Residual Zonal Flows for Non-Maxwellian Equilibrium Distribution Function

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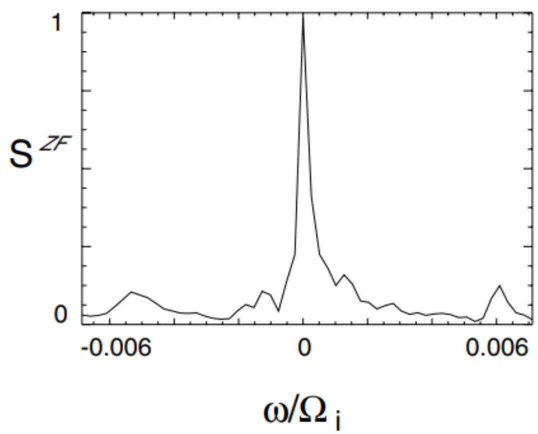
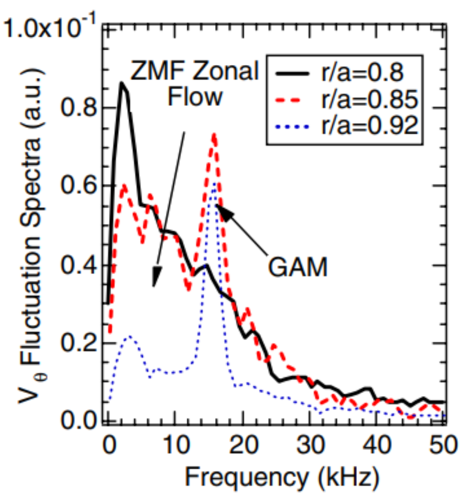
Outline

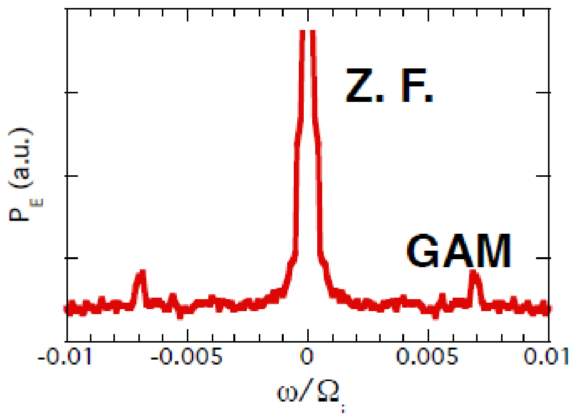
- Introduction
- Residual Zonal Flows and Polarization Shielding
- α-Particle Effects in ITER and Beyond
- Isotopic Dependence
- Conclusion

Zonal Flows in Magnetic Fusion Research

- Zonal flows regulate turbulence and transport.
- Turbulence in most cases produces zonal flows.
- Characteristics predicted by simulation and theory, have been confirmed from experiments







T.S.Hahm et al.,
PPCF (2000)
from GTC Simulation

D.K. Gupta et al.,
PRL(2006)
from Tokamak(DIII-D)

A. Fujisawa et al.
PRL(2004)
from Stellarator(CHS)

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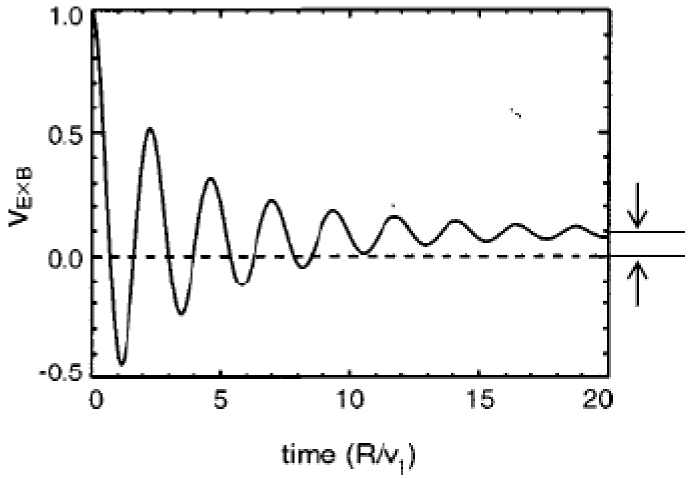
Residual Zonal Flows in Toroidal Geometry

- Based on gyro-Landau-fluid closure (up to mid 90's), ZF is completely damped even in collisionless plasmas

- Rosenbluth-Hinton [PRL '98] ZF undamped from Gyrokinetic theory



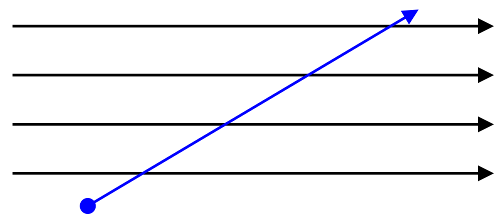
RH residual level : $\frac{1}{1+1.63q^2/\sqrt{\epsilon}}$

- Gyrokinetic codes are now benchmarked against the analytic results!
- Most transport models don't explicitly include zonal flows yet.
  (exceptions : M. Nunami et al, PoP (2013), E. Narita et al., PPCF (2018))

Nonlinear Gyrokinetics

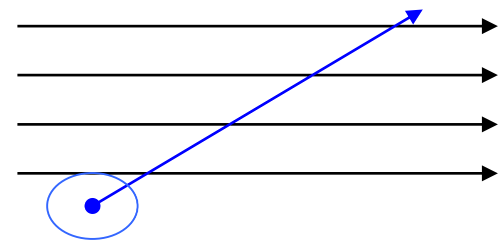
[Frieman and Chen, Phys. Fluids (1982)]

Guiding Center Motion $\vec{B}$
(Drift kinetics)



Point particle with
magnetic moment μ

Gyro Center Motion $\vec{B}$
(Gyrokinetics)



Ring with radius ρ
magnetic moment μ

Courtesy:
S.H. Ku

- Drift wave turbulence
  - (Wave vector k) (gyroradius) ~ O(1)
  - Wave frequency ω << gyrofrequency Ω
- Modern gyrokinetics via Lie-transform of phase-space Lagrangian
  keep intact the underlying symmetry and conservation laws
  $\rightarrow$ Gyrokinetics in firmer theoretical foundations.
  - T.S.Hahm, Physics of Fluids (1988), …
  - Review : A. Brizard and T.S. Hahm, Rev. of Mod. Phys. (2007)

$$f = f(\vec{x}, \mu, v_{||})$$

- Gyrokinetic ion (+ drift kinetic or adiabatic electron), and **GK Maxwell's** equation

Bounce Kinetics from the 2nd Adiabatic Invariant

* For $\omega << \Omega_\sigma$, and $\rho << L_B$,

"1st " $$\frac{d\mu}{dt} = 0$$ ➡ Gyrokinetics can be derived.

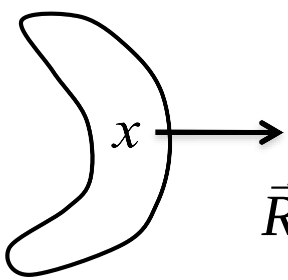
$F_{g.c.}(\vec{R}, \mu, v_{||}, t)$:
- gyro-angle ignorable
- μ : as a parameter

* For $\omega << \omega_{b\sigma} << \Omega_\sigma$, and $\Delta_{\mathrm{Banana}} << L_B$,

The 2nd adiabatic invariant, $J = \oint d\ell v_{||}$ is conserved.

➡ Bounce(-averaged) Kinetic Equation can be derived.

* Evolution of "Banana-Center"

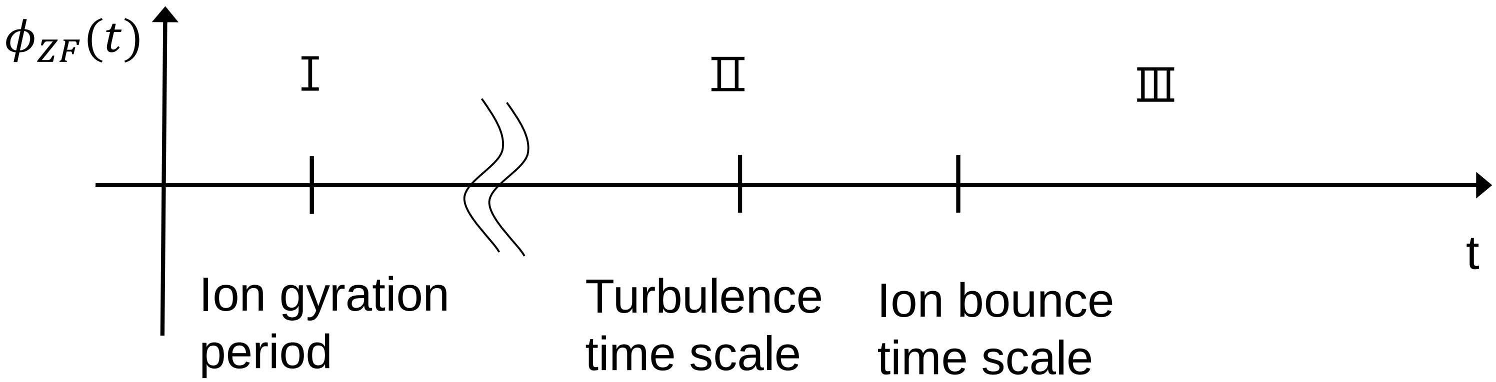


$$F_{B.C.}(\alpha, \beta; J, t)$$

$$\vec{R}_{B.C.} = \vec{R}(\alpha, \beta)$$

- bounce-angle ignorable in addition to gyro-angle
- J as a parameter in addition to μ

Disparate Temporal Scales in Residual Zonal Flow Problem



Ⅰ. Quasi-neutrality : $0 = n_e(\vec{x}) - n_i(\vec{x})$

Ⅱ. Polarization Shielding (Finite Larmor Radius effect) : from Gyrokinetics,

$t \gg \Omega_{ci}^{-1}$
$(\omega \ll \Omega_{ci})$

$$\chi_{cl} \frac{e\Phi_{ZF}(0)}{T_i} = \frac{n_{i,gc}(\vec{x}) - n_e(\vec{x})}{n_0}$$

Ⅲ. Neoclassical enhancement of polarization shielding (Finite Banana Orbit Width effect) :

from Bouncekinetics,

$t \gg \omega_{bi}^{-1}$
$(\omega \ll \omega_{bi})$

$$(\chi_{Neo} + \chi_{cl}) \frac{e\Phi_{ZF}(\infty)}{T_i} = \frac{n_{i,bc}(\vec{x}) - n_e(\vec{x})}{n_0}$$

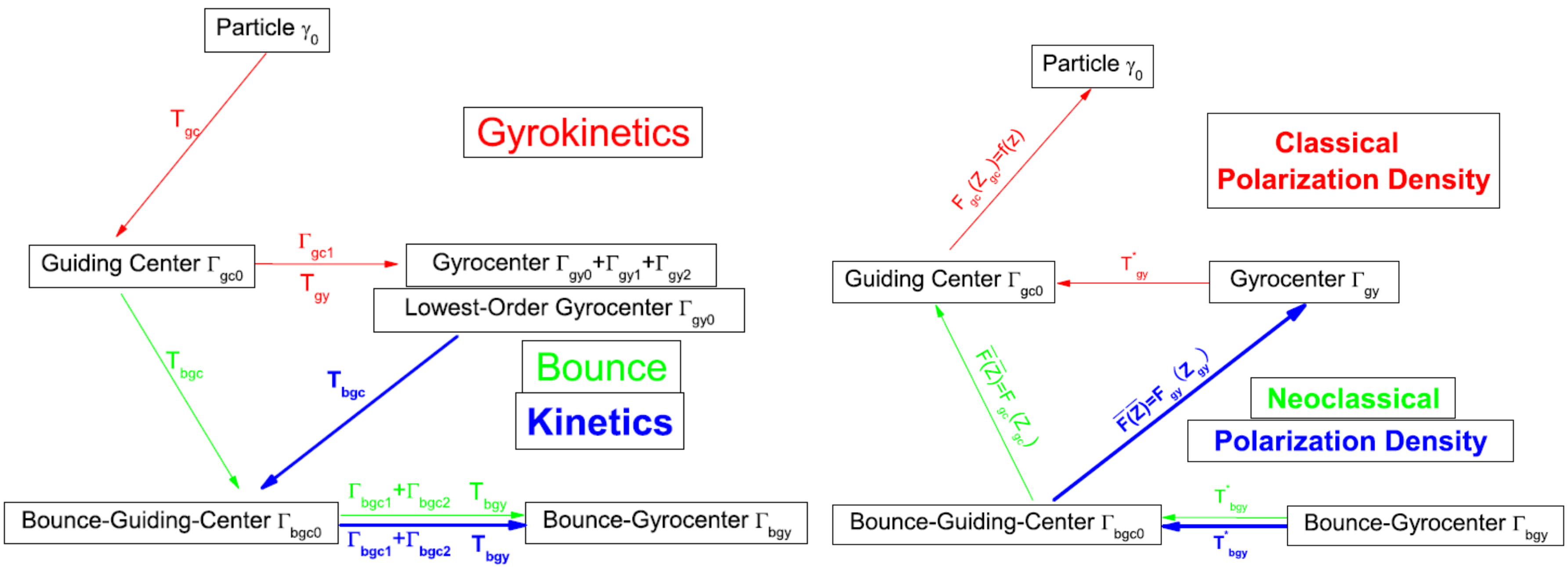
Neoclassical Polarization Density from Bounce-kinetic Approach [Fong & Hahm, PoP (1999)]

- Generalized Polarization Shielding :

$\chi_{total} = \chi_{Neo} + \chi_{cl}$ (Polarizability; Susceptibility)

Residual Zonal Flows determined by the Polarization Shielding

- $$R_{ZF} = \frac{\phi_{ZF}("\infty")}{\phi_{ZF}("0")} = \frac{\chi_{cl}}{\chi_{Neo}+\chi_{cl}}$$
  - Rosenbluth and Hinton PRL (1998)
  - Xiao and Catto PoP (2006)
- Modern GK/BK provides a systematic procedure of its calculation



Push-forward Lie Transform

Pull-back Transform

[L. Wang & T.S. Hahm PoP **16** 062309 (2009)]

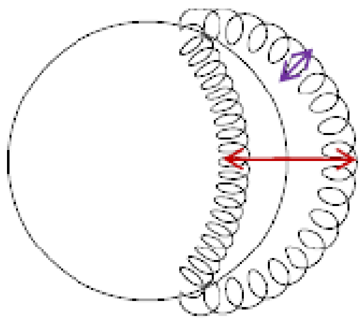
Generalized Polarization Shielding depends on k_r of ZF and Orbit Width.

$$\left\langle \widehat{\delta n} \right\rangle_{\psi} = -\frac{q}{T} \sum_k \delta\phi_k e^{iS(r)} \chi_k^{total}$$

(expressions for Maxwellian F_0)

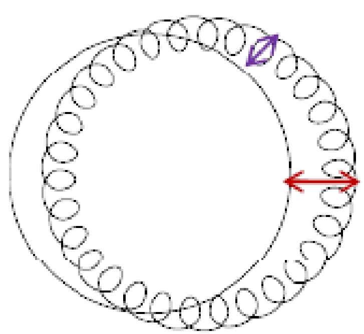
$$\chi_k^{total} = \chi_k^{cl} + \chi_k^{nc}, \qquad \chi_k^{cl} = 1 - \Gamma_0(b)$$

- For trapped particles:



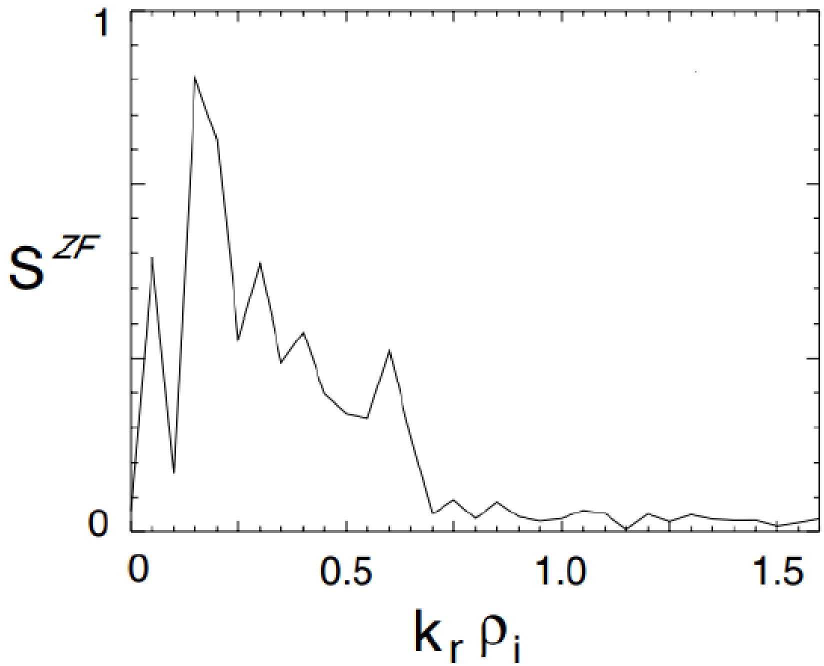
$$\chi_{k,nc}^{tr} \approx \left(\frac{2}{\pi}\right)^{3/2} \sqrt{\epsilon} \int_0^\infty dt^2\, t e^{-t^2} \underbrace{J_0^2(\sqrt{2} k_r \rho_T t)}_{FLR} \int_0^1 d\kappa^2\, K(\kappa) \left[1 - \underbrace{J_0^2(a\kappa)}_{FOW}\right]$$

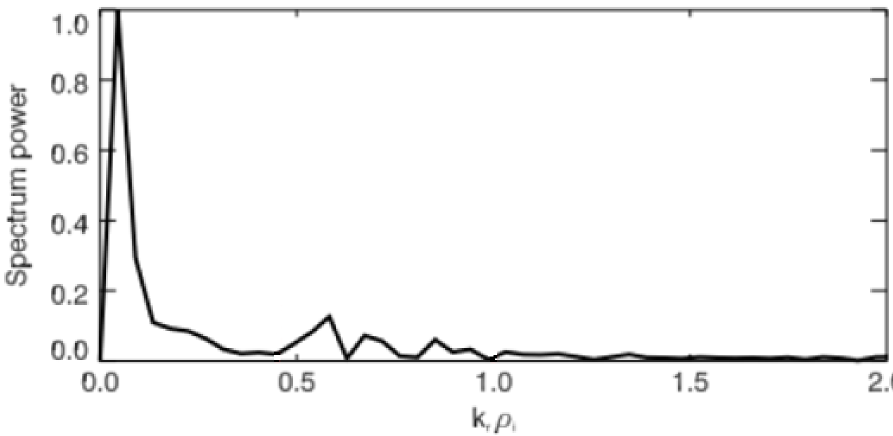
- For passing particles:



$$\chi_{k,nc}^{p} \approx \left(\frac{2}{\pi}\right)^{3/2} \sqrt{\epsilon} \int_0^\infty dt^2\, t e^{-t^2} \underbrace{J_0^2(\sqrt{2} k_r \rho_T t)}_{FLR} \int_1^{\sqrt{\frac{1+\epsilon}{2\epsilon}}} d\kappa\, K(\kappa^{-1}) \sum_{\sigma=\pm} \left(1 - \underbrace{\left| \left\langle e^{i\sigma a \sqrt{\kappa^2 - \sin^2(\hat{\xi}/2)}} \right\rangle_t \right|^2}_{FOW}\right)$$

Zonal Flow Spectra : ITG turbulence vs TEM turbulence





from T.S. Hahm et al.
GTC simulation of **ITG**
PPCF, vol. 42, A205-A210 (2000)

from J.M. Kwon et al.
GKPSP simulation of **TEM**
CPC, vol. 215 81-90 (2017)

- Both cases exhibit broad k_r spectra
- It extends further to higher k_r region for TEM

Further Research Progress on Residual Zonal Flows

- Rosenbluth and Hinton dealt with long wavelength zonal flows only with $k_r \rho_{\theta i} \ll 1$
- Extensions to shorter wavelength regime
  F. Jenko et al., PoP (2000), E.J. Kim, P.H. Diamond et al., PRL (2003)
  Y. Xiao et al., PoP (2006), O. Yamagishi et al., PPCF (2018)
- Stellarators : H. Sugama and T. Watanabe, PRL(2005), PoP (2006)
  P. Monreal et al., PPCF (2016)
- **Modern gyrokinetic/bouncekinetic approach for all wavelength regime**
  - L. Wang and T.S. Hahm, PoP (2009)
- **More accurate procedure outlined in**
  - F.X. Duthoit, A. Brizard and T.S. Hahm PoP (2014)
- Applications to
  - **Isotopic dependence of confinement**
  T.S. Hahm et al, NF (2013)
  - Impurity Effects
  W.X. Guo, L. Wang et al., NF(2017)
  - Effects of RMP on H-mode transition
  G.J. Choi and T.S. Hahm, NF(2018)

All considered Maxwellian F_0.

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α-particle Effects on Confinement of Burning Plasmas

- α-particle effects on Alfvenic energetic particle modes :
  L. Chen and F. Zonca, RMP (2016), Y. Todo, RMPP (2018)

- α-particle transport due to micro turbulence
  W. Zhang, Z. Lin and L. Chen et al, PRL (2008)
  C. Angioni and A. Peeters, PoP (2008)
  S. Yang, C. Angioni, T.S. Hahm et al., PoP (2018)

- α-particle effects on mean plasma rotation :
  (eg., α-particle's large orbit loss) found to be insignificant for ITER:
  M. N. Rosenbluth and F.L. Hinton, NF (1996)

⍟ No previous works on α-particle effects on Residual Zonal flows
(cf. preliminary attempt : K.P. Lee and T.S. Hahm, Proceeding of M&C 2017)

General Formulas of Polarizabilities for arbitrary F(Z)

Duthoit, Brizard and Hahm, PoP (2014)

- Polarization Density;

$$n_{cl} = \frac{Z|e|\delta\phi}{m} \int_0^\infty \int_{-\infty}^\infty 2\pi dv_\parallel d\mu \left(1 - \underbrace{J_0^2(k_r\rho_i)}_{\text{FLR effect}}\right)\left(-\frac{\partial}{\partial\mu}\right) F(Z_{gy})$$

- Neoclassical Polarization Density;

$$[n_{Neo}]_\psi = Z|e|\delta\phi \frac{B}{m} \int_{\kappa_i}^{\kappa_f} \int_0^\infty 4\pi R_\parallel \omega_\parallel d\mu d\kappa \frac{\omega_\parallel}{\omega_{b,t}}$$

$$\times \underbrace{J_0^2(k_r\rho_i)}_{\text{FLR effect}} \left(1 - \underbrace{\left|\left\langle e^{i\Delta\zeta}\right\rangle\right|_{b,t}^2}_{\text{FOW effect}}\right)\left(-\frac{1}{\omega_{b,t}}\frac{\partial}{\partial J}\right) F(Z_{bgy})$$

for arbitrary F(Z)

ρ_i : particle's Larmor radius

κ : pitch angle parameter

$\kappa \in [0,1)$ for trapped particles,

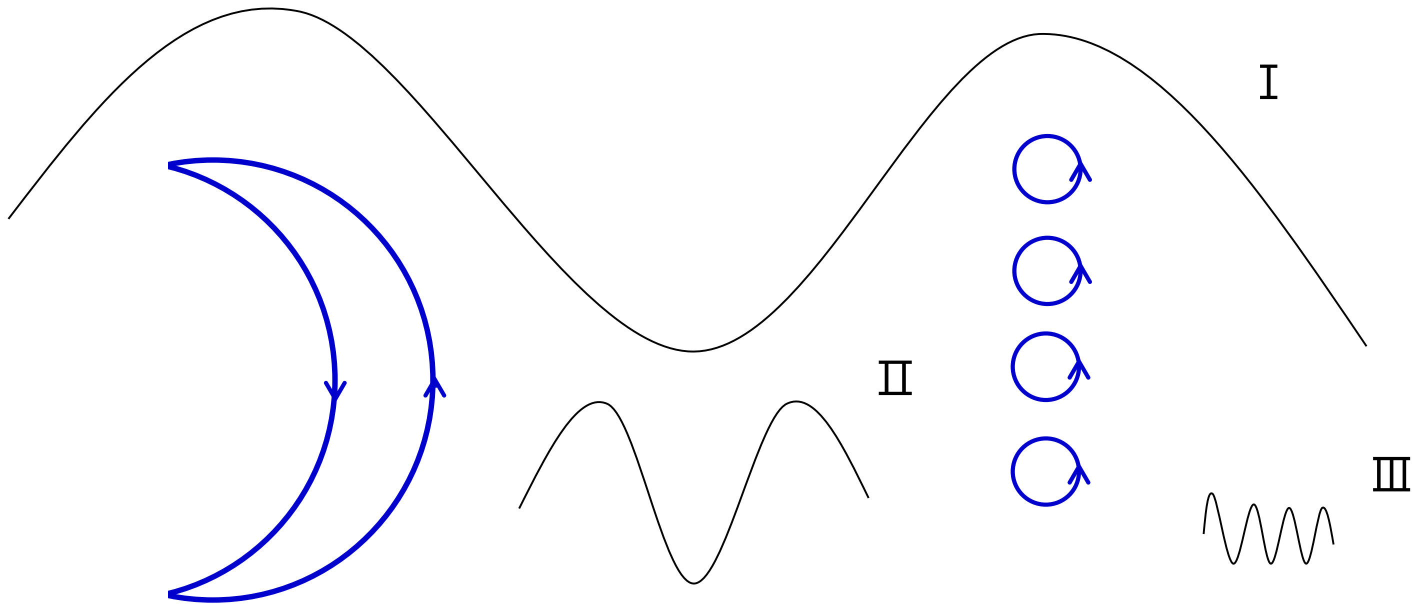
$\in \left[1, \frac{1+\epsilon}{2\epsilon}\right]$ for passing particles

$R_\parallel = qR$: connection length

$\omega_\parallel = \frac{p_{\parallel e}}{2\sqrt{\kappa} m R_\parallel}$: characteristic parallel frequency

ω_b : bounce frequency ω_t : transit frequency

Asymptotic Analyses for Different Wavelength Regimes



Integrations are possible in terms of various special functions for

Ⅰ. Long wavelength regime : $k_r \rho_{\theta i} \ll 1$

Ⅱ. Intermediate wavelength regime : $k_r \rho_i \lesssim 1 \lesssim k_r \rho_{\theta i}$

Ⅲ. Short wavelength regime : $1 \ll k_r \rho_i$

Classical Polarization in the long wavelength limit is independent of F(Z)

- $\frac{n_{cl}}{n_0} = \chi_{cl} \frac{Z|e|\delta\phi}{T}$

- $n_{cl}^{long} \simeq \frac{Z|e|\delta\phi}{T} \int_0^\infty \int_{-\infty}^\infty 2\pi dv_\parallel \frac{B d\mu}{m} \left(\frac{1}{2} k_r^2 \rho_i^2\right) \left(-\frac{T}{B}\frac{\partial}{\partial\mu}\right) F(Z)$

$$= \left(k_r \rho_i^T\right)^2 n_0 \frac{Z|e|\delta\phi}{T}$$

→ Familiar expression for polarization density in gyrokinetic-Possion equation

- $\chi_{cl} = \left(k_r \rho_i^T\right)^2, \rho_i^T$: Larmor radius at thermal velocity

Neo-Polarization in the long wavelength limit is identical for any isotropic F(Z)

- $$\left[n_{Neo,b}\right]_{\psi} = Z|e|\delta\phi \frac{T^{1/2}}{m^{3/2}} \int_0^{\infty} \int_0^{1} 8\sqrt{\epsilon y}\, d\kappa\, dy\, \alpha^2 \big(E(\kappa) - (1-\kappa)K(\kappa)\big) \left(-\frac{\partial}{\partial y}\right) F(y)$$

  $$\simeq 1.20\epsilon^{3/2}\left(k_r \rho_{\theta i}^{T}\right)^2 n_0 \frac{Z|e|\delta\phi}{T}$$

- $$\left[n_{Neo,t}\right]_{\psi} = Z|e|\delta\phi \frac{T^{1/2}}{m^{3/2}} \int_0^{\infty} \int_1^{(1+\epsilon)/2\epsilon} 8\sqrt{\epsilon y}\, d\kappa\, dy\, \alpha^2 \sqrt{\kappa} \left(E(\kappa^{-1}) - \frac{\pi^2}{4K(\kappa^{-1})}\right) \left(-\frac{\partial}{\partial y}\right) F(y)$$

  $$\simeq 0.43\epsilon^{3/2}\left(k_r \rho_{\theta i}^{T}\right)^2 n_0 \frac{Z|e|\delta\phi}{T}$$

  Here, $y = \frac{E}{T} \approx \frac{\mu B_0}{T}$, $\alpha = \sqrt{2\epsilon}\, k_r \rho_{\theta i}$

- $$\frac{n_{Neo}}{n_0} = \frac{n_{Neo,b} + n_{Neo,t}}{n_0} = 1.63\epsilon^{3/2}\left(k_r \rho_{\theta i}^{T}\right)^2 n_0 \frac{Z|e|\delta\phi}{T}$$ (for any isotropic distribution!)

$\therefore$ Residual Zonal Flow;

$$R_{ZF} = \frac{V_{E\times B}(t\to\infty)}{V_{E\times B}(t\to 0)} = \frac{n_{cl}}{n_{cl}+n_{Neo}} = \frac{\chi_{cl}}{\chi_{cl}+\chi_{Neo}} \simeq \left(1 + 1.63\frac{q^2}{\sqrt{\epsilon}}\right)^{-1},$$ the same as RH 98

which has been derived using Maxwellian F_0.

Residual Zonal Flows in the Presence of α-Particles

- We consider

  1) Slowing Down distribution of α-particles born at 3.5 MeV and background electrons with $T_e \sim 10keV$

  2) Equivalent Maxwellian distribution of α-particles with the same average kinetic energy

  3) Reference case without α-particles ($T_i = T_e \sim 10keV$)

  4) Finally,10% α-particle concentration with background consisting of equal amount of D and T ($T_i = T_e \sim 10keV$)

Y.W. Cho and T.S. Hahm, Nuclear Fusion **59** 066026 (2019).

Highlights of Results with Practical Interests

- α-particles enhance residual zonal flows, effect is maximum at $k_r \rho_{i,eff} \sim 10^{-1}$

  (for D 50%, T 50% mixture, $\rho_{i,eff} = \sum_a c m_a v_{Ta} / Z_a |e| B$).
- For 10% concentration, $\sim 10\%$ enhancement at $k_r \rho_{i,eff} \sim 10^{-1}$ is expected.
- Effects can be considerable for ITER, and significant for DEMO and reactors.
- $R_{ZF}(k_r \rho_{i,eff}) = \frac{\chi_{cl}}{\chi_{Neo} + \chi_{cl}}$

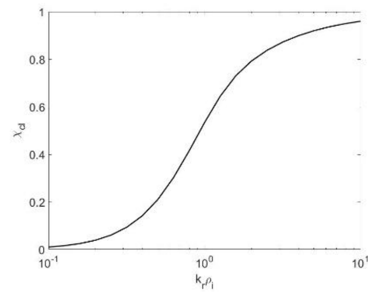
  i) χ_{cl} is a monotonically increasing in k_r, ($\sim tanh$-like shape)

  Transition occurs at lower k_r in the presence of energetic α's

  $(k_r \rho_{i,eff} \sim 10^{-1}, k_r \overline{\rho_\alpha} \sim 1)$

  ii) χ_{nc} peaks at similar k_r value and decreases as a function of k_r for higher k_r.

  i, ii) $\Rightarrow$ R_{ZF} is enhanced for $k_r \rho_{i,eff} \sim 10^{-1}$



Slowing Down Distribution Function

- $F_{SD}(v) = \frac{n_\alpha}{4\pi v_c^3 A_2} \frac{H(v_\alpha - v)}{1 + (v/v_c)^3}$

- $\frac{1}{2} m_\alpha v_\alpha^2 = 3.5MeV$

  $v_c^3 = 3\sqrt{\frac{\pi}{2}} \frac{m_e}{m_\alpha} Z_{eff} v_{th,e}^3$: slowing down critical velocity

- $n_\alpha \overline{E_{SD}} = \frac{1}{2} \int m_\alpha \mathrm{v}^2 F_{SD}(v) d^3 v \equiv \frac{3}{2} n_\alpha T_{SD} = \frac{A_4}{2A_2} n_\alpha T_c$

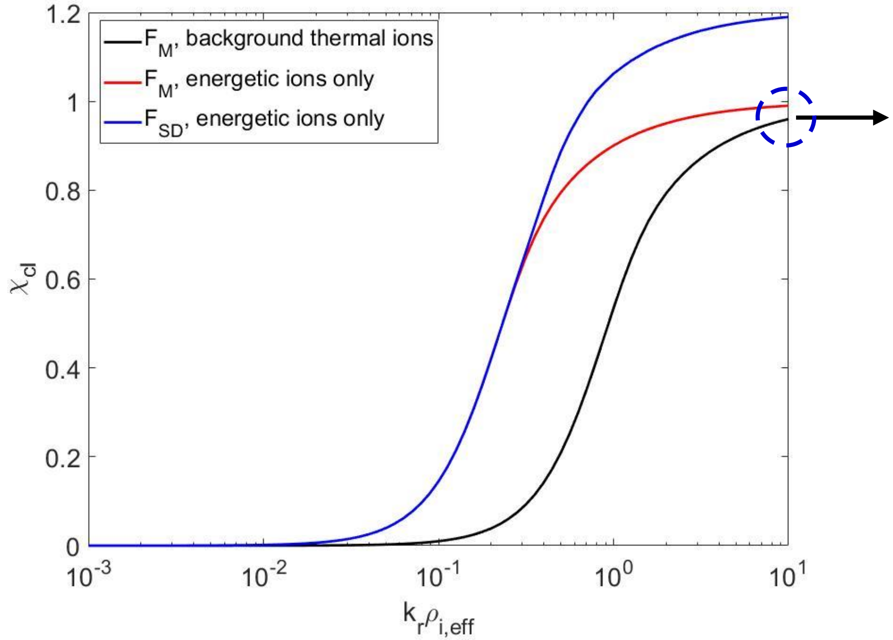
- $A_n\left(\frac{v_\alpha}{v_c}\right) = \int_0^{v_\alpha/v_c} \frac{x^n}{1+x^3} dx$: appear in various expressions.

  $T_c = m_\alpha v_c^2$

Classical Polarization in the short wavelength limit

- $\frac{n_{cl}^{short}}{n_0} \simeq \frac{Z|e|\delta\phi}{T}\frac{A_4}{3A_2}\left[\frac{A_0}{A_2} - \frac{1}{2A_2 k_r \rho_c}\right]$
- ρ_c : Larmor radius with critical velocity v_c
- Asymptotic value > 1, for $T_e \lesssim 16keV$, < 1 for $T_e \gtrsim 16keV$

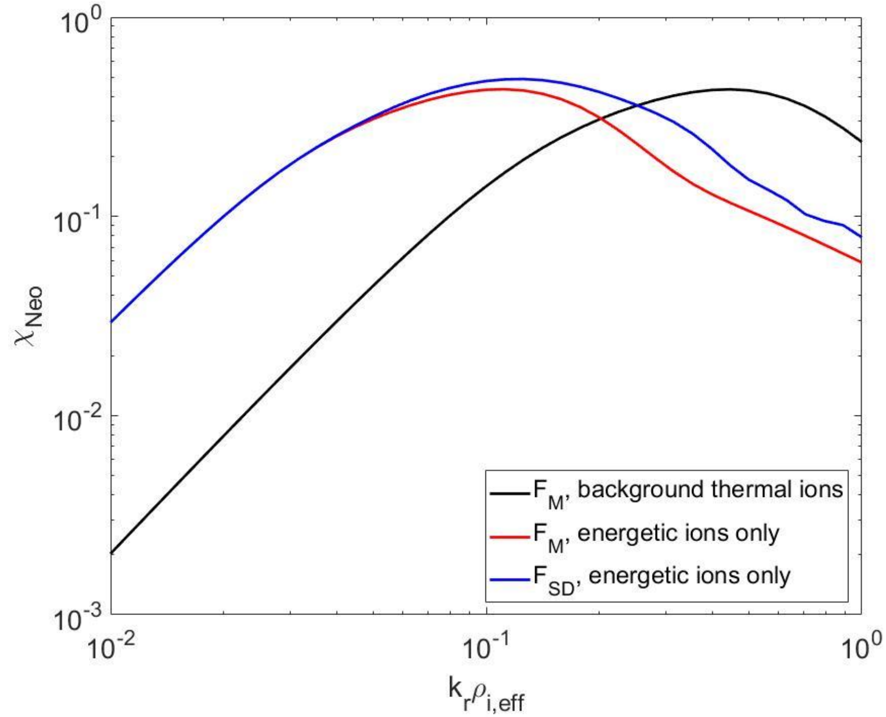
  and, $= 1$ for Boltzmann (adiabatic) response for Maxwellian F_0 ($\propto 1 - \Gamma_0(b_i)$)
- Asymptotic behaviors are similar for cases considered, but transition occurs at different k_r value.



"$1 - \Gamma_0(k_r^2 \rho_i^2)$ "
for Maxwellian

for $T_e = 10keV$

χ_{Neo} peaks when Finite Orbit Width enhancement and FLR-reduction balance



$q = 2.0$
$\epsilon = 0.1$
$T_e = 10keV$

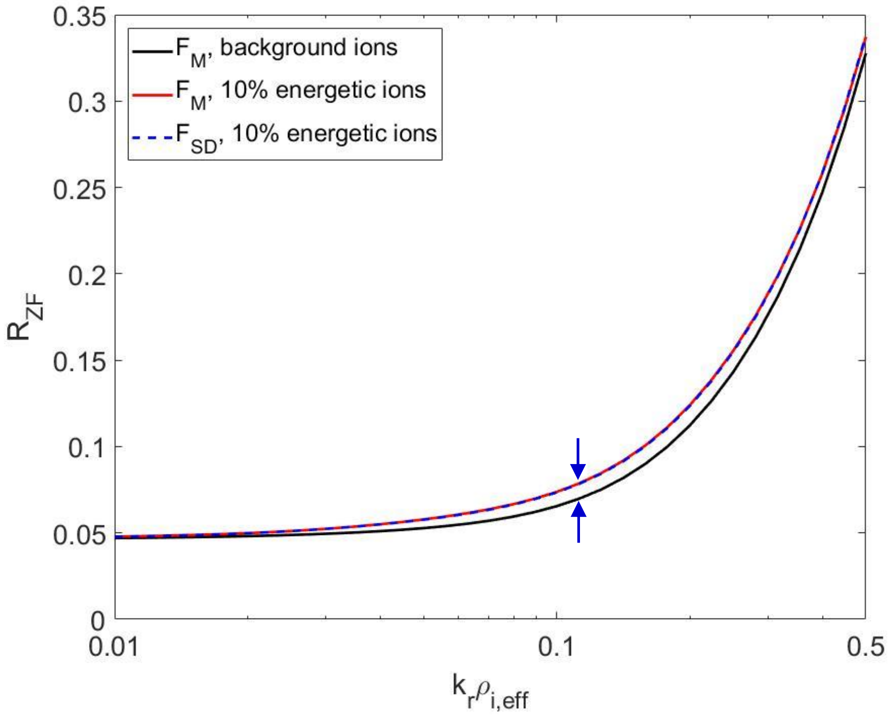
- $n_{Neo,b} = Z|e|\delta\phi \frac{\sqrt{\epsilon T}}{m^{3/2}} \int_0^\infty \int_0^1 2\sqrt{2}\pi d\kappa dx \frac{\omega_\parallel}{\omega_b} x J_0^2(k_r\rho_i^T x)\{1 - J_0^2(\alpha a_1(\kappa))\}\left(-\frac{\partial}{\partial x}\right) F(x)$
- $n_{Neo,t} = Z|e|\delta\phi \frac{\sqrt{\epsilon T}}{m^{3/2}} \int_0^\infty \int_1^{(1+\epsilon)/2\epsilon} 2\sqrt{2}\pi d\kappa dx \frac{\omega_\parallel}{\omega_t} x J_0^2(k_r\rho_i^T x)$ (in green) : decreasing in k_r

$$\times \{1 - J_0^2(\alpha b_2(\kappa))\}\left(-\frac{\partial}{\partial x}\right) F(x)$$

(in red) : increasing in k_r

where $a_1(\kappa) = 2\frac{\omega_b}{\omega_\parallel} \mathrm{sech}\left[0.5\pi \frac{K(1-\kappa)}{K(\kappa)}\right]$ $\quad b_2(\kappa) = \frac{\omega_t}{\omega_\parallel} \mathrm{sech}\left[\pi \frac{K(1-\kappa^{-1})}{K(\kappa^{-1})}\right]$

α-Particle Effects can be considerable in ITER



$q = 2.0$
$\epsilon = 0.1$
$T_e = 10keV$

- 10% enhancement at $k_r\rho_{i,eff} \sim 10^{-1}$.

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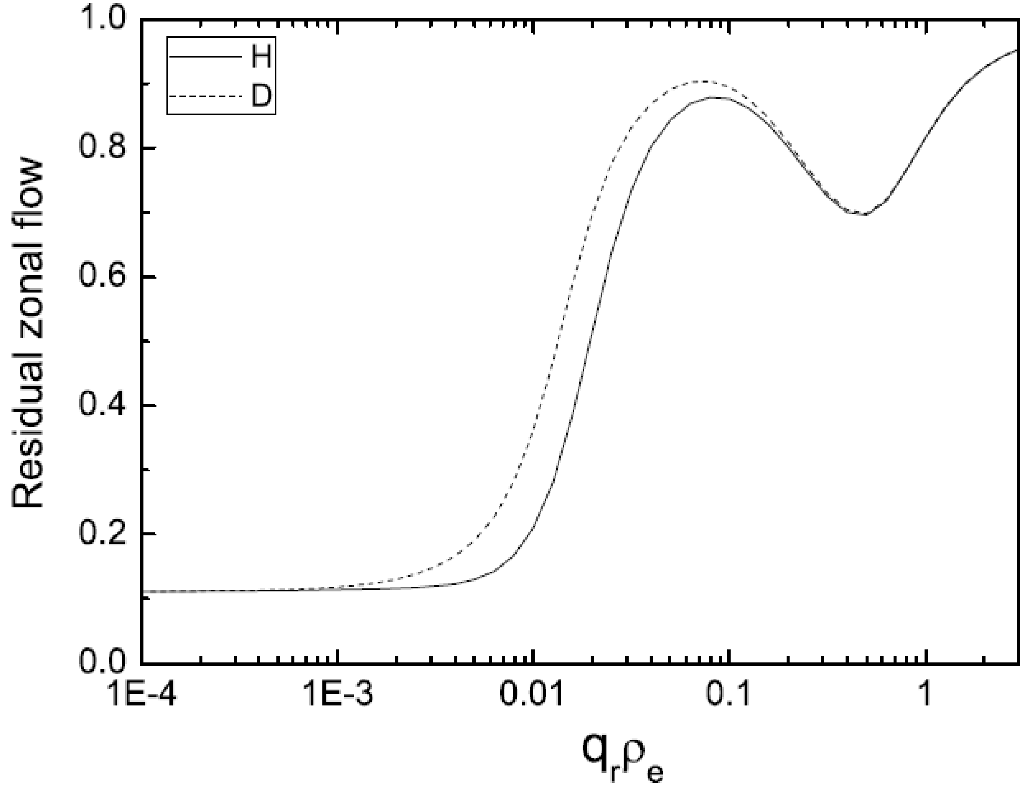
Isotopic Dependence of Fine-scale Residual Zonal Flow

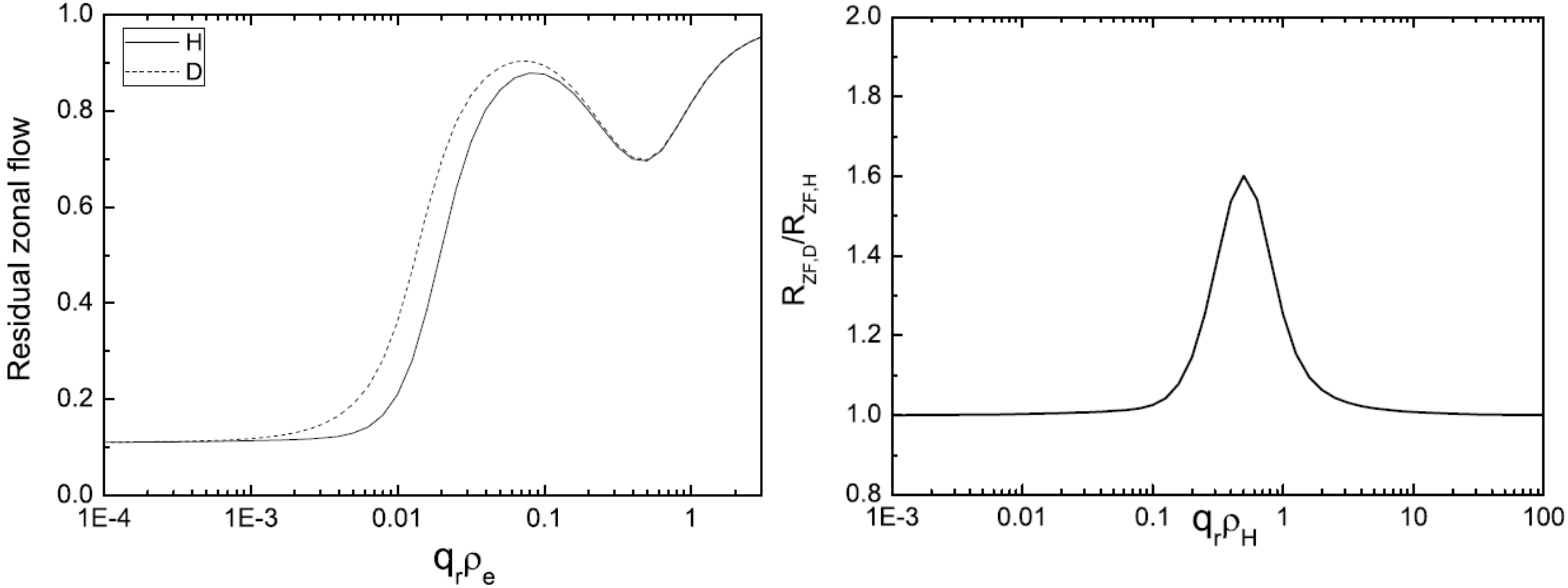
[T.S. Hahm, L. Wang, W.X. Wang et al., NF 53, 072002 (2013)]



- For the same temperature, fast increasing of residual Zonal Flow for D plasmas starts at lower q_r than that of H plasma

Isotopic Dependence is appreciable at $q_r \rho_H \sim 0.5$

[T.S. Hahm, L. Wang, W.X. Wang et al., NF 53, 072002 (2013)]



- Residual Zonal Flow for D is higher than that for H for $q_r \rho_H$ around 0.5

  ---> This mechanism works better for
  moderately short wavelength drift wave turbulence (eg. TEM)

Trapped Electron Mode (TEM)

- Nonlinear theory [Similon-Diamond, PF ‘84]
  ---> Neo-Alcator scaling: $\tau_E \propto n_e a R^2$

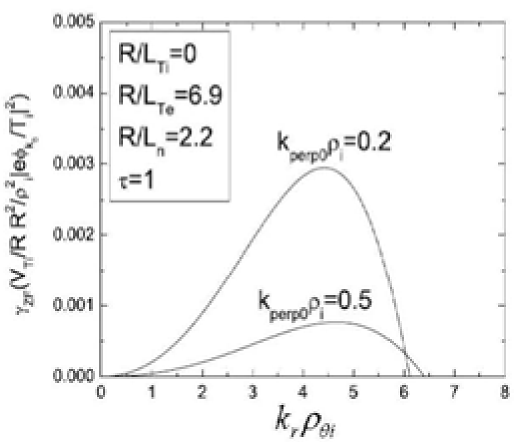
  Derived with no hint from gyrokinetic simulations, or movies

- But, C-Mod validation using TEM-ITG was initially unsatisfactory for electron thermal transport [Lin et al., Phys. Plasmas ‘09]

- Consideration of impurities improved the comparison, [M. Porkolab, private communications ‘12].

- Fine scale($k_r \rho_i \sim 0.5$) ZF can be strong

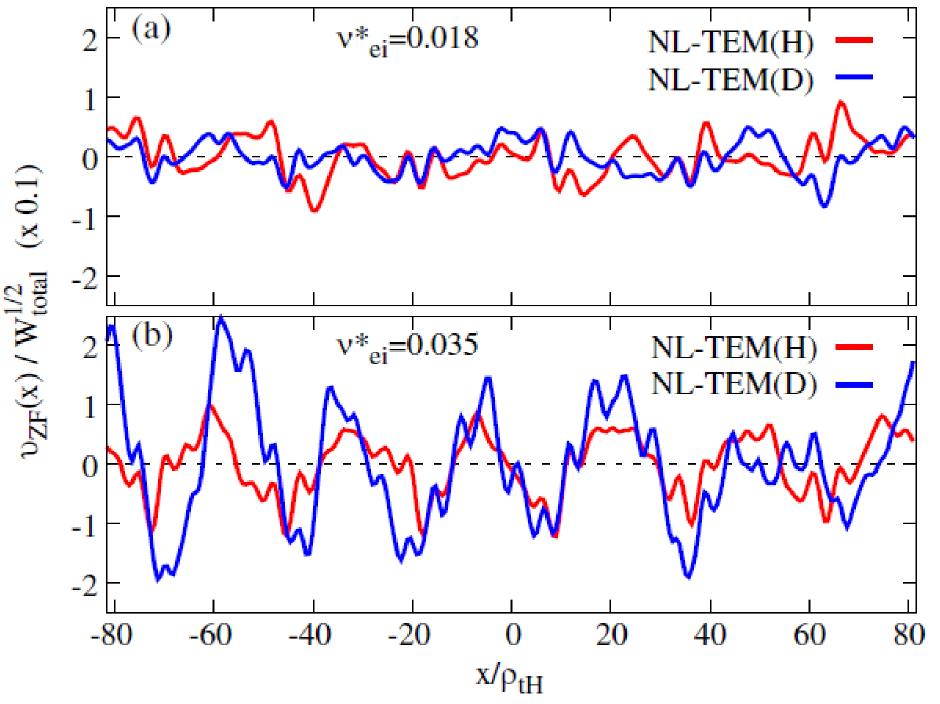
  [Lu Wang and Hahm, PoP ’09;
  Xiao et al., PRL ‘09;
  W.X. Wang et al., PoP ‘10].



[Lu Wang and Hahm, PoP ‘09]

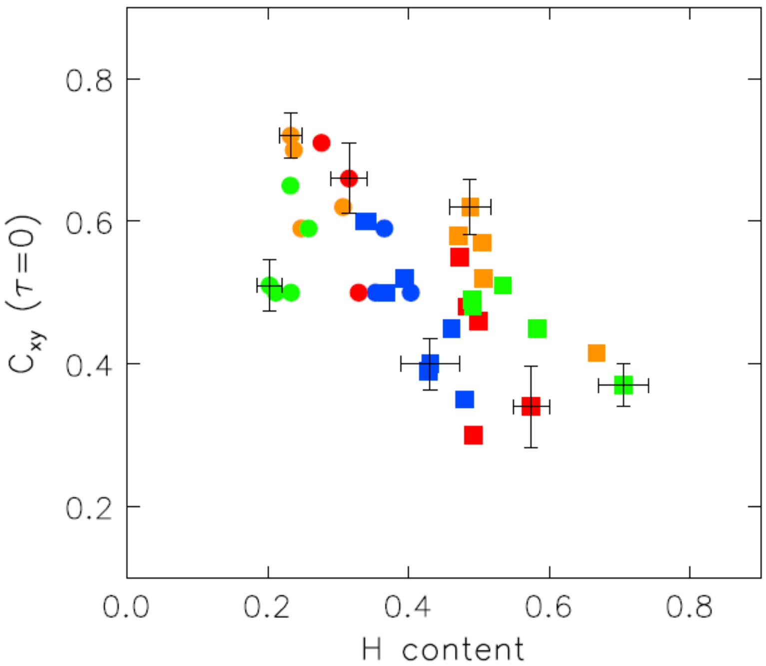
Isotopic Dependence of Zonal Flow from Gyrokinetic Simulations

- “Nonlinear Gyrokinetic Simulations exhibit isotopic dependence”
  [J.Garcia et al., Nucl. Fusion **57**, 014007 (2017)]



- Zonal Flow structures
  from [M. Nakata et al., Phys. Rev. Lett. **118** 165002 (2017)]

Experimental Evidence for Isotopic Dependence of ZF?



Y. Xu, C. Hidalgo, I. Shesterikov et al., [Phys. Rev. Lett. 110, 265005 (2013).]

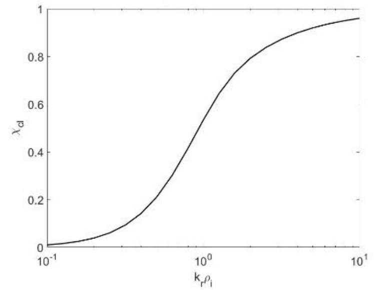
Long range correlation in toroidal direction decreases as H/D content increases

from TEXTOR

Also, B. Liu et al., [Nucl. Fusion, **55** 112002 (2015)] from TJ-II.

Conclusions I.

- α-particles enhance residual zonal flows with $k_r \rho_{i,eff} \sim 10^{-1}$.
- For 10% concentration, $\sim 10\%$ enhancement at $k_r \rho_{i,eff} \sim 10^{-1}$ is expected.
- So effects can be considerable for ITER, and significant for DEMO and reactors.



- $R_{ZF}\left(k_r \rho_{i,eff}\right) = \frac{\chi_{cl}}{\chi_{Neo}+\chi_{cl}}$

  i) χ_{cl} is a monotonically increasing in k_r, ($\sim tanh$-like shape)

  Transition occurs at lower k_r in the presence of energetic α's

  $(k_r \rho_{i,eff} \sim 10^{-1}, k_r \overline{\rho_\alpha} \sim 1)$

  ii) χ_{nc} peaks at similar k_r value and decreases as a function of k_r for higher k_r.

  i, ii) $\Rightarrow$ R_{ZF} is enhanced for $k_r \rho_{i,eff} \sim 10^{-1}$

Conclusions II.

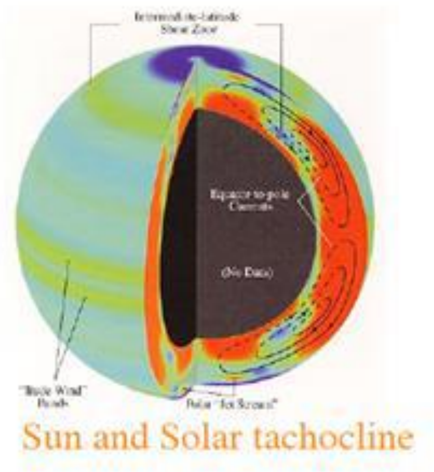
- Underlying physical mechanism behind the isotopic dependence of confinement is not known yet.
- We studied isotopic dependence of fine scale residual Zonal Flows by using our generalized polarization shielding formula.
- We found stronger residual ZF for D plasma than that for H plasma for $q_r \rho_H$ around 0.5
- This can possibly lead to lower turbulence and transport and better confinement of D plasmas than H plasmas, in qualitative agreement with experimental results.

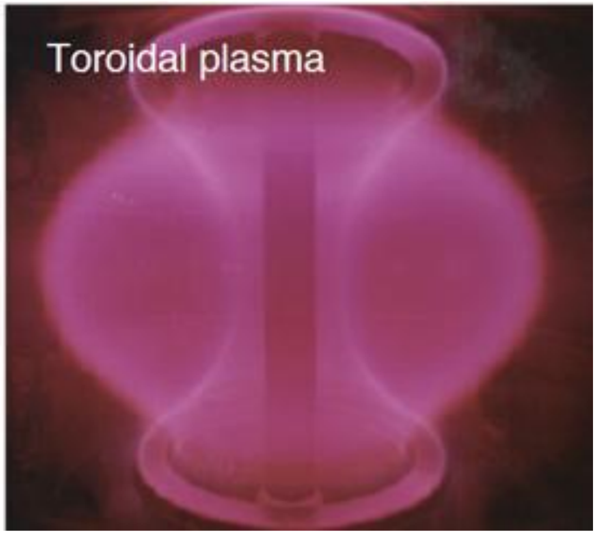
  [T.S. Hahm et al., NF 53, 072002 (2013)]
- Fine-resolution nonlinear gyrokinetic simulations and fluctuation measurements from experiments addressing this mechanism are on-going.

Self-Organized Structures in Nature

Courtesy : K. Itoh

Zonal Flow, an example of meso-scale structure, which is far from the system size, is self-sustained





- New Review Paper on Meso-scale Physics :
  T.S. Hahm and P.H. Diamond, J. Korean Phys. Soc. **73**, 747 (2018)