Stability of dangerous low frequency modes

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Presented at the IAEA Technical Meeting on Energetic Particles, Shizuoka, Japan
September 2, 2019
Fast-ion driven low frequency modes are dangerous

- Modes with frequencies below RSAEs are often unstable on existing devices
- Empirically, they are correlated with substantial fast-ion transport
- Theoretically, this is expected since $n\Delta E = \omega\Delta P \phi$ implies that, if a low-frequency mode effectively resonates with a fast ion, the spatial transport is large.

**Goal**
Understand the stability of beta-induced Alfvén eigenmodes (BAE) and beta-induced Alfvén-acoustic eigenmodes (BAAE)
BAEs were discovered on DIII-D

- $f_{BAE} \approx 0.5f_{TAE}$
- Large fast-ion losses were measured
- MHD theory (GATO) calculated an eigenmode in the BAE gap

Heidbrink, PRL 71 (1993) 855
The correct theoretical identification of BAEs was controversial for many years.

What is the “beta-induced Alfvén eigenmode?”

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(Received 5 March 1998; accepted 14 December 1998)

An instability with a lower frequency than the toroidicity-induced Alfvén eigenmode was initially identified as a beta-induced Alfvén eigenmode (BAE). Instabilities with the characteristic spectral features of this “BAE” are observed in a wide variety of tokamak plasmas, including plasmas with negative magnetic shear. These modes are destabilized by circulating beam ions and they transport circulating beam ions from the plasma core. The frequency scalings of these “BAEs” are compared to theoretical predictions for Alfvén modes, kinetic ballooning modes, ion thermal velocity modes, and energetic particle modes. None of these simple theories match the data. © 1999 American Institute of Physics. [S1070-664X(99)00204-9]

- Observed on ASDEX-Upgrade, EAST, FTU, H-1NF, HL-2A, J-TEXT, KSTAR, Tore Supra
- Driven by fast ions, electrons & MHD
BAEs occur near an accumulation point of the ideal MHD spectrum

- $f_{BAE} \approx f_{GAM}$
- Alfvénic polarization

X. Wang, PPCF 52 (2010) 115005

Classen, PPCF 53 (2011) 124018
BAEs occur near an accumulation point of the ideal MHD spectrum

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Classen, PPCF 53 (2011) 124018

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Note: Some experimental “BAE” may be Energetic Particle Modes [PoP 6 (1999) 1147]
BAAEs were first identified on JET & NSTX

- Even lower frequency than BAE
- MHD code (NOVA) finds coupling between acoustic & Alfvénic branches
- Mixed polarization

We claimed BAAEs cause substantial fast-ion transport in DIII-D.

- BAAEs appear in “Christmas lights” pattern
- Reductions in neutron rate & FIDA signal when only BAAEs were detected.

Gorelenkov PoP 16 (2009) 056107
The theory of BAAE remains unsettled

- Zonca & Chen predict “that EPs preferentially excite the BAE over the BAAE branch due to the stronger wave-EP interaction.” [PoP 24 (2017) 072511]
- Large ion Landau damping often predicted unless $T_e \gg T_i$
- Non-perturbative simulations* do find both BAEs and BAAEs

*e.g., Yaqi Liu, NF 57 (2017) 114001
Outline

- **Motivation & Background:** Modes are ubiquitous & dangerous but poorly understood
- **2019 Experiment:** "What drives BAEs? BAAEs?"
- Stability trends in > 1000 shots
- Preliminary modeling results
Outline

- Motivation & Background
- 2019 Experiment: "What drives BAEs? BAAEs?"
- Stability trends in > 1000 shots
- Preliminary modeling results
Inject 80 keV D$^0$ beams into reproducible L-mode plasma

- Interferometers, electron cyclotron emission (ECE), beam emission spectroscopy (BES), and magnetics measure the modes
The current ramp phase of the discharge scans $q_{\text{min}}$
RSAEs, BAEs, and BAAEs are all unstable during the selected phase of the discharge.
The modes are located near $q_{\text{min}}$
The modes are located near $q_{\text{min}}$

- BES $\delta n_e$ profile also peaks near $q_{\text{min}}$
The BAAEs do not show up on magnetics

- Consistent with greater acoustic polarization
- BAE frequency varies ~ 5 kHz in time
- Typical BAE toroidal mode number is n=2,3
Both BAEs and BAAEs appear transiently → discrete values of $q_{\text{min}}$

- $q_{\text{min}}$ evolution reproducible shot-to-shot
- BAEs appear at nearly the same time & frequency on every shot
- Similar behavior for BAAEs
Probe driving gradients at a fixed time in the discharge (Reference shot)

- Points are modes found by peak-finding program
- Lines separate BAAE/BAE/ RSAE frequency bands
Substitution of perpendicular beams for tangential beams stabilizes BAEs

- BAAEs unaffected
- Slowing-down time: $\sim 100$ ms
Substitution of 3 perpendicular beams for 2 tangential beams destabilizes higher frequency BAEs

- BAEs disappear from original frequency band
- New band of BAEs appears at higher frequency
Distribution function becomes more perpendicular

- Spatially averaged “classical” distribution at 1200 ms (calculated by NUBEAM)
Beam-angle dependence of BAEs consistent with resonant condition

- 87 kHz $n=2$ mode resonates with many injected parallel beam ions

Resonance when:
1) $\omega = n\omega_\phi - p\omega_\theta$
2) Orbit traverses eigenfunction

Deposition of tangential beam ions
Beam-angle dependence of BAEs consistent with resonant condition

- 87 kHz n=2 mode resonates with few ⊥ beam ions
Beam-angle dependence of BAEs consistent with resonant condition

- Mode frequency increases to resonate with $\perp$ beam ions
Surprise! High-energy beam ions do **not** drive BAAEs

- BAAEs persist during beam notch
- Frequency drops as rotation decreases
- RSAEs & BAEs are suppressed
BAAEs are unstable at high $T_e$ but modest $\beta_p$

- Database from all 20 shots of the 2019 experiment
- Dashed lines show trajectories for 2 cases
- No correlation with fast-ion parameters
BAEs are driven by fast ions but BAAEs are not.

Correlation of mode power with classical beam beta

<table>
<thead>
<tr>
<th></th>
<th>Par. $\beta_p$</th>
<th>Perp. $\beta_p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>BAAE</td>
<td>-0.14</td>
<td>-0.06</td>
</tr>
<tr>
<td>BAE</td>
<td>0.32</td>
<td>0.25</td>
</tr>
<tr>
<td>RSAE</td>
<td>0.04</td>
<td>0.22</td>
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</table>
Motivation & Background

2019 Experiment: (a) BAEs driven by tangential high-energy ions (b) BAAEs occur when $T_e$ is high but $\beta_p$ is modest

Stability trends in $> 1000$ shots

Preliminary modeling results
Categorize modes at selected times during the current ramp in 1112 discharges

- Survey ECE, BES, interferometer, & magnetics data
- Process overlooks some unstable modes

**Conditions span**

\[ I_p \leq 1.6 \text{ MA} \]
\[ 0.5 \leq B_T \leq 2.1 \text{ T} \]
\[ 0.1 \leq \beta_N \leq 3.2 \]
\[ 1.1 \leq \kappa \leq 2.2 \]
\[ -0.4 \leq \delta \leq 1.0 \]
\[ 0.4e19 \leq \overline{n_e} \leq 5.0e19 \text{ m}^{-3} \]
\[ T_e(0) \leq 7.6 \text{ keV} \]
\[ T_i(0) \leq 11.4 \text{ keV} \]
BAEs are more likely at high beta

- In same discharge, appear at certain times (=q) but not others

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta_p )</td>
<td>0.34</td>
</tr>
</tbody>
</table>

Little correlation with:
- Shape
- Density
- Temperature

#178631
BAAEs occur when electron temperature is large but total pressure is modest

- “Christmas light” BAAE pattern
- Less ion Landau damping at low ion beta?

<table>
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<tr>
<th>Parameter</th>
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</thead>
<tbody>
<tr>
<td>$T_e$</td>
<td>0.35</td>
</tr>
<tr>
<td>$T_e/T_i$</td>
<td>0.08</td>
</tr>
</tbody>
</table>

Little correlation with
- Shape
- Beams
- Density
Summary: Theory should explain these features

**BAE Stability**
- Strong q-profile dependence
- More unstable at higher beta
- Beam-angle dependence

**BAAE Stability**
- Strong q-profile dependence
- More unstable at higher $T_e$
- More unstable at lower beta
- Weak beam dependence
Outline

• Motivation & Background
• 2019 Experiment: (a) BAEs driven by tangential high-energy ions (b) BAAEs occur when $T_e$ is high but $\beta_p$ is modest
• Large database confirms trends found in dedicated experiment
• Preliminary modeling results
Three complementary codes* are analyzing the 2019 experiment

<table>
<thead>
<tr>
<th>Code</th>
<th>Description</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>FAR3D</td>
<td>Gyrofluid</td>
<td>Maxwellian fast ions</td>
</tr>
<tr>
<td>LIGKA</td>
<td>Gyrokinetic linear global</td>
<td>Maxwellian fast ions (so far)</td>
</tr>
<tr>
<td>GTC</td>
<td>Gyrokinetic linear (so far)</td>
<td>Maxwellian fast ions (so far)</td>
</tr>
</tbody>
</table>

*Additional collaborators welcome
GTC finds an unstable $n=3$ BAE with properties beginning to resemble experiment.

- Lower $q_{\text{min}}$ is closer to experiment.
- "Classical" fast-ion profile is closer than one from equilibrium fitting.

**ALCON:**
ideal MHD eigenvalue code

**Thick:** Alfvénic continua
**Thin:** acoustic continua
FAR3D finds unstable modes with frequencies similar to experimental BAAEs

- Scan $q_{\min}$
- Map $q_{\min} \rightarrow$ time & overlay unstable modes
- $n=2$ (white)
- $n=3$ (black)

Scans of $q_{\min}$ & $T_e$ to understand stability are planned
Conclusions

**BAE Stability**
- Strong q-profile dependence (low-order rational qmin crossings)
- More unstable at higher beta (larger BAE gap & suppressed RSAE/TAE activity)
- Beam-angle dependence (resonance condition)

**BAAE Stability**
- Strong q-profile dependence (low-order rational qmin crossings)
- More unstable at higher $T_e$ (reduced ion Landau damping)
- More unstable at lower beta (reduced ion Landau damping)
- No observed beam dependence (What drives these modes?)

Zonca & Chen predict “that EPs preferentially excite the BAE over the BAAE branch due to the stronger wave-EP interaction.”
Conclusions

**BAE Stability**
- Strong q-profile dependence (low-order rational q_{min} crossings)
- More unstable at higher beta (larger BAE gap & suppressed RSAE/TAE activity)
- Beam-angle dependence (resonance condition)

**BAAE Stability**
- Strong q-profile dependence (low-order rational q_{min} crossings)
- More unstable at higher \( T_e \) (reduced ion Landau damping)
- More unstable at lower beta (reduced ion Landau damping)
- No observed beam dependence (What drives these modes?)

Possible implication*: BAAEs not dangerous in ITER!

Great case for validation studies

* If high-energy alphas also not resonant
BAAE resonance depends very weakly on energy

Transit resonances: weak $E$ dependence

\[ \Omega_{\text{res}} = n\omega_\zeta - m\omega_\theta \]

\[
\begin{align*}
\text{Coordinates used:} & \\
& \frac{r}{a} = 0.24, 0.32 \\
& E = 30 - 80 \text{ keV}
\end{align*}
\]

Resonant Fourier harmonics on orbit:
- BAE band: $m/n = 5/3$
- BAAE band: $m/n = 3/2$
Profiles in Reference Shot

- Density ($10^{19}$ m$^{-3}$)
  - $n_e$
  - Classical $n_f$

- Temperature (keV)
  - $T_e$
  - $T_i$

- $Z_{eff}$
  - $q$

- Rotation (kHz)
Comparison of Reference Shot to ITPA Benchmark
Comparison of Reference Shot to ITPA Benchmark
Plausible determination of BAAE toroidal mode numbers for reference shot

- Find frequencies & times that appear clearly on both BES & ECE
- $f_{\text{rot}}(t)$ & $q_{\text{min}}(t)$ also known
- Assume all modes have similar $f_{\text{BAAE}}$ in lab frame
- $n = (f_{\text{lab}} - f_{\text{BAAE}}) / f_{\text{rot}}$
- Peak spacing consistent with $f_{\text{rot}}$
- Graph shows $n$ numbers beside measured mode
BES data are noisy but available

- Get clean BES signals by averaging over 3 radial channels, then computing crosspower & crossphase of matched vertical pairs
- Graph shows BES crosspower near qmin. Click on mode to analyze (*)
5) Why did we claim BAAEs cause significant fast-ion transport?

- The statement was based on the data in this figure, corroborated by the inferred fast-ion pressure profile. This shot did have lots of BAAE activity.

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FIG. 8. (Color online) Measured FIDA energetic particle density profile (experiment) compared to the FIDA density profile calculated using the classical fast ion distribution function from TRANSP (theory). The “FIDA density” is the radiance over blueshifted wavelengths that correspond to energies of $E_A = 30–60$ keV, divided by the local density of injected neutrals for DIII-D shot No. 132710 at $t = 1571$ ms.
5) Why did we claim BAAEs cause significant fast-ion transport?

- Maybe RSAEs and TAEs were above sensitive bandwidth at 1575 ms
- First sawtooth ~ 1920 ms
5) The neutron deficit is modest at 1575 ms

- Assuming no deficit when sawteeth begin, the neutron ratio is ~ 92%
5) The FIDA data on the published shot look good

Red: net signal @ 1875 ms
Light blue: net signal @ 1575 ms
Yellow: FIDASIM @ 1875 ms
Dark blue: FIDASIM @ 1575 ms
Vertical lines: 30-60 keV $E_{\lambda}$

- The data hardly change
- FIDASIM predicts more signal at 1575 ms because the density is lower
5) There is a FIDA deficit but it’s not as large as the published deficit

Points: data @ 1575 (red) & 1875 (blue)
Lines: FIDASIM

- Assuming FIDA is classical at 1875, the average FIDA signal for R<198 cm is $67 \pm 4\%$ of the prediction at 1575 ms
- No evidence of energy dependence to this deficit in the FIDA spectra
Frequency formulas

\[ f_{TAE} = \frac{v_A}{4\pi q R} \]  \hspace{2cm} (1)

\[ f_{GAM} = \frac{1}{2\pi} \frac{c_s}{R} \sqrt{\left(2 + \frac{1}{q^2}\right) \frac{2}{\kappa^2 + 1}} \]  \hspace{2cm} (2)

where \( c_s = \sqrt{\gamma_{kin} T_e / m_i} \) and

\[ \gamma_{kin} = 1 + \frac{3}{4} \frac{p_i}{p_e + p_i} \]  \hspace{2cm} (3)

\[ f_{BAAE} = \sqrt{\frac{\gamma \beta / 2}{q}} \frac{v_A}{R} \frac{1}{1 + \sqrt{(1 + 2q^2) \sqrt{\gamma \beta / 2}}} \]  \hspace{2cm} (4)
Care is required in application of the frequency formulas

\[ f_{TAE} = \frac{v_A}{4\pi q R} \]  

\[ f_{GAM} = \frac{1}{2\pi} \frac{c_s}{R} \sqrt{\left( 2 + \frac{1}{q^2} \right) \frac{2}{\kappa^2 + 1}} \]  

where \( c_s = \sqrt{\gamma_{kin} T_e / m_i} \) and

\[ \gamma_{kin} = 1 + \frac{3}{4} \frac{p_i}{p_e + p_i} \]  

\[ f_{BAAE} = \frac{\sqrt{\gamma\beta/2}}{q} \frac{v_A}{R} \frac{1}{1 + \sqrt{(1 + 2q^2) \sqrt{\gamma\beta/2}}} \]  

• Need to know \( q \) accurately
• Need toroidal mode number to include Doppler shift
  • Chen & Zonca BAAE formula: What to use for \( \Lambda \)?
  • Bierwage & Lauber (2017) say none of the formulas agree with LIGKA