Relativistic guiding-center motion of runaway electrons

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Runaway electrons

- Runaway electrons (REs): collisionless acceleration of electrons in magnetized plasma
  - Continuous external electric field in Ohmic plasmas [Dreicer mechanism]
  - Plasma disruption $\rightarrow$ secondary generation mechanism [Rosenbluth and Putvinskii (1997)]
  - REs easily go to relativistic regime with $v_{\parallel} \sim c$
  - Once generated, they cause significant damage on surrounding plasma facing components
    $\rightarrow$ serious concern in reactor-scale experiments
Relativistic formulations

- Covariant vs. non-covariant formulations of RE orbit:
  - Covariant formulation uses four vector coordinates [Boghosian (1987)]
  - Non-covariant formulation is more applicable and easy to see nonrelativistic limit. [Cary & Brizard (2009) Rev. Mod. Phys., White & Gobbin (2014) PPPL-5078]

- RE speed approaches to $\sim c$, necessary to develop singularity-free formulation
  - A certain Lie-like transform to avoid $B_{//}^* = 0$ singularity. [Burby & Ellison (2017) PoP 24, 110703]
  - For this work, try to extend the formulation to relativistic guiding-center theory.
  - Theoretical modeling of its generation & termination is one of important subjects in ITER physics
  - Study guiding-center motion of an RE
Outline

• Simulation model
  ✓ Relativistic non-canonical guiding-center phase space
    Lagrangian
  ✓ Toroidally regularized relativistic guiding-center Lagrangian

• Simulation results

• Conclusion and future works
Relativistic guiding-center Lagrangian

- **Non-canonical relativistic guiding-center Lagrangian**

\[\mathcal{L}_{rgc} = \left[ \frac{e}{c} A(X,t) + p_{\parallel} b(X,t) \right] \cdot X - J \dot{\xi} - H_{rgc}\]

\[H_{rgc} = \gamma mc^2 + e\Phi(X,t)\]

\[\gamma = \sqrt{1 + \frac{2\mu B(X,t)}{mc^2} + \frac{p_{\parallel}^2}{m^2 c^2}}\]

- **Equations of motion**

\[\frac{dp_{\parallel}}{dt} = e \frac{B^*}{B_{\parallel}^*} \cdot E^*\]

\[B^* = B + \frac{c}{e} p_{\parallel} \nabla \times b\]

\[B_{\parallel}^* = B^* \cdot b\]

\[E^* = E - \frac{\mu}{e\gamma} \nabla B\]

\[\frac{dX}{dt} = \frac{p_{\parallel}}{m\gamma} \frac{B^*}{B_{\parallel}^*} + E^* \times \frac{cb}{B_{\parallel}^*}\]

Assume \(\frac{\partial b}{\partial t} = 0\)

- **Nonrelativistic correspondence**

\[p_{\parallel} = \gamma m v_{\parallel} \rightarrow p_{\parallel} = m v_{\parallel}\]

\[\gamma m \rightarrow m\]

\[\frac{\mu}{\gamma} \rightarrow \mu\]

\[\therefore \mu = \frac{m\gamma^2 v_\perp^2}{2B}\]

- **Phase space volume and conserved quantities**

\[d^3q d^3p = m^2 B_{\parallel}^* d^3X dp_{\parallel} d\mu d\xi\]

\[p_\phi = \frac{\partial \mathcal{L}_{rgc}}{\partial \dot{\phi}} = \frac{c}{e} A_\phi + p_{\parallel} b_\phi\]

\[H_{rgc} = \gamma mc^2\]
Toroidally regularized guiding-center Lagrangian

- Lie-like transform

\[
G = -\frac{p_{\parallel}}{eB^\phi} \nabla \phi \times b \quad \mathbf{X} = \mathbf{X} + G
\]

\[
\Gamma_0 = \frac{e}{c} \mathbf{A} \cdot \mathbf{dX} \quad H_0 = e\Phi \quad \Upsilon_0 = \Gamma_0 \quad K_0 = H_0
\]

\[
\Gamma_1 = p_{\parallel} \mathbf{b} \cdot \mathbf{dX} \quad H_1 = \gamma mc^2 \quad \Upsilon_1 = p_{\parallel} \mathbf{b} + e\mathbf{G} \times \mathbf{B} + dS_1 = p_{\parallel} \frac{B}{B^\phi} \nabla \phi \quad K_1 = H_1 - \mathbf{G} \cdot \nabla H_0
\]

- Toroidally regularized guiding-center Lagrangian

Assume \( B^\phi \neq 0 \)

\[
\mathcal{L}_{rgc}^* = \left[ \frac{e}{c} \mathbf{A}(\mathbf{X},t) + p_{\parallel} \frac{B}{B^\phi} \nabla \phi \right] \cdot \dot{\mathbf{X}} - J \dot{\mathbf{\xi}} - H_{rgc}^* \quad H_{rgc}^* = \gamma mc^2 + e\Phi(\mathbf{X},t) - e\mathbf{G} \cdot \mathbf{E} \quad p_{\parallel}^* = p_{\parallel} \frac{B}{R_0 B^\phi}
\]

- Equations of motion

\[
\frac{dp_{\parallel}^*}{dt} = e \frac{\mathbf{B}}{R_0 B^\phi} \cdot \mathbf{E}^* \quad \mathbf{E}^* = \mathbf{E} - \frac{mc^2}{e} \nabla \gamma + \frac{p_{\parallel}^*}{e} \nabla \left( \mathbf{b} \cdot \frac{\mathbf{E} \times R_0 \nabla \phi}{B} \right)
\]

\[
\frac{d\mathbf{X}}{dt} = \frac{\mathbf{B}}{R_0 B^\phi} \frac{\partial H_{rgc}^*}{\partial p_{\parallel}^*} + c\mathbf{E}^* \times \frac{R_0 \nabla \phi}{R_0 B^\phi} \quad \gamma = \sqrt{1 + \frac{2\mu B(\mathbf{X},t)}{mc^2} + \left( \frac{p_{\parallel}^*}{mc} \right)^2 \left( \frac{R_0 B^\phi}{B^2} \right)^2}
\]
Guiding-center orbits and full Lorentz orbit are agreed well for not so high kinetic energies.

10MeV $v_{//}/v=-0.9$ relativistic passing electron in a KSTAR equilibrium magnetic field [#21576 13.81s]

10MeV $v_{//}/v=-0.8$ relativistic and non-relativistic electron orbit comparison [KSTAR #10902 2.0s]
High kinetic energy cases

- Guiding-center orbits starts to deviate for high kinetic energy case.

50MeV, 70MeV and 100MeV (left to right) passing (v_\parallel/v_\perp=-0.9) electron orbits in a KSTAR equilibrium magnetic field [#21576 13.81s] 100MeV case, the transformed coordinates fitted well for the gyro-averaged particle orbit, but it is not always.
Orbit classification

KSTAR equilibrium magnetic field [#10902 2.0s]

20MeV electron

10MeV electron

50MeV electron

70MeV electron

100MeV electron

∃ minimum energy for the orbit confinement

PP-L : Parallel Passing Loss
PP-C : Parallel Passing Confined
AP-L : Anti-parallel Passing Loss
AP-C : Anti-parallel Passing Confined
T-L : Trapped Loss
T-C : Trapped Confined
Po : Potato

confined region disappears
Concentric circular magnetic field

\[ \mathbf{B} = -\frac{B_0 z}{q_0 R} \hat{R} - \frac{B_0 R_0}{R} \hat{\phi} + \frac{B_0}{q_0 R} R - R_0 \hat{z} \]

Concentric circular \( \mathbf{B} \) field in cylindrical \( R, \phi, z \) coordinates

\[ B_{\parallel}^* = 0 \]

singular case

Standard GC theory fails

\( B_0 = 1 \text{T}, q_0 = 1.414, R_0 = 1 \text{m} \)
200keV (\( v_{\parallel}/v = -0.21 \)) trapped ion orbits

\( B_0 = 1 \text{T}, q_0 = 0.1, R_0 = 1 \text{m} \)
1MeV (\( v_{\parallel}/v = -0.7 \)) passing ion orbits
$B_{\|} = 0 \quad \text{singularity}$

For large $p_{\|}$ and/or small twist length $1/b \cdot \nabla \times b$ $B_{\|} = 0$ singularity could take place [Wimmel, Boozer, etc.]

For the concentric circular magnetic field case ($B_0=1\,\text{T}$), the minimum electron velocity which makes $B_{\|} = 0$ can be written as

$$u_{\|/\min} = \frac{e}{m} \frac{q_0 R_0 B_0}{2}$$

Figure shows corresponding kinetic energy as function of the safety factor $q_0$
Conclusions

- Two guiding-center equations of motion are implemented.
  - Toroidally regularized guiding-center theory applied to remove potential singularity for the $v_{\parallel} \sim c$ case
  - Caveats: the coordinate transformations do not guarantee accurate orbits
  - The higher order correction is unclear.
  - The applicability of the toroidally regularized case may increases when the safety factor is small, such as reversed field pinch (RFP) system.

- On-going and future works:
  - What causes orbit deviation and how to remedy it.
  - How RE orbits modify when some perturbations given outside.
  - Application to other system, space, RFP, etc.