Multiplicity of axi-symmetric global Alfven eigenmodes in tokamaks

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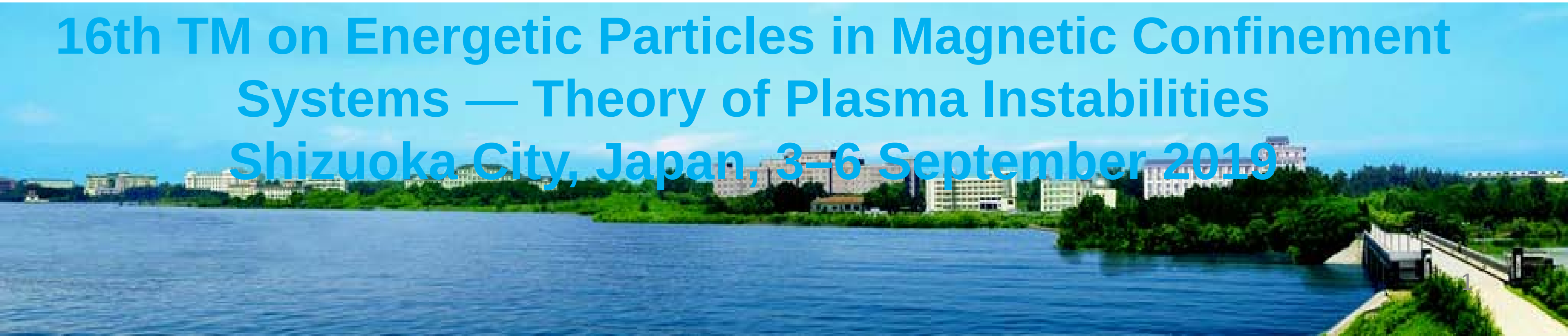

OUTLINE

- Background
- The electron drift effect on the axi-symmetric global Alfven eigenmodes
- An MHD analysis of the multiplicity for the mode
- Summary

Background

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Experiment observations:

- The axi-symmetric global Alfven eigenmodes (AS-GAE) observed in JET with maximum amplitude near the edge region [1].

- AS-GAEs observed in TFTR with frequency well-separated two branches near the edge of the device [2].

- AS-GAEs observed in MAST Ohmic heating plasmas [3].

[1] H. Oliver et al., Phys. Plasmas 24, 122505 (2017)
[2] Z. Zhang et al., Nucl. Fusion 35, 1459 (1995).
[3] A. Sykes, Plasma Phys. Control. Fusion 43 A127 (2001)

Background(2):Mode features and theory work

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- Mode frequency scales as Alfvenic waves, usually larger than that of TAEs.
- Mode number n=0, poloidally standing wave with m=1, 2.
- Maximal amplitude near edge region, but the mode may extend to the internal region.
- Modes may exist in NBI or ICRF discharge, and also in purely Ohmic discharges.
- Numerically found the mode frequency related to edge density[4]; two-fluid simulation proposed that Aws be excited by low frequency MHD[5]; ellipticity causes the splitting of the cylindrical continuum[1]

[4] L. Villard et al., Nucl. Fusion 37, 351 (1997).
[5] K. McClements et al., Nucl. Fusion 42, 1155 (2002).

Background(3):what is this work

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- Study the axi-symmetric Alfvenic modes using two-fluid model to show that:

  (1) For typical tokamak profiles the extrema exist near the edge region;

  (2) the drift effect due to the electron temperature gradient causes the splitting of the continuum of AS-GAEs

- Study the coupling effect between the SWs and the AS-GAEs, and the coupling between GAMs and AS-GAEs

The electron drift effect on AS-GAEs (1)

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Starting equations: two-fluid cold ion equations

$$0=-\nabla\delta p_e - en_e\left(\delta E+\delta u_e\times B+u_{*e}\times\delta B\right)$$

$$m_i n_i\frac{\partial\delta u_i}{\partial t}=en_i\left(\delta E+\delta u_i\times B\right)$$

$$\frac{\partial\delta n_e}{\partial t}+\nabla\cdot\left(n_e\delta u_e\right)-\frac{1}{e}\nabla\cdot\delta j_{\|}=0$$

$$\nabla\cdot\left(\delta j_\perp+\delta j_{\|}\right)=0$$

A closure system

$$\delta E=-\nabla\delta\phi-(\partial\delta\psi/\partial t)b \quad \delta p_e=T_e\delta n_e$$

$$\delta B=-b\times\nabla\delta\psi \quad \delta j_\perp=en_i\delta u_i-en_e\delta u_e=\frac{n_i m_i}{B^2}\frac{\partial\delta E_\perp}{\partial t}+\frac{1}{B^2}B\times\nabla\delta p_e$$

$$-b\cdot\nabla\left[\frac{i}{\omega}u_{*ne}\cdot\nabla\delta\phi-\frac{T_e}{\mu_0 V_A{}^2 e^2 n_e}\nabla_\perp{}^2\delta\phi\right]-b\cdot\nabla\delta\phi+i\omega\delta\psi-u_{*e}\cdot\nabla\delta\psi=0 \quad (1)$$

$$i\frac{\omega}{V_A{}^2}\nabla_\perp{}^2\delta\phi+\nabla\times\left(\frac{\vec{B}}{B^2}\right)\cdot\nabla\left[\frac{T_e}{i\omega}\nabla\times\left(\frac{n_e\vec{B}}{B^2}\right)\cdot\nabla\delta\phi+\frac{T_e}{\mu_0 V_A{}^2 e}\nabla_\perp{}^2\delta\phi\right]-\nabla\cdot\left(b\nabla_\perp{}^2\delta\psi\right)=0 \quad (2)$$

The electron drift effect on AS-GAEs (2)

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[1]Concentric circular cross section

[2] Only curvature coupling kept

[3] $\nabla \times \left(\frac{\vec{B}}{B^2} \right) \simeq \frac{2}{B^2} B \times \kappa \simeq -\frac{2|\kappa|}{B} \left(\hat{\theta} \cos\theta + \hat{r} \sin\theta \right)$

[4] $\delta\phi = \sum_{l=0,\pm1,\pm2} \delta\phi_l e^{-i(\omega t + l\theta)}$

From eq.(1), we have

$$\delta\psi_0 = 0$$

$$qR_0 \omega \left(\omega + l\omega_{*e} \right) \delta\psi_l = \left(l\omega \rho_s^2 \Delta_l - l\omega - l^2 \omega_{*ne} \right) \delta\phi_l$$

$$\Delta_l = \frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial}{\partial r} - \frac{l^2}{r^2}$$

The electron drift effect on AS-GAEs (3)

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From eq.(2):

$$\frac{\omega^2}{V_A^2}\frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial}{\partial r}\delta\phi_0-\frac{C_1}{r^2}\left(1+r\frac{d}{dr}\right)\delta\phi_c+\frac{C_2}{r}\Delta_1\left(1+\frac{d}{dr}\right)\delta\phi_s=0$$

$$\left[\frac{\omega^2}{V_A^2}-\frac{\omega^2-\omega_{*e}\omega_{*ne}}{q^2R_0^2\left(\omega^2-\omega_{*e}^2\right)}\right]\Delta_1\delta\phi_c+\frac{\omega\omega_{*Te}}{q^2R_0^2\left(\omega^2-\omega_{*e}^2\right)}\Delta_1\delta\phi_s=0$$

$$-\frac{C_2}{r}\frac{d}{dr}\Delta_0\delta\phi_0+\frac{\omega\omega_{*Te}}{q^2R_0^2\left(\omega^2-\omega_{*e}^2\right)}\Delta_1\delta\phi_c+\left[\frac{\omega^2}{V_A^2}-\frac{\omega^2-\omega_{*e}\omega_{*ne}}{q^2R_0^2\left(\omega^2-\omega_{*e}^2\right)}\right]\Delta_1\delta\phi_s=0$$

$$C_1=\frac{\kappa e n_e u_{*ne}}{B}$$

$$C_2=\frac{\kappa e n_e \omega\rho_s^2}{B}$$

$$\delta\phi_c=\delta\phi_{+1}+\delta\phi_{-1}$$

$$\delta\phi_s=\delta\phi_{+1}-\delta\phi_{-1}$$

Dropping finite Lamor radius effect

$$\left(\omega^2-\omega_s^2\right)^2\left(\omega^2-\omega_{*e}^2\right)-2\left(\omega^2-\omega_s^2\right)\left(\omega^2-\omega_{*e}\omega_{*ne}\right)+\omega^2-\omega_{*ne}^2-\frac{1}{4}\omega_s^4=0$$

DR of continuum, with frequency normalized by Alfvenic wave

The electron drift effect on AS-GAEs (4)

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Iteratively solving the DR since $\omega^2 \gg \omega_{*e}{}^2 \sim \omega_s{}^2$

[1] A low frequency DW branch

$$\omega^2 \simeq \omega_{*ne}{}^2 + \frac{1}{4}\omega_s{}^2 - 2\omega_{*ne}\omega_{*Te}\left(\omega_{*ne}{}^2 - \omega_s{}^2\right)$$

[2] Two high frequency AW branches

$$\omega^2 \simeq 1 + \omega_s{}^2 + \omega_{*e}\omega_{*Te} \pm \omega_{*Te}{}^2$$

!! Temperature gradient is necessary to make splitting of the AW continuum

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$$n_e / T_e = n_0 / T_0 \left[1 - \left(\frac{r}{a}\right)^2 + \Delta\right]^\alpha$$

$$q = 1 + \Delta q \left(\frac{r}{a}\right)^2$$

(a) $\Delta q = 3.5 \quad \Delta = 0.1$

(b) $\Delta q = 3.5 \quad \Delta = 0.6$

(c) $\Delta q = 1.2 \quad \Delta = 0.6$

(d) $\Delta q = 1.2 \quad \Delta = 0.1$

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$$T_e = T_0 \frac{1 + \Delta_T - e^{\eta(x^2-1)}}{1 + \Delta_T - e^{-\eta}}$$

$$n_e = n_0 \left[1 - \left(\tfrac{r}{a}\right)^2 + \Delta_n\right]$$

(a) $\Delta q = 1.5 \quad \Delta_n = 0.2 \quad \Delta_T = 0.001$

(b) $\Delta q = 1.5 \quad \Delta_n = 0.3 \quad \Delta_T = 0.3$

(c) $\Delta q = 1.1 \quad \Delta_n = 0.2 \quad \Delta_T = 0.001$

(d) $\Delta q = 1.1 \quad \Delta_n = 0.3 \quad \Delta_T = 0.3$

The electron drift effect on AS-GAEs (5)

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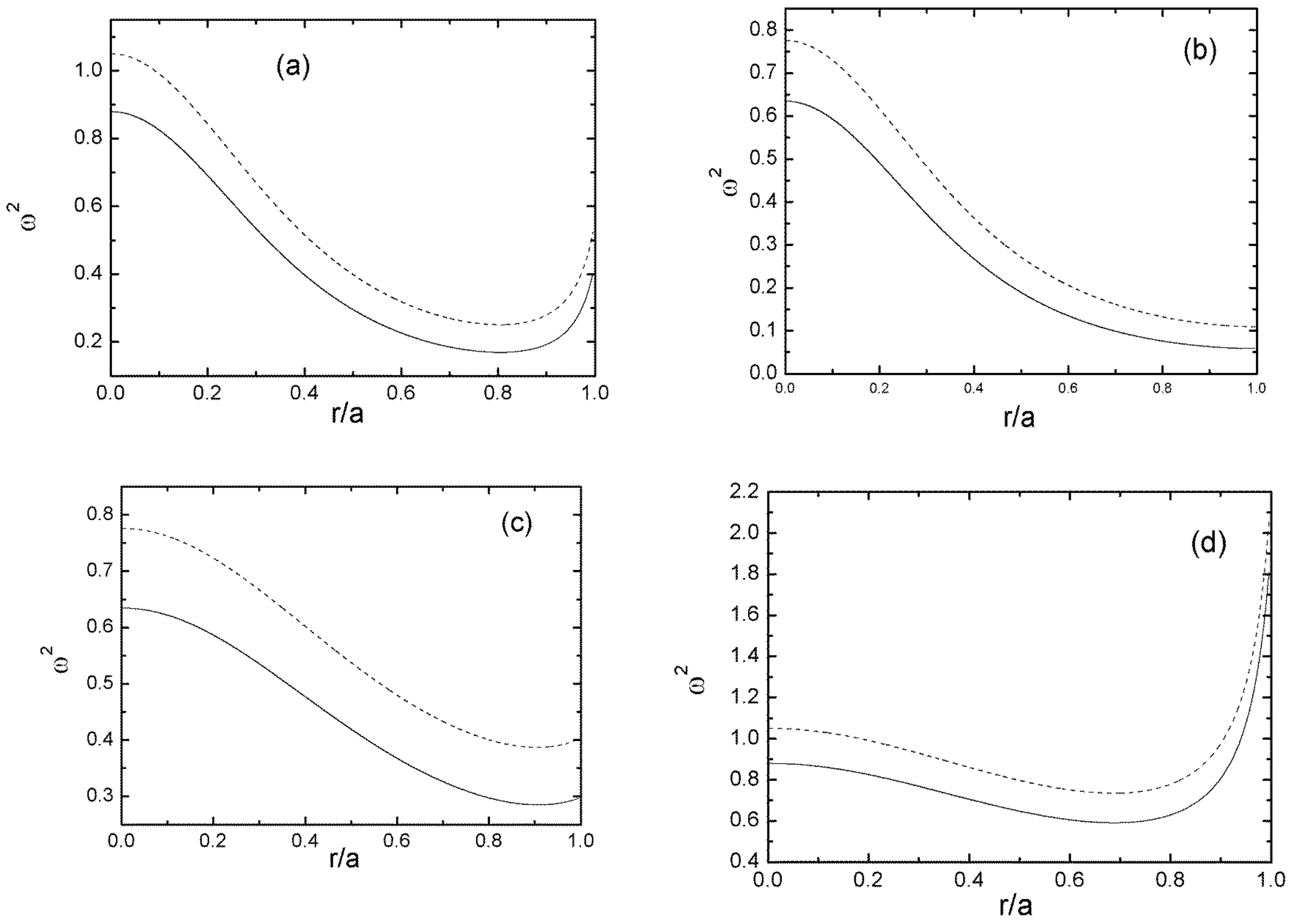

The electron drift effect on AS-GAEs (6)

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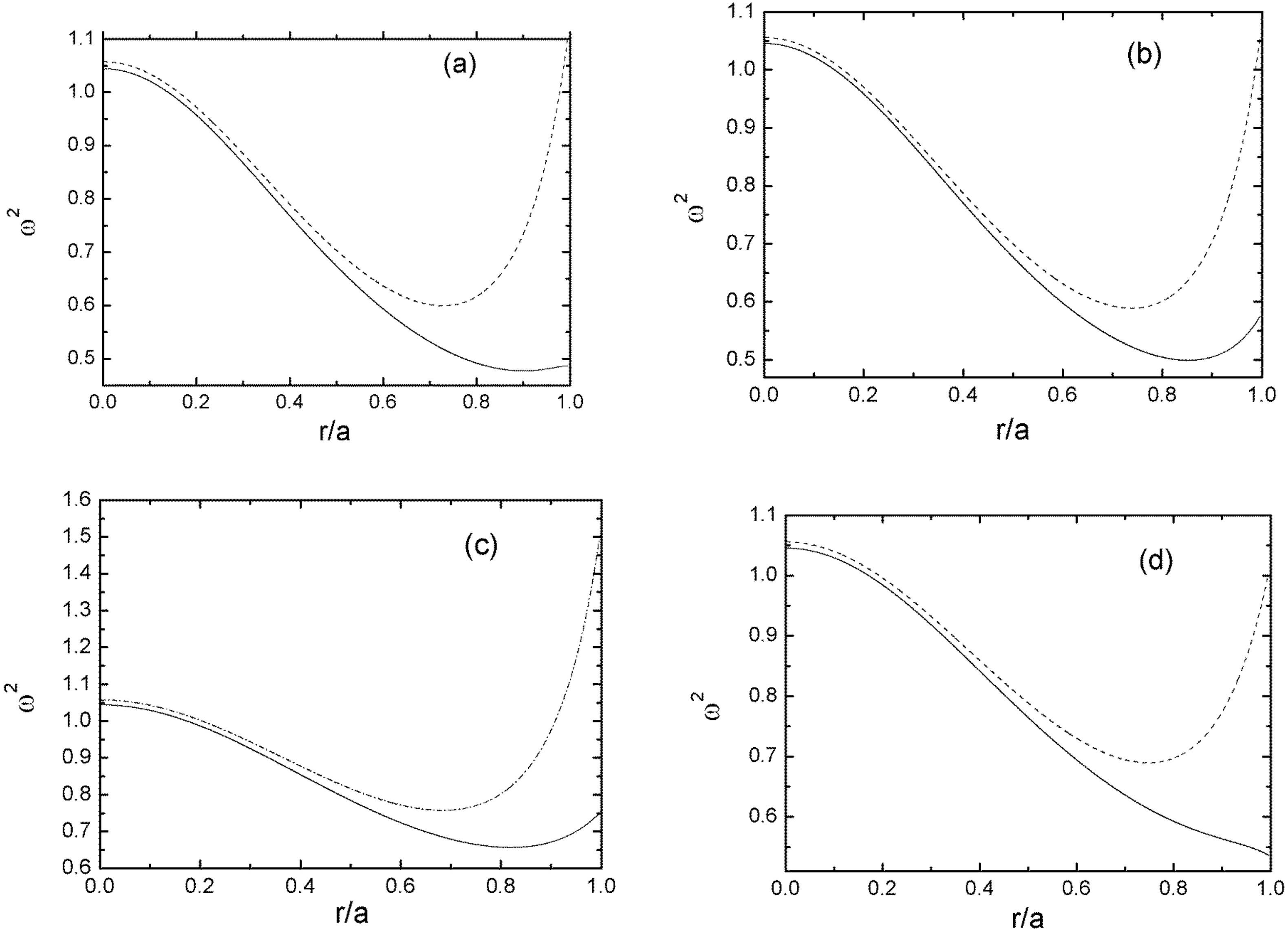

The MHD AS-GAEs (1)

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Ideal MHD description

$$\boldsymbol{\xi} = \frac{1}{B^2}\mathbf{B}\times\nabla\Phi + \xi_{\parallel}\mathbf{b}$$

$$\delta\mathbf{B} = -\mathbf{b}\times\nabla\nabla_{\parallel}\Phi$$

$$\delta p = -\boldsymbol{\xi}\cdot\nabla p + \delta p_{comp}$$

$$\delta p_{comp} = -\gamma p\nabla\cdot\boldsymbol{\xi}$$

$$\rho\frac{\partial^2\xi_{\parallel}}{\partial t^2} = -\nabla_{\parallel}\delta p_{comp}$$

$\Rightarrow$

$$\frac{\partial^2\delta p_{comp}}{\partial t^2} - c_s^{\,2}\mathbf{B}\cdot\nabla\frac{1}{B}\nabla_{\parallel}\delta p_{comp} = \frac{2\gamma p}{B^2}\mathbf{B}\times\boldsymbol{\kappa}\cdot\nabla\frac{\partial^2\Phi}{\partial t^2} \qquad (1)$$

$\nabla\cdot\delta\mathbf{j} = 0 \Rightarrow$

$$\nabla\cdot\left(\frac{1}{V_A^{\,2}}\nabla_{\perp}\frac{\partial^2\Phi}{\partial t^2}\right) - \mathbf{B}\cdot\nabla\frac{1}{B}\nabla_{\perp}^{\,2}\nabla_{\parallel}\Phi + \mathbf{B}\times\nabla\left(\frac{j_{\parallel}}{B^2}\right)\cdot\nabla_{\perp}\nabla_{\parallel}\Phi$$
$$-\nabla\times\frac{\mathbf{B}}{B^2}\cdot\nabla\left(\frac{\mathbf{B}\times\nabla p}{B^2}\cdot\nabla\Phi\right) + \nabla\times\frac{\mathbf{B}}{B^2}\cdot\nabla\delta p_{comp} = 0 \qquad (2)$$

(1) (2) form a closure system for δp_{comp} Φ

The MHD AS-GAEs (2)

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[1] Large aspect ratio circular cross section

[2] Only curvature coupling kept

[3] $\Phi = \sum_{l=0,\pm1,\pm2} \delta\Phi_l e^{-i(\omega t + l\theta)} \qquad \delta p_{comp} = \sum_{l=0,\pm1,\pm2} \delta p_l e^{-i(\omega t + l\theta)}$

$$\left(\frac{\omega^2}{V_A^2} - 2C_1\right)\frac{d^2}{dr^2}\delta\Phi_0 - \frac{2C_1}{r}\frac{d}{dr}\delta\Phi_0 + \frac{\beta'}{rR_0}\left(\frac{1}{r} + \frac{d}{dr}\right)\delta\Phi_{1c} + C_1\left(\frac{d^2}{dr^2} + \frac{3}{r}\frac{d}{dr}\right)\delta\Phi_{2c} = 0$$

$$\left(\frac{\omega^2}{V_A^2} - \frac{1}{q^2R_0^2} - C_2\right)\left(\frac{d^2\delta\Phi_{1c}}{dr^2} - \frac{\delta\Phi_{1c}}{r^2}\right) - \frac{C_2}{r}\frac{d\delta\Phi_{1c}}{dr} + \frac{2\beta'}{rR_0}\left(\frac{2}{r} + \frac{d}{dr}\right)\delta\Phi_{2c} = 0$$

$$\left(\frac{\omega^2}{V_A^2} - \frac{1}{q^2R_0^2} - \frac{2\gamma p}{B_0^2R_0^2} - C_2\right)\left(\frac{d^2}{dr^2} - \frac{1}{r^2}\right)\delta\Phi_{1s} - \left(C_2 + \frac{2\gamma p}{B_0^2R_0^2}\right)\frac{1}{r}\frac{d\delta\Phi_{1s}}{dr} + \frac{2\beta'}{rR_0}\left(\frac{2}{r} + \frac{d}{dr}\right)\delta\Phi_{2s} = 0$$

$$2C_1\left(\frac{d^2}{dr^2} - \frac{1}{r}\frac{d}{dr}\right)\delta\Phi_0 + \left(\frac{\omega^2}{V_A^2} - \frac{4}{q^2R_0^2} - C_1\right)\left(\frac{d^2}{dr^2} - \frac{4}{r^2}\right)\delta\Phi_{2c} + \frac{2\beta'}{rR_0}\left(\frac{1}{r} - \frac{d}{dr}\right)\delta\Phi_{1c} = 0$$

$$\left(\frac{\omega^2}{V_A^2} - \frac{4}{q^2R_0^2} - C_1\right)\left(\frac{d^2}{dr^2} - \frac{4}{r^2}\right)\delta\Phi_{2s} + \frac{2\beta'}{rR_0}\left(\frac{1}{r} - \frac{d}{dr}\right)\delta\Phi_{1s} = 0$$

The MHD AS-GAEs (3)

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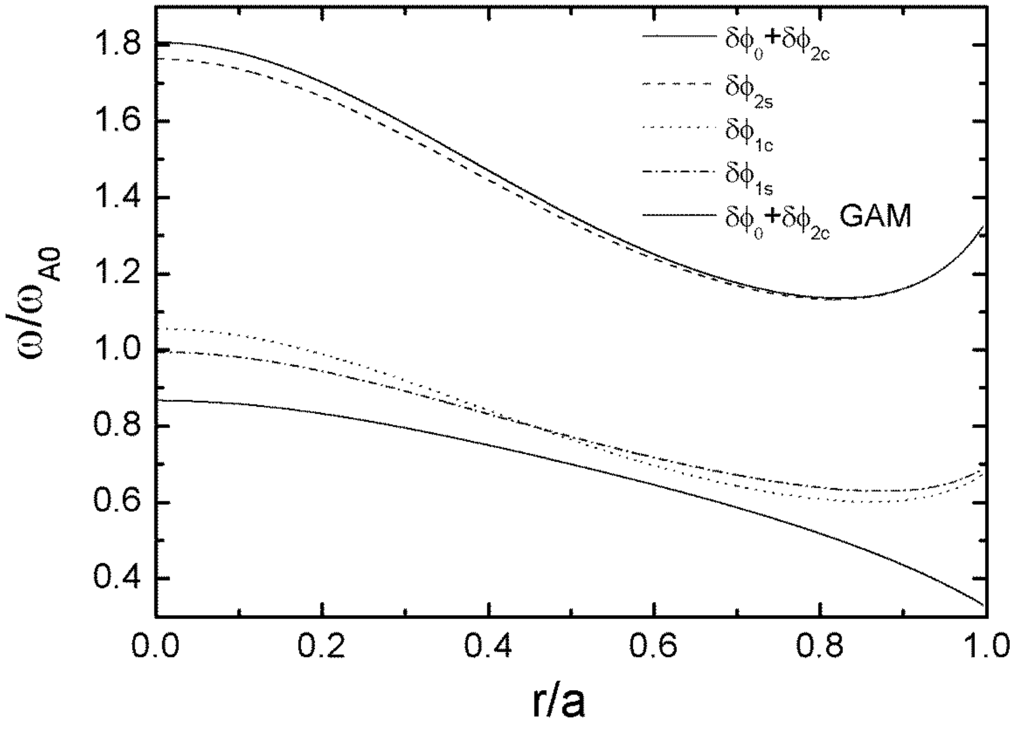

$$n_e / T_e = n_0 / T_0 \left[1 - \left(\frac{r}{a} \right)^2 + \Delta \right]$$

$$q = 1 + \Delta q \left(\frac{r}{a} \right)^2$$

$$\Delta q = 2.0$$

$$\Delta = 0.25$$

$$\beta_0 = 0.04$$

Five branches of the continuum： one AW splits into two due to coupling with SW, typically the extremum appears near the edge.

Summary

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- The axi-symmetric Alfvenic eigenmodes were observed in tokamaks near the edge and with multiple frequency.
- From the 2-fluid model, the drift effect due to the electron temperature gradient causes the splitting of the continuum of AS-GAEs.
- Five branches of the axisymmtric continuum found from MHD, one of which is GAM. one AW with m=1 or 2 can split into two due to coupling with SW, typically the extremum appears near the edge.