HALO : A full-orbit GPU code for calculating the non-linear evolution of Eigenmodes in Tokamaks

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HALO (HAGIS-LOCUST) is a new code which uses quasi Monte-Carlo techniques to solve the perturbative Vlasov-Maxwell problem in a Tokamak plasma. This enables predictions of the stability of ideal MHD eigenmodes in the presence of fast particle populations from various heating sources. As well as calculations of the linear growth rates, HALO enables full-orbit predictions of the non-linear saturation of these modes. HALO is built on top of the LOCUST GPU code to enable rapid tracking of millions of markers over the mode saturation timescale.

Formalism

The fields in the plasma are represented by a static background magnetic equilibrium with a set of mode fields superimposed. The total mode electric field is represented as follows (with an equivalent relation for $\boldsymbol{B}$):

$$\delta\boldsymbol{E}(\boldsymbol{x},t) = \sum_i Re\{\boldsymbol{E}_i(\boldsymbol{x},t)\} = \sum_i Re\{A(t;\omega_i)\boldsymbol{e}(\boldsymbol{x};\omega_i)e^{-i\omega_i t}\}$$

Where $\boldsymbol{e}(\boldsymbol{x};\omega_i)$ is an ideal MHD eigenmode with frequency $\boldsymbol{\omega_i}$. If the growth rates of the modes are small compared with their frequencies the evolution of the mode amplitudes satisfies:

$$\frac{\dot{A}(t;\omega_j)}{A(t;\omega_j)} = -\frac{1}{2\delta W A_j^2}\int d\boldsymbol{x}\,\boldsymbol{E}_j^{\dagger}(\boldsymbol{x};t)\delta\boldsymbol{J}_{\boldsymbol{fast}}(\boldsymbol{x},t)$$

Where δW is a constant and $\boldsymbol{\delta J}$ is the fast particle current.

$$\delta\boldsymbol{J}_{\boldsymbol{fast}}(\boldsymbol{x},t) = \int d\boldsymbol{v}\,f(\boldsymbol{x},\boldsymbol{v},t)Ze\boldsymbol{v}$$

To numerically solve this equation the fast particle distribution is represented in the code by a Monte-Carlo marker population. A further simplification is made by splitting f into equilibrium and perturbed parts and noting that the equilibrium part yields 0 by axisymmetry. This *delta-f scheme* significantly reduces the noise in the numerical integrations.

$$\frac{\dot{A}(t;\omega_j)}{A(t;\omega_j)} \approx -\frac{1}{2\delta W A_j^2}Ze\sum_i \boldsymbol{E}_j^{\dagger}(\boldsymbol{x_i}(t))\cdot\boldsymbol{v}_i\left[F_0(\boldsymbol{x_i}(t),\boldsymbol{v_i}(t)) + \delta f_i(t)\right]\Delta^3x_i\Delta^3v_i$$

$$\therefore \frac{\dot{A}(t;\omega_j)}{A(t;\omega_j)} \approx -\frac{1}{2\delta W A_j^2}Ze\sum_i \boldsymbol{E}_j^{\dagger}(\boldsymbol{x_i}(t))\cdot\boldsymbol{v}_i\delta f_i(t)\Delta^3x_i\Delta^3v_i$$

The evolution of δf_i along the trajectories of the markers is governed by the following differential equation:

$$\frac{d}{dt}\delta f_i(t) = -\frac{Ze}{m}\left(\boldsymbol{v}_i(t)\times\delta\boldsymbol{B}(\boldsymbol{x_i}(t),t) + \delta\boldsymbol{E}(\boldsymbol{x_i}(t),t)\right)\cdot\left(\frac{\partial F_0}{\partial\boldsymbol{v}}\right)_{\boldsymbol{x}}(\boldsymbol{x_i}(t),\boldsymbol{v_i}(t))$$

Markers are initially distributed uniformly across phase space ($\boldsymbol{x}$,$\boldsymbol{v}$) according to a Halton scheme in order to minimise statistical noise. The full-orbit trajectories of the markers in the presence of the modes are then determined via solution of the Lorentz equations of motion. This calculation is performed on GPUs using the Boris-Leapfrog scheme. After a set period of time has elapsed the average power transfer of each marker is calculated and a reduction is performed to determine the total contribution to the above integral. The wave amplitudes are then updated according to the above equation before being broadcast back to the GPUs. This scheme is efficient as the wave growth is slow compared to the particle gyro-motion so many particle steps are taken per reduction.

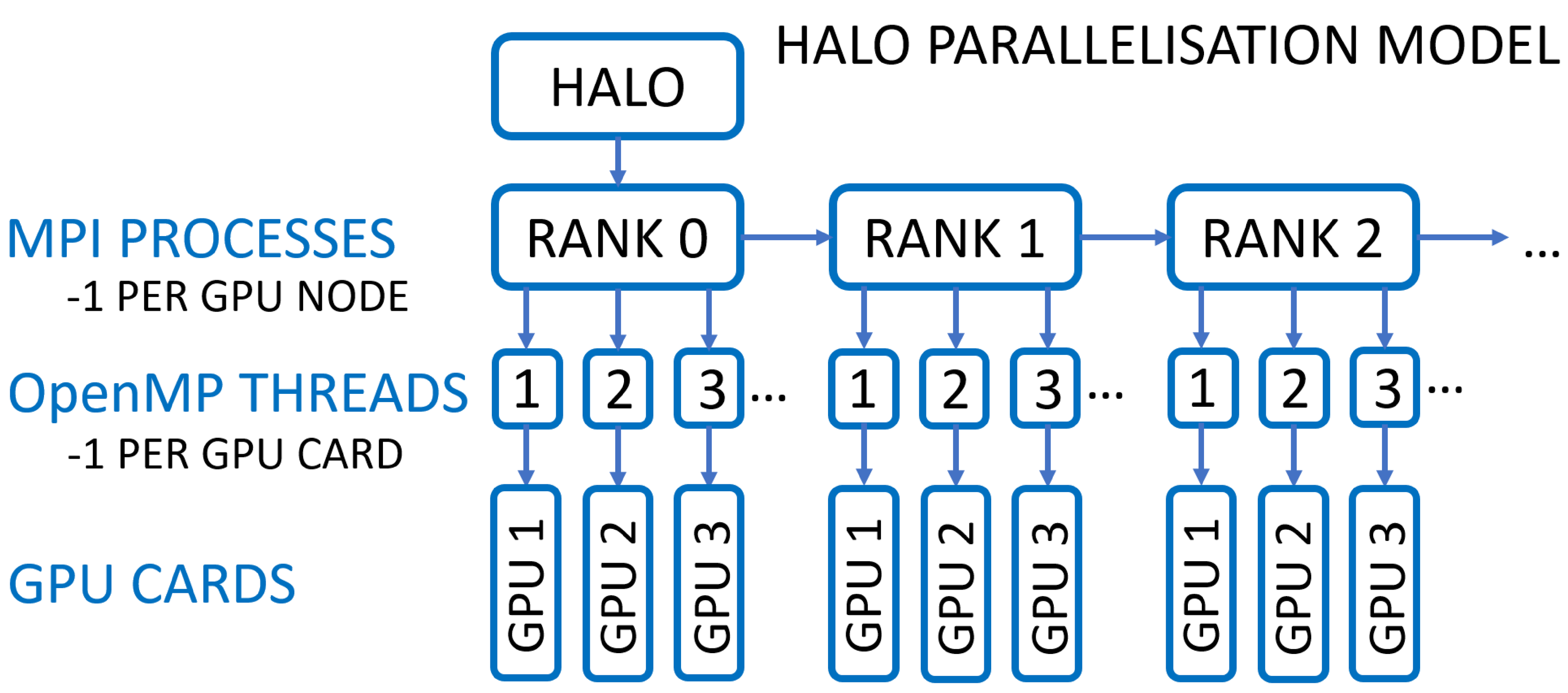

Figure 1: The HALO parallelisation model. HALO uses MPI to communicate across multiple GPU nodes on a cluster. Each node spawns one OpenMP thread per GPU card. CUMULUS at UKAEA has 5 GPU nodes each of which has 8 NVIDIA TESLA P100 cards with 3584 cores.

Conclusions

HALO has been successfully benchmarked against HAGIS for a single *n=6* mode. It correctly predicts the linear growth rate and non-linear saturation of the mode including finite Larmor radius effects. In the future the effect of collisions and sources/sinks needs to be included.

Benchmarking

HALO has been benchmarked against the existing guiding-centre HAGIS code.

1) Resonant particles were followed in both codes and Poincare plots were made showing the correlation between the wave phase and the radial excursions of the particles. Similar island structures can be seen in both codes.

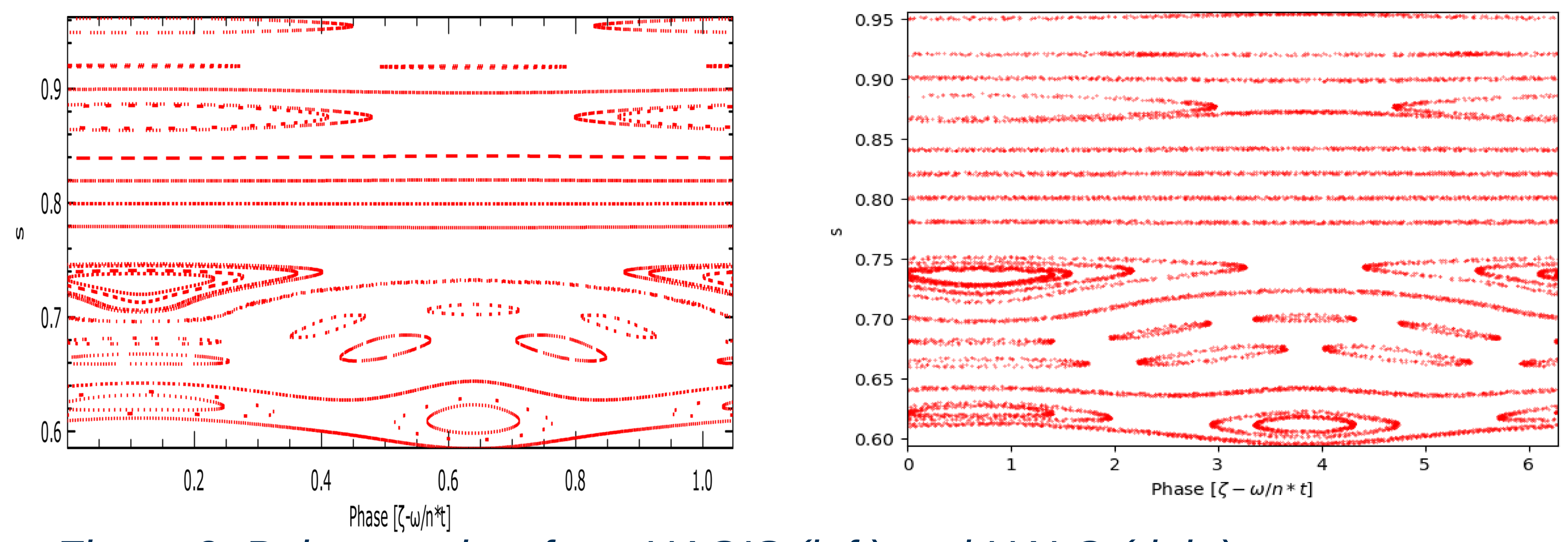

Figure 2: Poincare plots from HAGIS (left) and HALO (right)

2) In the absence of any modes the energy and toroidal canonical momentum of each particle, P_φ, are conserved. In the presence of a single mode toroidal symmetry is broken meaning E and P_φ are no longer conserved. However a new invariant $K = E - \frac{\omega}{n}P_\varphi$ can be constructed. The plots below show K is well conserved in HALO (right) compared with P_φ (left).

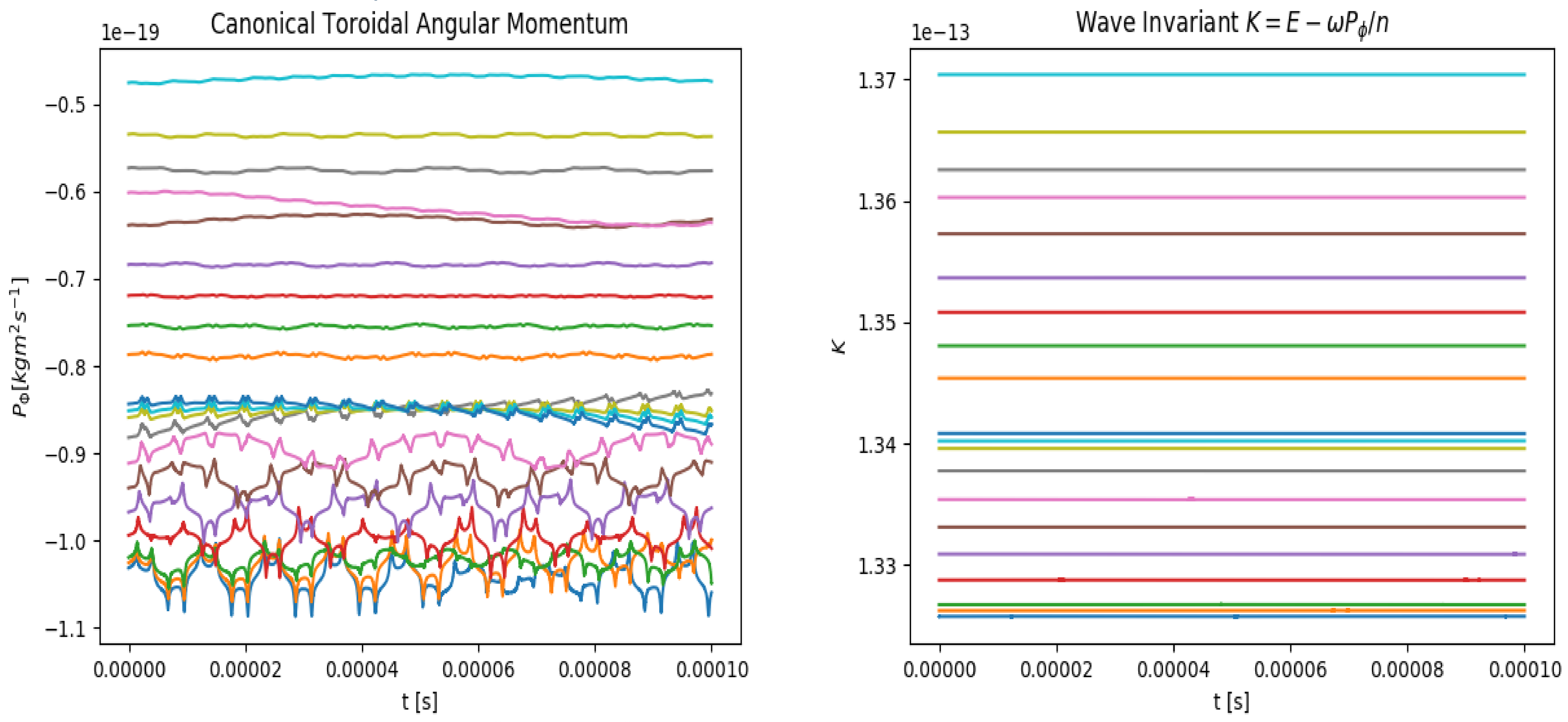

Figure 3: Variation of P_φ (left) and K (right) along resonant particle orbits.

3) The linear growth rate of the mode was calculated in both codes. When HALO uses the guiding centre location of the particles to evaluate the fields the agreement is good. The full-orbit predictions are lower as expected by $J_0(k_\perp\rho)$ =0.57. Applying a manual damping close to the linear drive the code exhibits frequency chirping consistent with 1D Berk-Breizman theory.

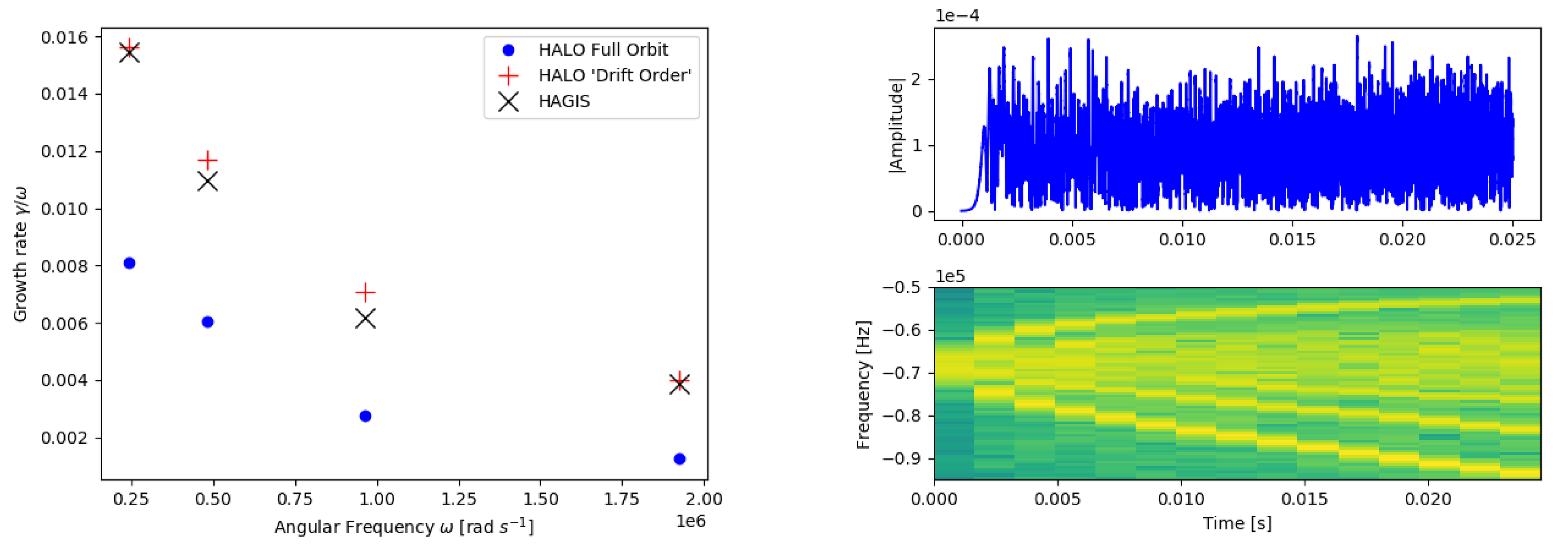

Figure 4: Left: Comparison of linear growth rates between HAGIS and HALO. Right : Frequency chirping observed in the presence of damping.

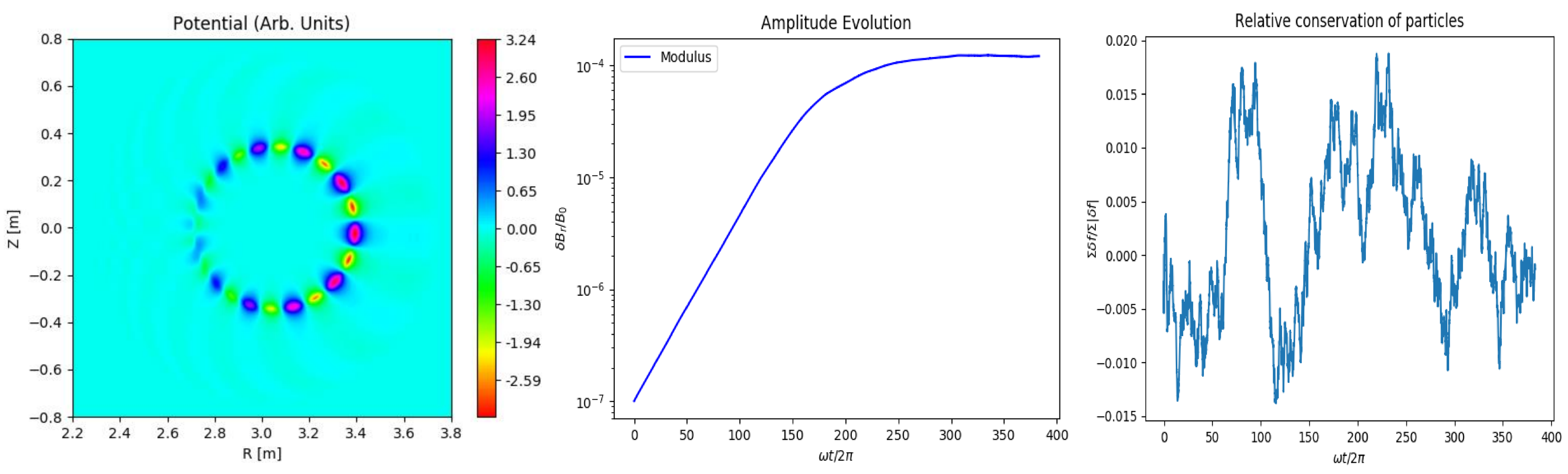

Figure 5: An n=6 ideal MHD mode and its predicted evolution under an isotropic slowing-down distribution of neutral beam particles. The rightmost plot shows the relative conservation of particles during the simulation $\frac{\sum\delta f}{\sum|\delta f|}$ which is ~2%.

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