Relativistic guiding-center motion of runaway electrons

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Runaway electrons

- Runaway electrons (REs) : collisionless acceleration of electrons in magnetized plasma
  - Continuous external electric field in Ohmic plasmas [Dreicer mechanism]
  - Plasma disruption ⟶ secondary generation mechanism [Rosenbluth and Putvinskii (1997)]
  - REs easily go to relativistic regime with $v_{//} \sim c$
  - Once generated, they cause significant damage on surrounding plasma facing components

  ⟶ serious concern in reactor-scale experiments

Photo credit: MunSeong Cheon (NFRI)

Relativistic formulations

- Covariant vs. non-covariant formulations of RE orbit :
  - ➢ Covariant formulation uses four vector coordinates [Boghosian (1987)]
  - ➢ Non-covariant formulation is more applicable and easy to see nonrelativistic limit. [Cary & Brizard (2009) Rev. Mod. Phys., White & Gobbin (2014) PPPL-5078]
- RE speed approaches to $\sim c$, necessary to develop singularity-free formulation
  - ➢ A certain Lie-like transform to avoid $B_{//}{}^{*} = 0$ singularity. [Burby & Ellison (2017) PoP 24, 110703]
  - ➢ For this work, try to extend the formulation to relativistic guiding-center theory.
  - ➢ Theoretical modeling of its generation & termination is one of important subjects in ITER physics
  - ➢ Study guiding-center motion of an RE

Outline

- Simulation model
  - ✓ Relativistic non-canonical guiding-center phase space Lagrangian
  - ✓ Toroidally regularized relativistic guiding-center Lagrangian
- Simulation results
- Conclusion and future works

Relativistic guiding-center Lagrangian

- **Non-canonical relativistic guiding-center Lagrangian**

$$\boxed{\mathcal{L}_{rgc} = \left[\frac{e}{c}\mathbf{A}(\mathbf{X},t) + p_{//}\mathbf{b}(\mathbf{X},t)\right]\cdot\dot{\mathbf{X}} - J\dot{\xi} - H_{rgc}} \qquad H_{rgc} = \gamma mc^2 + e\Phi(\mathbf{X},t) \qquad \gamma = \sqrt{1 + \frac{2\mu B(\mathbf{X},t)}{mc^2} + \frac{p_{//}^{\ 2}}{m^2c^2}}$$

- **Equations of motion**

$$\boxed{\frac{dp_{//}}{dt} = e\frac{\mathbf{B}^*}{B_{//}^{\ *}}\cdot\mathbf{E}^*} \qquad \mathbf{B}^* = \mathbf{B} + \frac{c}{e}p_{//}\nabla\times\mathbf{b} \qquad B_{//}^{\ *} = \mathbf{B}^*\cdot\mathbf{b} \qquad \mathbf{E}^* = \mathbf{E} - \frac{\mu}{e\gamma}\nabla B$$

$$\boxed{\frac{d\mathbf{X}}{dt} = \frac{p_{//}}{m\gamma}\frac{\mathbf{B}^*}{B_{//}^{\ *}} + \mathbf{E}^*\times\frac{c\mathbf{b}}{B_{//}^{\ *}}}$$

Assume $\frac{\partial \mathbf{b}}{\partial t} = 0$

- **Nonrelativistic correspondence**

$$p_{//} = \gamma m v_{//} \rightarrow p_{//} = m v_{//} \qquad \gamma m \rightarrow m \qquad \frac{\mu}{\gamma} \rightarrow \mu \quad \left(\because\ \mu = \frac{m\gamma^2 v_\perp^{\ 2}}{2B}\right)$$

- **Phase space volume and conserved quantities**

$$d^3\mathbf{q}\,d^3\mathbf{p} = m^2 B_{//}^{\ *} d^3\mathbf{X}\,dp_{//}\,d\mu\,d\xi \qquad p_\phi = \frac{\partial \mathcal{L}_{rgc}}{\partial \phi} = \frac{c}{e}A_\phi + p_{//}b_\phi \qquad H_{rgc} = \gamma mc^2$$

Toroidally regularized guiding-center Lagrangian

- **Lie-like transform**

$$\mathbf{G} = -\frac{p_{//}}{eB^{\phi}}\nabla\phi\times\mathbf{b} \qquad \mathbb{X} = \mathbf{X} + \mathbf{G}$$

$$\Gamma_0 = \frac{e}{c}\mathbf{A}\cdot d\mathbf{X} \qquad H_0 = e\Phi \qquad \longrightarrow \qquad \Upsilon_0 = \Gamma_0 \qquad K_0 = H_0$$

$$\Gamma_1 = p_{//}\mathbf{b}\cdot d\mathbf{X} \qquad H_1 = \gamma mc^2 \qquad \Upsilon_1 = p_{//}\mathbf{b} + e\mathbf{G}\times\mathbf{B} + dS_1 = p_{//}\frac{B}{B^{\phi}}\nabla\phi \qquad K_1 = H_1 - \mathbf{G}\cdot\nabla H_0$$

- **Toroidally regularized guiding-center Lagrangian** Assume $B^{\phi} \neq 0$

$$\boxed{\mathcal{L}_{rgc}{}^{*} = \left[\frac{e}{c}\mathbf{A}(\mathbb{X},t) + p_{//}\frac{B}{B^{\phi}}\nabla\phi\right]\cdot\dot{\mathbb{X}} - J\dot{\xi} - H_{rgc}{}^{*}} \qquad H_{rgc}{}^{*} = \gamma mc^2 + e\Phi(\mathbb{X},t) - e\mathbf{G}\cdot\mathbf{E} \qquad p_{//}{}^{*} = p_{//}\frac{B}{R_0 B^{\phi}}$$

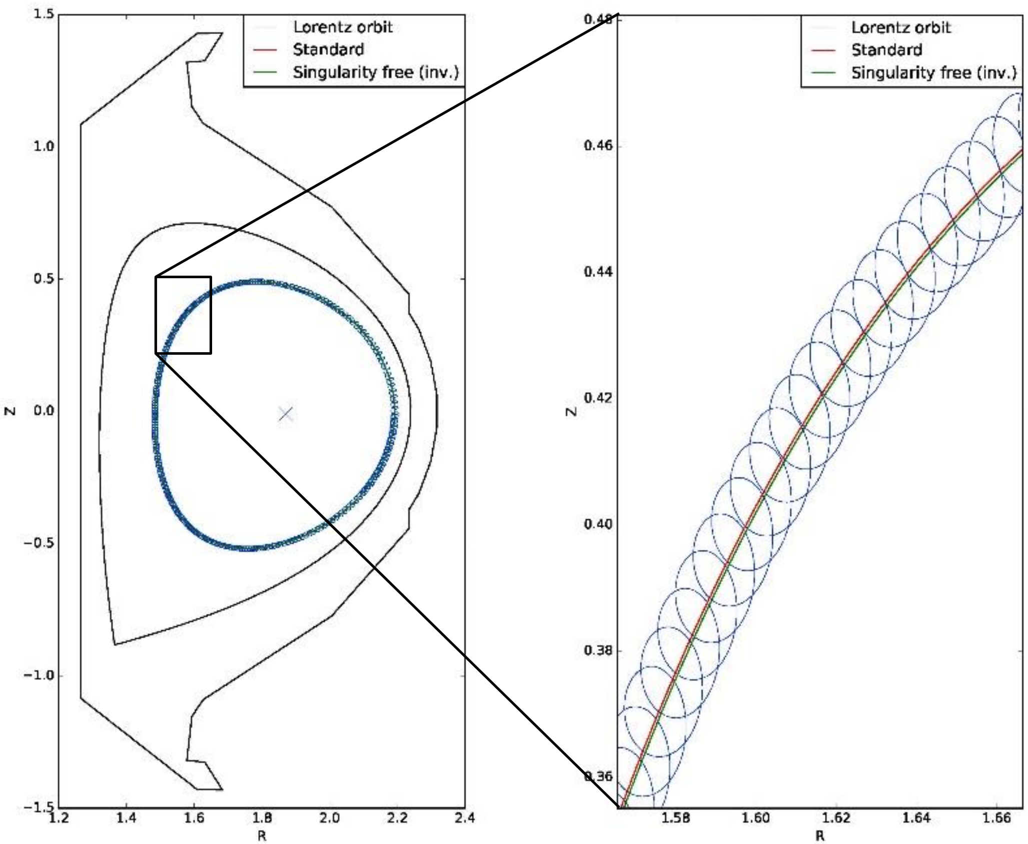
- **Equations of motion**

$$\boxed{\frac{dp_{//}{}^{*}}{dt} = e\frac{\mathbf{B}}{R_0 B^{\phi}}\cdot\mathbf{E}^{*}} \qquad \mathbf{E}^{*} = \mathbf{E} - \frac{mc^2}{e}\nabla\gamma + \frac{p_{//}{}^{*}}{e}\nabla\left(\mathbf{b}\cdot\frac{\mathbf{E}\times R_0\nabla\phi}{B}\right)$$

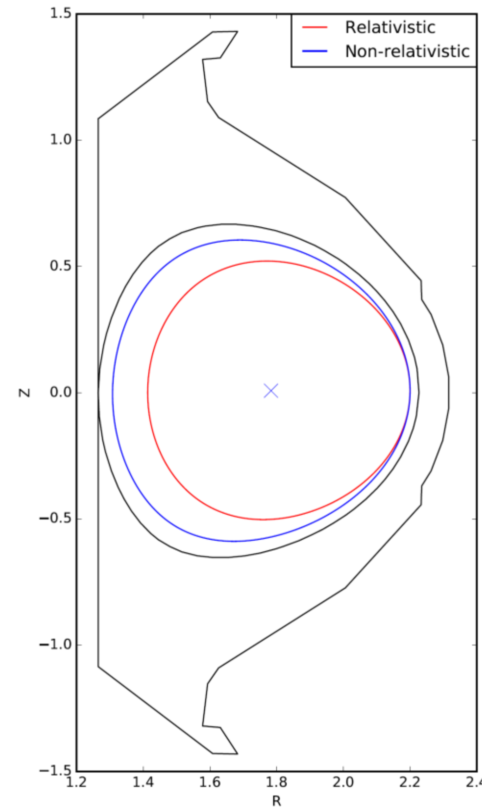
$$\boxed{\frac{d\mathbb{X}}{dt} = \frac{\mathbf{B}}{R_0 B^{\phi}}\frac{\partial H_{rgc}{}^{*}}{\partial p_{//}{}^{*}} + c\mathbf{E}^{*}\times\frac{R_0\nabla\phi}{R_0 B^{\phi}}} \qquad \gamma = \sqrt{1 + \frac{2\mu B(\mathbb{X},t)}{mc^2} + \frac{(p_{//}{}^{*})^2}{m^2c^2}\frac{(R_0 B^{\phi})^2}{B^2}}$$

Orbit calculation

- Guiding-center orbits and full Lorentz orbit are agreed well for not so high kinetic energies.



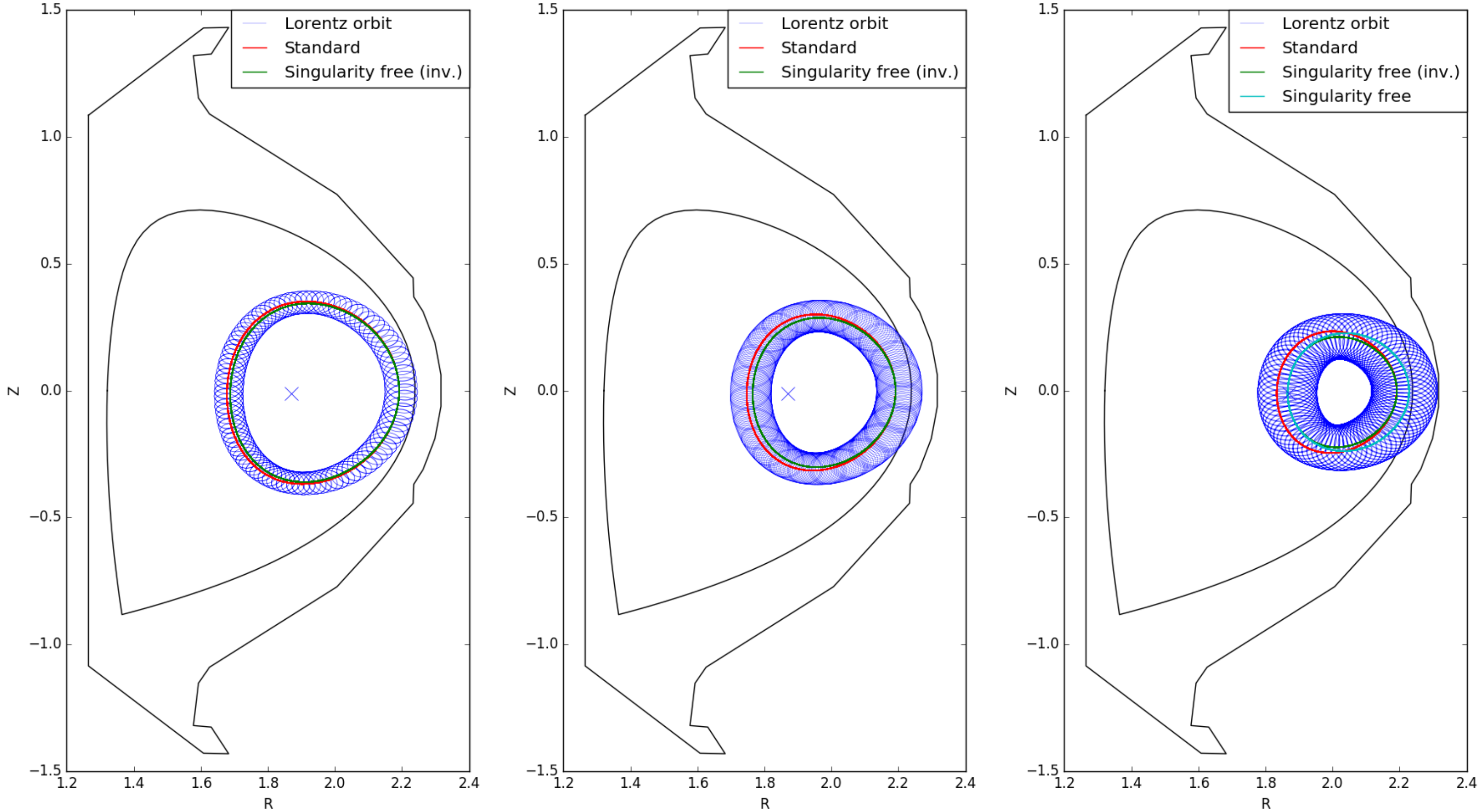
10MeV $v_{//}/v$=-0.9 relativistic passing electron in a KSTAR equilibrium magnetic field [#21576 13.81s]



10MeV $v_{//}/v$=-0.8 relativistic and non-relativistic electron orbit comparison [KSTAR #10902 2.0s]

High kinetic energy cases

- Guiding-center orbits starts to deviate for high kinetic energy case.



50MeV, 70MeV and 100MeV(left to right) passing ($v_{//}/v$=-0.9) electron orbits in a KSTAR equilibrium magnetic field [#21576 13.81s]
100MeV case, the transformed coordinates fitted well for the gyro-averaged particle orbit, but it is not always.

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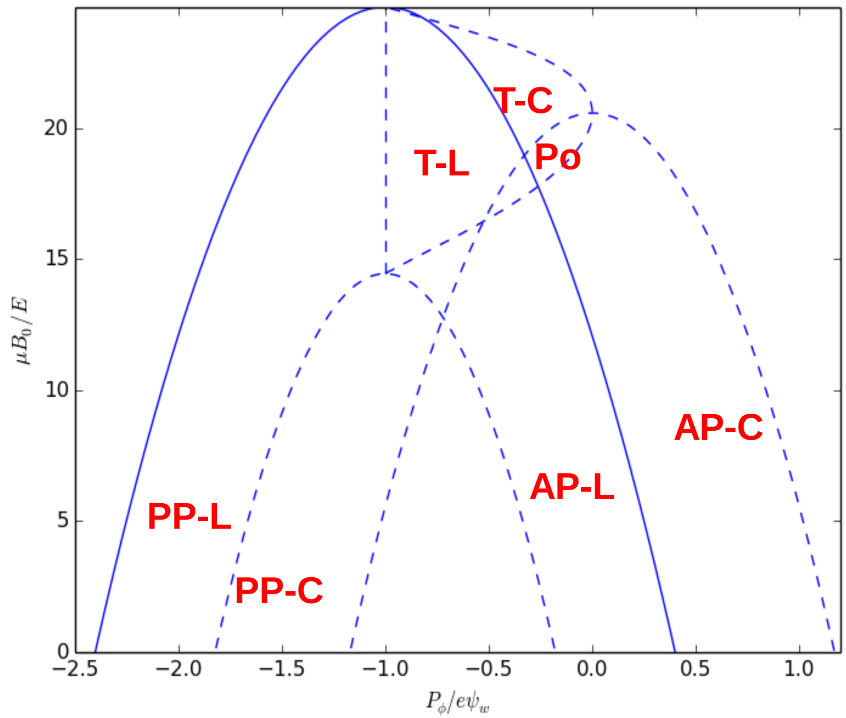
Orbit classification

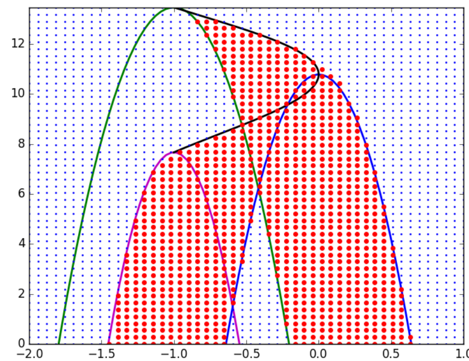
20MeV electron



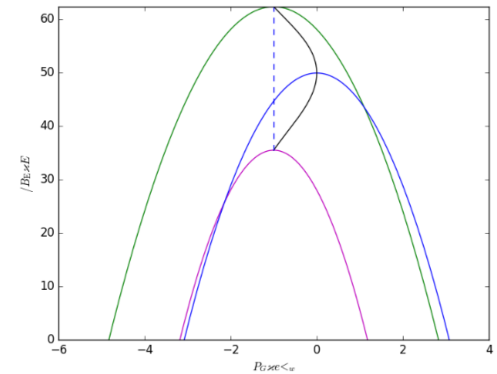
PP-L : Parallel Passing Loss
PP-C : Parallel Passing Confined
AP-L : Anti-parallel Passing Loss
AP-C : Anti-parallel Passing Confined
T-L : Trapped Loss
T-C : Trapped Confined
Po : Potato

KSTAR equilibrium magnetic field [#10902 2.0s]

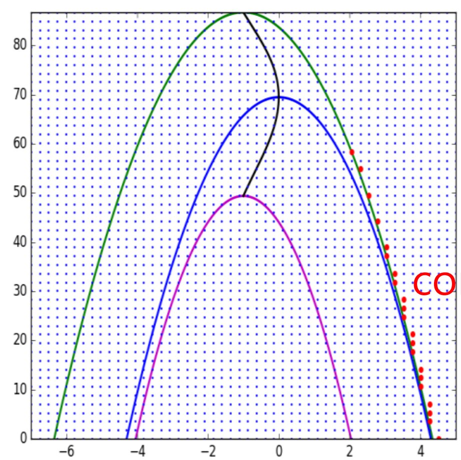
10MeV electron



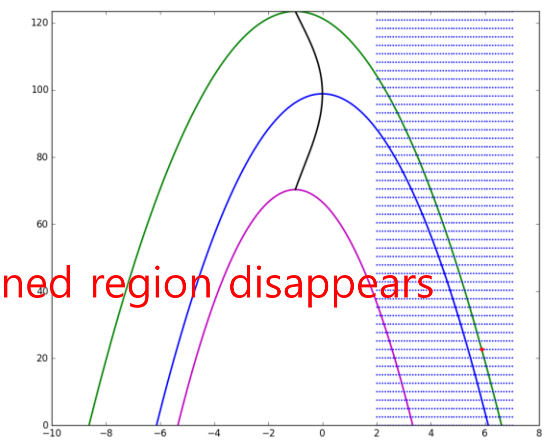
50MeV electron



70MeV electron



100MeV electron



confined region disappears

∃ minimum energy for the orbit confinement

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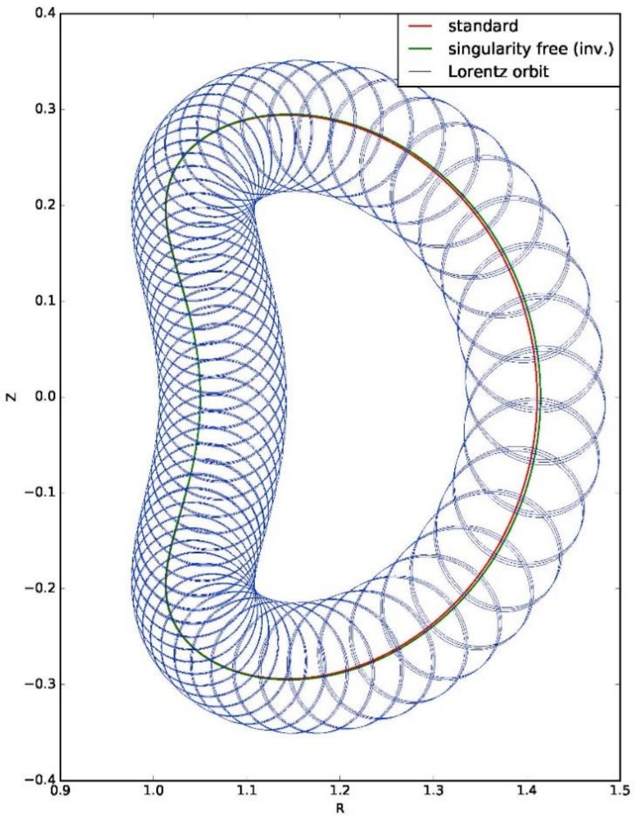
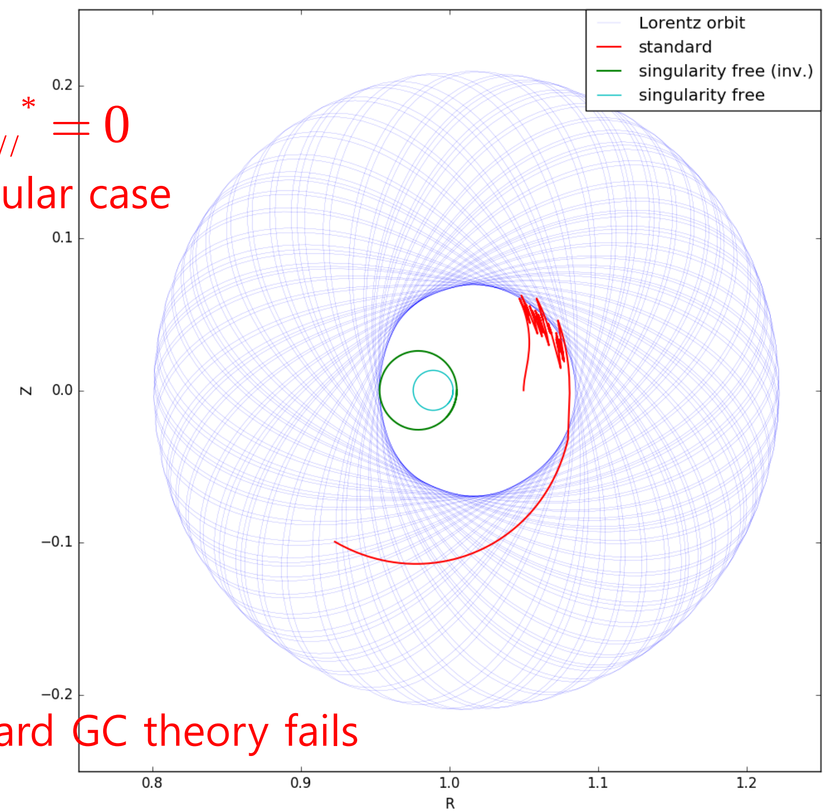
Concentric circular magnetic field

$$\mathbf{B} = -\frac{B_0 z}{q_0 R}\hat{R} - \frac{B_0 R_0}{R}\hat{\phi} + \frac{B_0}{q_0 R}\frac{R - R_0}{}\hat{z}$$

Concentric circular $\mathbf{B}$ field in cylindrical R, ϕ, z coordinates





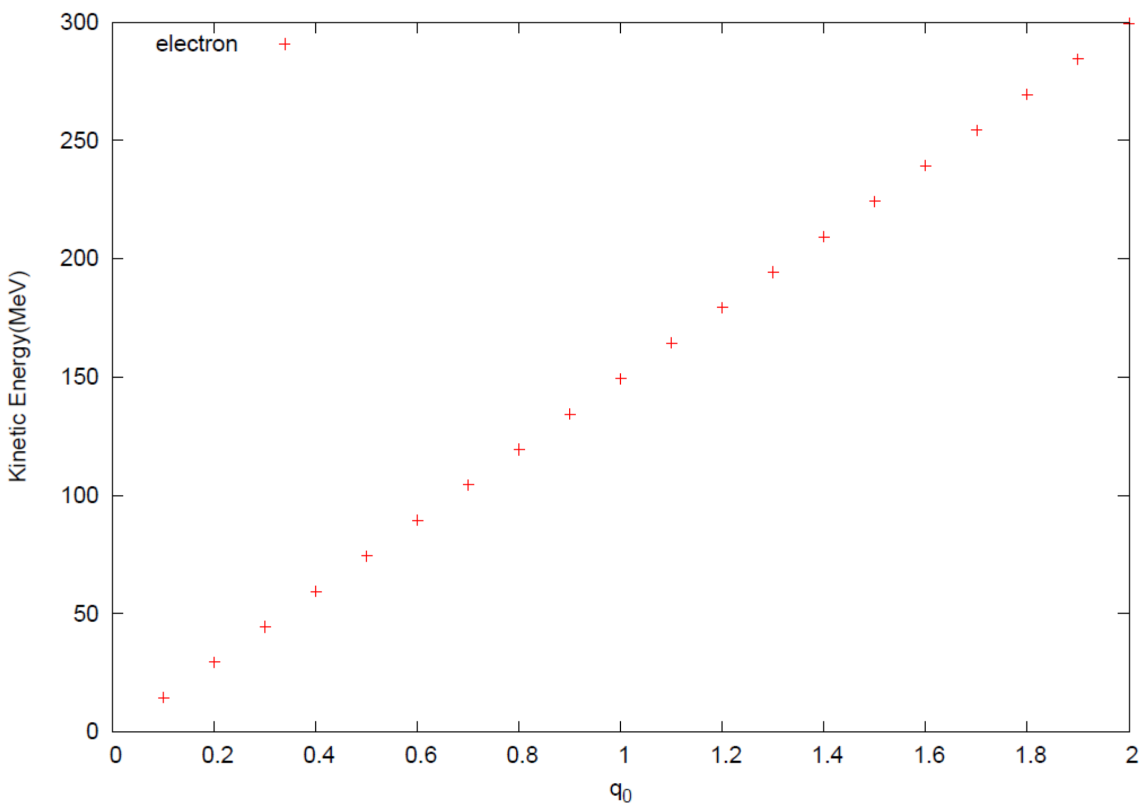
B_0=1T, q_0=1.414, R_0=1m
200keV ($v_{//}/v$=-0.21) trapped ion orbits

B_0=1T, q_0=0.1, R_0=1m
1MeV ($v_{//}/v$=-0.7) passing ion orbits

$B_{//}^{*}=0$ singularity

$$B_{//}^{*}=B+\frac{c}{e}p_{//}\ \mathbf{b}\cdot\nabla\times\mathbf{b}$$

For large $p_{//}$ and/or small twist length $1/\mathbf{b}\cdot\nabla\times\mathbf{b}$ $B_{//}^{*}=0$ singularity could take place [Wimmel, Boozer, etc.]



For the concentric circular magnetic field case (B_0=1T), the minimum electron velocity which makes $B_{//}^{*}=0$ can be written as

$$u_{//\min}=\frac{e}{m}\frac{q_0R_0B_0}{2}$$

Figure shows corresponding kinetic energy as function of the safety factor q_0

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Conclusions

- Two guiding-center equations of motion are implemented.
  - ✓ Toroidally regularized guiding-center theory applied to remove potential singularity for the $v_{//} \sim c$ case
  - ✓ Caveats : the coordinate transformations do not guarantee accurate orbits
  - ✓ The higher order correction is unclear.
  - ✓ The applicability of the toroidally regularized case may increases when the safety factor is small, such as reversed field pinch (RFP) system.

- On-going and future works :
  - ✓ What causes orbit deviation and how to remedy it.
  - ✓ How RE orbits modify when some perturbations given outside.
  - ✓ Application to other system, space, RFP, etc.