu s$ providing an ensemble of values over the whole space of:

$$\Delta E(E, P_\zeta, \mu)$$

$$\Delta P_\zeta(E, P_\zeta, \mu)$$

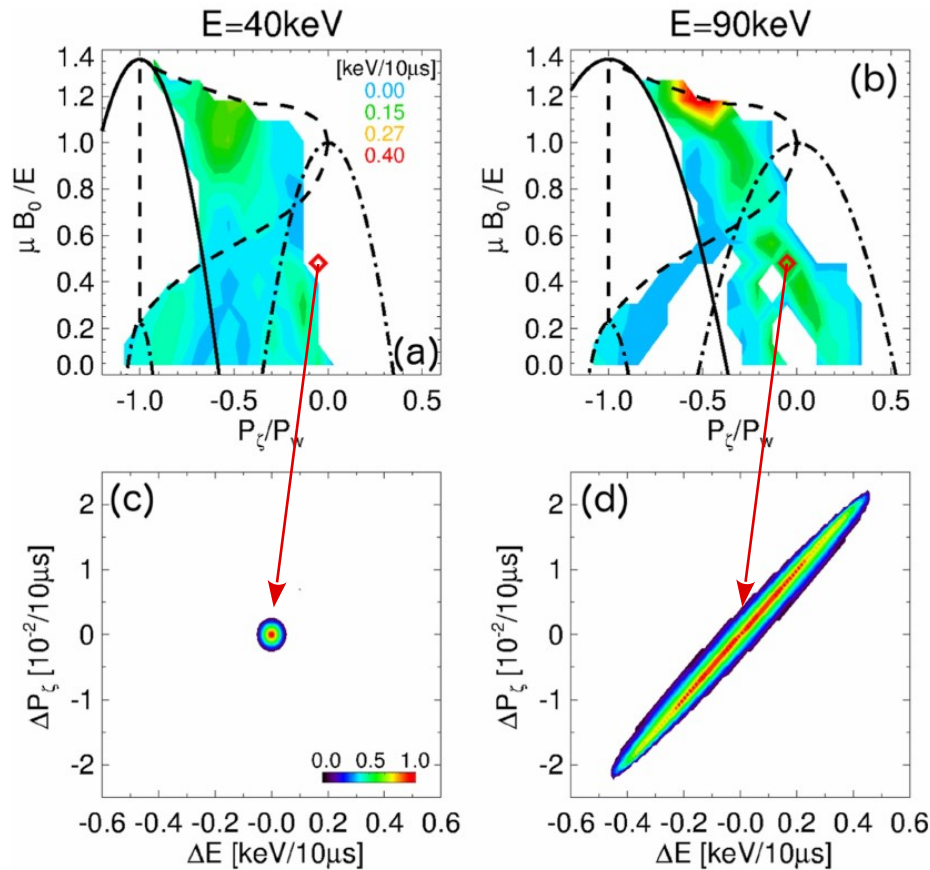
In this phase, atomic processes are excluded.





Kicks probability examples

Podestà M, Gorelenkova M, Gorelenkov NN, White RB. Computation of Alfvén eigenmode stability and saturation through a reduced fast ion transport model in the TRANSP tokamak transport code. Plasma Physics and Controlled Fusion. 2017 Sep 1;59(9):095008.



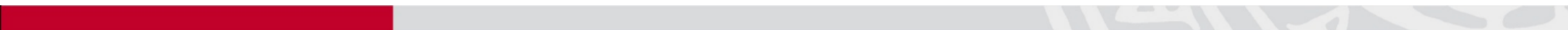
Root mean square of the energy kicks in the (P_ζ, μ) plane for two representative fast ion energies.

Kicks shown in this figure are associated with a $n = 4$ TAE mode.

Examples of kick probabilities $p(\Delta P_\zeta, \Delta E)$ corresponding to the (P_ζ, E, μ) bins indicated by a red diamond in panels (a), (b).



Step 2: evolving the FID



Evolving the FID: an overview

Wave-particle interaction processes are distilled (through ORBIT modeling) into kick probability matrices $p(\Delta E, \Delta P_\zeta | P_\zeta, E, \mu)$

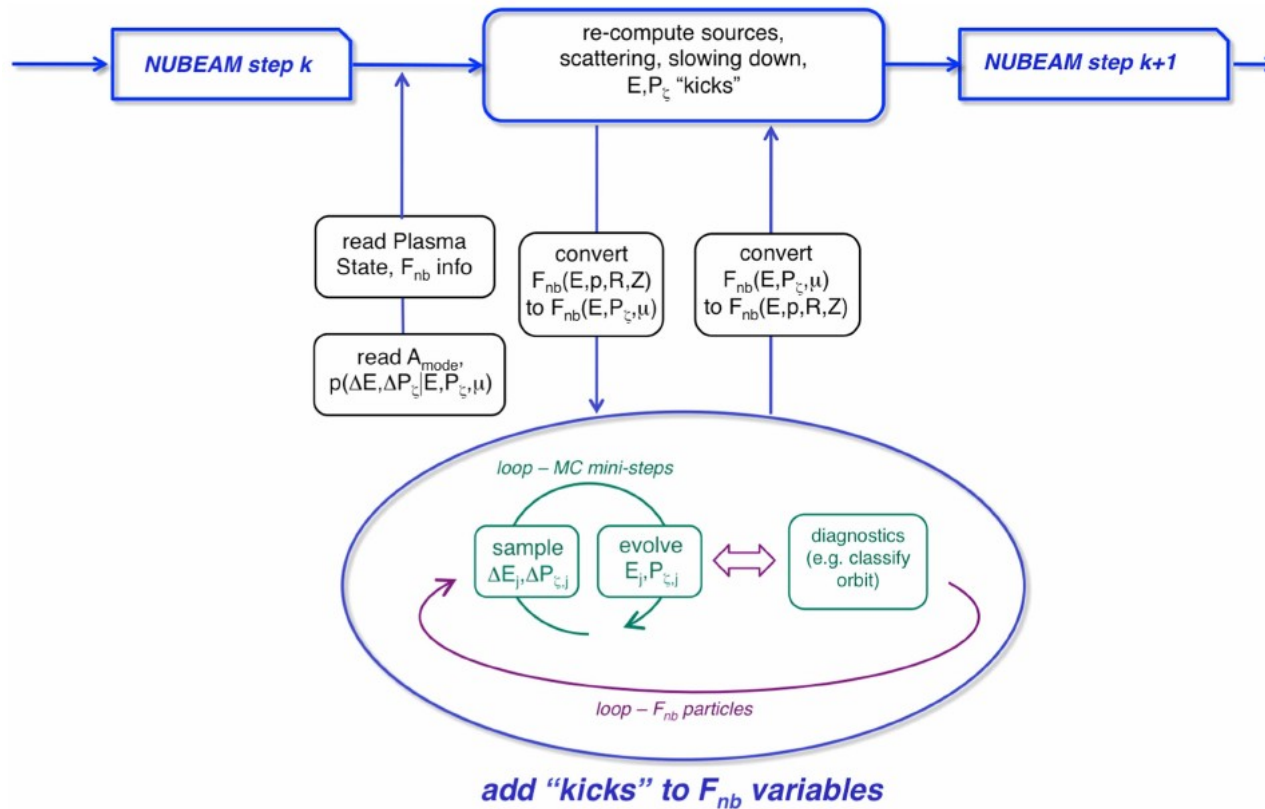
Up to 10 matrices and associated time-dependent kick scaling factors can be used in a TRANSP/NUBEAM simulation.

Each probability can represent a single perturbation or a set of perturbations with similar temporal evolution.

NUBEAM evolves the FID in user-defined steps (of the order of ms); between step k and $k+1$:

- the FID is updated based (primarily) on sources, such as FI originating from NB injection, and sinks, such as losses outside the last closed flux surface, re-neutralization and thermalization
- particle variables are mapped on phase space variables to compute the kick model corrections to the particle's orbit
- kicks are sampled randomly from each active (i.e., associated mode amplitude >0) probability and applied
- the evolution of the fast ion ensemble under (neo-)classical and kick model effects continues until the end of step k , at which time quantities such as fast ion density, power from thermalization, NB-driven current are computed.
- based on those updated terms, TRANSP parameters are updated (e.g. by recomputing the magnetic equilibrium based on total current evolution) and made available for the following NUBEAM step.

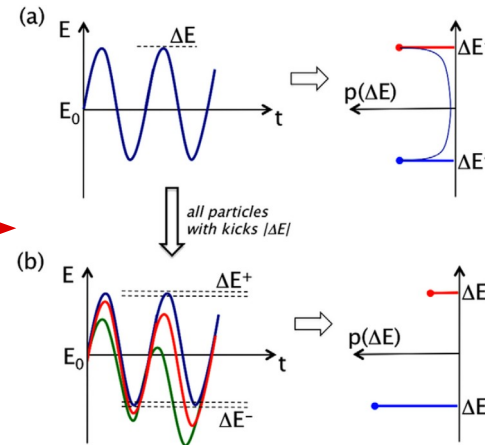
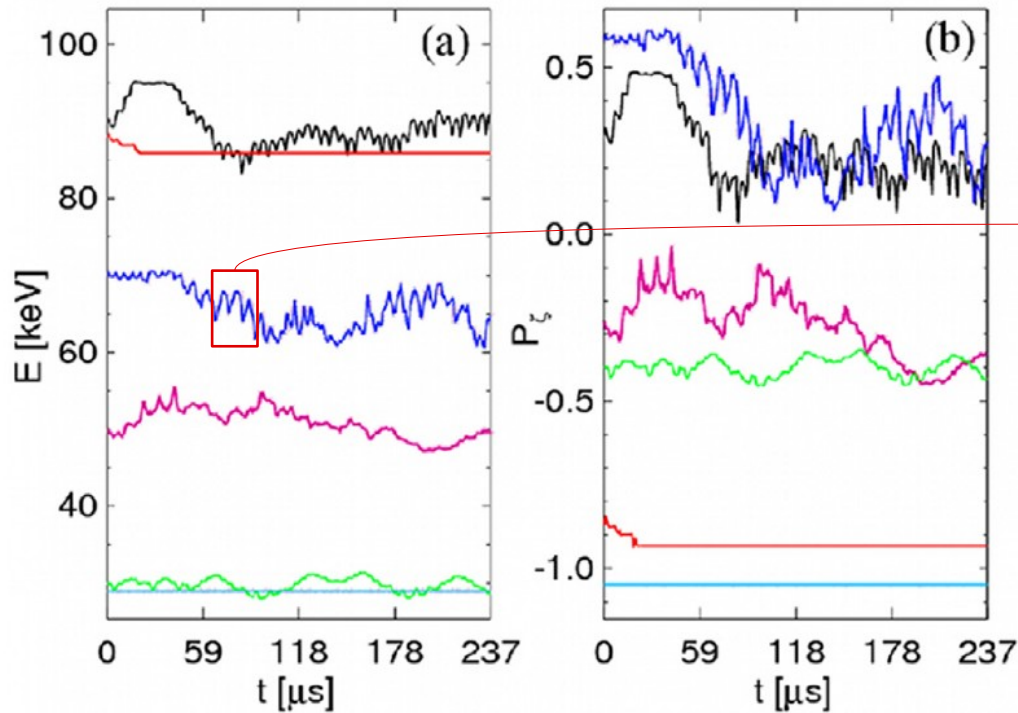
Schematics of the FID evolution



Podestà M, Gorelenkova M, Fredrickson ED, Gorelenkov NN, White RB. Effects of energetic particle phase space modifications by instabilities on integrated modeling. Nuclear Fusion. 2016 Nov 1;56(11):112005.

Accumulating changes in energy and toroidal canonical momentum

E and P_ζ time evolution during a NUBEAM time step $\delta t = 1$ ms during which ΔE and ΔP_ζ are calculated on time steps $\delta t_\Delta \ll \delta t$

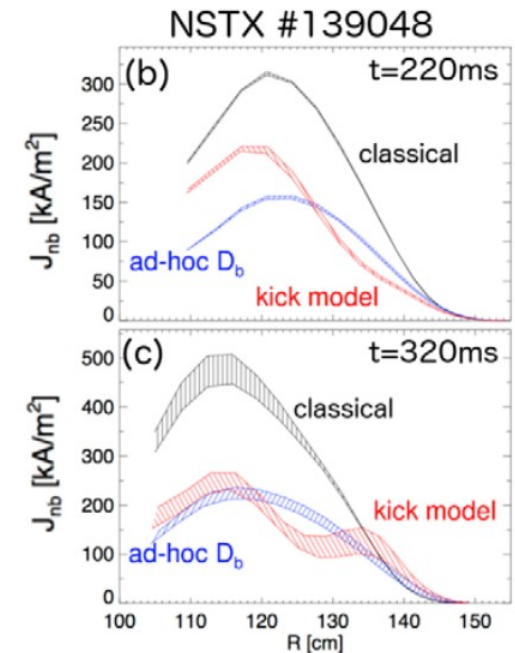
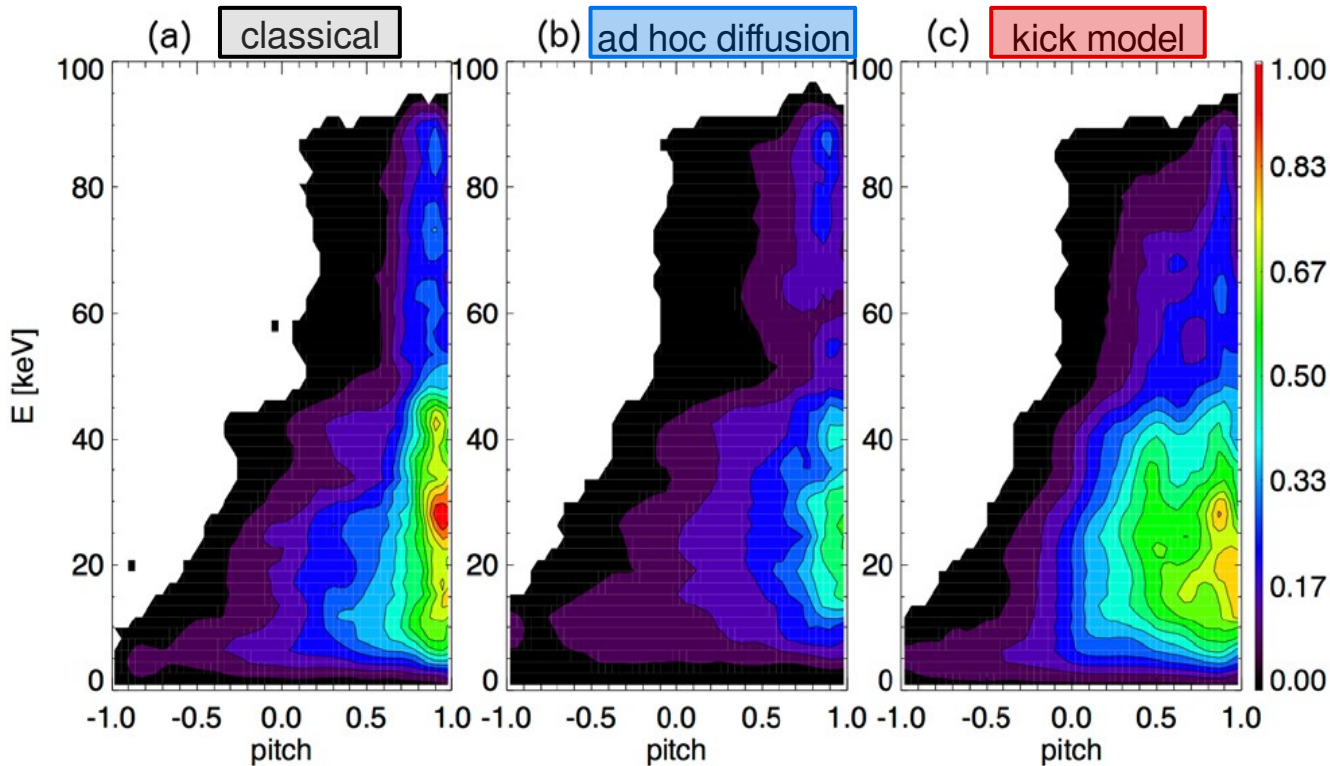


$$\overline{\Delta E} \approx \sum_{k=1}^{N_\Delta} S_{r,k} \Delta E A_{m,k}$$

$S_{r,k}$ is the sign of a random number uniformly extracted from $[-p_-/(p_+ + p_-), p_+/(p_+ + p_-)]$ at each step

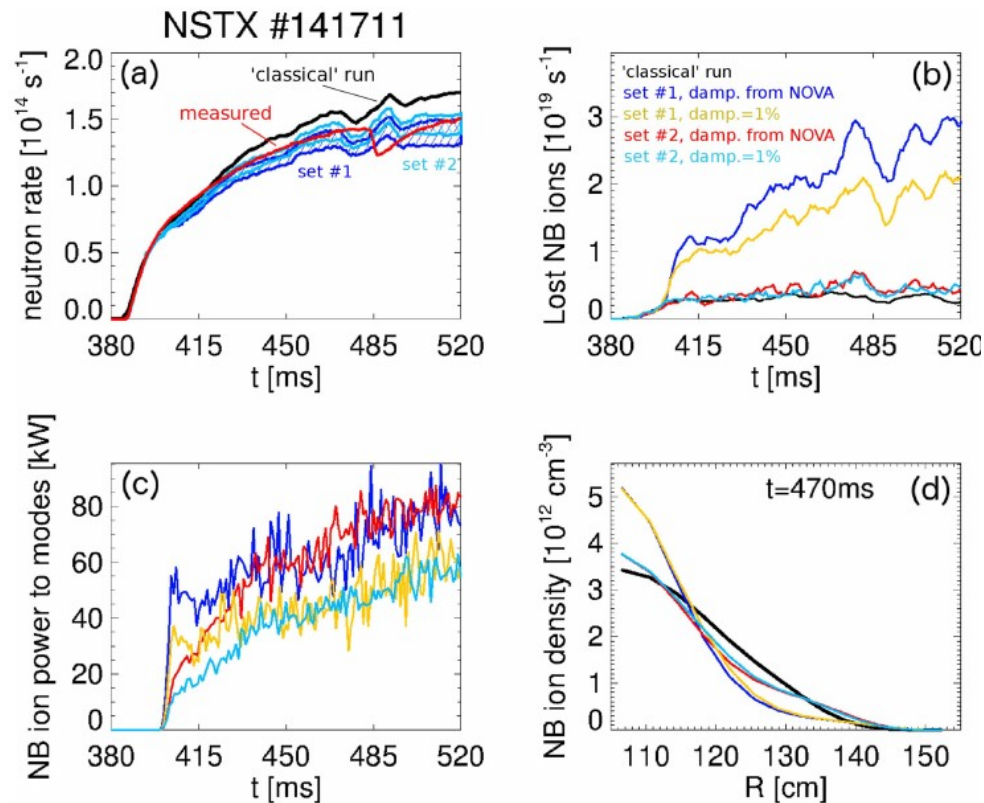
Some results from Mario's previous work

Fast ion distribution functions around $r/a \sim 0.5$



Podestà M, Gorelenkova M, Fredrickson ED, Gorelenkov NN, White RB. Effects of energetic particle phase space modifications by instabilities on integrated modeling. Nuclear Fusion. 2016 Nov 1;56(11):112005.

Some results from Mario's previous work



Comparison with FC rates remains the main reality check.

We can do more with the NC profiles since the spatial distribution is very different between AFID and kick-model results.

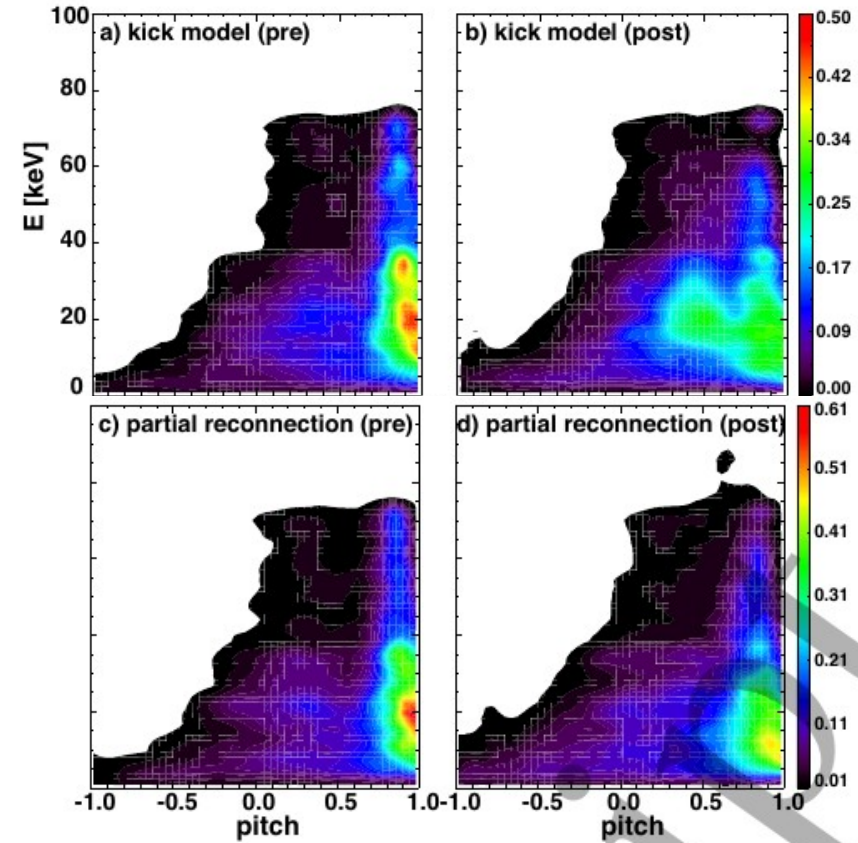
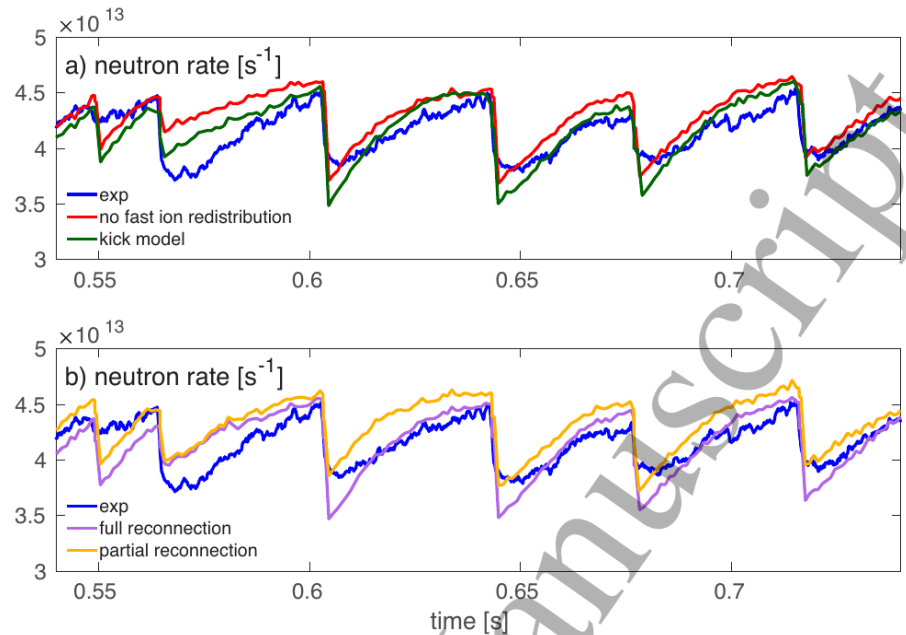
If agreement is not found:

- adjust perturbation (eigenvalue and/or eigenfunction)
- re-evaluate the kick-probability
- repeat

Podestà M, Gorelenkova M, Gorelenkov NN, White RB. Computation of Alfvén eigenmode stability and saturation through a reduced fast ion transport model in the TRANSP tokamak transport code. Plasma Physics and Controlled Fusion. 2017 Sep 1;59(9):095008.

Kick-model applied to sawteeth in NSTX

- $m = 1, n = 1$ mode perturbations to represent sawtooth instability
- the mode amplitude of each crash determined by comparing with the relative change in neutron yield with measurements

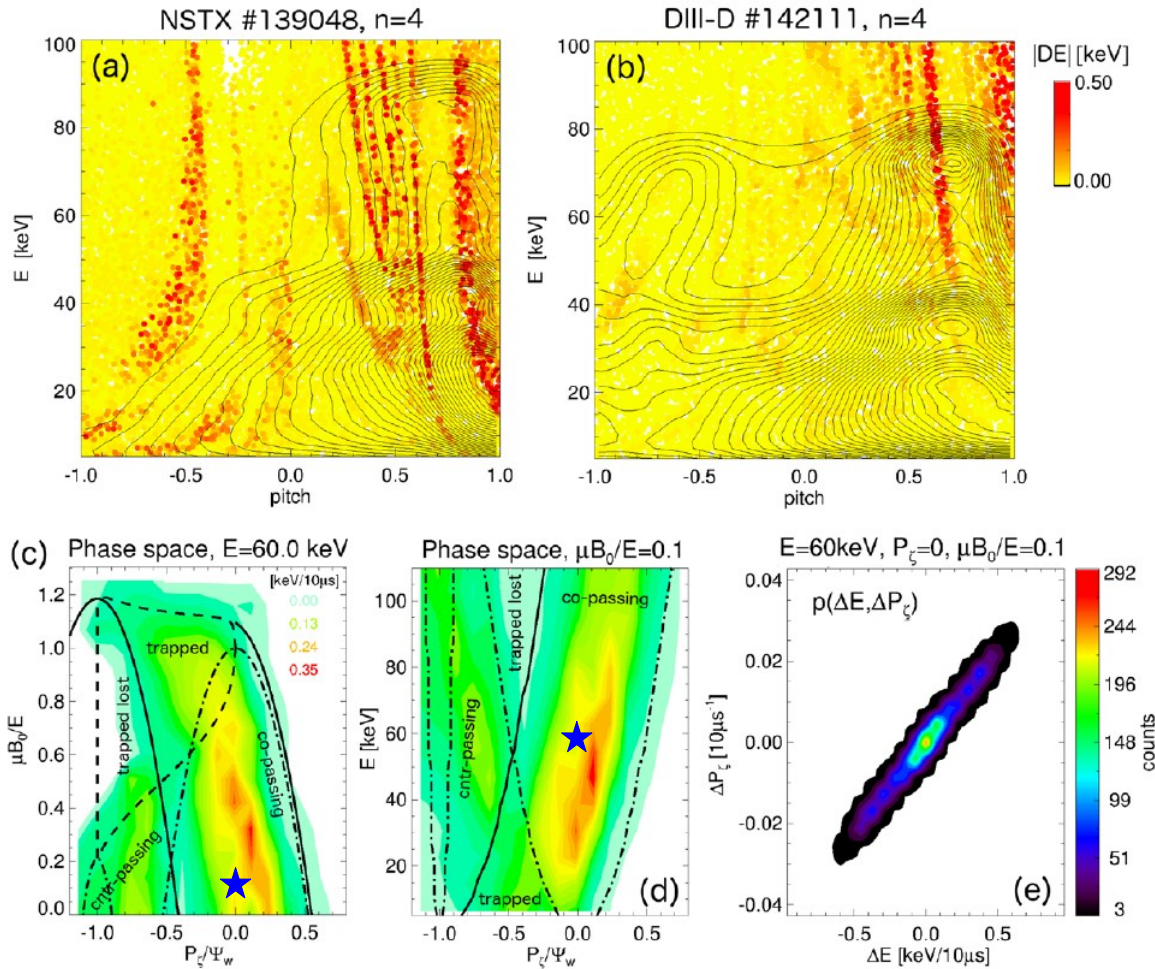


Doohyun Kim et al 2019 Nucl. Fusion in press <https://doi.org/10.1088/1741-4326/ab1f20>
Investigation of fast particle redistribution induced by sawtooth instability



Kicks probability examples

Podestà M, Gorelenkova M, Fredrickson ED, Gorelenkov NN, White RB. Effects of energetic particle phase space modifications by instabilities on integrated modeling. Nuclear Fusion. 2016 Nov 1;56(11):12005.



Resonant particles initialized at mid-plane, whose energy is modified over 0.5 ms at $r/a \approx 0.4$ (location of peak mode amplitude)

A single mode with multiple poloidal harmonics is used in ORBIT.

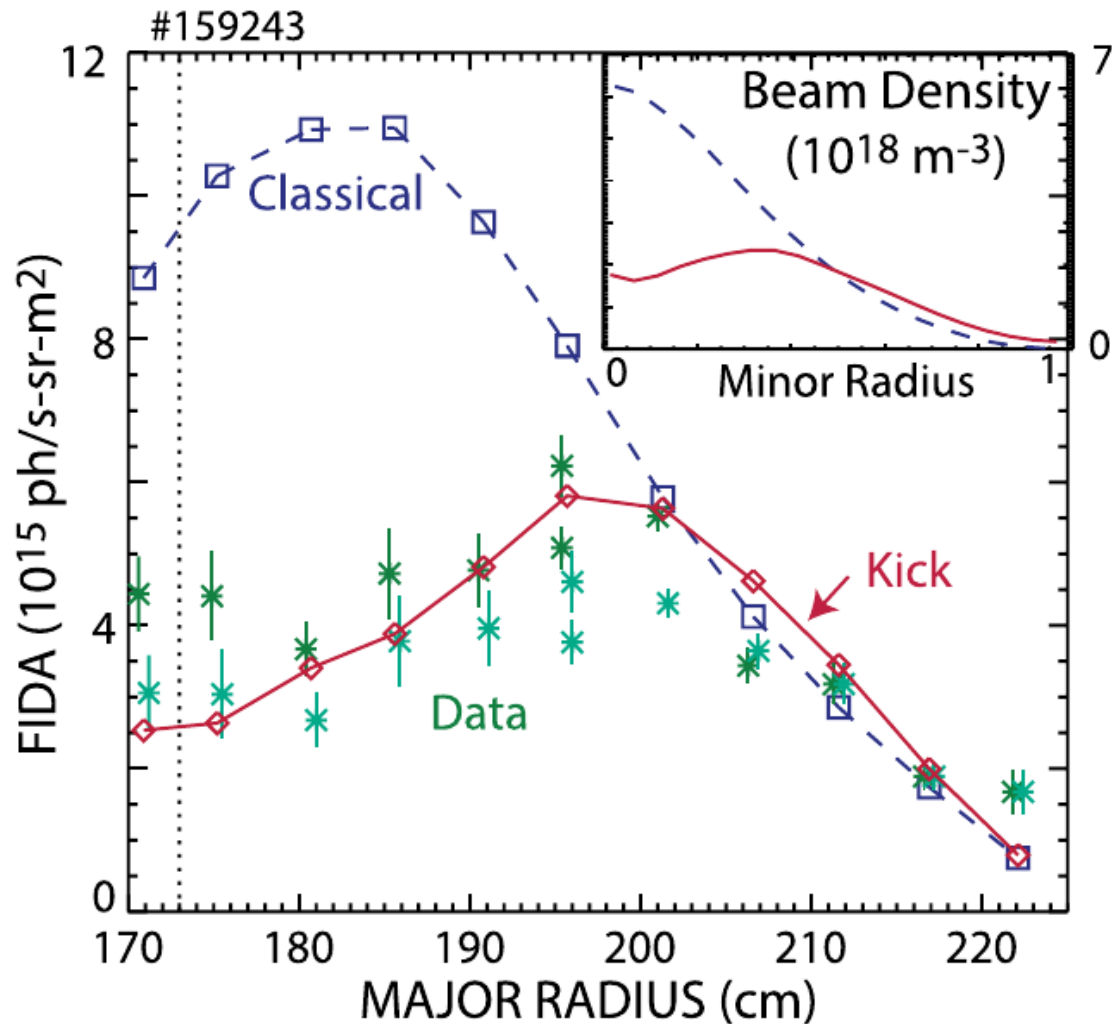
Contour lines show the FID as computed for classical TRANSP runs.

Phase-space representations of panel (a) showing the root-mean-square energy kicks as a function of constants of motion.

Kick probability $p(\Delta P_\zeta, \Delta E)$ from (c) and (d) for co-passing particles.

Fast ion transport in DIII-D due TAEs modelled with the kick model

Good match between FIDA measurements and FIDASIM:



TRANSP predicted fast ion and neutron profile rates: AFID vs “kick-model”

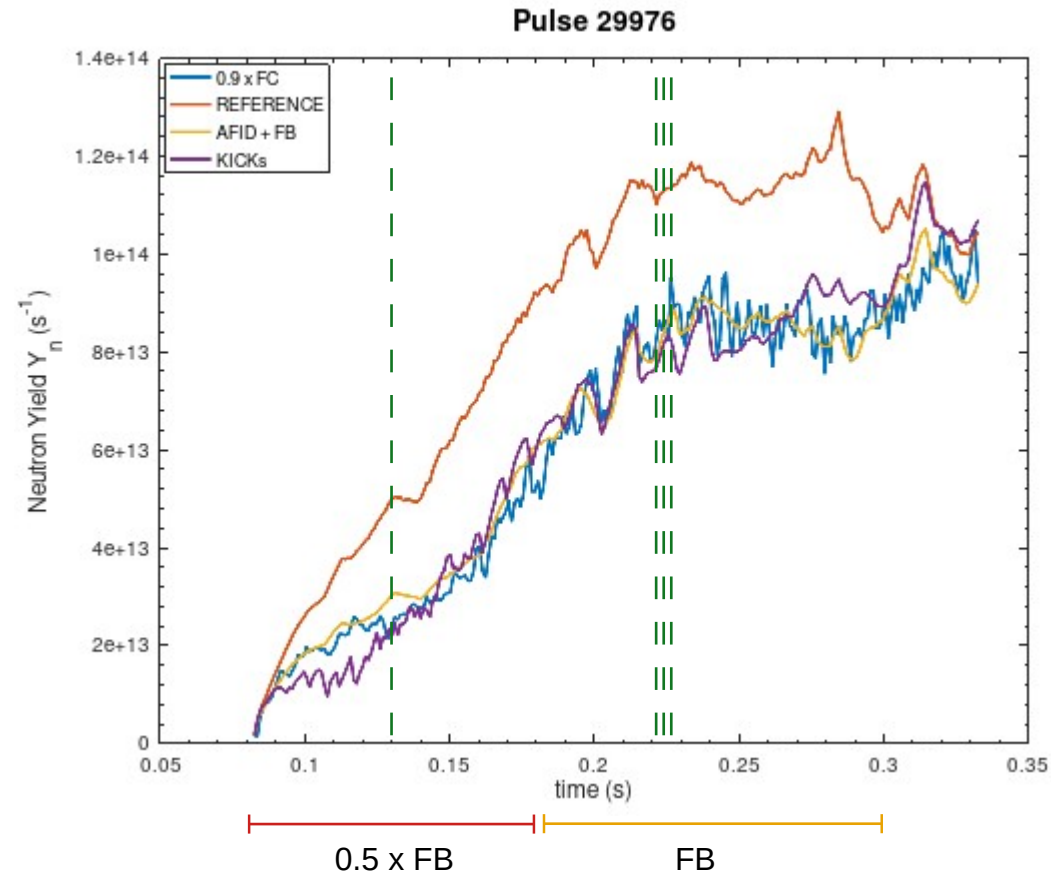
Both models can reproduce the fission chamber:

- Reference run: no AFID, no Kicks
- AFID and no Kicks
- Kicks and no AFID

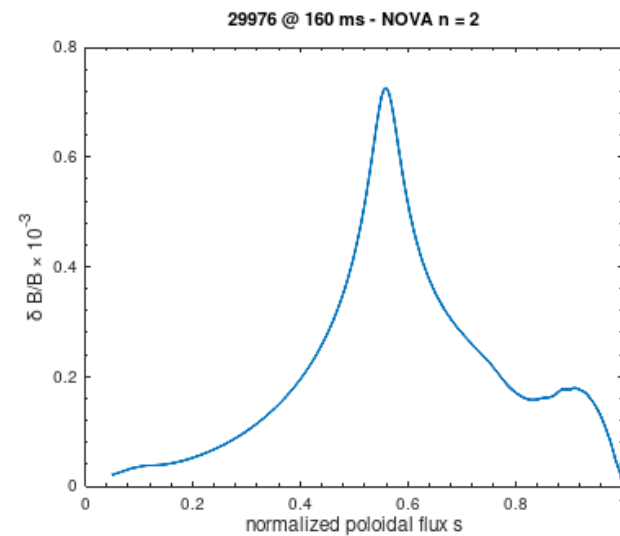
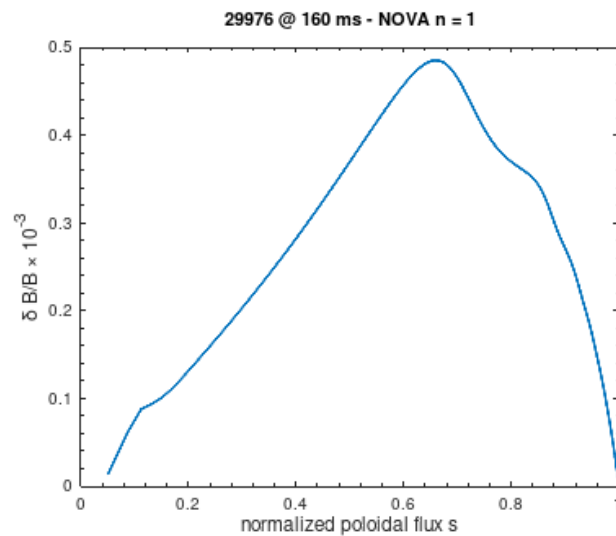
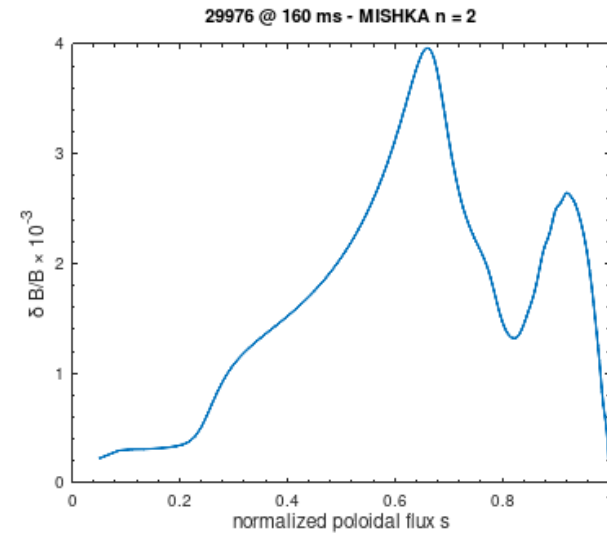
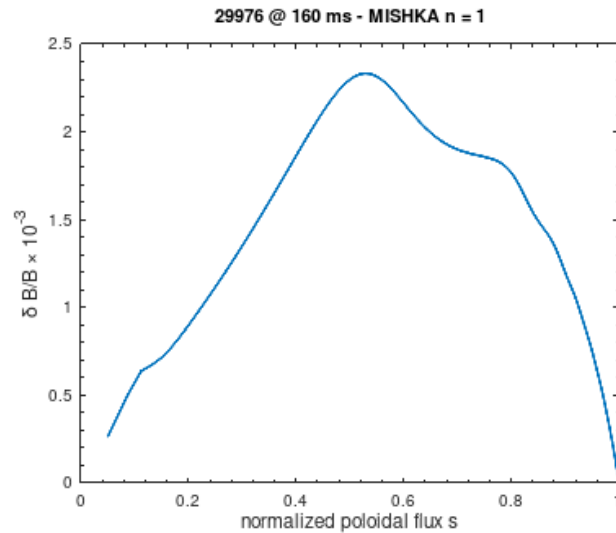
High statistics runs with FI and non-flux averaged neutron emissivities at selected times (FLR included):

0.130, 0.212, 0.214, 0.217 (s)

for profile comparison with FIDA and NC in addition to comparison with global quantities.



TAE eigenfunctions: MISHKA vs NOVA-K



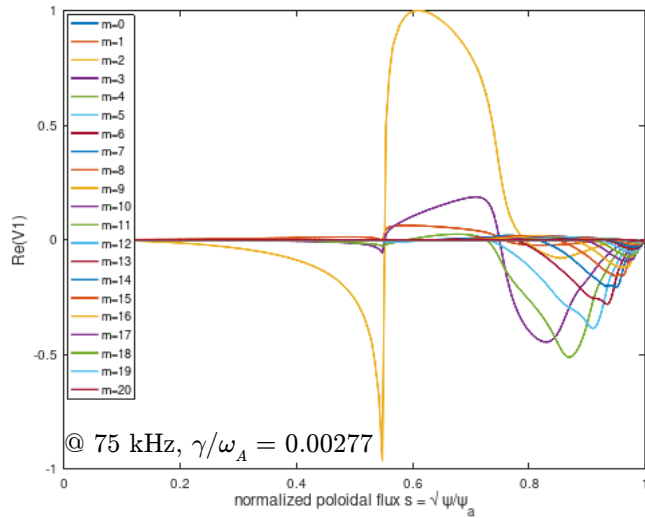
Modelling the perturbation for the calculation of the kicks pdf at $t = 130, 210$ ms

NOVA-K calculations with $m = 2$ dominant (all m used in the kick matrix).

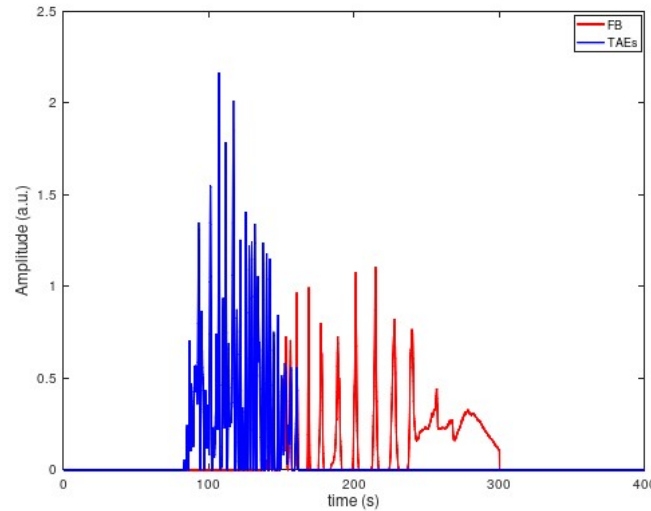
Time evolution of the perturbation based on the RMS of a Mirnov coil signal.

Simple analytical approximation for the $(m,n) = (1,1)$ kink mode

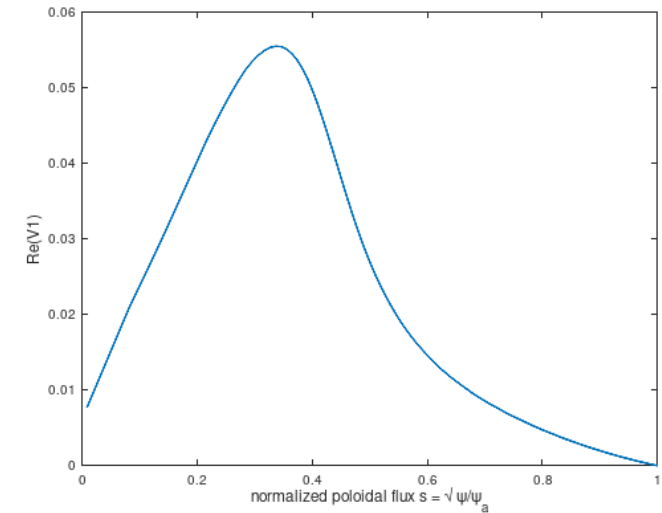
NOVA-K 29976 @ 130 ms - n = 1 modes



29976 MAST - MODES



Hat-like displacement @ 210 ms - n = 1 mode



NB: the eigenfunction zero at the LCFS (NOVA/MISHKA limits) while the perturbation is measured outside the LCFS! Difficult to constrain the eigenfunction amplitude with external measurements.

file:///home/marco/Documents/MAST/TRANSP/kick_model.m

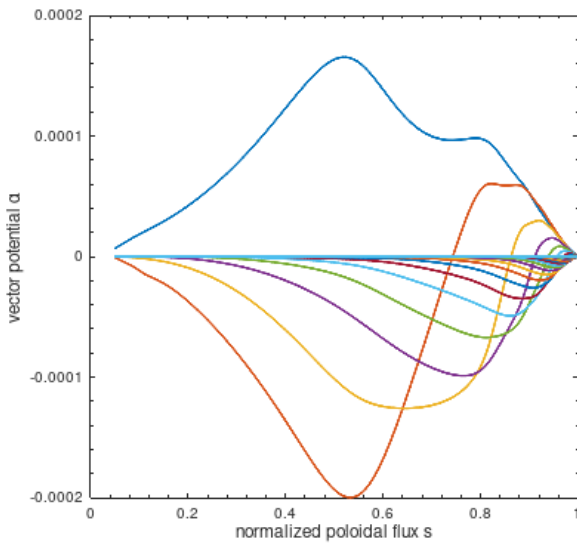
file:///home/marco/Documents/MAST/TRANSP/Kick_Model/ORBIT_calculations/nova.m

Perturbations for 29976 at 160 ms

Based on EFIT++ equilibrium with MSE constraints

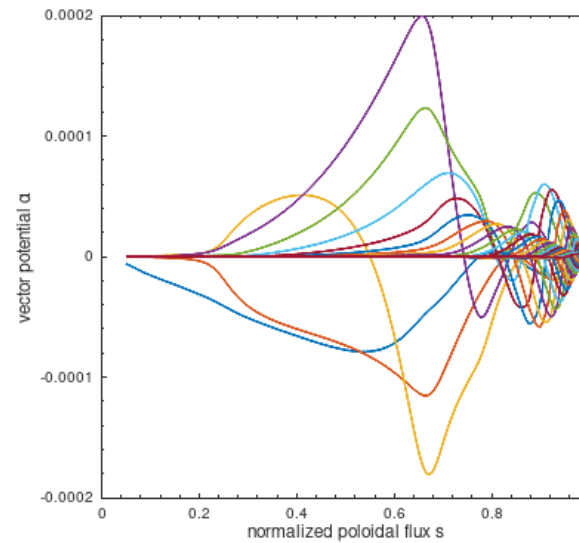
MISHKA eigenfunctions

$n = -1$



$$\frac{\omega}{\omega_A} = -0.362$$

$n = -2$

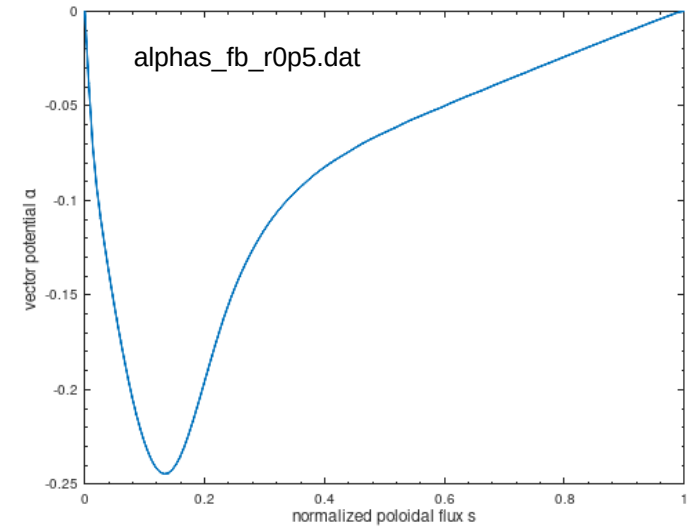


$$\frac{\omega}{\omega_A} = -0.335$$

Fishbone

$(m,n) = (1,1)$

29976 @ 210 ms - FB

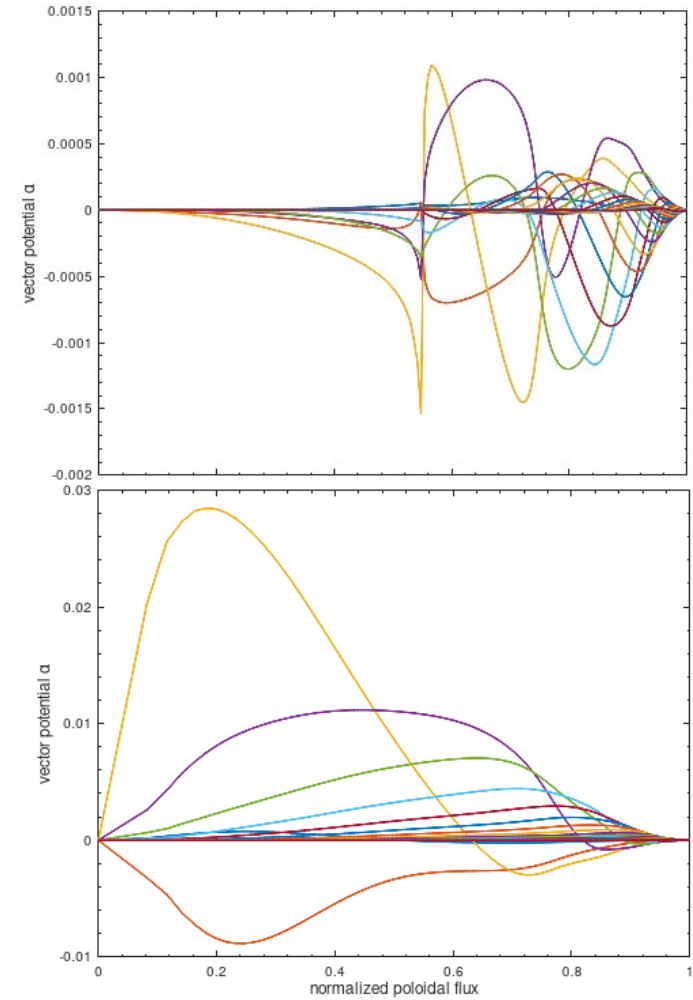
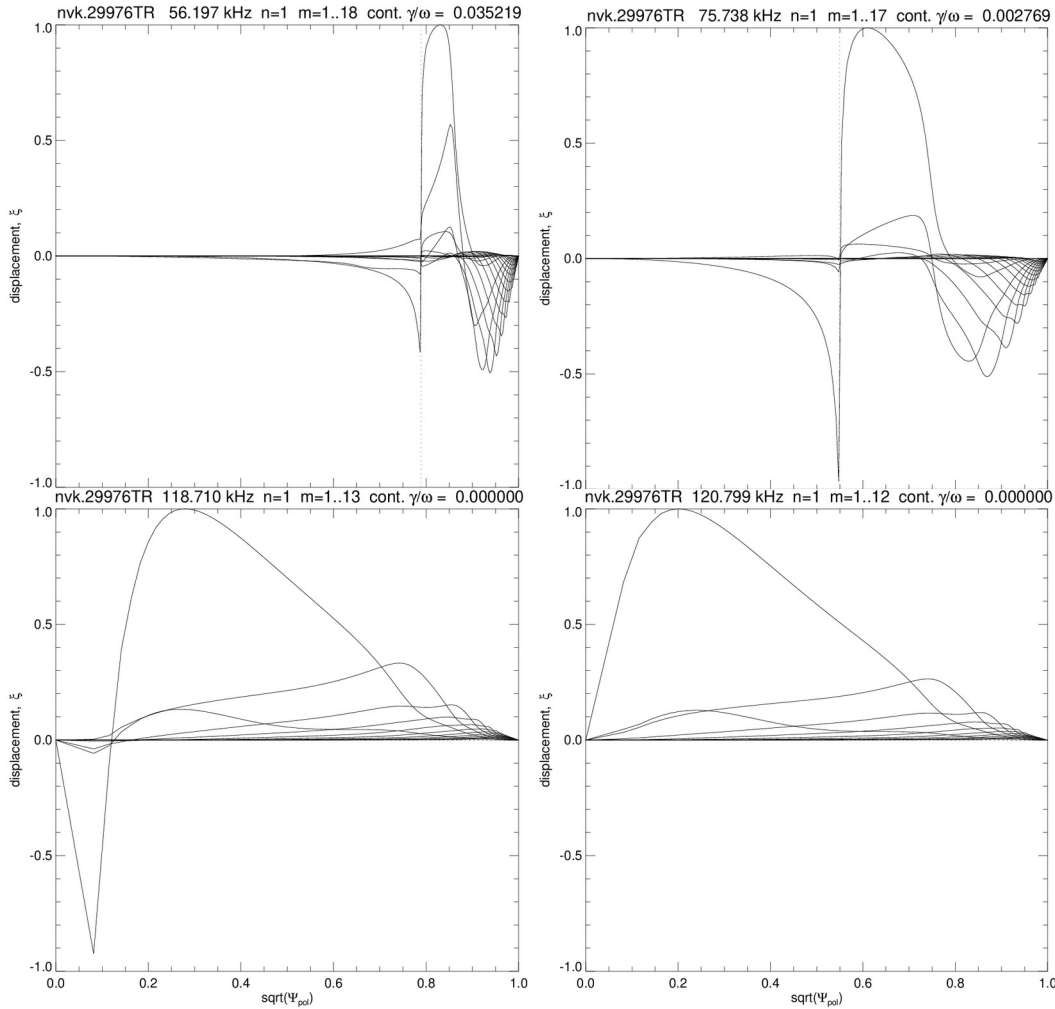


$$\delta b = \delta \times \alpha B$$

NOVA-K eigenfunctions for 29976 at 160 ms

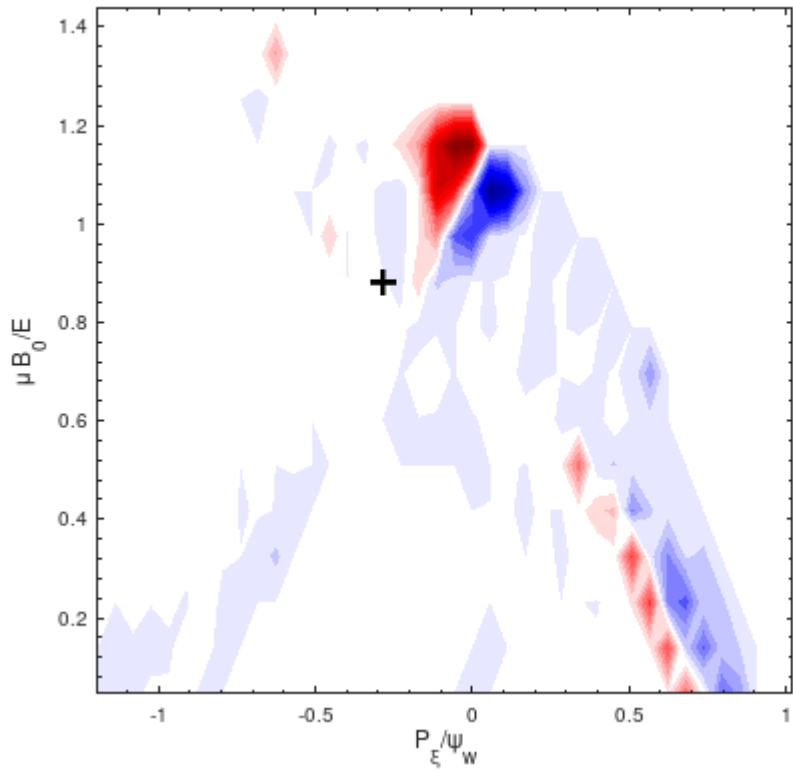
Based on TRANSP TEQ equilibrium

Normalized displacement ξ

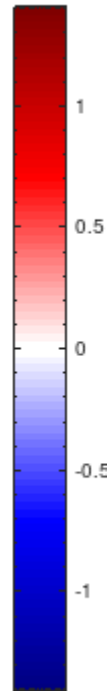


NOVA-K eigenfunctions for 29976 at 160 ms

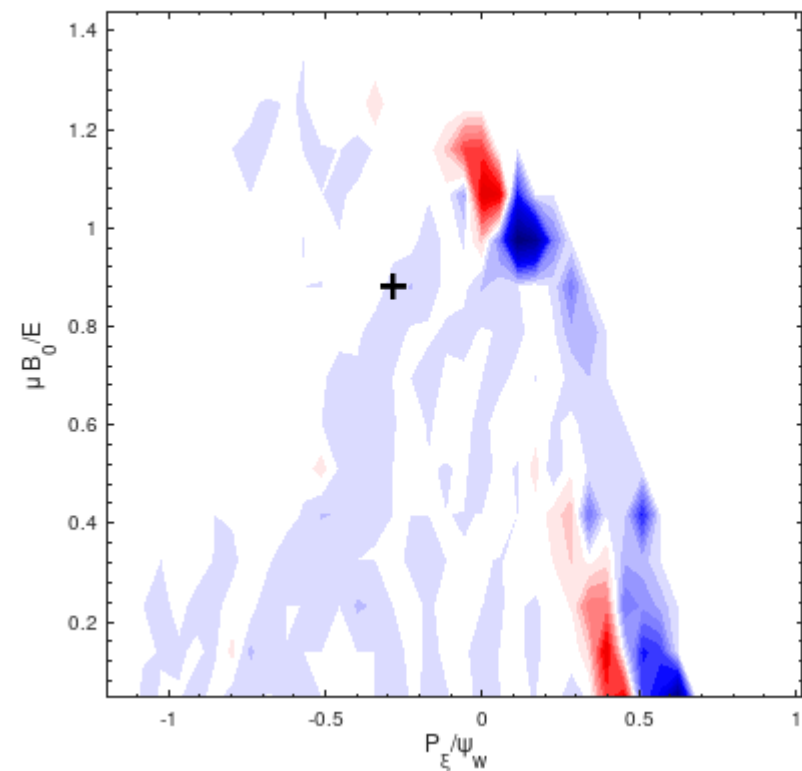
Phase space at $E = 69.125$ (keV)



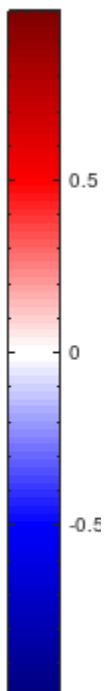
ΔE (keV)



Phase space at $E = 32.792$ (keV)



ΔE (keV)



Time evolution of the perturbations in the kick-model

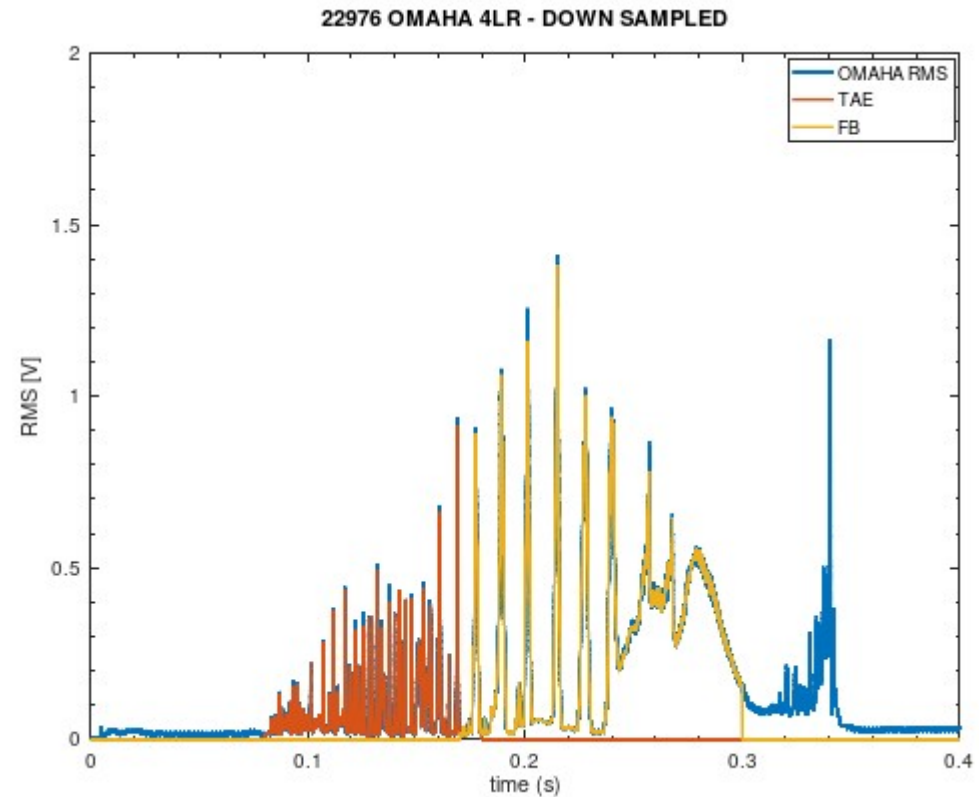
RMS window width = 512 samples

Time intervals for

- TAE: [80, 170] ms
- FB: [170, 300] ms

Set to zero outside and
downsampled (0.128 ms time step)

FB_Spectrogram_29976.m
29976_Spectrogram_RMS.dat
create_fileamode.m



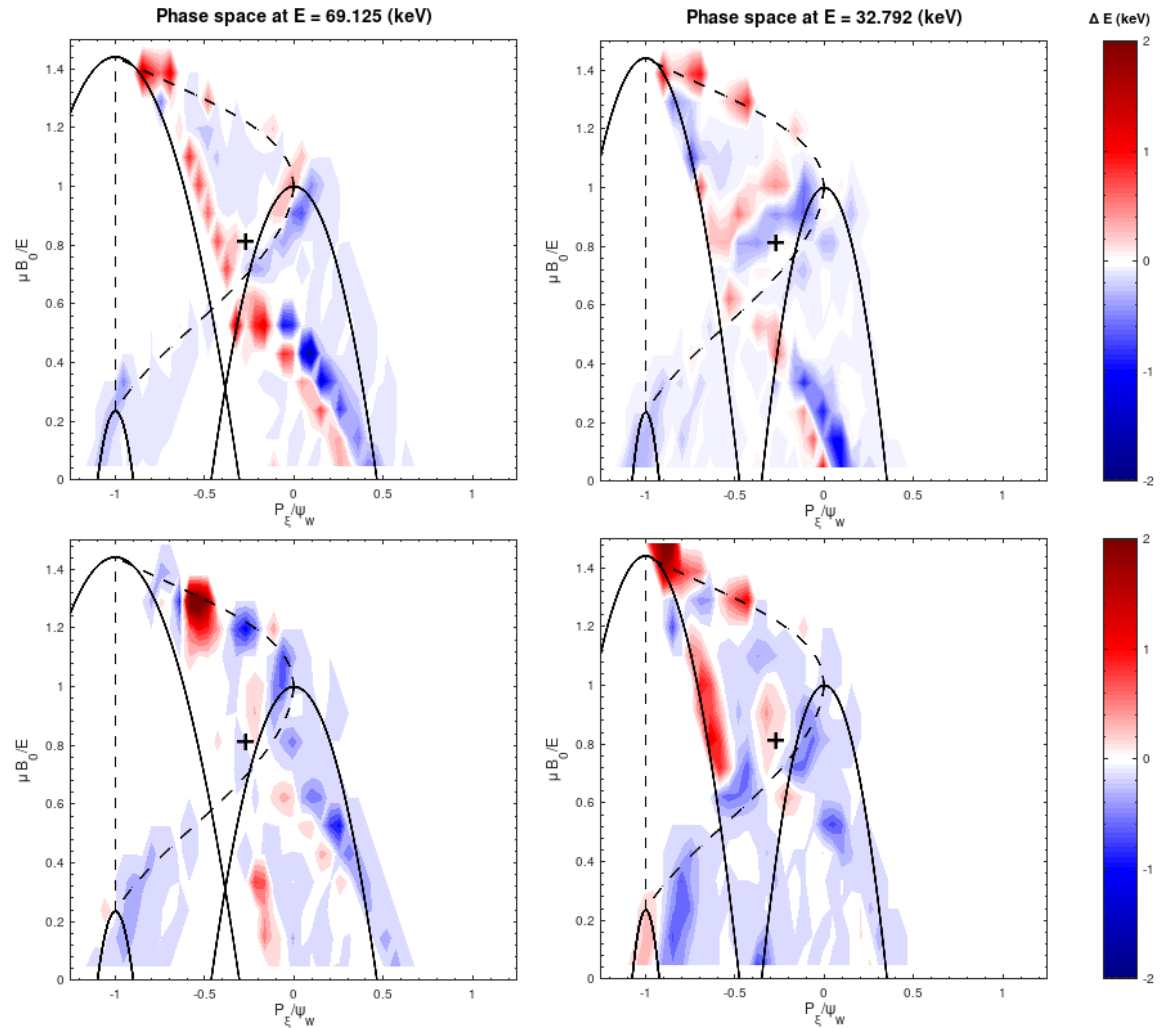
MISHKA based kicks probabilities matrices ...

$n = -1$

n-1_0.13143

$n = -2$

n-2_0.11268



TAE kick probability matrix pdf at $t = 130$ ms

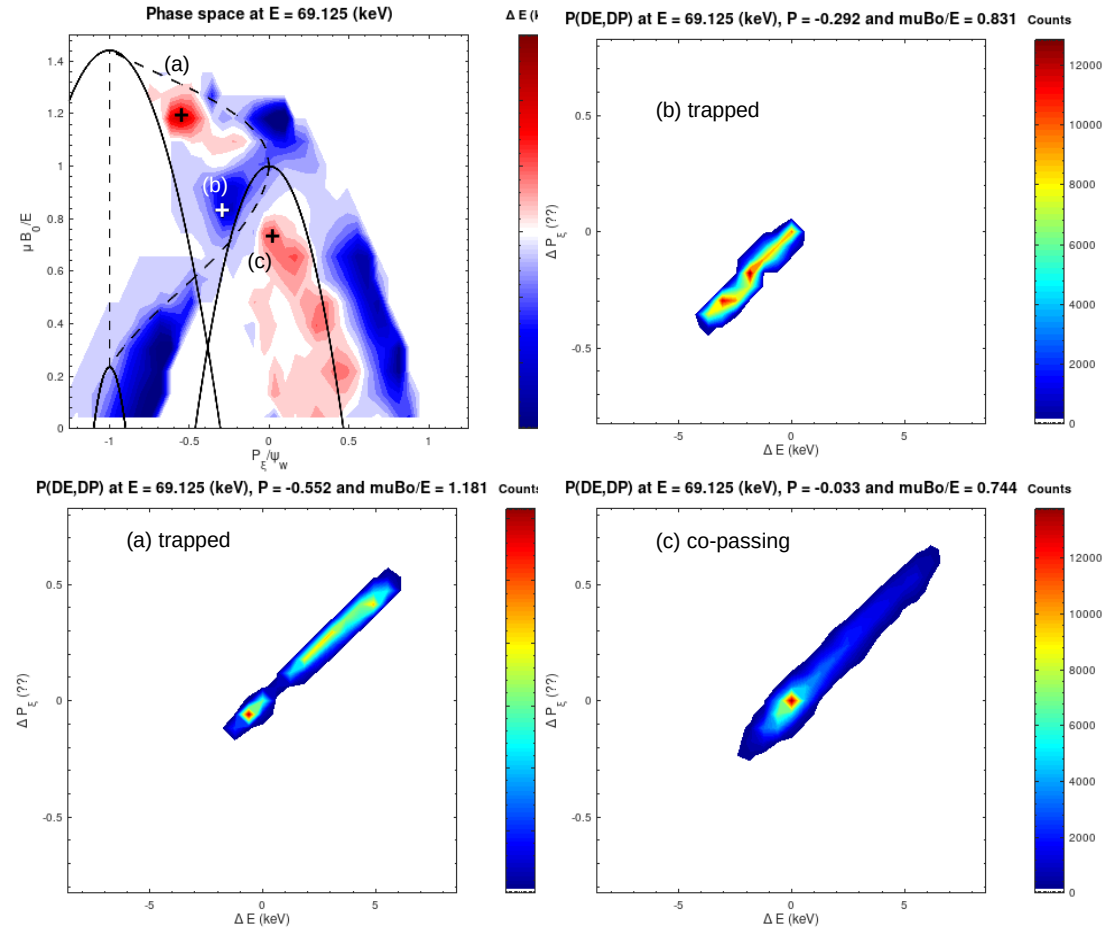
$$p(\Delta E, \Delta P_\xi | E, P_\xi, \mu)$$

ORBIT calculations:

- 29×29 bins $\Delta E \times \Delta P_\xi$, 1 ms simulation time
- $12 E \times 40 P_\xi \times 16 \mu$
- 2000 particles per run uniformly samped in CoM phase space (30 runs).

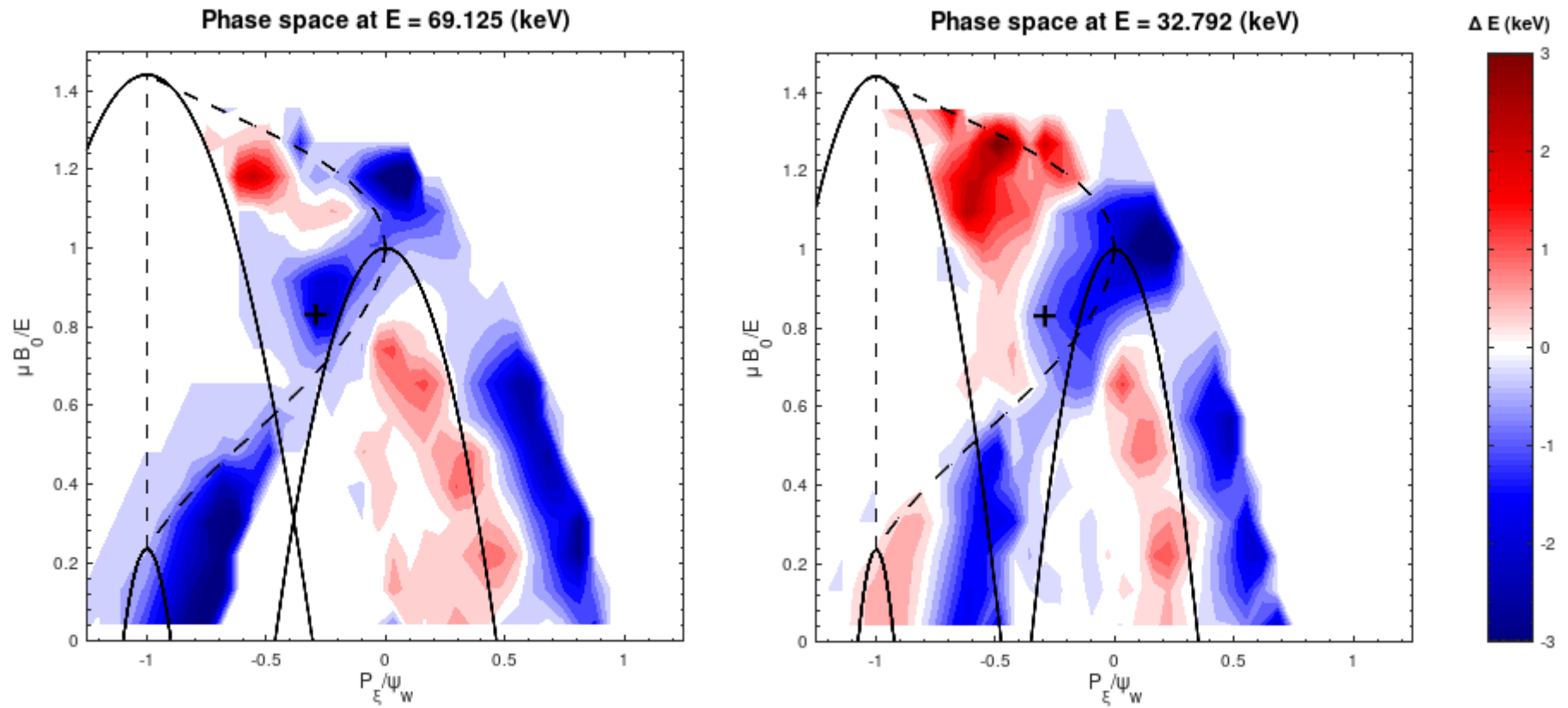
$$\Delta E_{\text{RMS}} = \frac{\sum_i |\Delta E_i| S(\Delta E_i | E, P_\xi, \mu)}{\sum_i S(\Delta E_i | E, P_\xi, \mu)}$$

$$S(\Delta E_i | E, P_\xi, \mu) = \sum_j S(\Delta E_i, \Delta P_{\xi,j} | E, P_\xi, \mu)$$



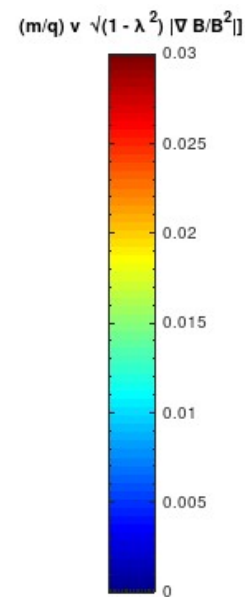
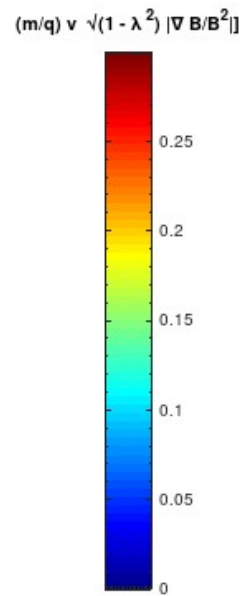
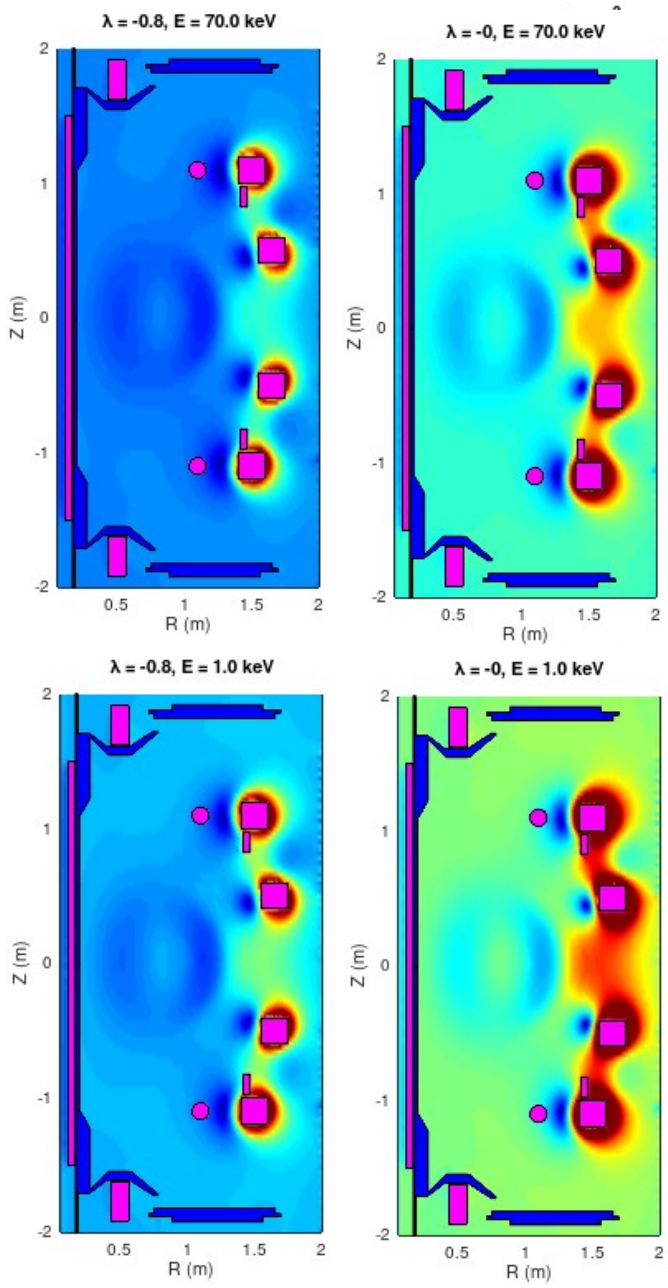
TAE kick probability matrix pdf at $t = 130$ ms

$$p(\Delta E, \Delta P_\zeta | E, P_\zeta, \mu)$$



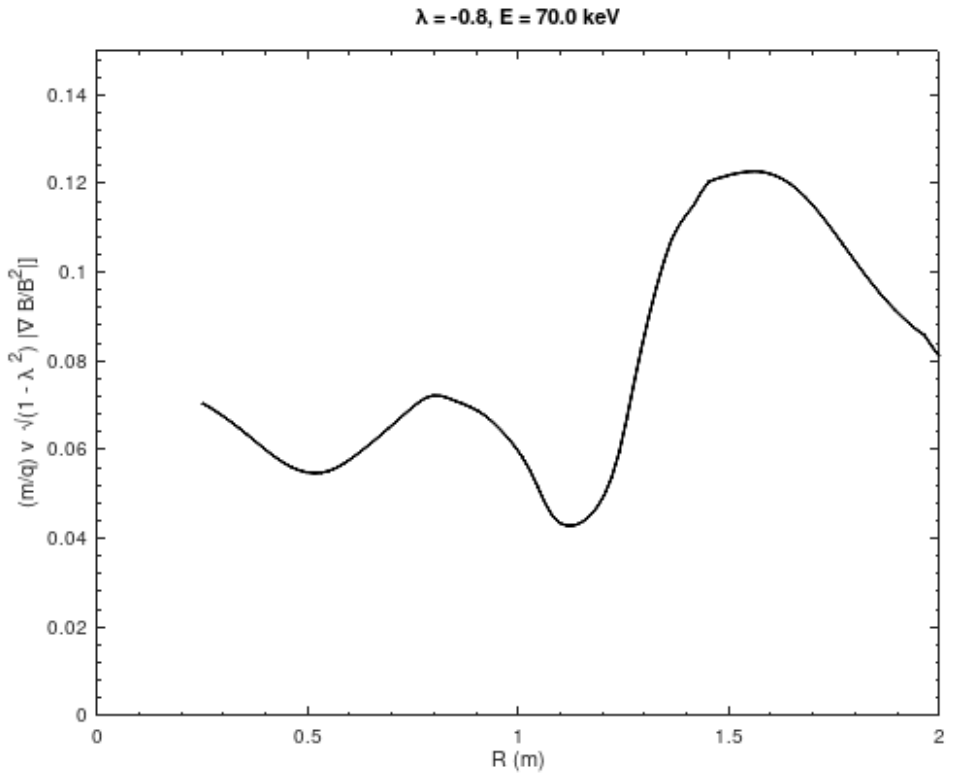
file:///home/marco/Documents/MAST/TRANSP/ufile_read_AEP_kick.m
file:///home/marco/Documents/MAST/TRANSP/PDEDP_plot.m

How well is the magnetic moment conserved in MAST?



$$r_L \left| \frac{\nabla B}{B} \right| \ll 1 \quad \frac{\omega}{\omega_{c,i}} \left| \frac{\tilde{B}}{B} \right| \ll 1$$

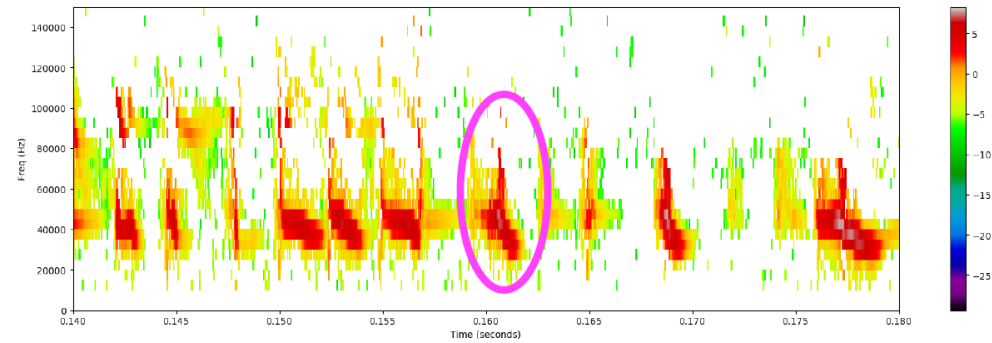
$$\left(\frac{m}{q} \right) \sqrt{\frac{2E}{m}} \sqrt{1 - \lambda^2} \frac{|\nabla B|}{B} \ll 1$$



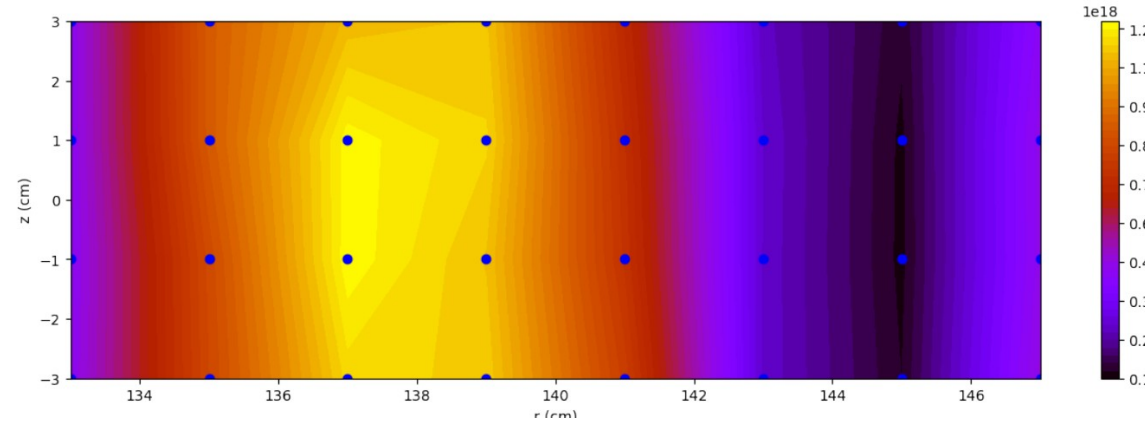
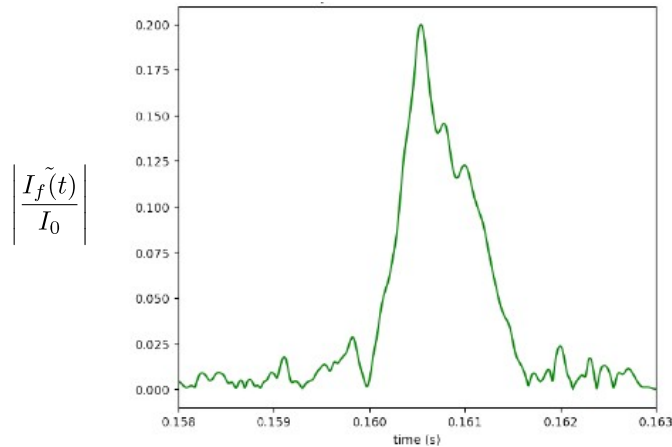
BES fluctuations at 160 ms #29976

- Select the chirp for $n = 1$ at 160ms.
- Apply the mask to the STFT of BES and OMAHA signals
- “Inverse” STFT to retrieve the signals from the selected chirp in time domain
- BES and OMAHA are located in different toroidal positions: correlation analysis to eliminate uncorrelated parts
- Low pass filter amplitudes of the fluctuation associated to the TAE
- No measurements available for 29976 for $R < 1.3$ m

29976
Bz signals (for n = 1 only)

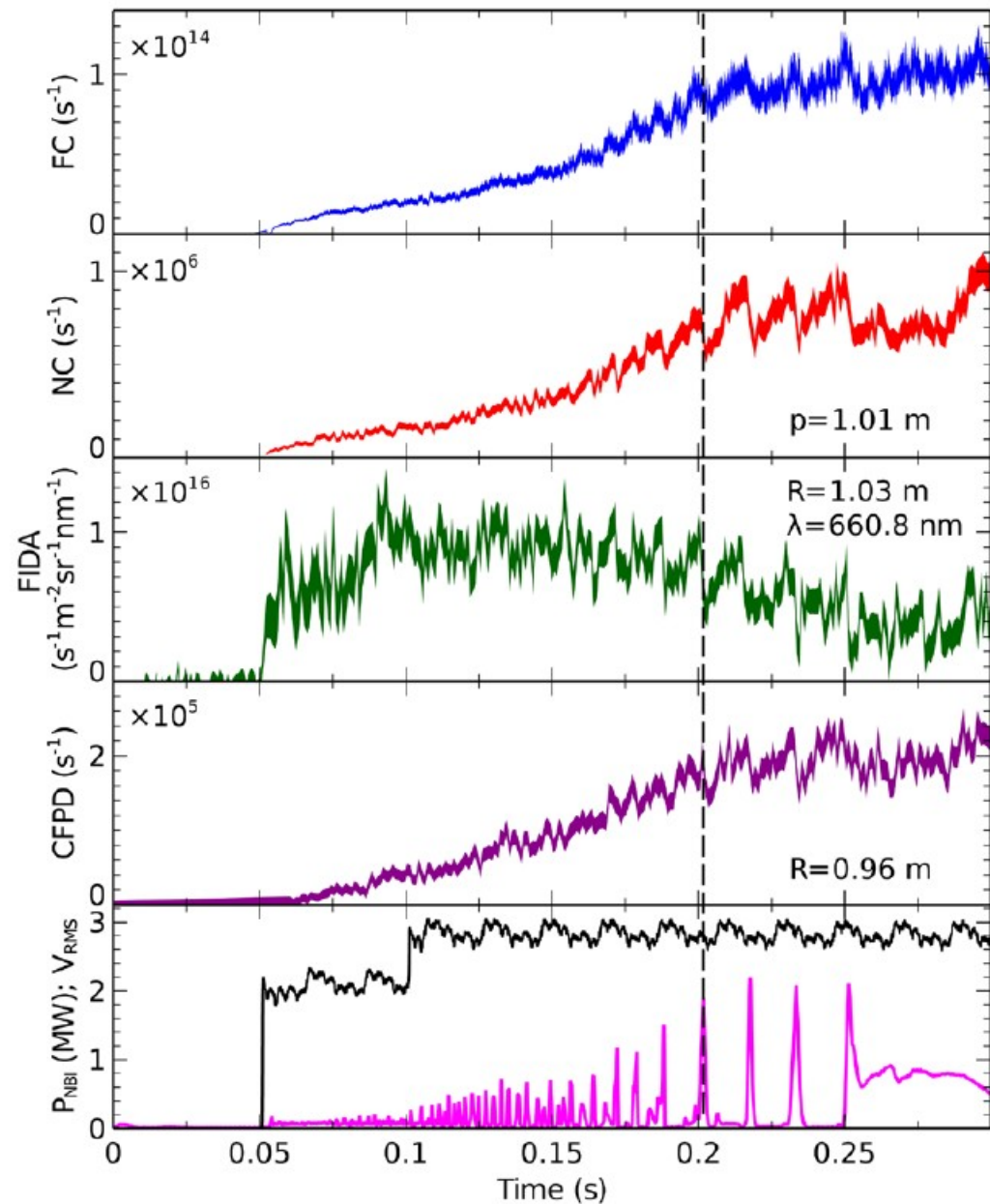
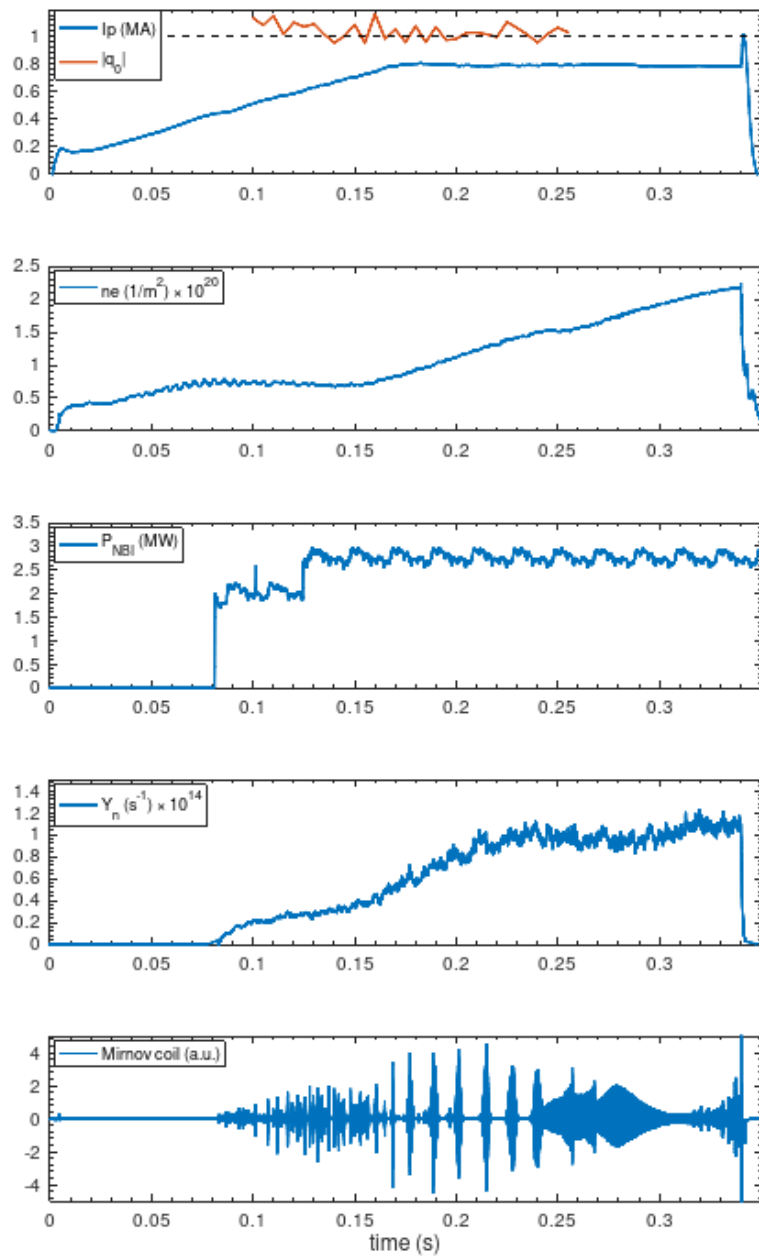


The time where low-pass filtered OMAHA signal at peaks is chosen.

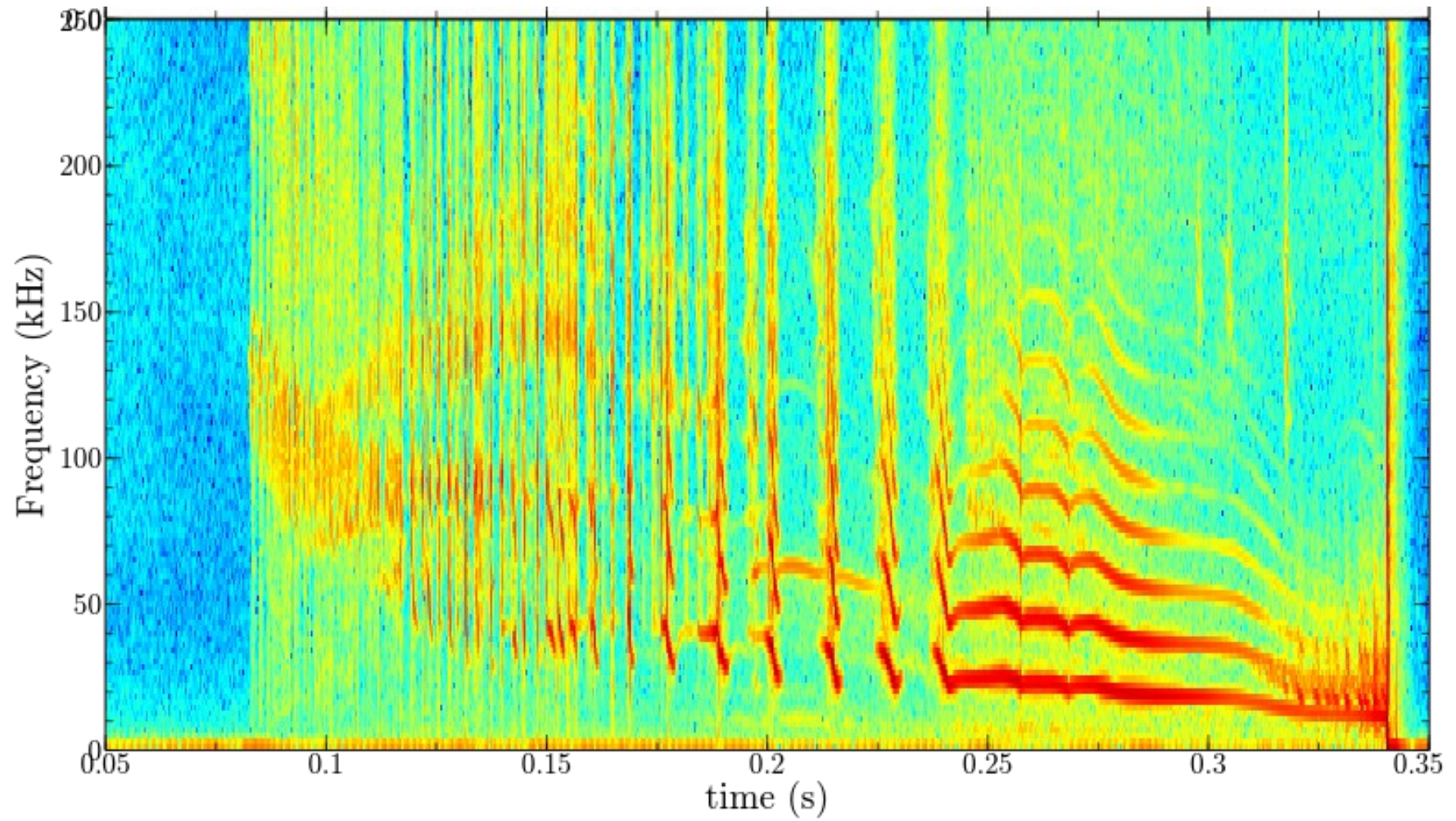


$$\tilde{n} = |(\tilde{I}_f(t)/I_0)_{TAE}| * n(r)$$

Global plasma parameters for 29976 and 29980



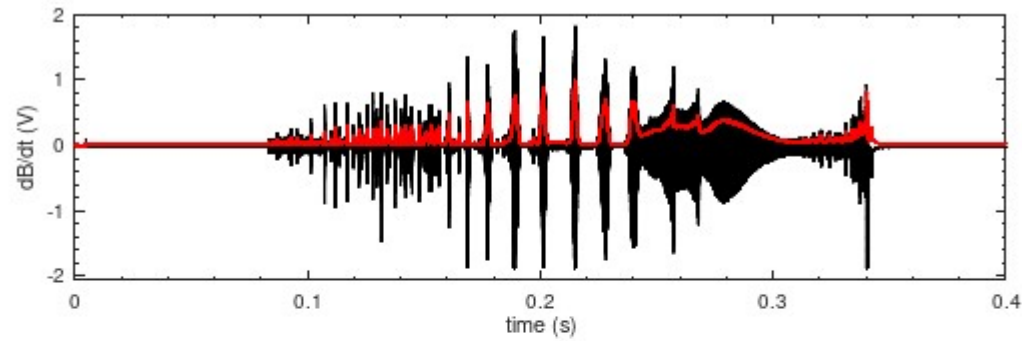
MHD activity in 29976



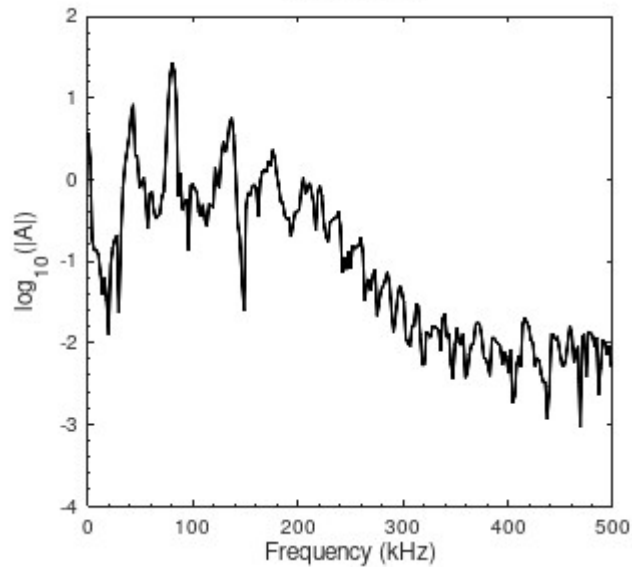
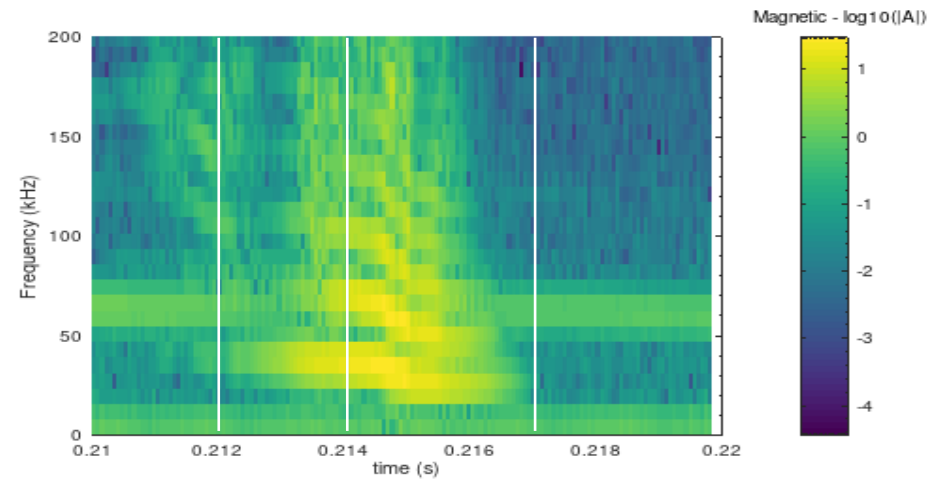
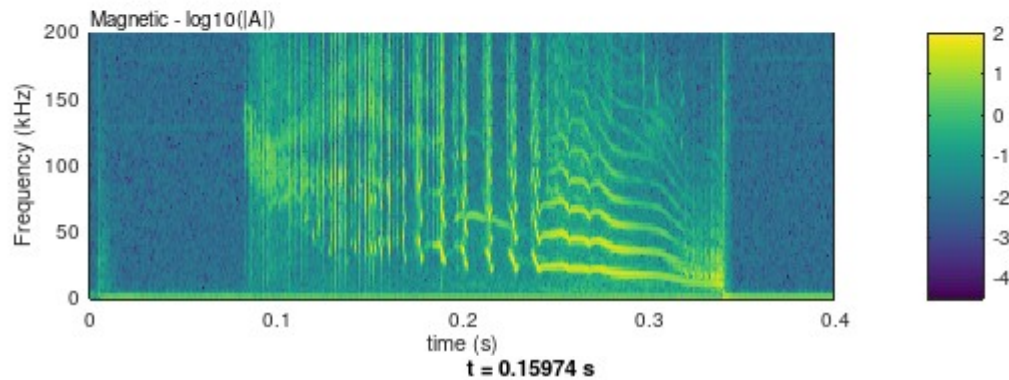
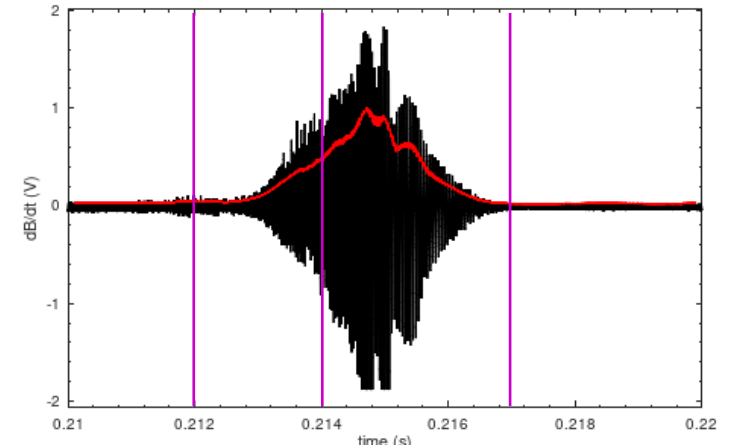


MHD activity in 29976

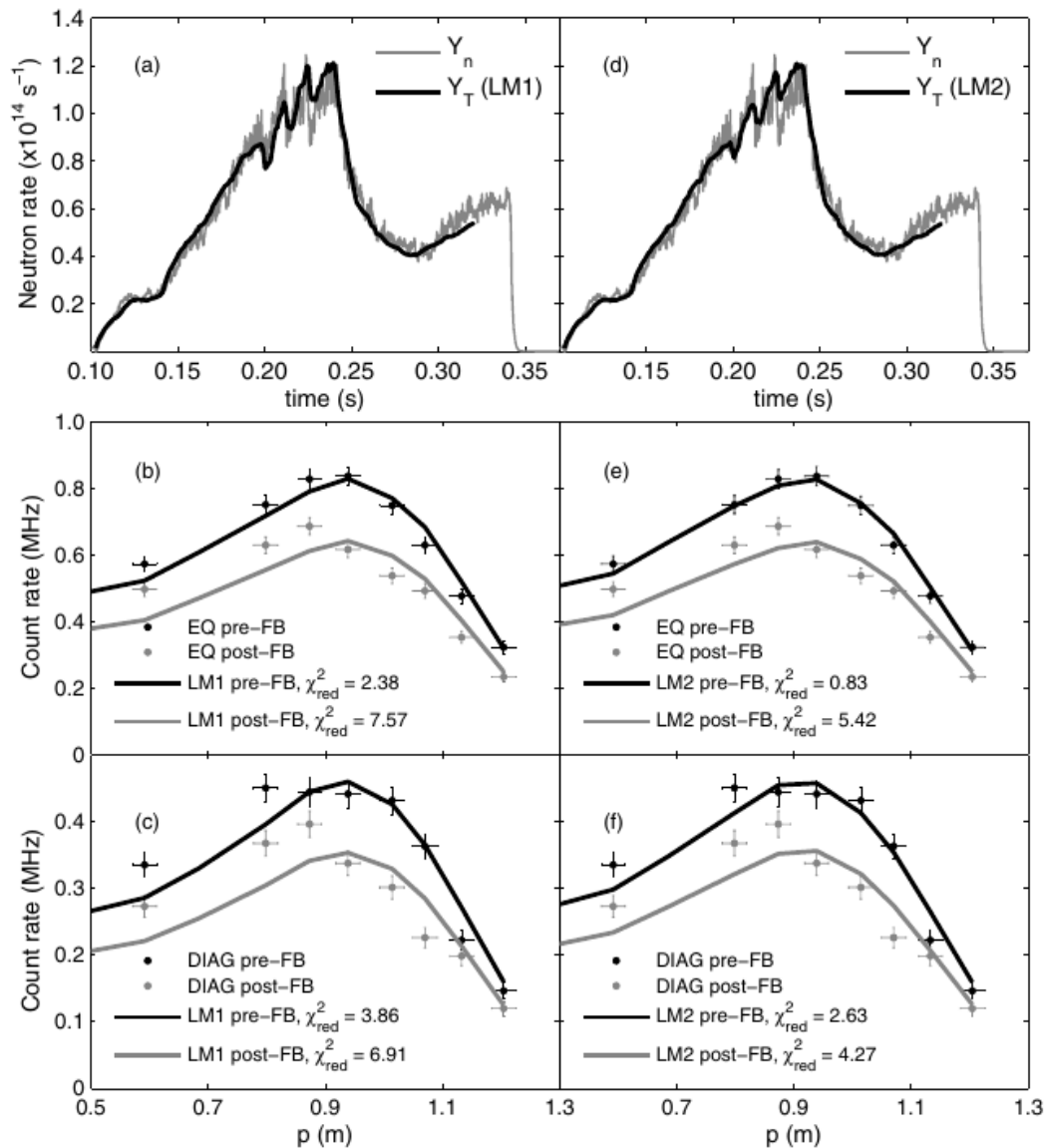
29976 OMAHA 4LR



29976 OMAHA 4LR



AFID and FB model combination



Reasonably good agreement between TRANSP/NUBEAM predicted neutron rates and FC and NC measurements, but:

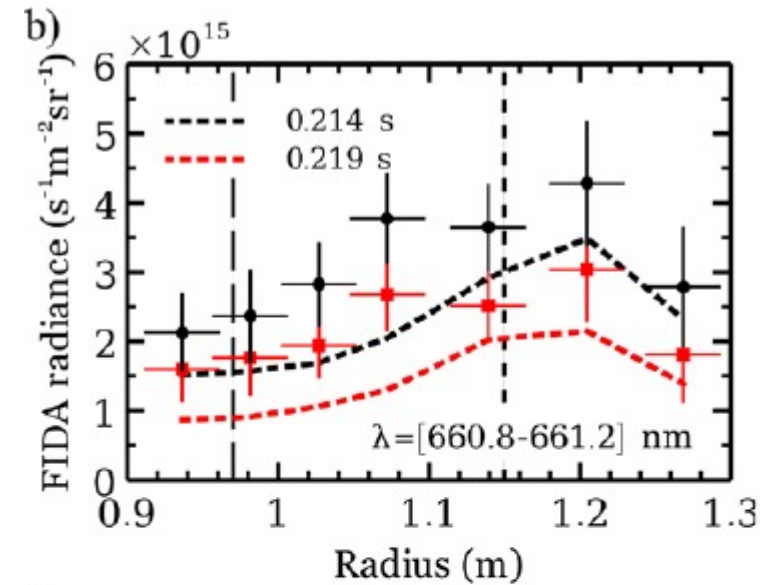
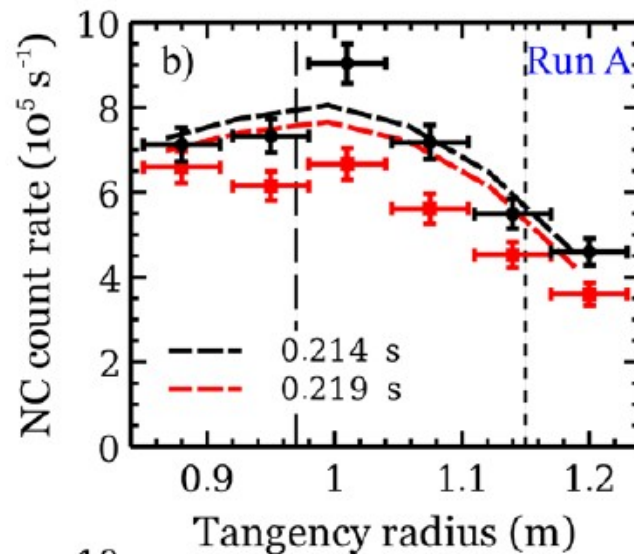
- the same underlying FI distribution does not match FIDA and NC spatial profiles simultaneously, and
- no physics model nor justification for the “ad hoc” adjustments.

Can we do any better?

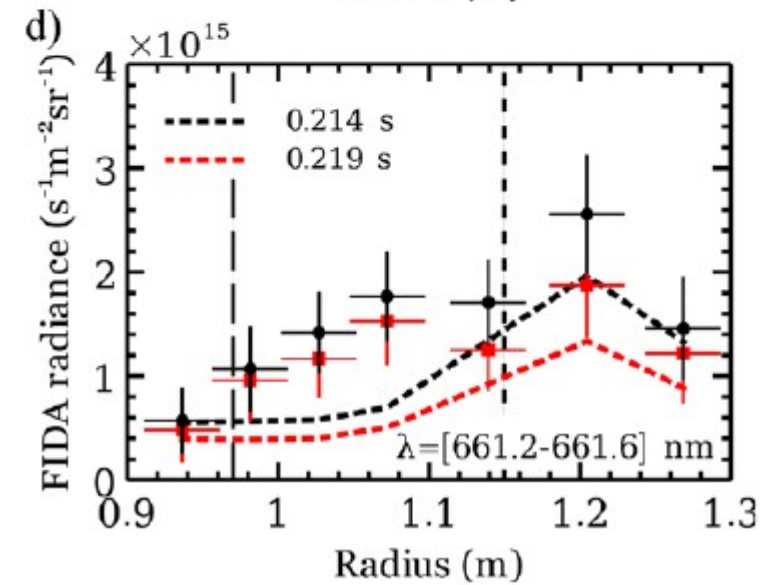
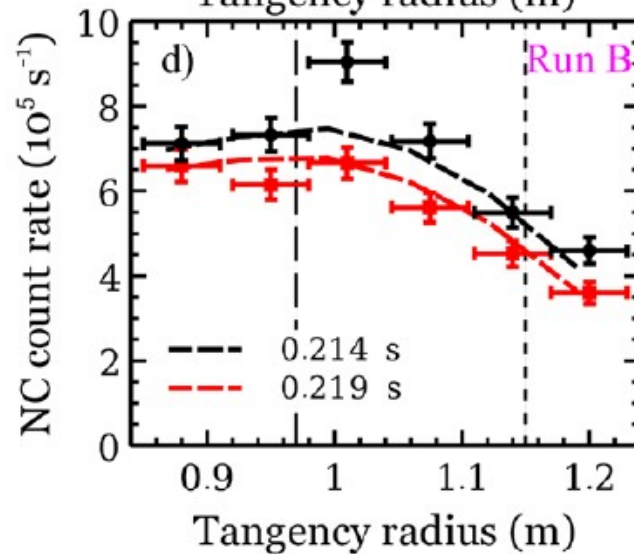
Klimek I, Cecconello M, Gorelenkova M, Keeling D, Meakins A, Jones O, et al. TRANSP modelling of total and local neutron emission on MAST. Nuclear Fusion. 2015 Feb 1;55(2):023003.

FIDA and NC profiles before/after a FB vs two different FB model implementations

In **run A**, FIs with $50 \text{ keV} \leq E \leq 75 \text{ keV}$ and $0.69 \leq p \leq 0.93$ were removed by the fishbones; these values of pitch correspond to co-passing FIs.

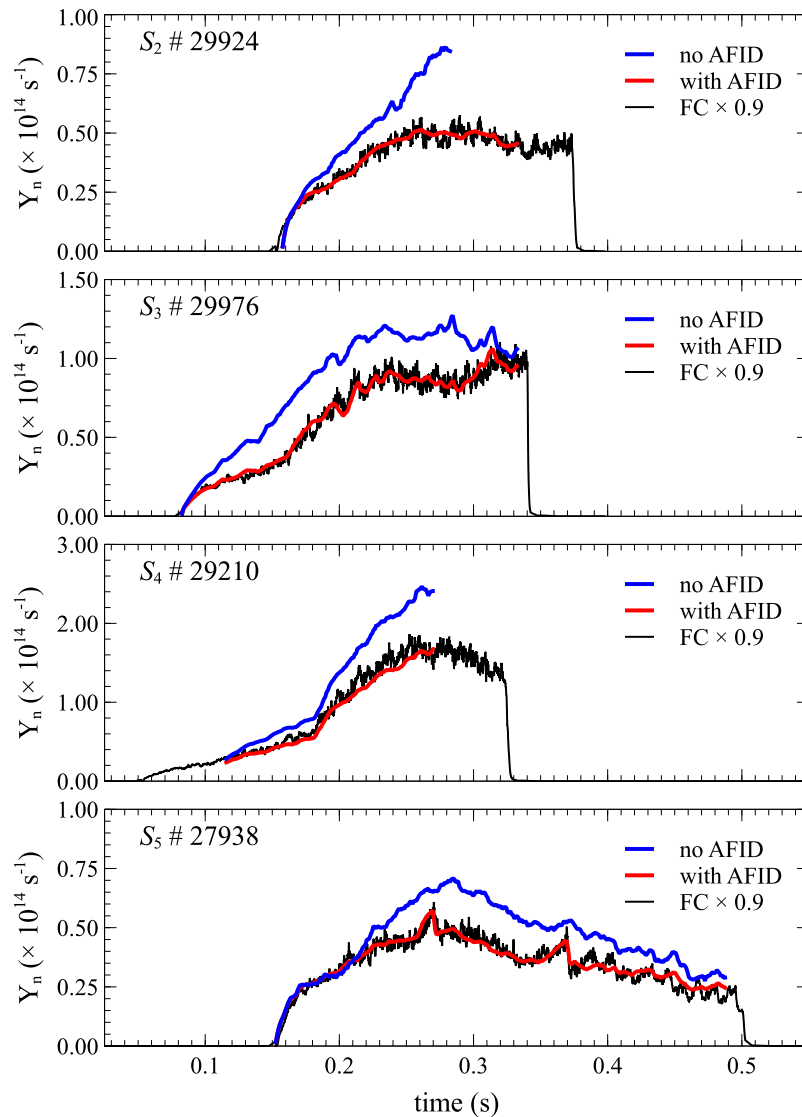


In **run B**, these ranges were $60 \text{ keV} \leq E \leq 70 \text{ keV}$ and $0 \leq p \leq 0.7$, corresponding to trapped and co-passing FIs.





Anomalous fast ion diffusion coefficient



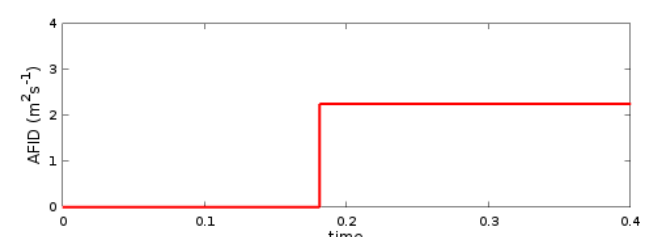
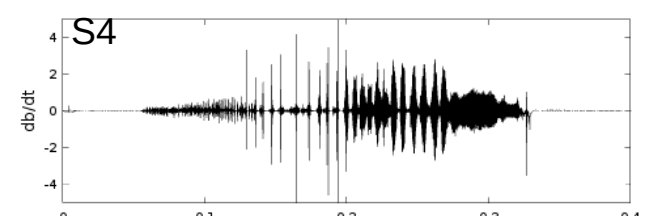
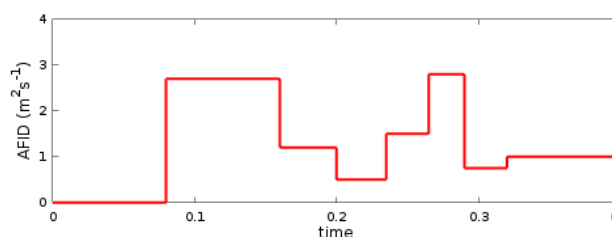
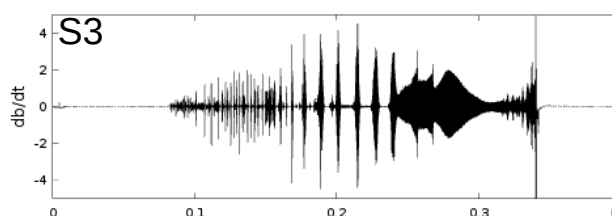
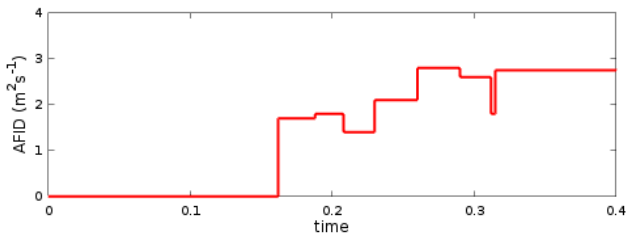
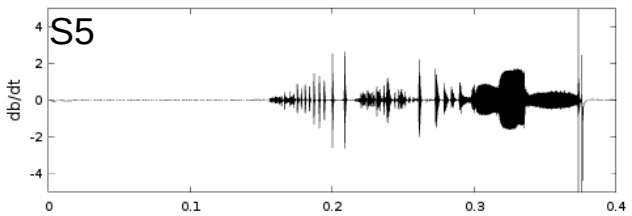
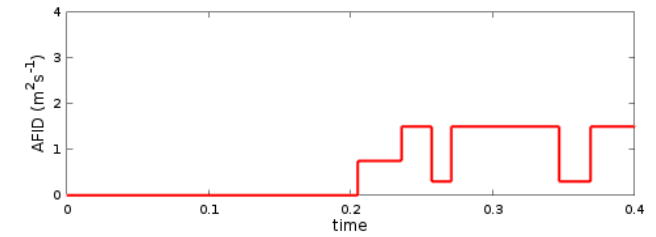
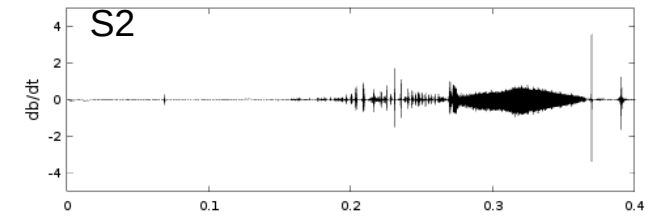
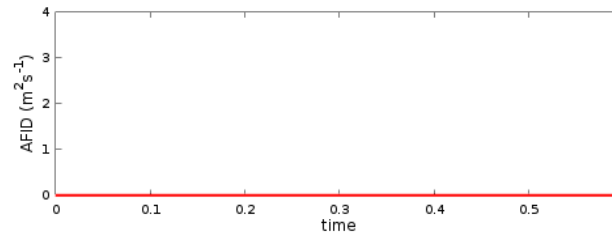
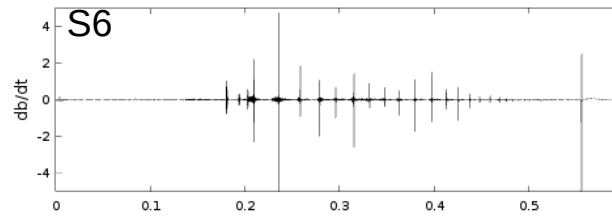
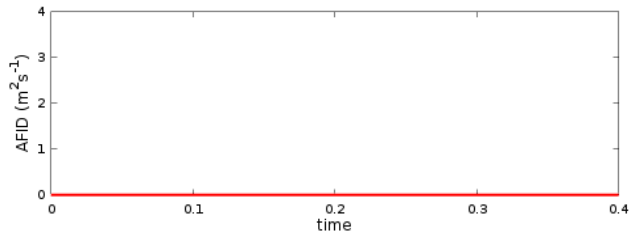
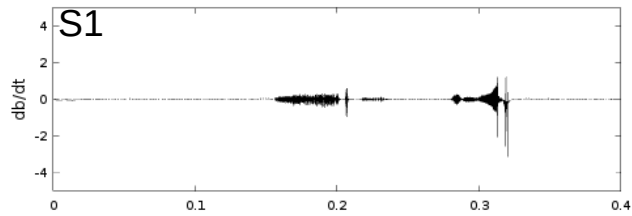
M9 FPP 01: Intermediate MHD activity (low density)
1.5 MW NBI, on-axis, $I_p = 800 \text{ kA}$
#29922 – 29931
AFID = 0.0 – 2.0 m^2s^{-1}

M9 FPP 01: Large fishbones series
2.75 MW NBI, on-axis, $I_p = 800 \text{ kA}$
#29975 – 29980
AFID = 0.0 – 2.8 m^2s^{-1}
FB model: $E > 50 \text{ keV}$, $\lambda = [0.69, 0.93]$

M9 SOL 003: Large fishbones, high NBI power series
3.4 MW NBI, on-axis, $I_p = 1 \text{ MA}$
#29132 – 29359
AFID = 0.0 – 2.2 m^2s^{-1}

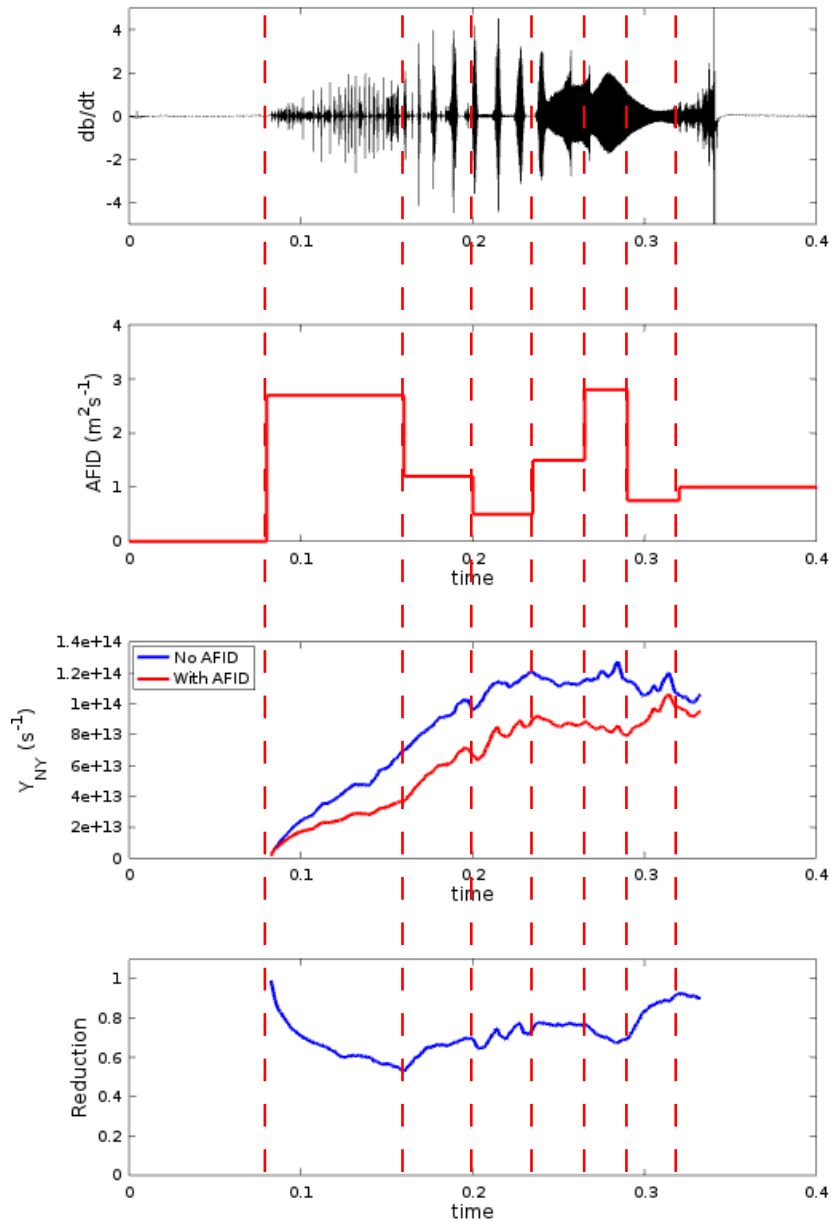
M8 IPS 004: quiescent MHD with LLM, low NBI power series
1.5 MW NBI, off-axis, $I_p = 630 \text{ kA}$
#27932 – 27938
AFID: 0.0 – 1.5 m^2s^{-1}
FB model: $E > 50 \text{ keV}$, $\lambda = [-0.50, 0.60]$

Anomalous fast ion diffusion coefficient: what is the justification?



Anomalous fast ion diffusion coefficient: what is the justification?

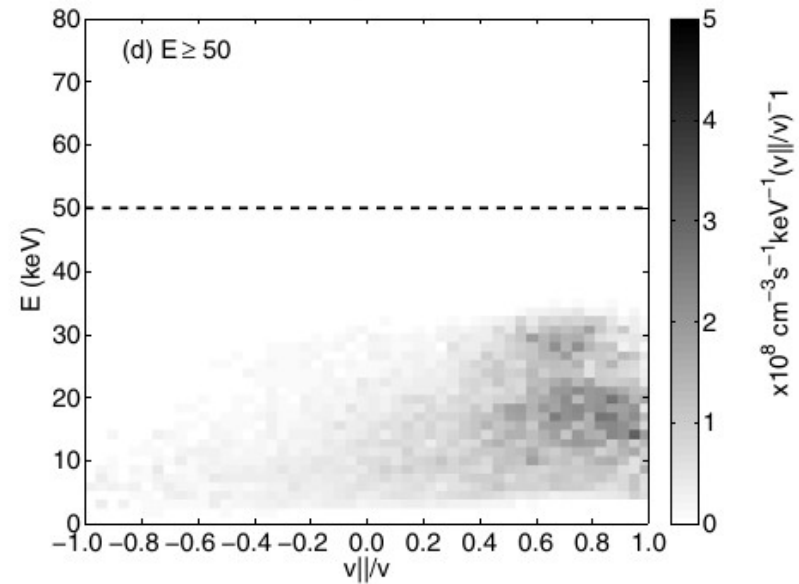
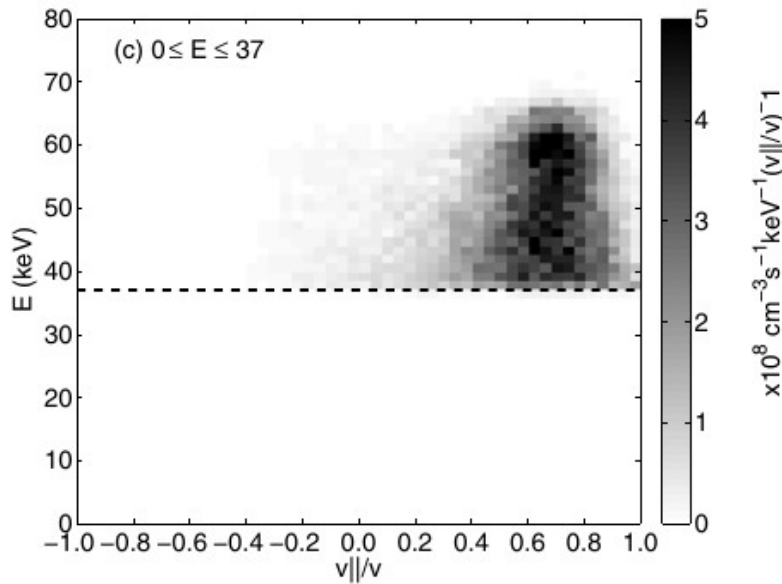
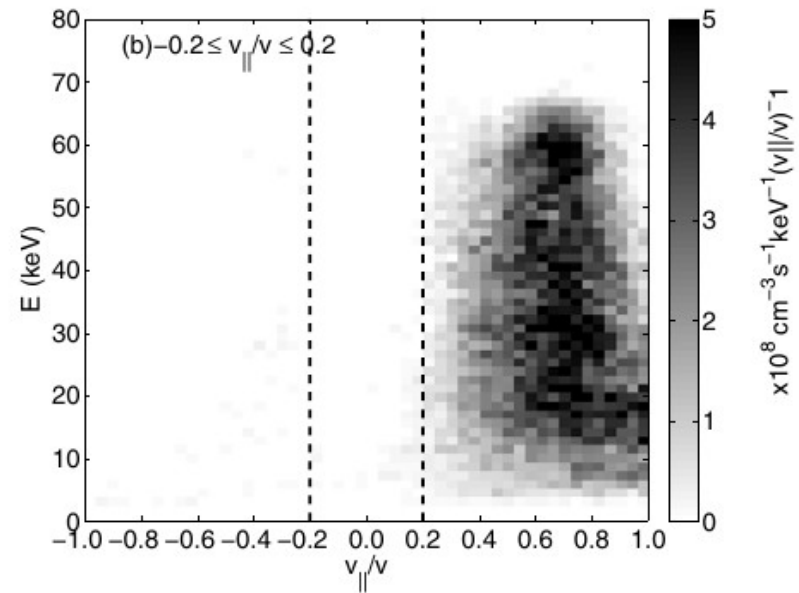
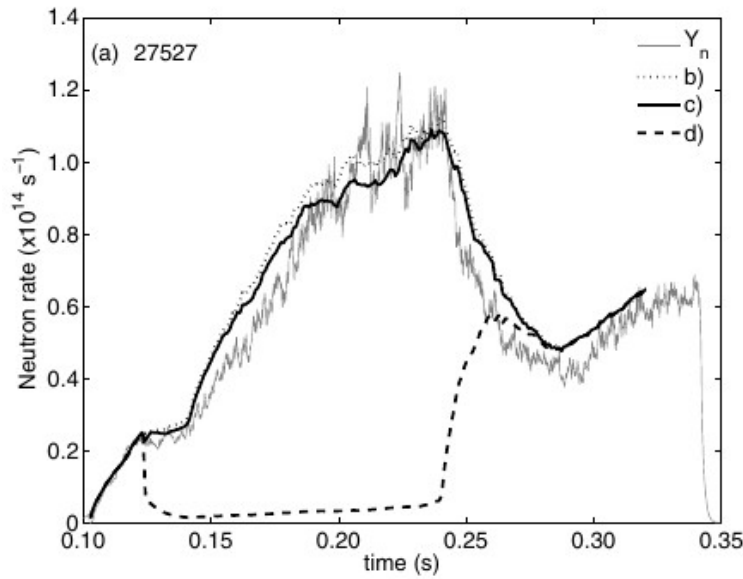
S3, # 29976



- Level of AFID used inconsistent with the level of MHD activity alone.
- Frequency spectrum and mode numbers of the MHD instabilities

The NUBEAM fishbones “model”

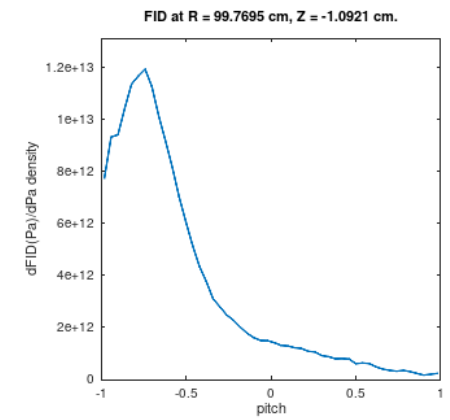
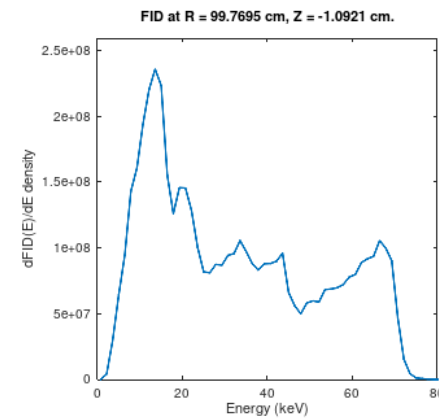
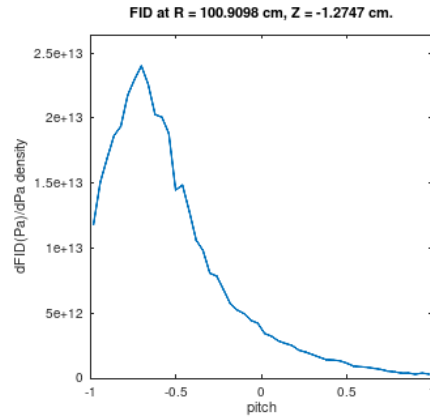
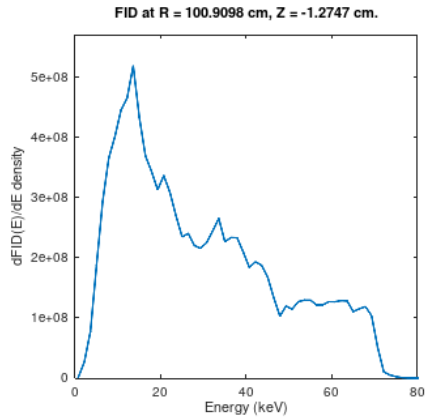
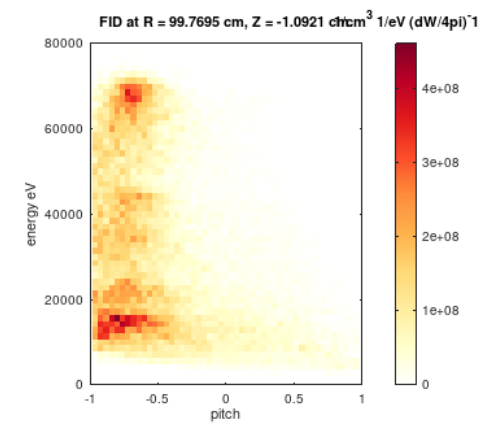
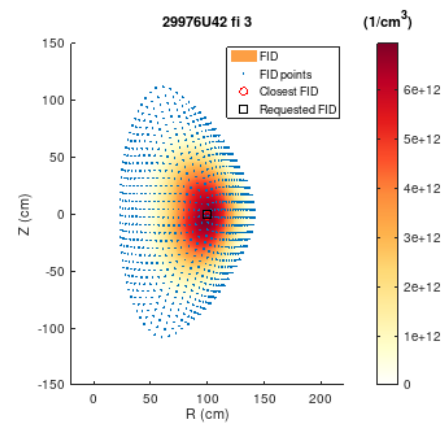
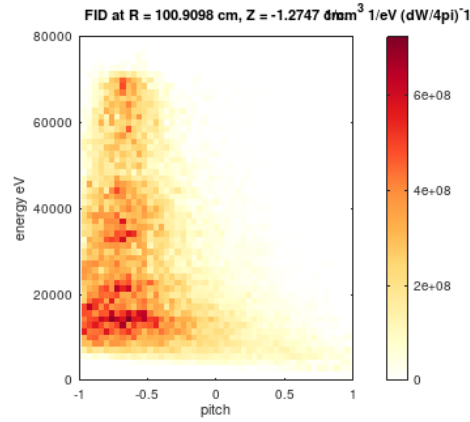
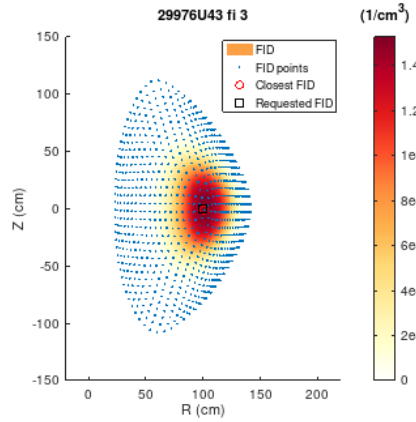
Klimek I, Cecconello M, Gorelenkova M, Keeling D, Meakins A, Jones O, et al. TRANSP modelling of total and local neutron emission on MAST. Nuclear Fusion. 2015 Feb 1;55(2):023003.



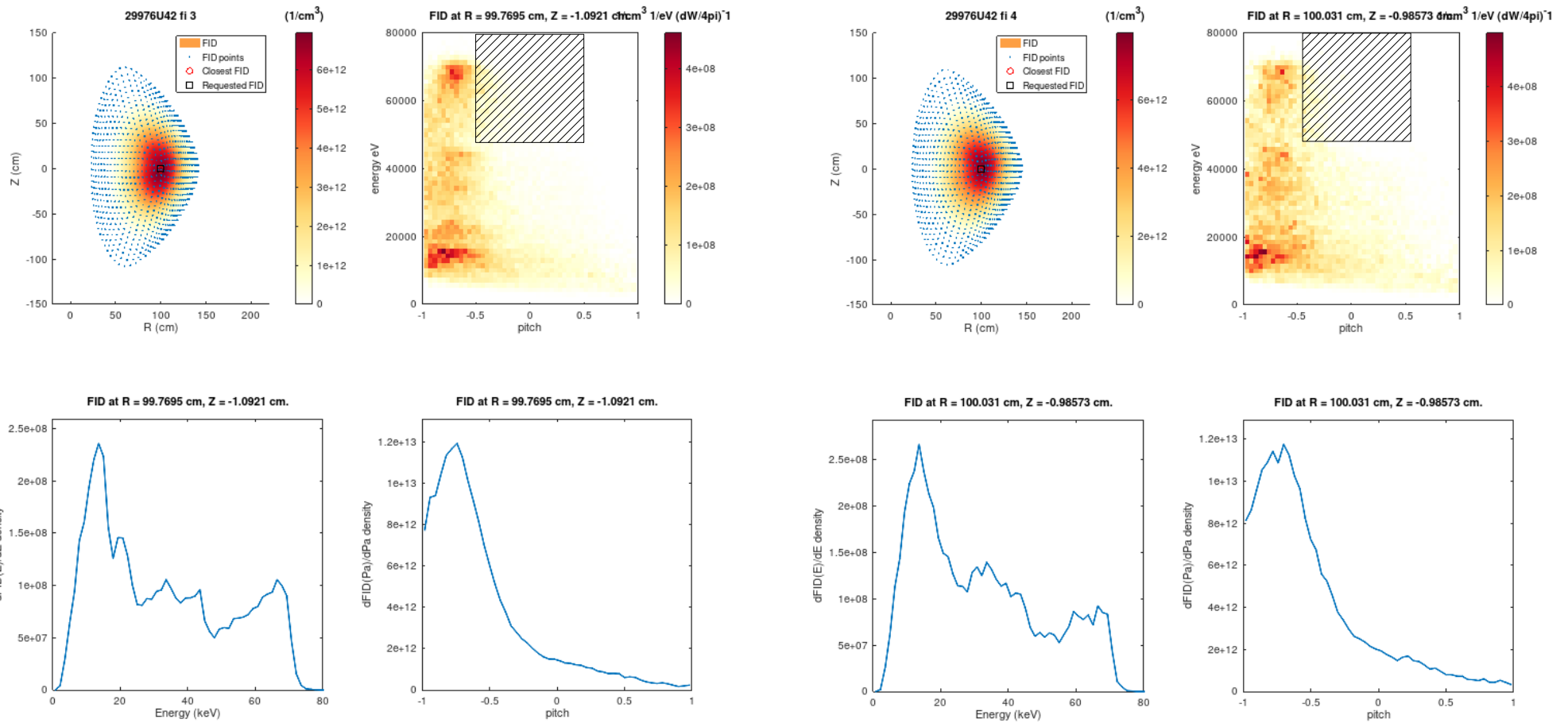
FI distributions with and without AFID

without

with



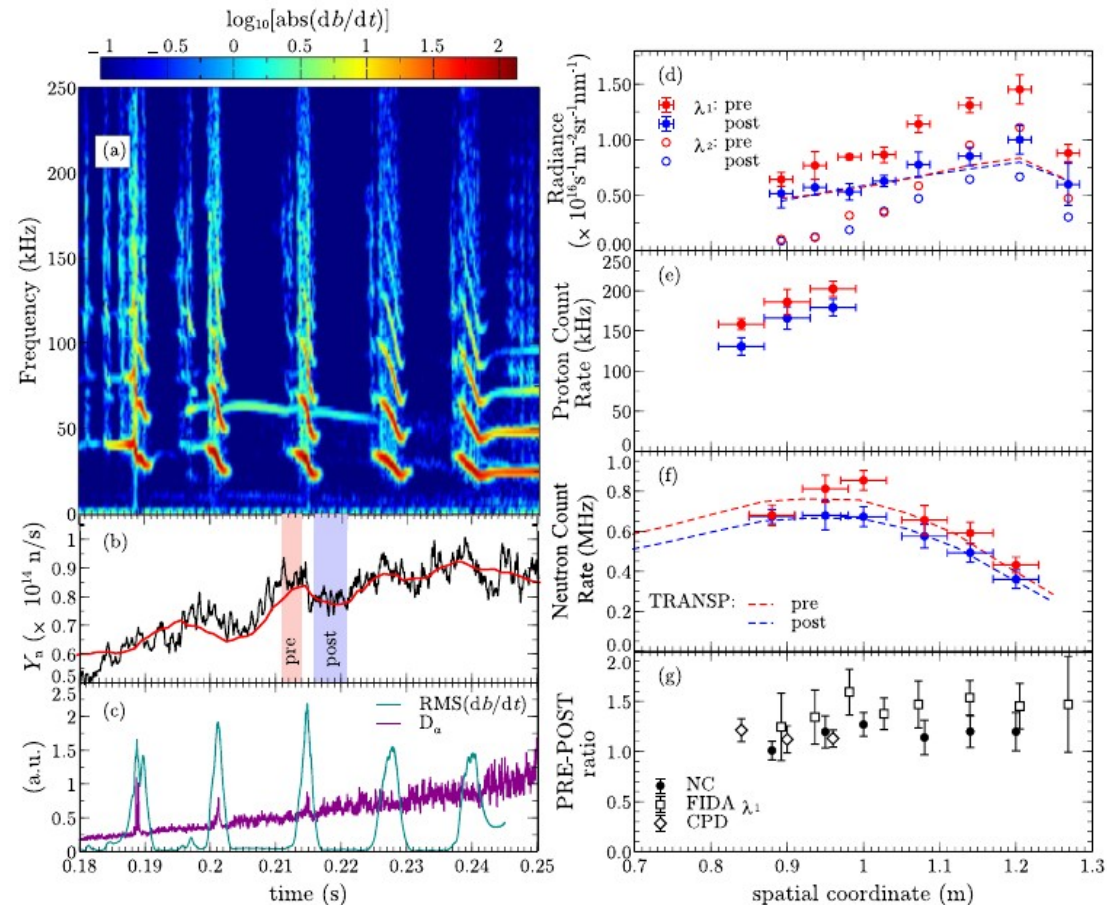
FI distribution: AFID and FB



Kick-model applied to MAST measurements

Summary of the work done at PPPL:

- the entire kick-model workflow has been discussed with Mario, Werner and Alexander
- the FB event at $t = 210$ ms of pulse 29976 has been analysed by Mario before our visit
- Werner, Alexander and I worked our way through Mario's simulation by processing our own simulation
- account on PPPL cluster secured and first runs of ORBIT with perturbations carried out
- Kick-model available only on the PPPL cluster (but nothing prevent us to replicate the framework using other tools such as LOCUST...)
- kick-model matrix generated for both FBs and TAEs
- test TRANSP runs submitted with the kicks included
- discussed way ahead



Cecconello M et al. Energetic ion behaviour in MAST. Plasma Physics and Controlled Fusion. 2015 Jan 1;57(1):014006.

MHD spectroscopy for q profile reconstruction

Modes used to identify radial position of resonant surfaces:

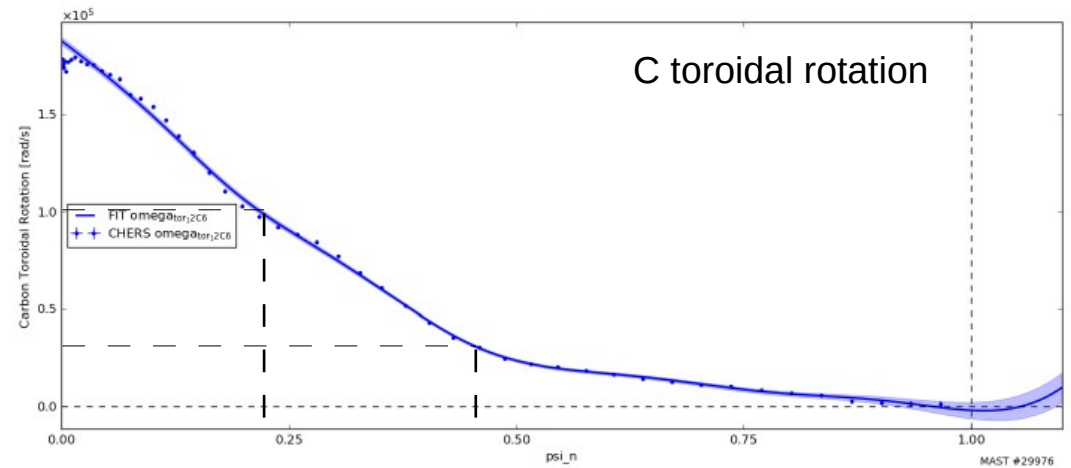
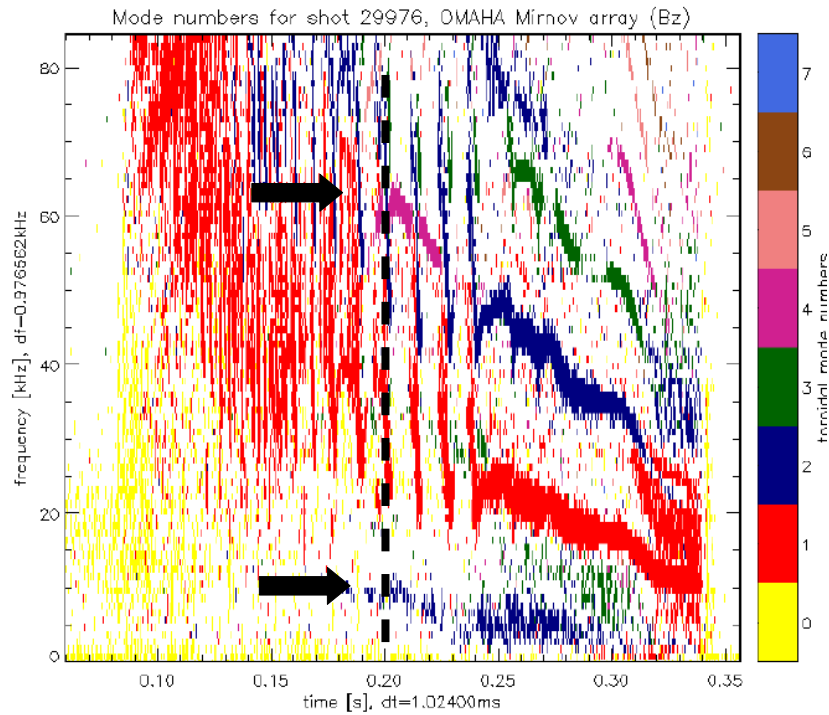
$$n_{1,2} = 2, 4$$

$$q(s_1) = \frac{m_1}{4} \quad @ 60 \text{ kHz}$$

$$q(s_2) = \frac{m_2}{2} \quad @ 10 \text{ kHz}$$

@ 60 kHz

@ 10 kHz



$$\omega(s_1) \approx 9.4 \times 10^4 \text{ rad/s}$$

$$s_1 \approx 0.24$$

$$q(s_1) = \frac{5}{4}$$

$$\omega(s_2) \approx 3.14 \times 10^4 \text{ rad/s}$$

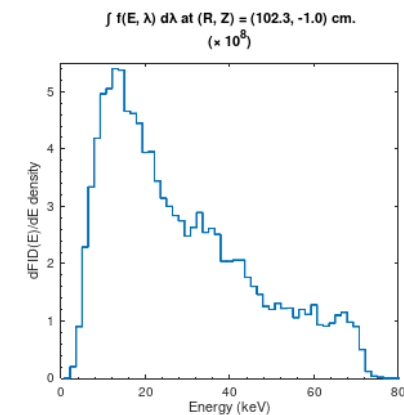
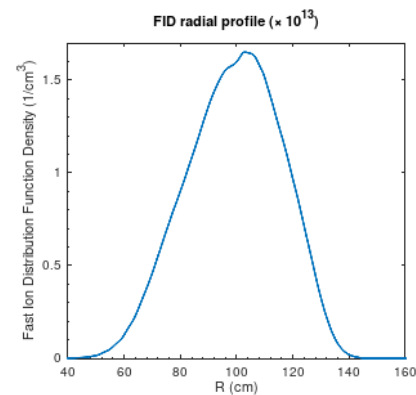
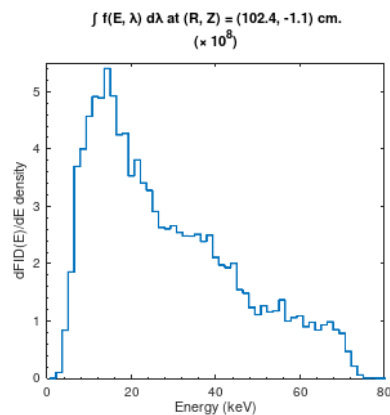
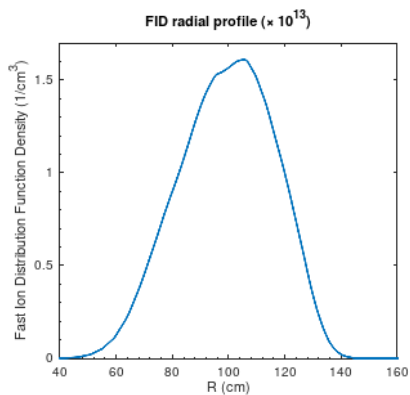
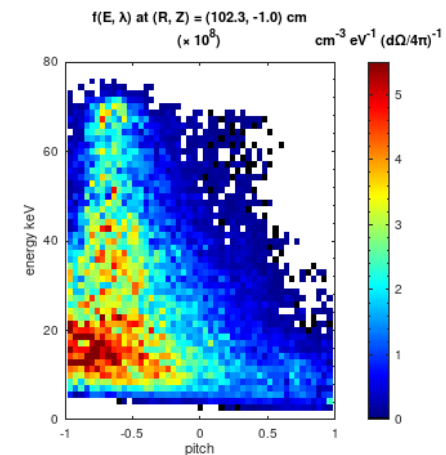
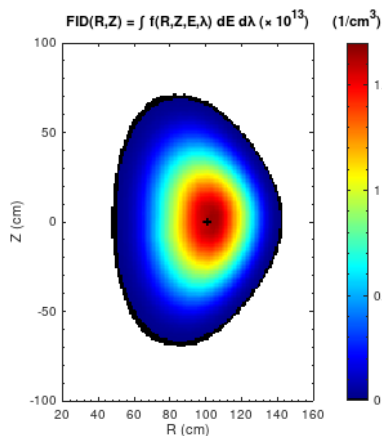
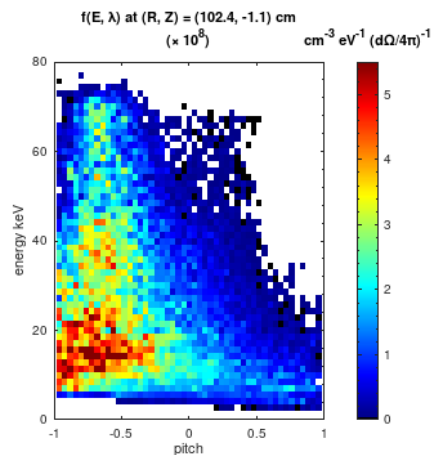
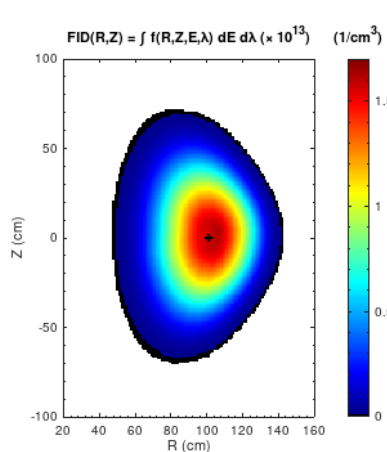
$$s_2 \approx 0.45$$

$$q(s_2) = \frac{3}{2}$$

FID for reference TRANSP run (no AFID/Kicks)

$t = 212$ ms

$t = 217$ ms



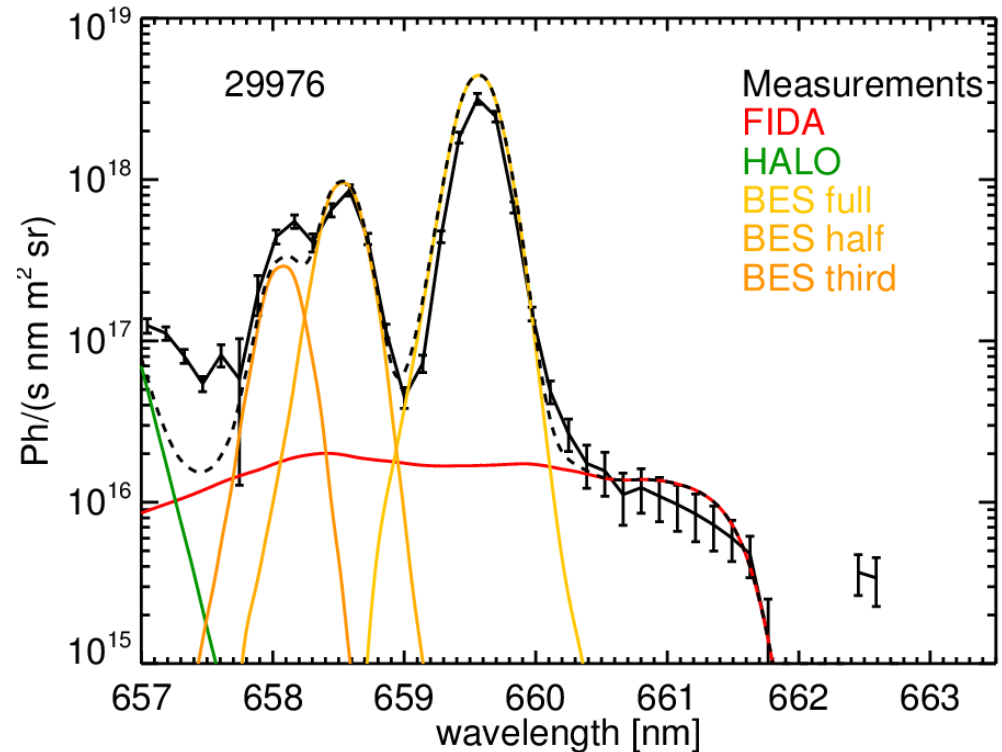
/media/marco/WDPassport/marco/Documents/MAST/TRANSP/RUNS/29976/U57/29976U57 fi 7.cdf

/media/marco/WDPassport/marco/Documents/MAST/TRANSP/RUNS/29976/U57/29976U57 fi 9.cdf



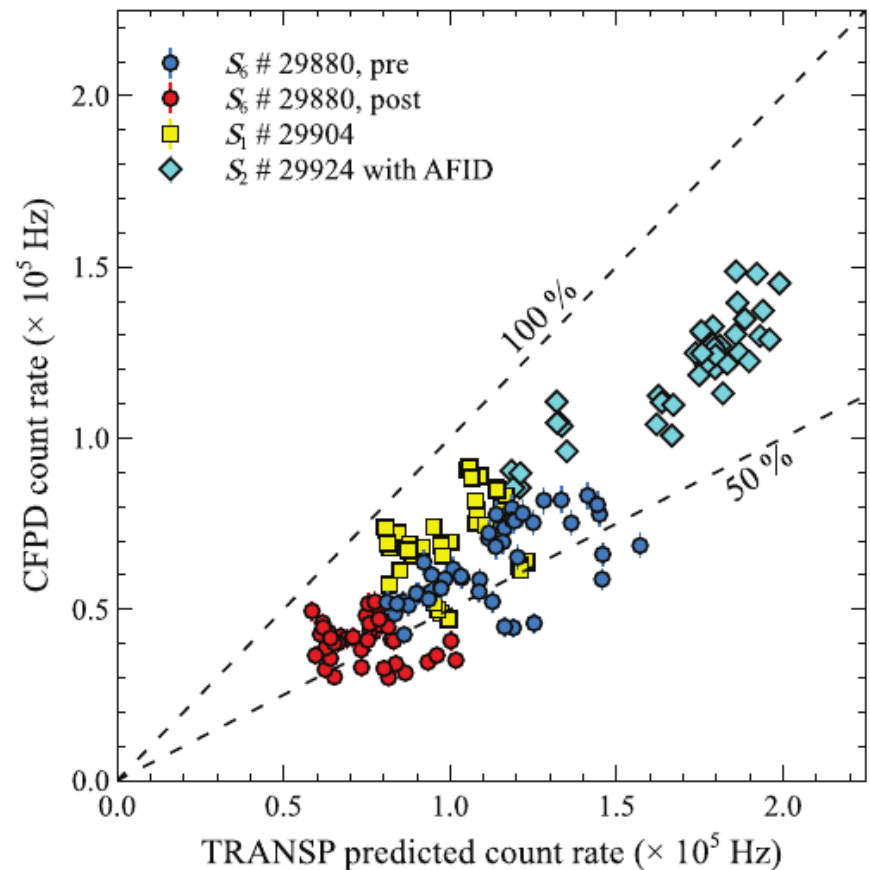
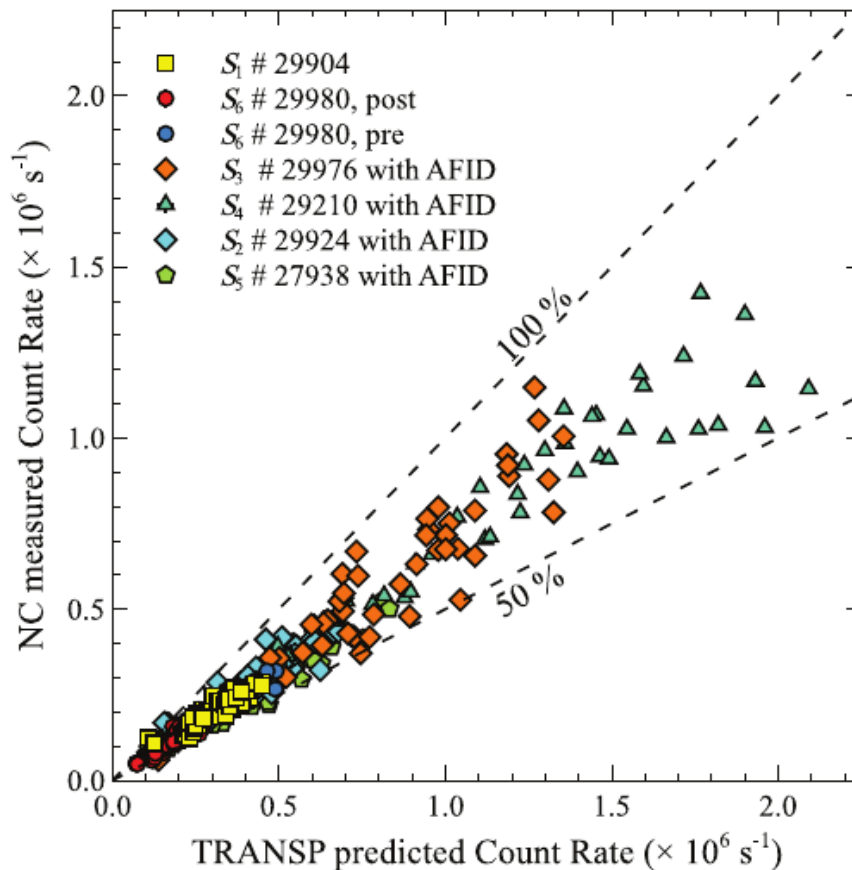
FIDA/FIDASIM comparison

- Experimental measurements are scaled up by factor 1.8 to better match beam emission peaks.
- The intensity between 660.8 nm and 661.4 nm is integrated to obtain radial profiles.
- Experimental measurements are active minus passive. FIDASIM is only the simulated active FIDA.
- Experimental measurements are averaged over 1 ms.



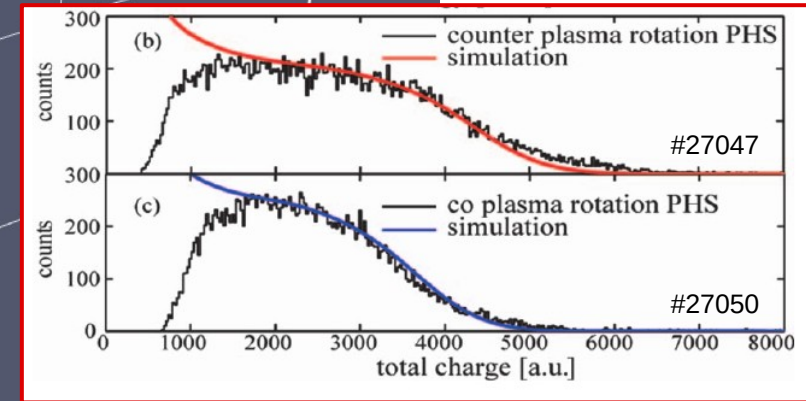
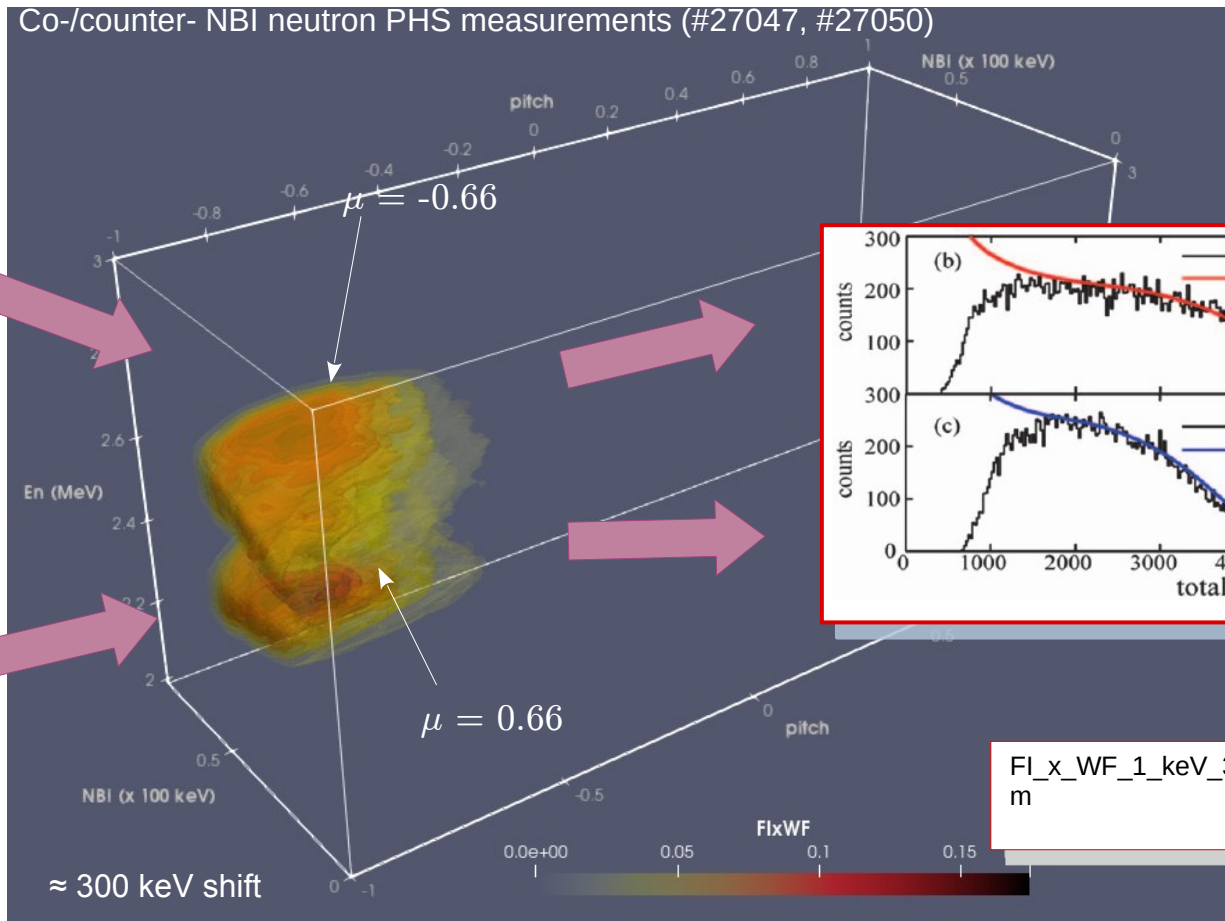
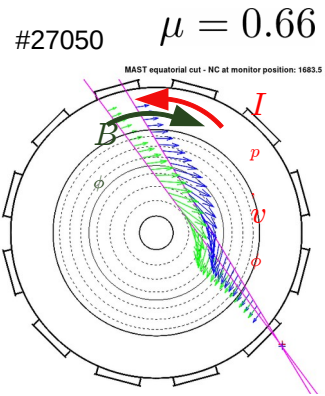
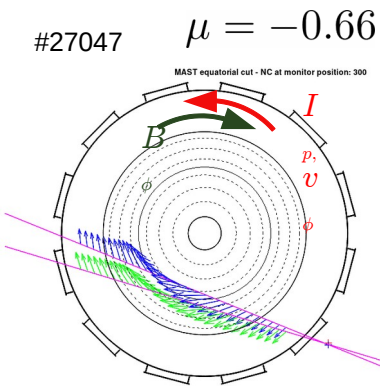
Neutron discrepancy on MAST

TRANSP predicted DD rates need to be multiplied by a factor 0.6 on average to match neutron camera and CFPD measured rates.



Neutron camera weight functions

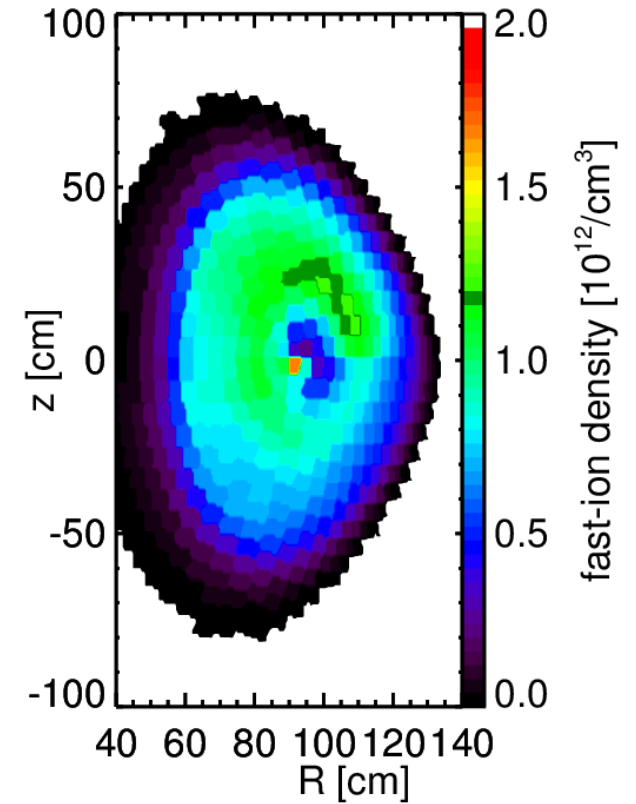
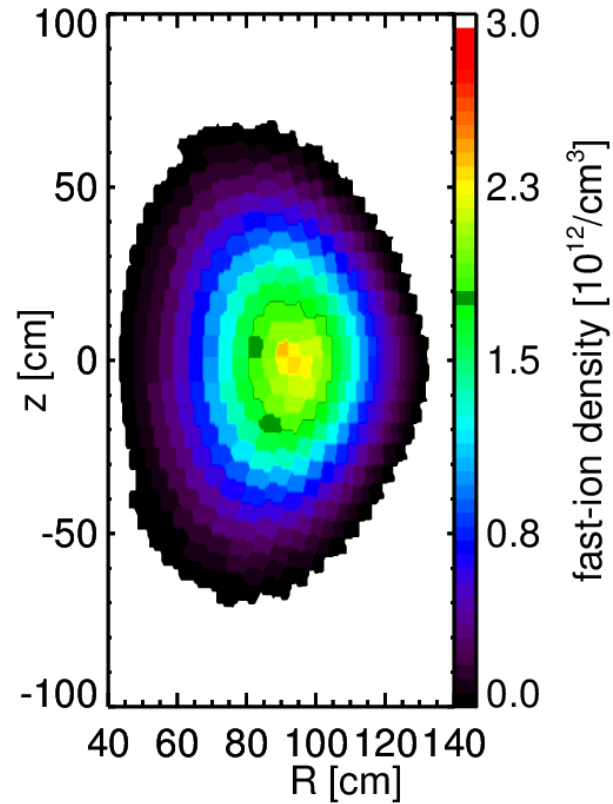
$$\omega(\lambda_i, E_{D,j}, \mu_k, E_n) \times f(t_m, R_n, Z_p, \lambda_i, E_{D,j})$$



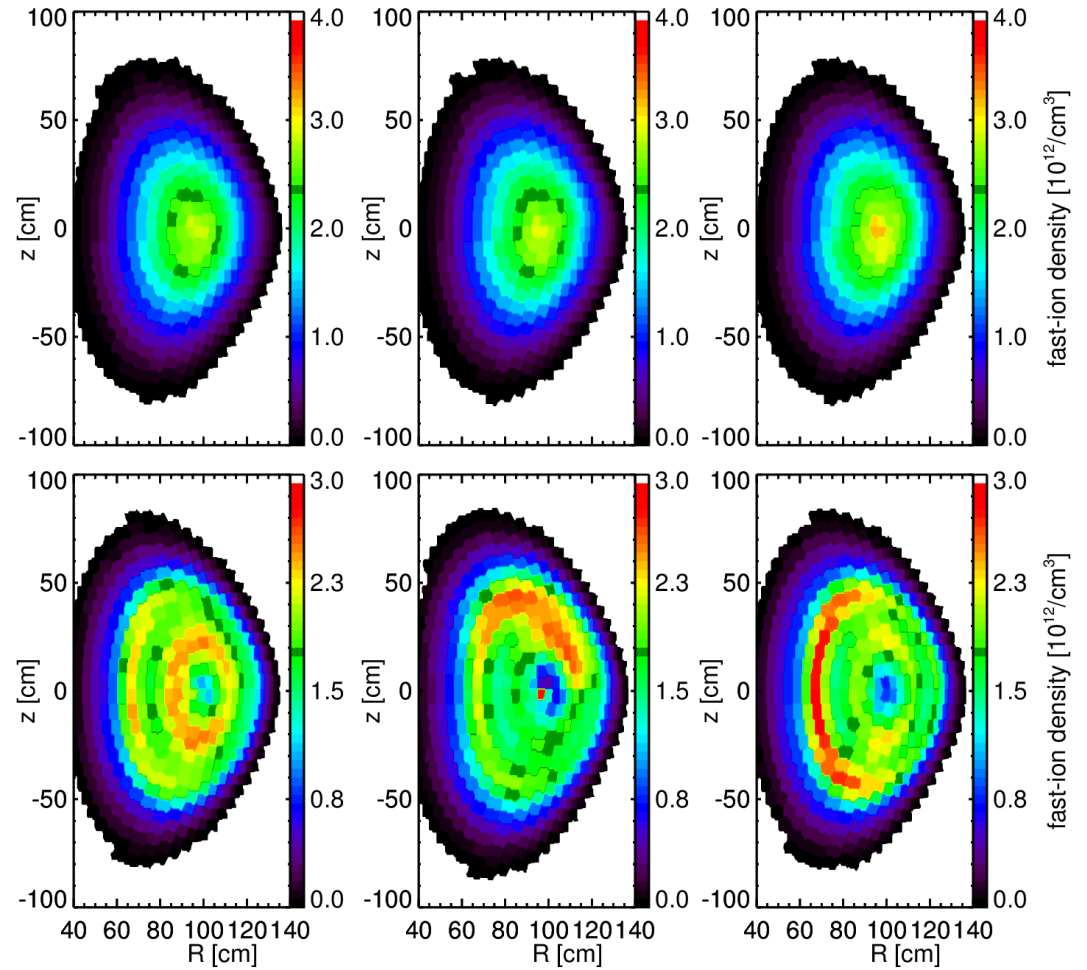
240 keV
shift

Fl_x_WF_1_keV_300k_vrot_anti_clockwise.pvs
m

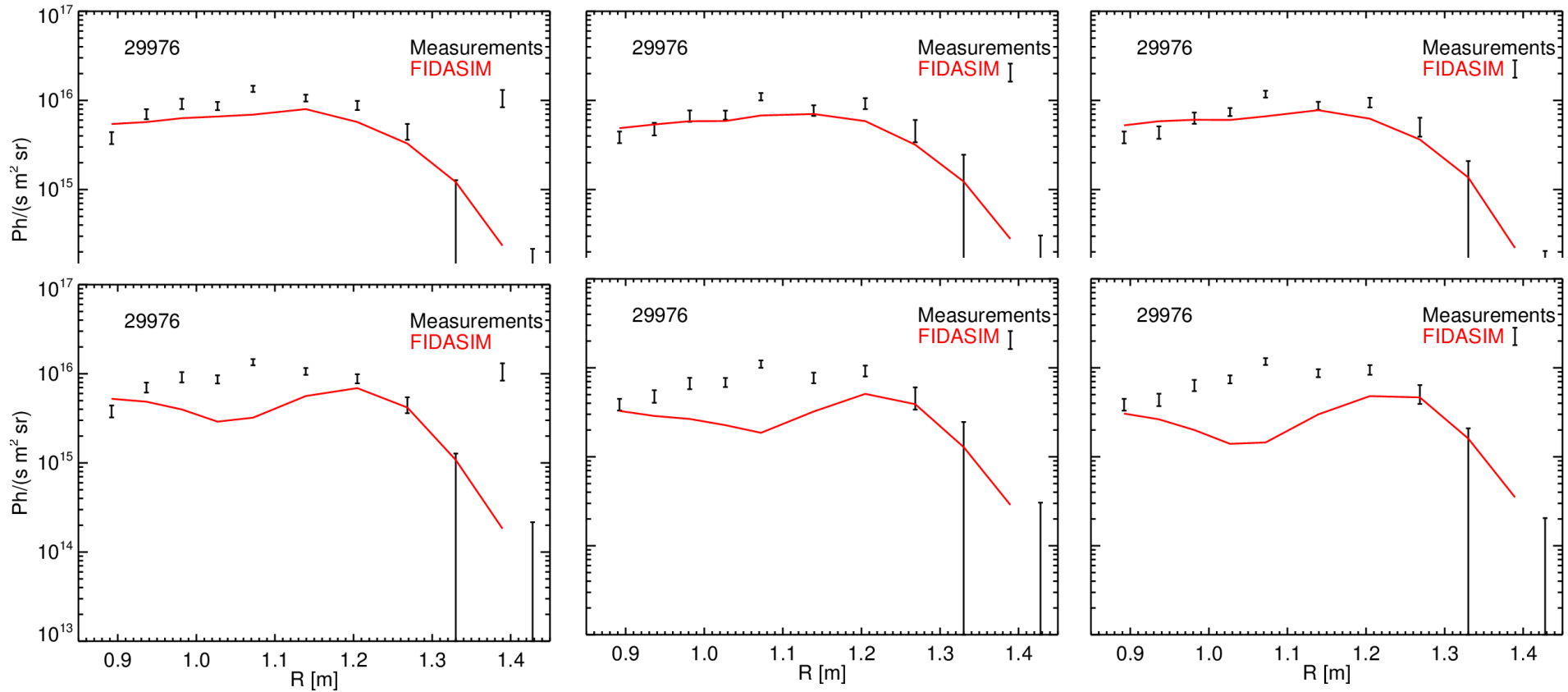
Fast ion density for TAE at 130 ms



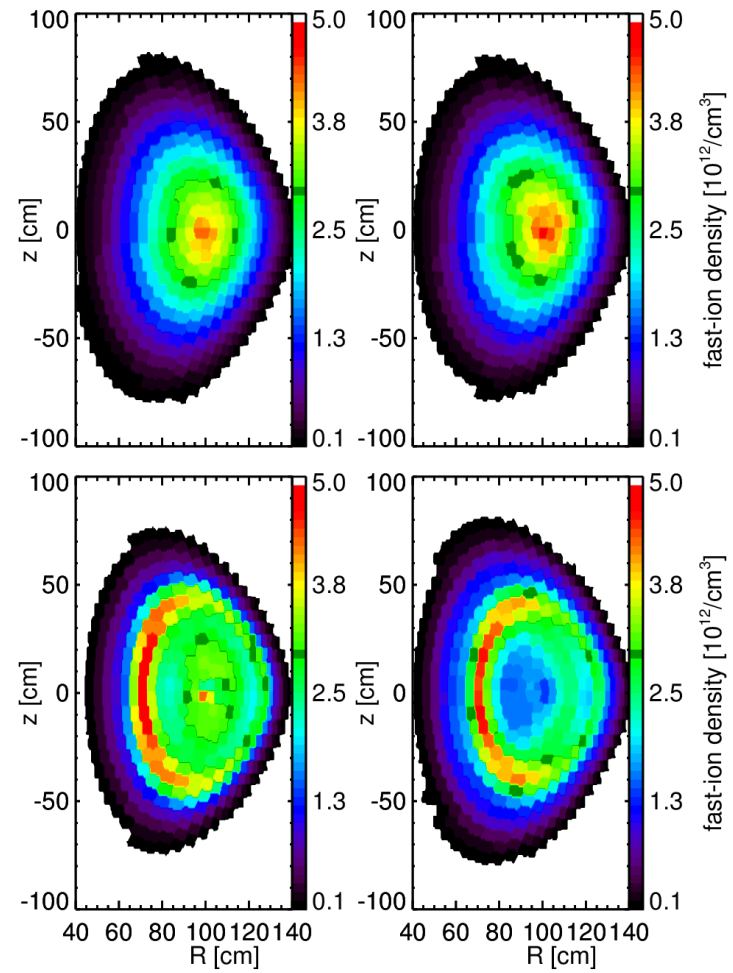
Fast ion density for TAE at 160 ms



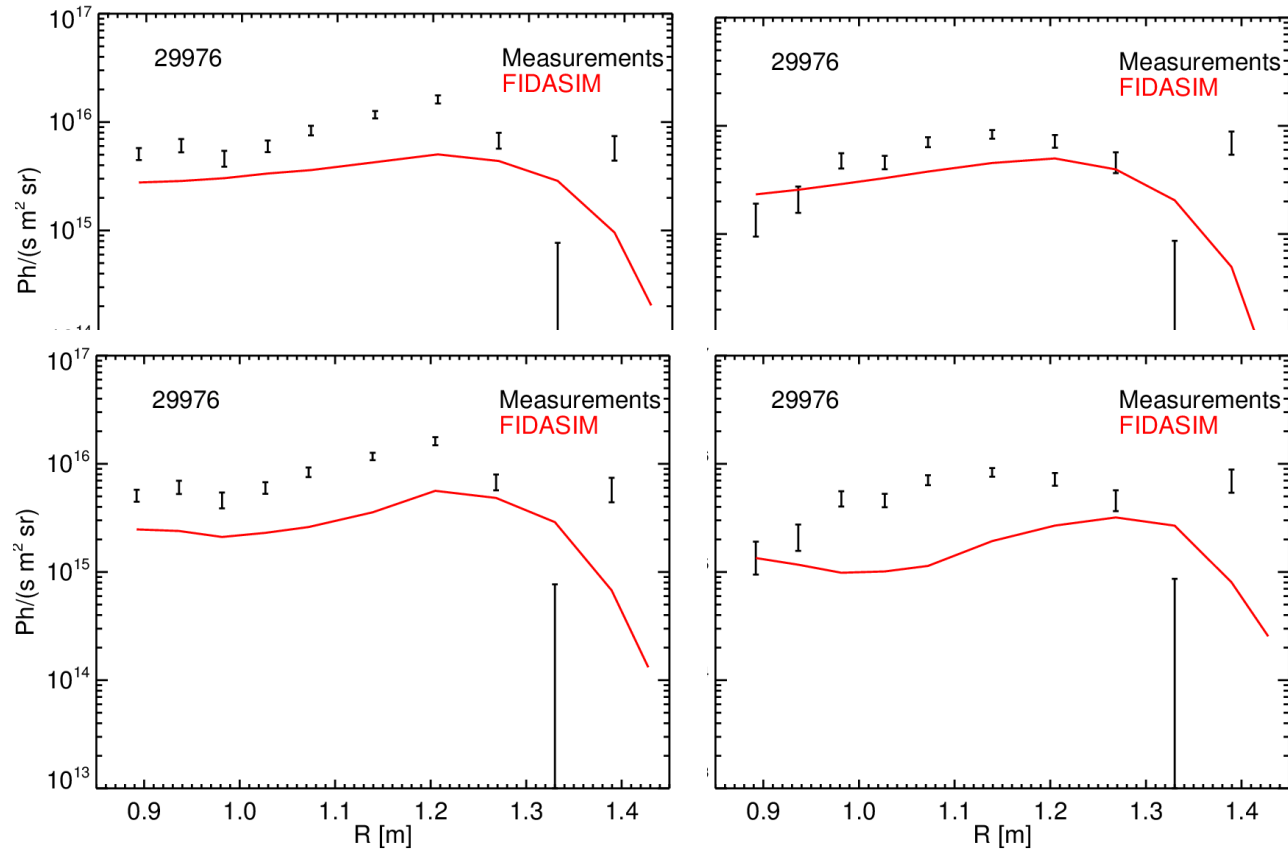
FIDASIM - FIDA comparison for TAE at 160 ms



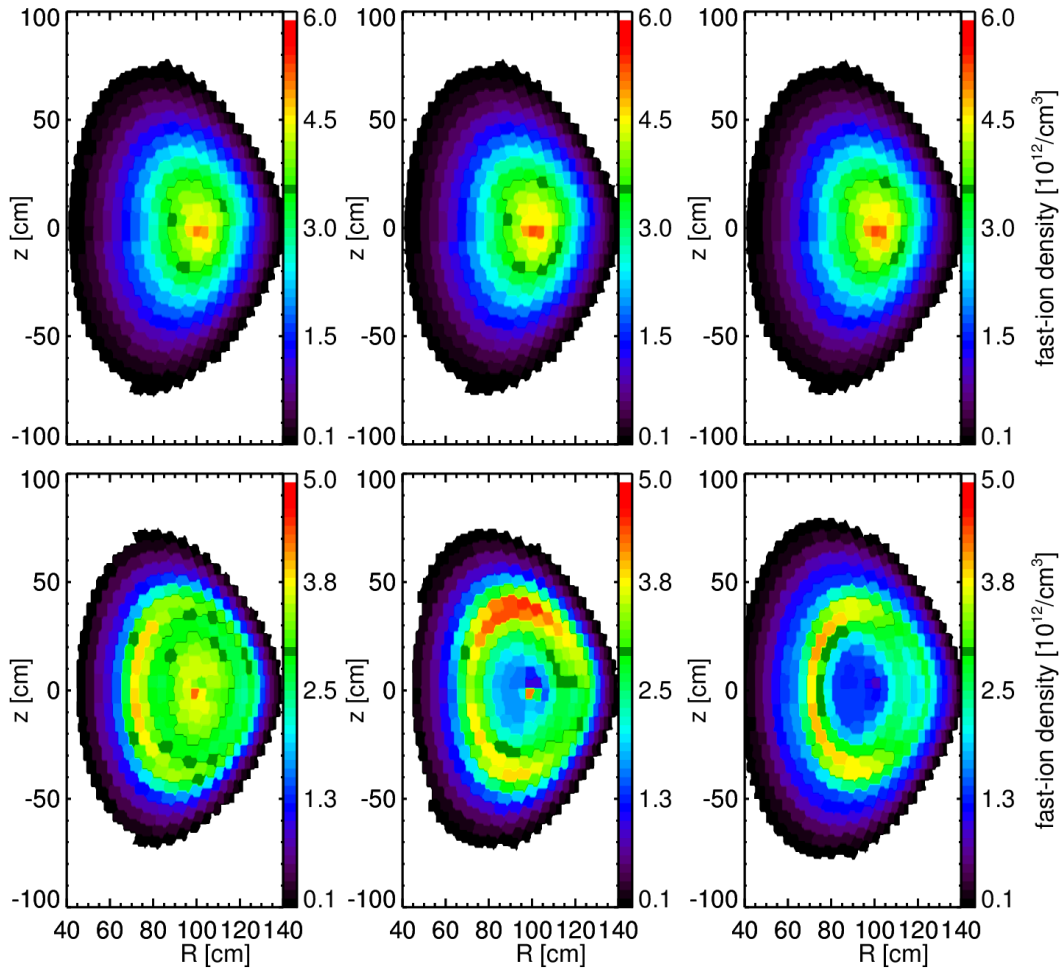
Fast ion density for FB at 200 ms



FIDASIM - FIDA comparison for FB at 200 ms

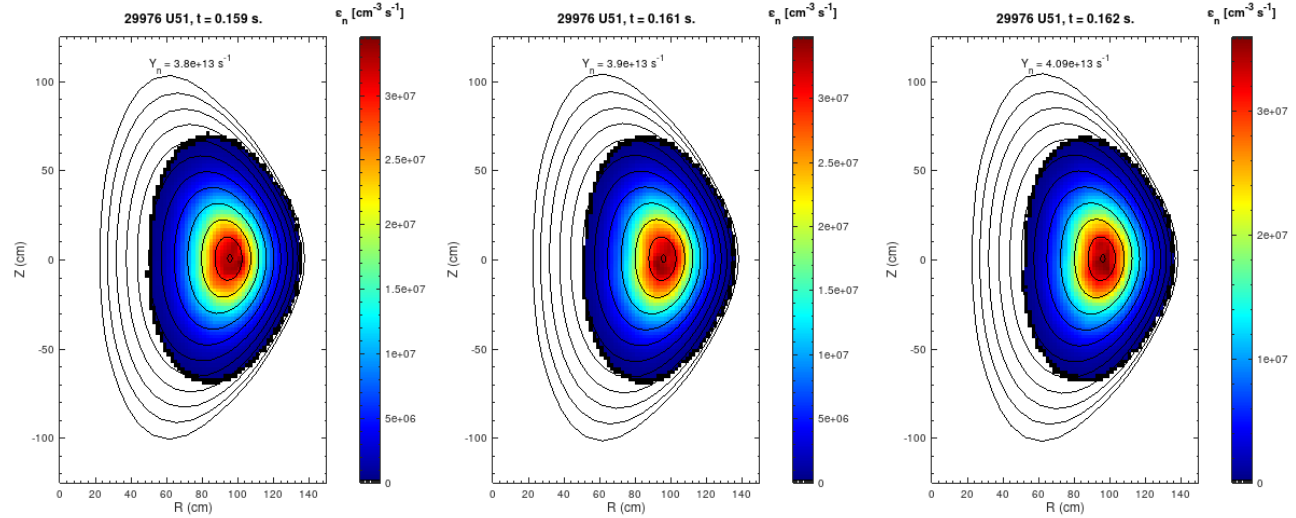


Fast ion density for FB at 210 ms

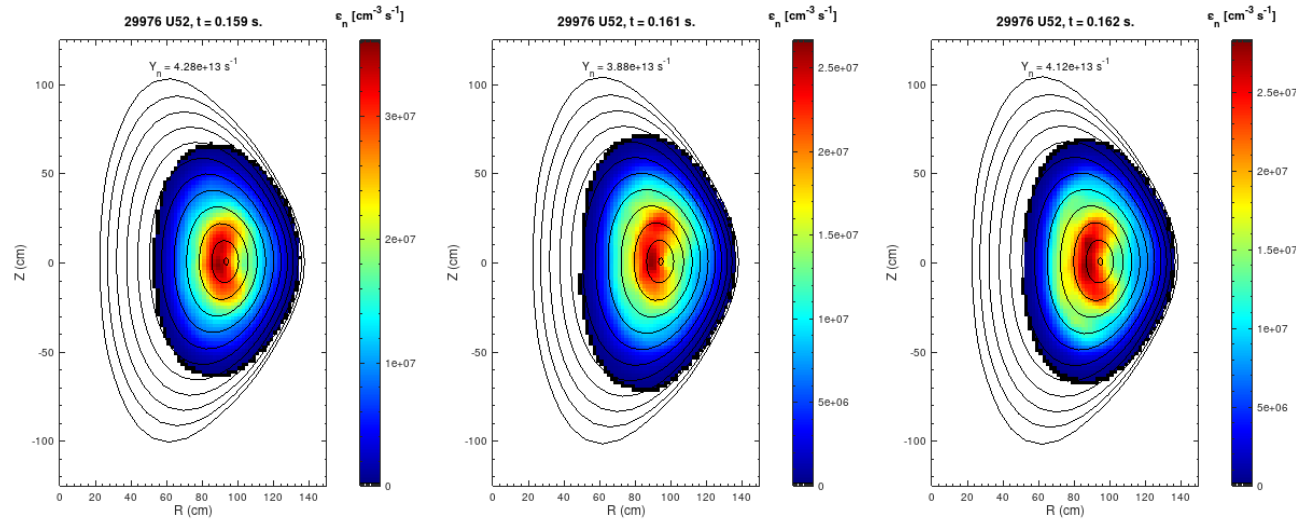


Neutron emissivity for TAE at 160 ms

AFID + FB

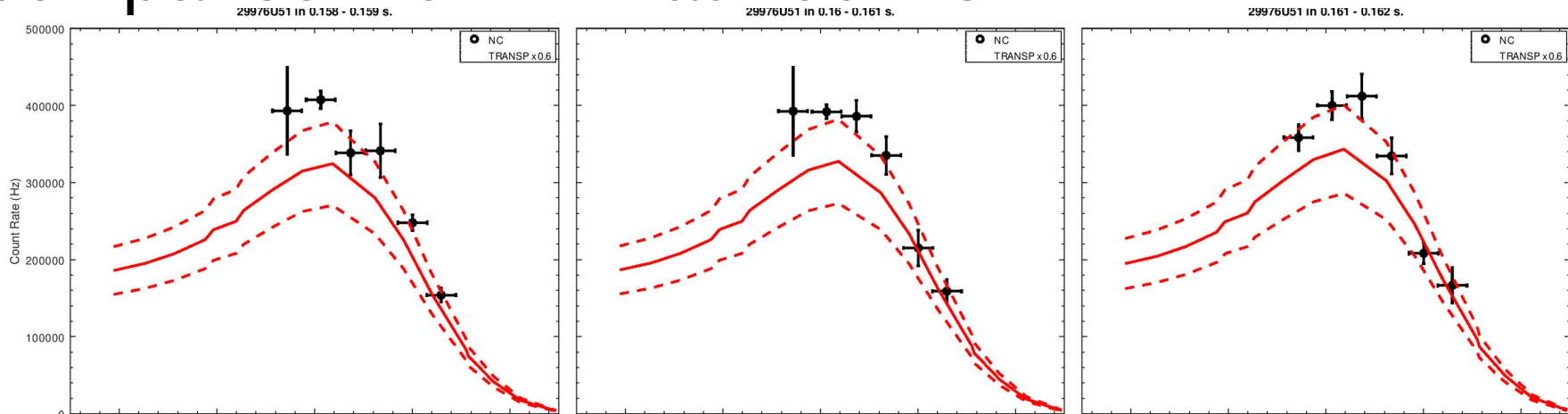


Kick-model

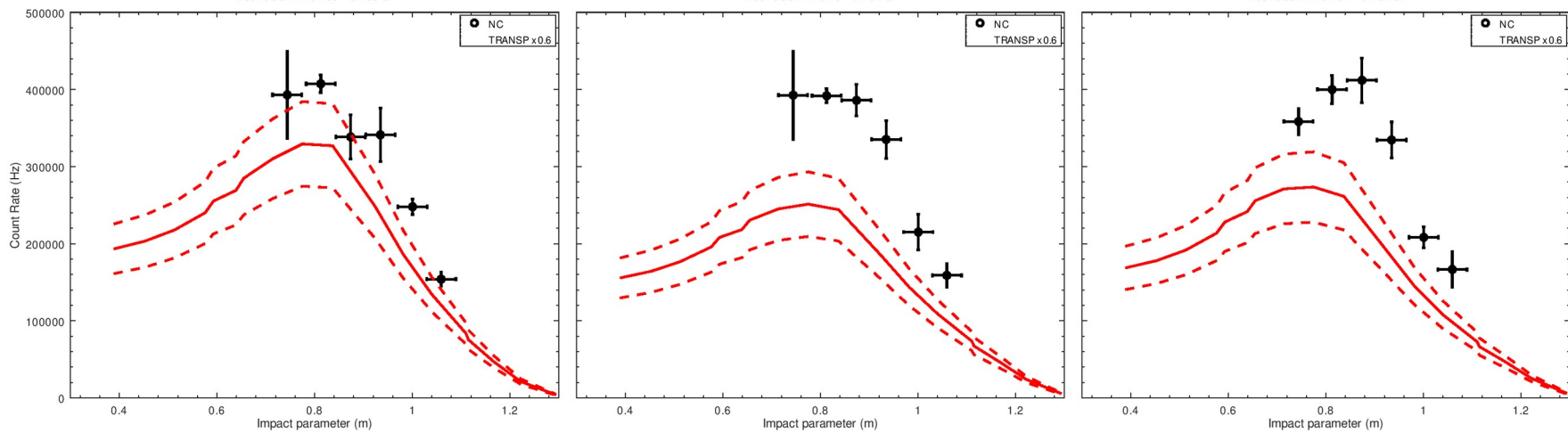


TRANSP - NC comparison for TAE at 160 ms

AFID + FB

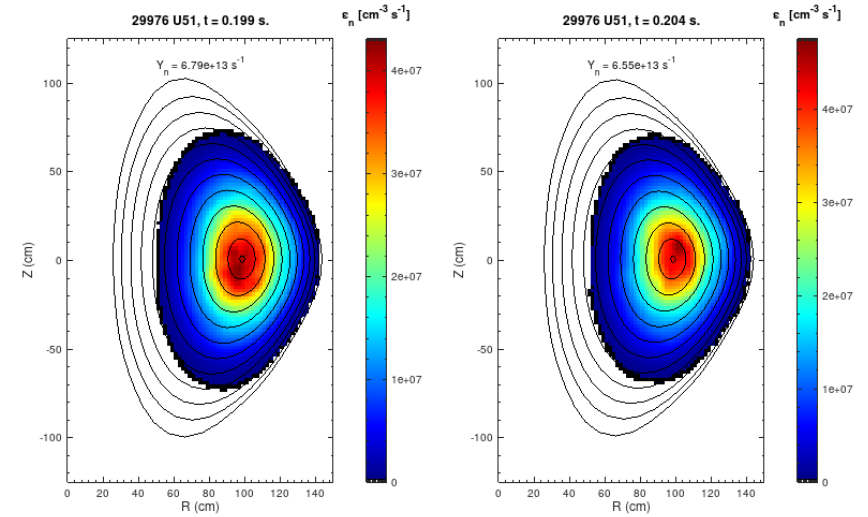


Kick-model

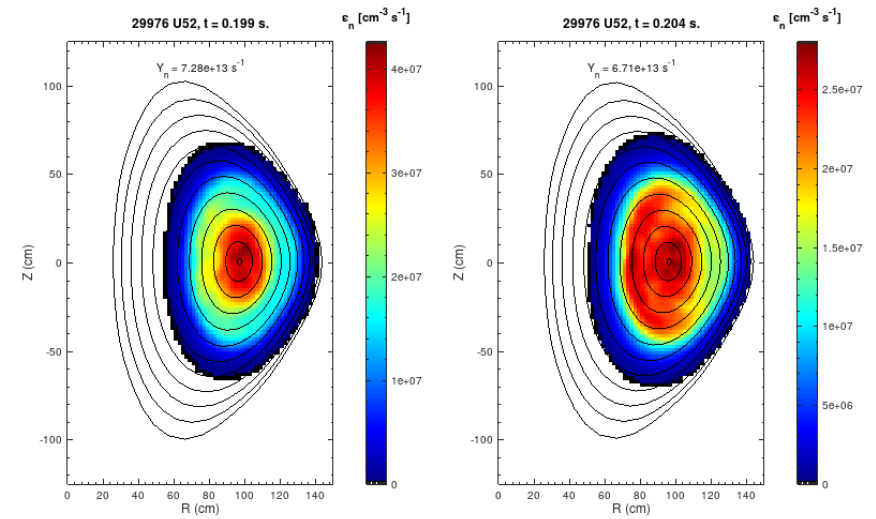


Neutron emissivity for FB at 200 ms

AFID + FB



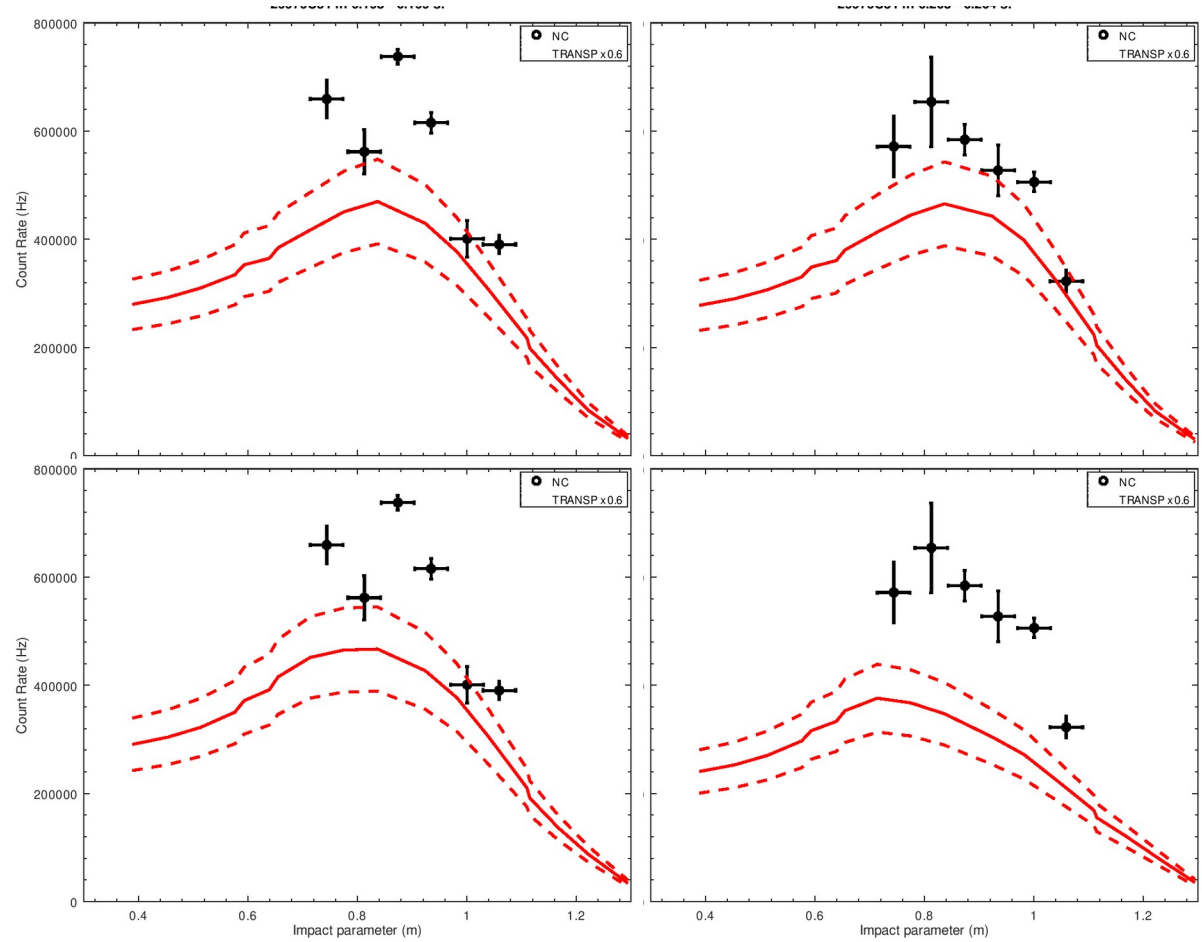
Kick-model



TRANSP – NC comparison for FB at 200 ms

AFID + FB

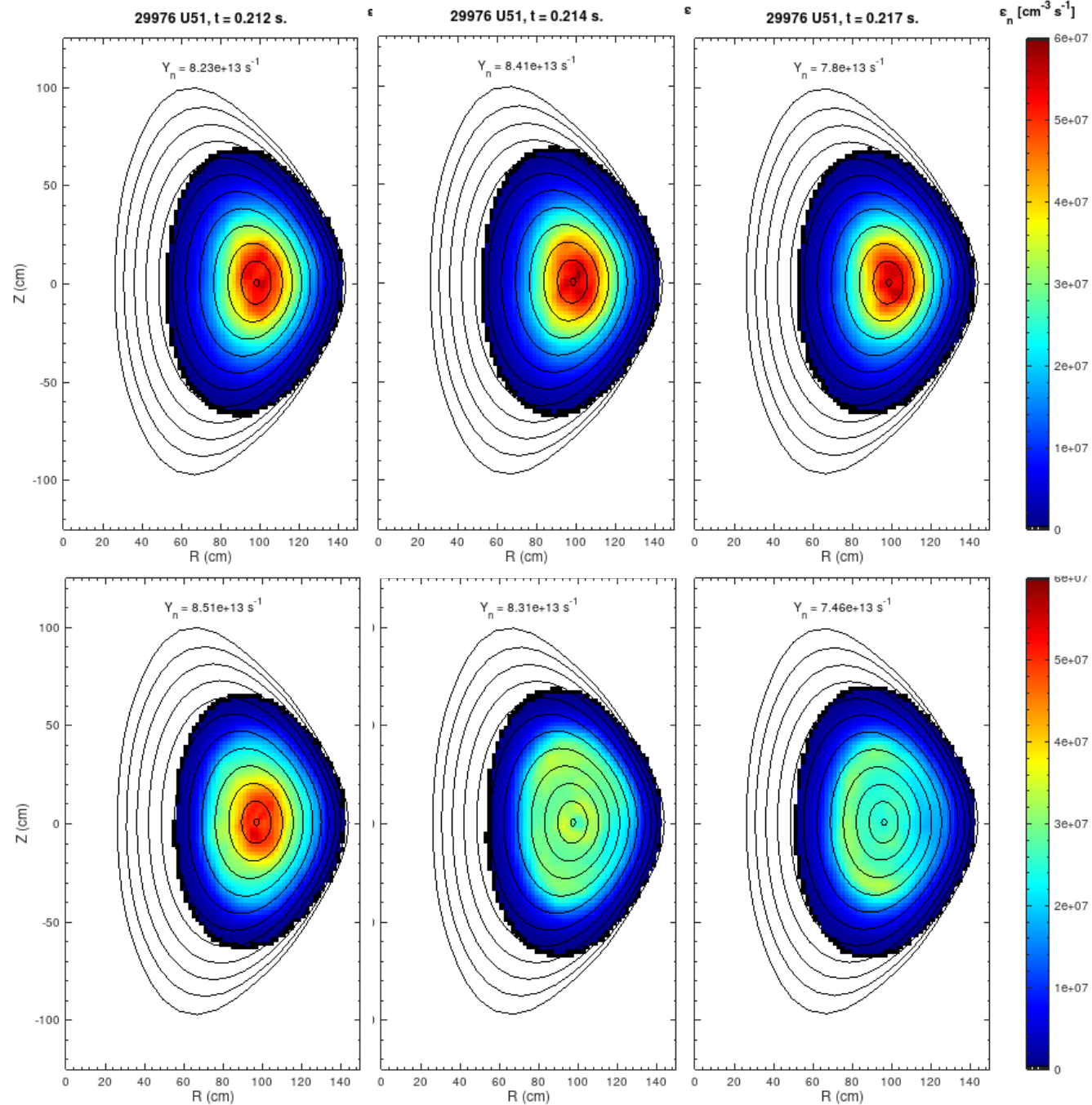
Kick-model



Neutron emissivity for FB around 214 ms

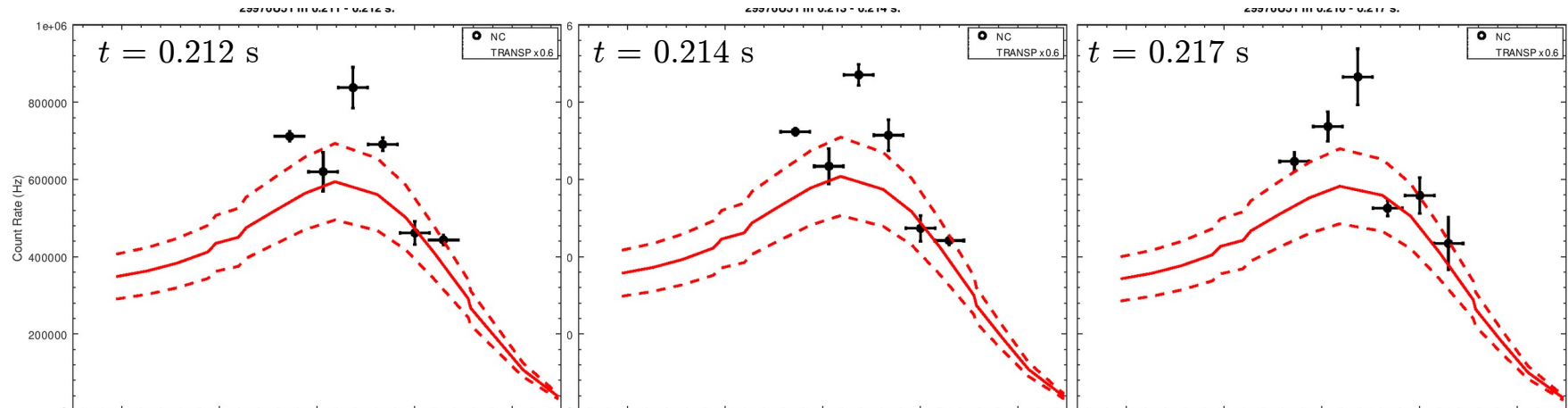
AFID + FB

Kick-model

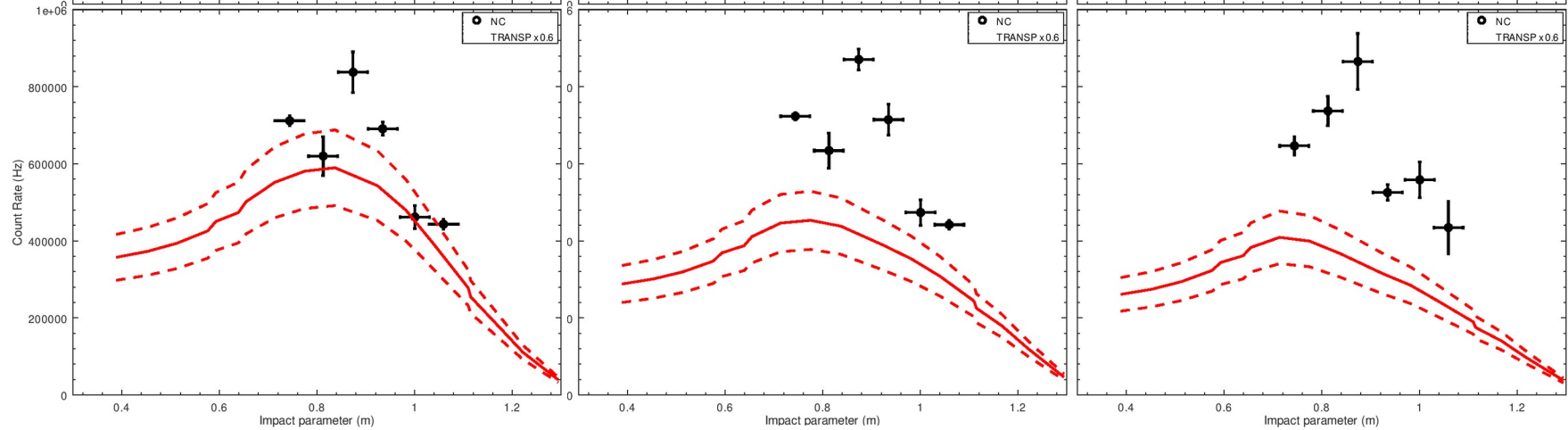


TRANSP/NC comparison for FB at 214 ms

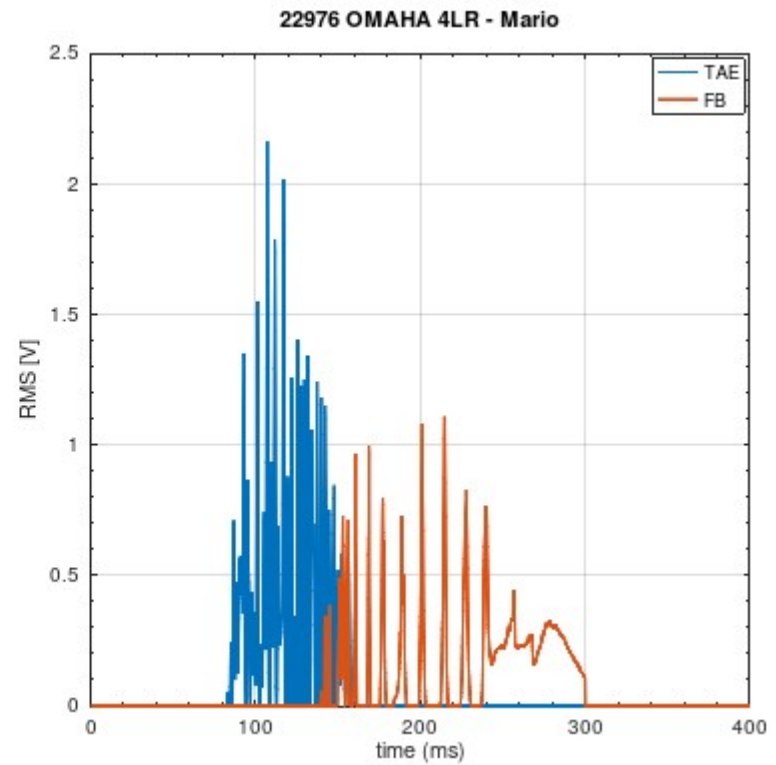
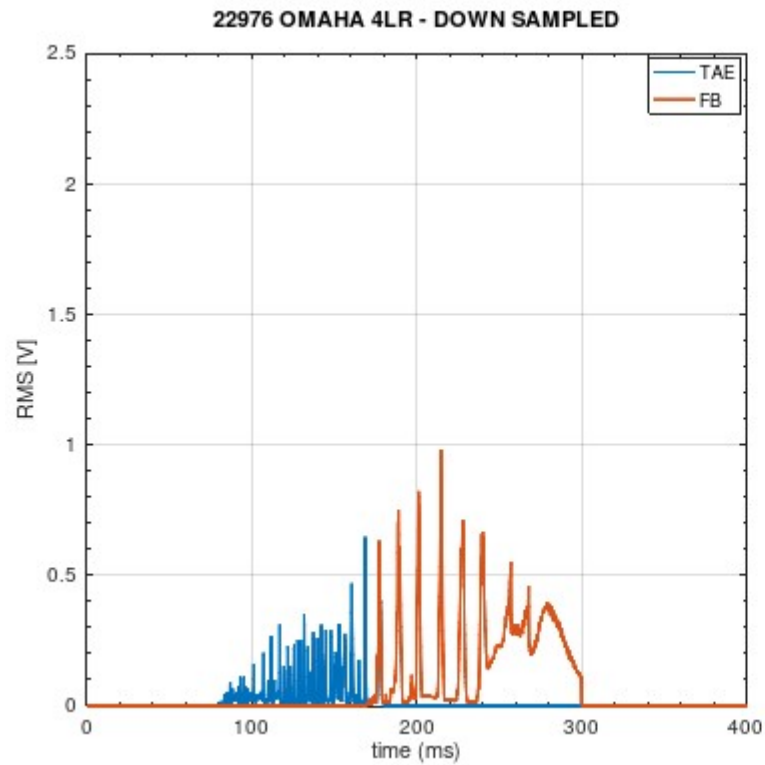
AFID + FB



Kick-model

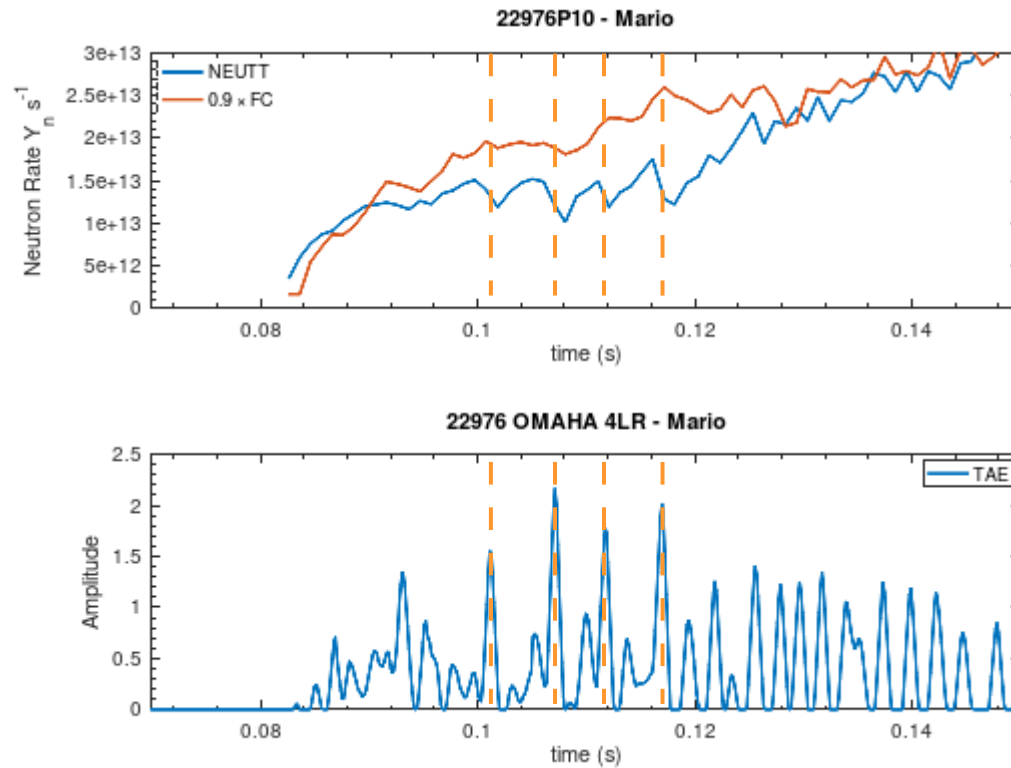


Comparison with Mario's FILEAMODE_X.AEP



FB_Spectrogram_29976.m
29976_Spectrogram_RMS.dat
create_fileamode.m

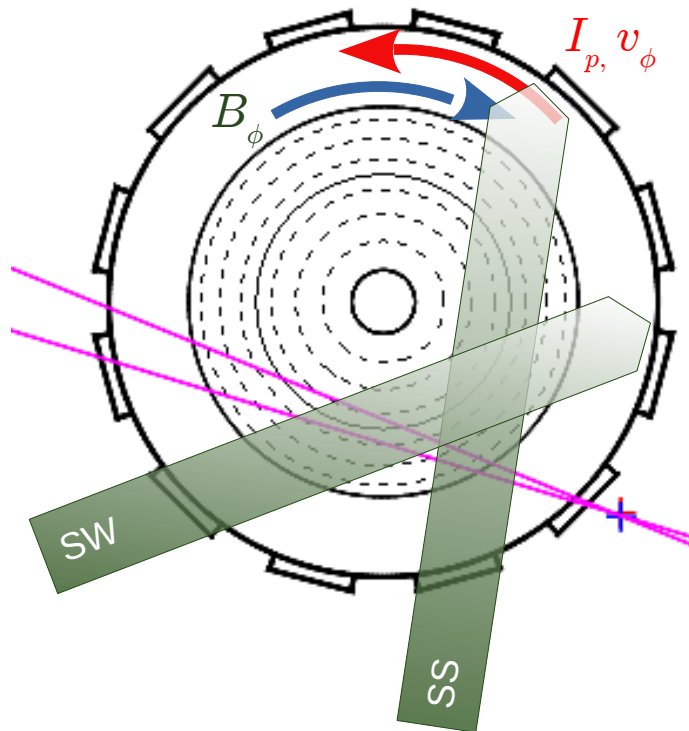
Effect of bursting TAEs in kick-model on the neutron rates



fileamode_neutron_rate.m

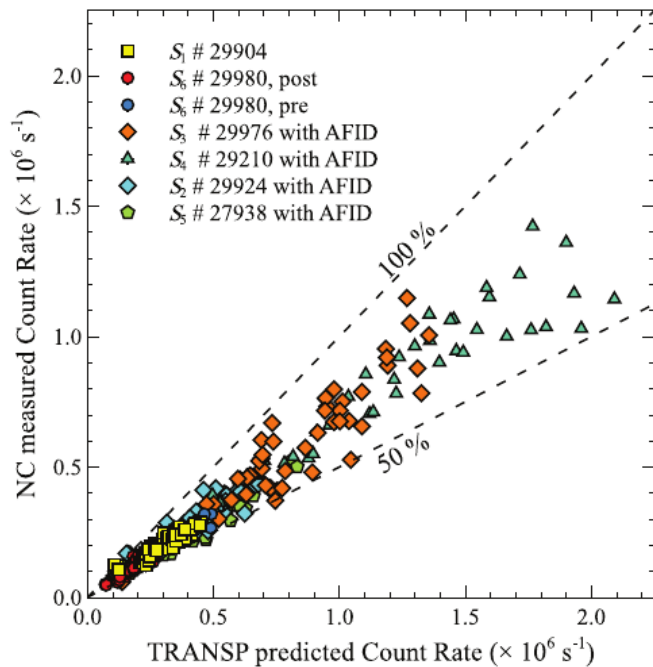
Progress and status ...

1. MISHKA calculations for 29976 at 160 ms based on EFIT++ equilibrium
2. Kick-matrices for MISHKA eigenfunctions calculated
3. TRANSP runs on MISHKA – based kick matrices in progress...
4. Two TRANSP runs using 2013 equilibria redone with new output times, higher statistics and larger number of zones:
 - i. U51 with AFID + FB
 - ii. U52 with NOVA-K based kick matrices (re-run of Mario's P10 run)
 - iii. FIDA analysis of U51 and U52 done
 - iv. NC analysis to be done (before IAEA meeting)
 - v. CFPD data analysis later (after IAEA meeting)
5. Implementation of kick-matrices in HALO: in progress.
6. BES for 29976 at 160 ms by H. Wong
7. FIDA and NC analysis of MISHKA based kick-probabilities for the TAE at 160 ms to be done after point 3 above completed
8. Presentation at IAEA on the 5th of September. Aiming a rehearsal towards the end of next week with all points above completed. Hopefully understanding what it all means before the talk...

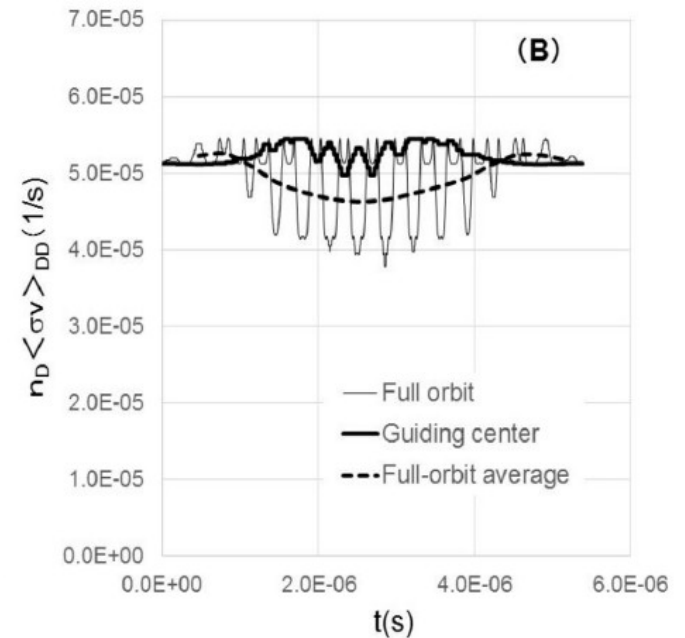
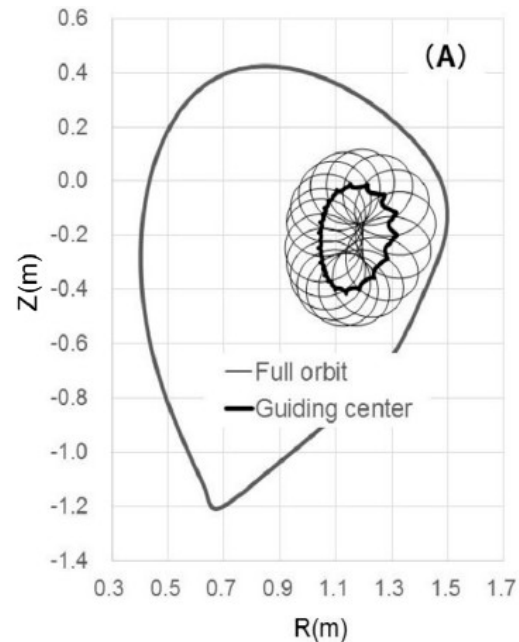


Neutron rate deficit

Is the neutron deficit observed with respect to the TRANSP predictions due to guiding-centre approximation used in NUBEAM?

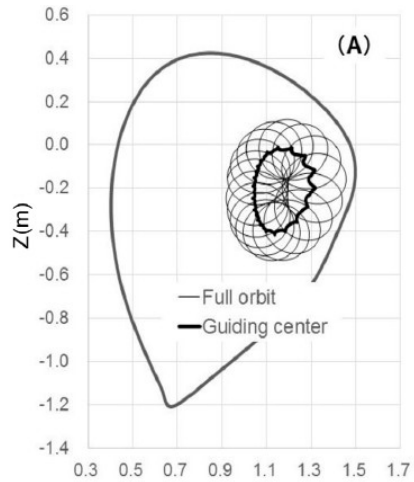


Cecconello M, Boeglin W, Keeling D, Conroy S, Klimek I, Perez RV, et al. Discrepancy between estimated and measured fusion product rates on MAST using TRANSP/NUBEAM. Nucl Fusion. 2019 Jan 1;59(1):016006.

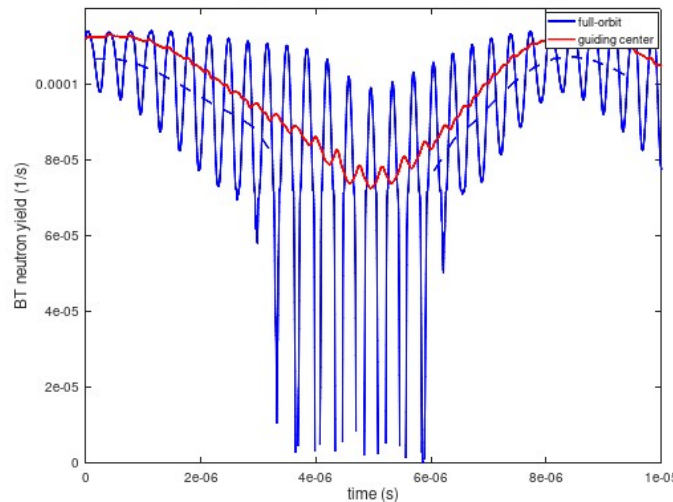
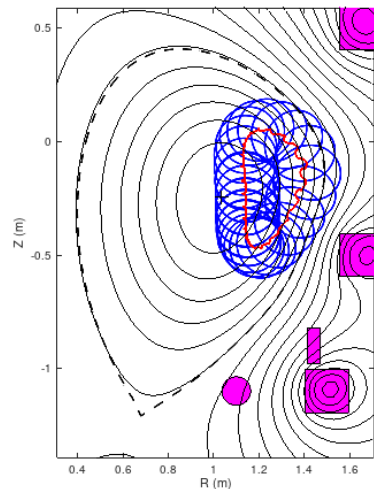
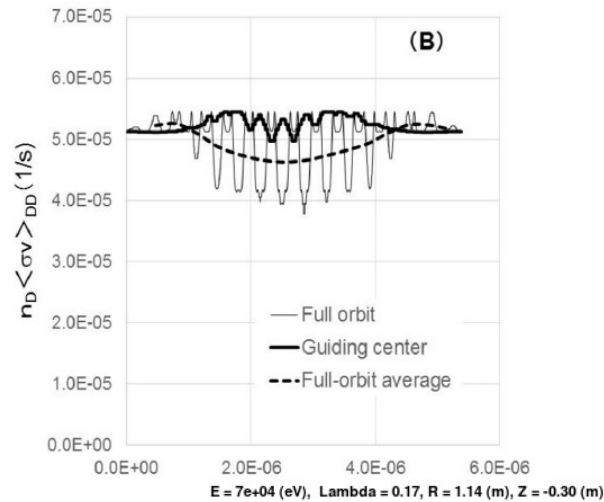


Tani K, Shinohara K, Oikawa T, Tsutsui H, McClements KG, Akers RJ, et al. Application of a non-steady-state orbit-following Monte-Carlo code to neutron modeling in the MAST spherical tokamak. Plasma Physics and Controlled Fusion. 2016 Nov 1;58(10):105005.

Repeating Tani-san's simulations



Poloidal Projection - Pulse 30086, at 0.365 s.



Full orbit and instantaneous guiding center calculated using the FLOCK code for pulse 30086 at time 0.370 s using the IDAM EFIT equilibrium (so no kinetic pressure included) and the TRANSP run 30086K09.

Full orbit in my case extending beyond the LCFS but not in Tani's case.

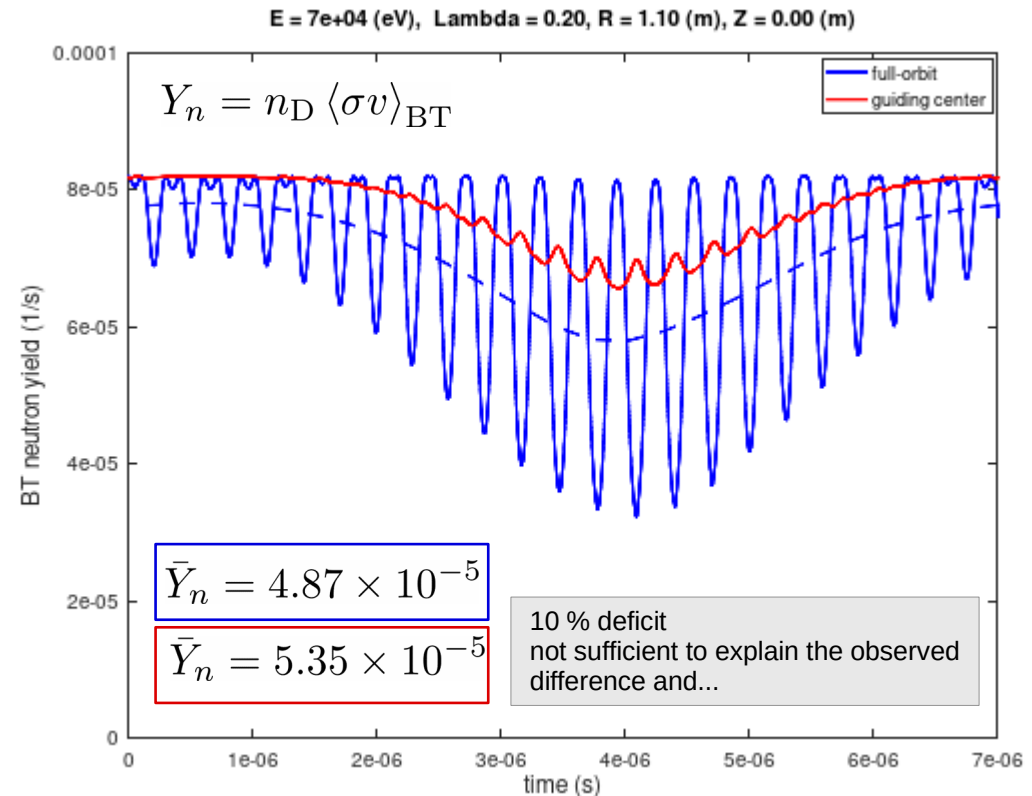
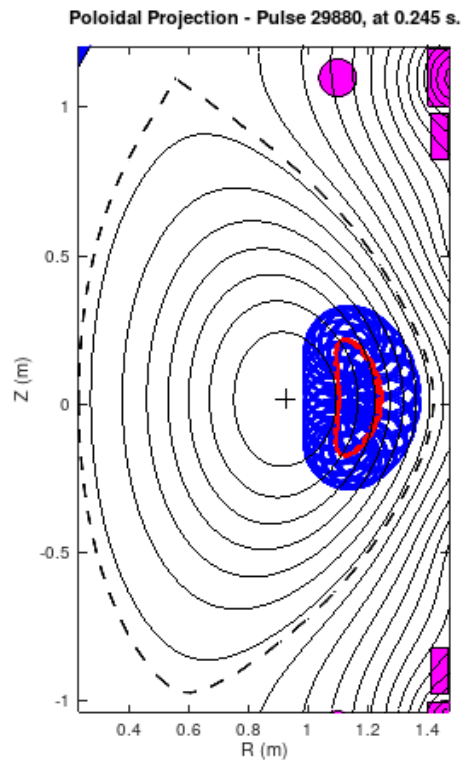
Similar B field (0.392 T in my case, 0.386 T in Tani's).

Neutron rate difference: density is similar, temperature is 20 % higher in my case but does not make much of a difference (800 eV vs 1 keV compared to 71 keV).

Most importantly I don't see the difference in between GC and FO.

Reproducing Tani-san's results (almost...)

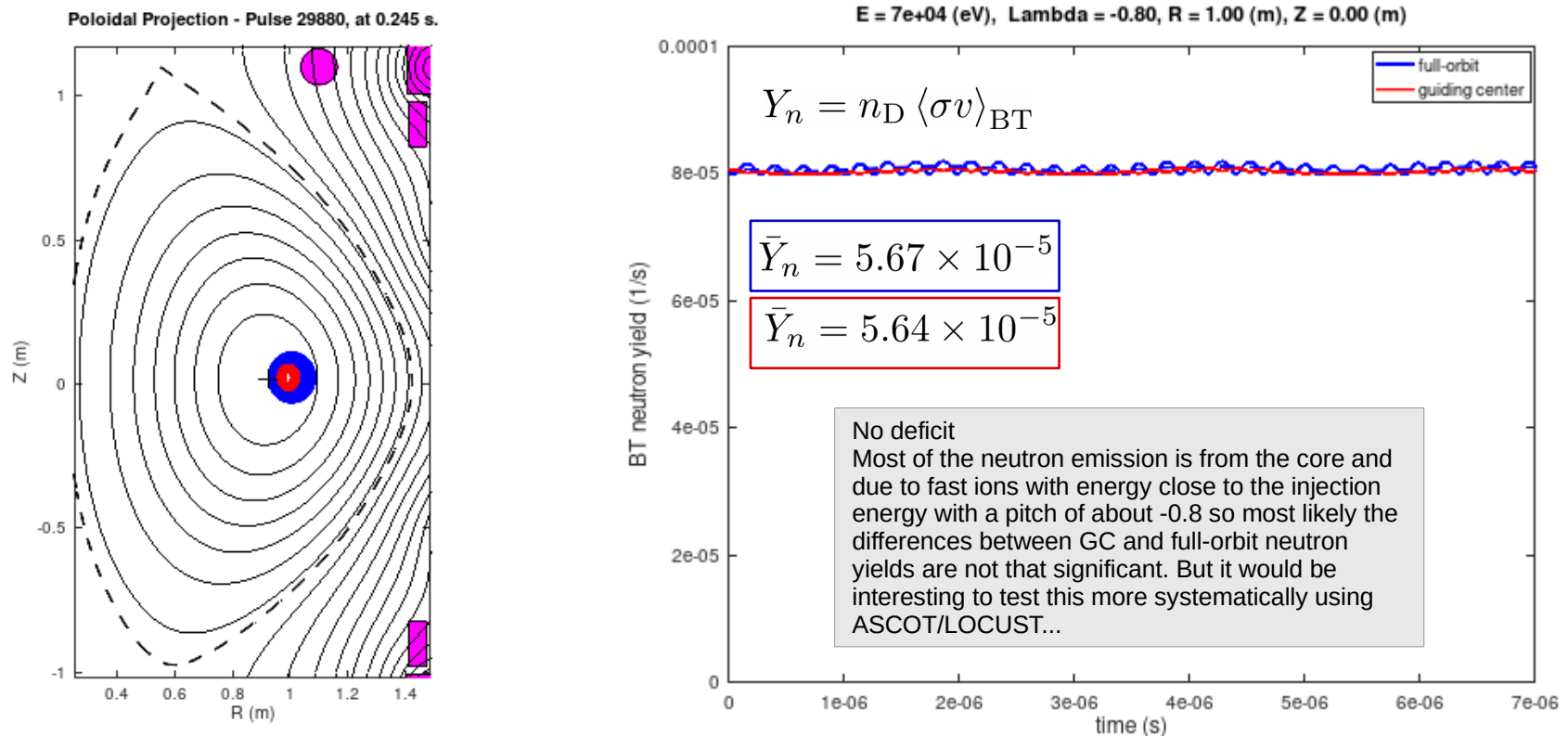
Full orbit and instantaneous guiding center calculated using the FLOCK code for pulse 29881 at time 0.245 s using the IDAM EFIT equilibrium and the TRANSP run 29980U32:



BT reactivity calculated according to Mikkelsen DR. Approximation for non-resonant beam target fusion reactivities, Nuclear Fusion. 1989;29(7):1113 and H S Bosch, G M Hale, Improved formulas for fusion cross-sections and thermal reactivities Nuclear Fusion, 1992, 32, 611

For more typical fast ions...

Full orbit and instantaneous guiding center calculated using the FLOCK code for pulse 29881 at time 0.245 s using the IDAM EFIT equilibrium and the TRANSP run 29980U32:



FO vs GC beam-thermal neutron rate

Pulse 29881 at $t = 0.258$ s

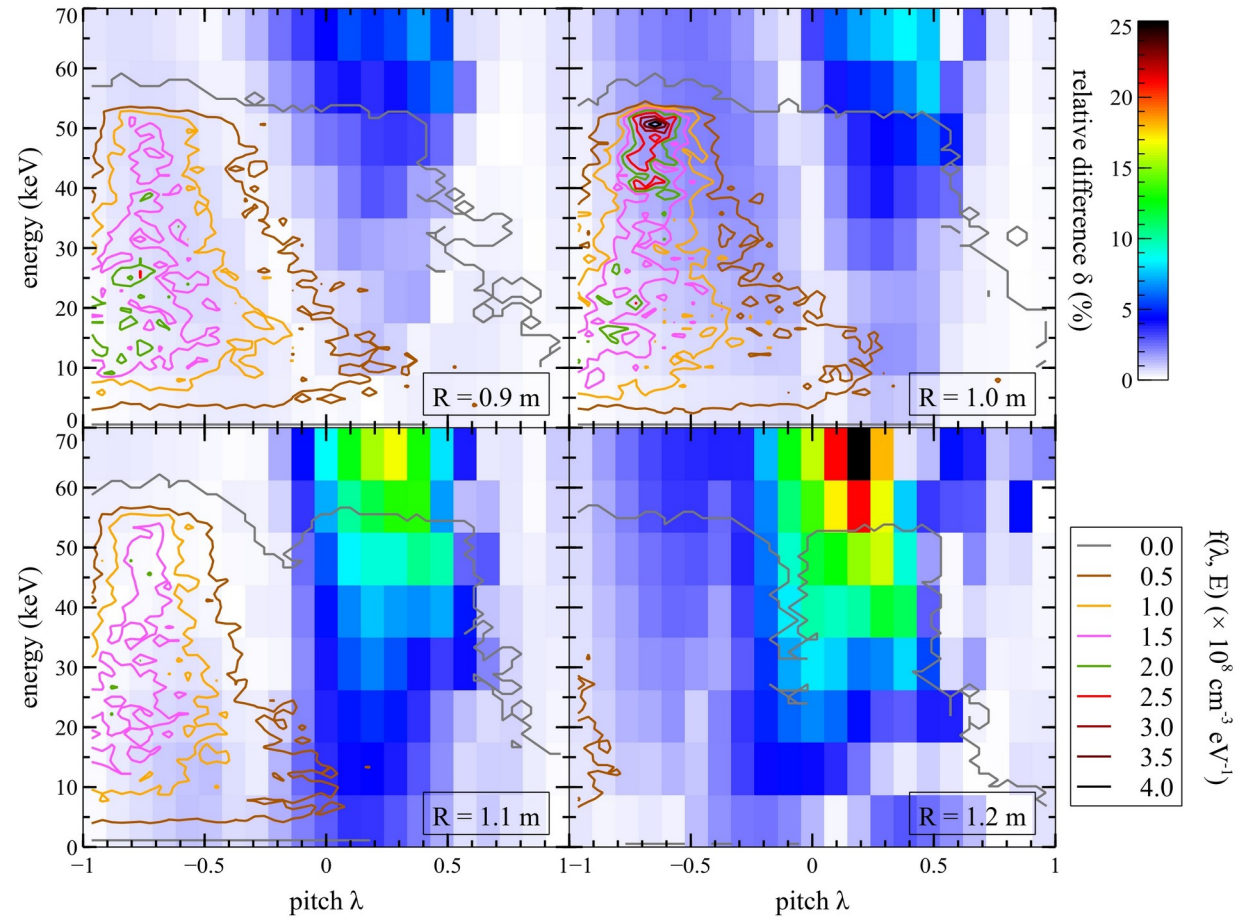
FI distribution at $Z = 0$ m from
TRANSP run 29980U32_fi_3.cdf

Relative difference between the
average BT neutron rate along the Full
Orbit and the Guiding Center:

$$\delta(E, \lambda) = 1 - \frac{\int \langle \sigma v \rangle_{GC} dt}{\int \langle \sigma v \rangle_{FO} dt}$$

As expected, the relative difference is
stronger where the FI population is
negligible and the other way around.

FO/GC difference can not explain the
neutron deficit observed on MAST.



Calculations done using: `.../home/andrea/Documents/MASTOrbit/Orbit_Neutron_Yield_plot_EP.m`

Some definitions

Slowing down time (on electrons): $\tau_{s,e} \propto \frac{T_e^{3/2}}{Z_{\text{eff}}^2 n_e}$

Toroidal canonical momentum:

Larmor radius: $r_L = \frac{mv_{\perp}}{qB}$

$$P_{\phi} = mRv_{\phi} - q\psi$$

Gyro-frequency: $\omega = \frac{qB}{m}$

$$v_{\phi} = R \frac{d\phi}{dt}$$

Pitch: $\lambda = \frac{v_{\parallel}}{v}$

but in NUBEAM: $\lambda_N = \sigma \frac{\bar{\mathbf{v}} \cdot \bar{\mathbf{B}}}{|\bar{\mathbf{v}}| |\bar{\mathbf{B}}|}$ and in MAST $\sigma = -1$

Magnetic moment: $\mu = \frac{1}{2} \frac{mv_{\perp}^2}{B}$

$$\Lambda = \mu \frac{B_0}{E}$$

$$v_{\phi} = -v_{xy,x} \sin \left[\arctan \left(\frac{y}{x} \right) \right] + -v_{xy,x} \cos \left[\arctan \left(\frac{y}{x} \right) \right]$$

$$\tau_{s,e} \propto \frac{T_e^{3/2}}{n_e}$$

At lower density, an increase in plasma electron temperature for a given heating power will also be expected which will increase the slowing down time of the FIs. These effects will both act to increase the FI pressure and hence increase the radial gradient of the FI distribution function.

The longer slowing down time increases the probability that a fast ion will undergo a fusion reaction with a thermal ion,