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Linear simulation of trapped energetic particle effects on double tearing modes

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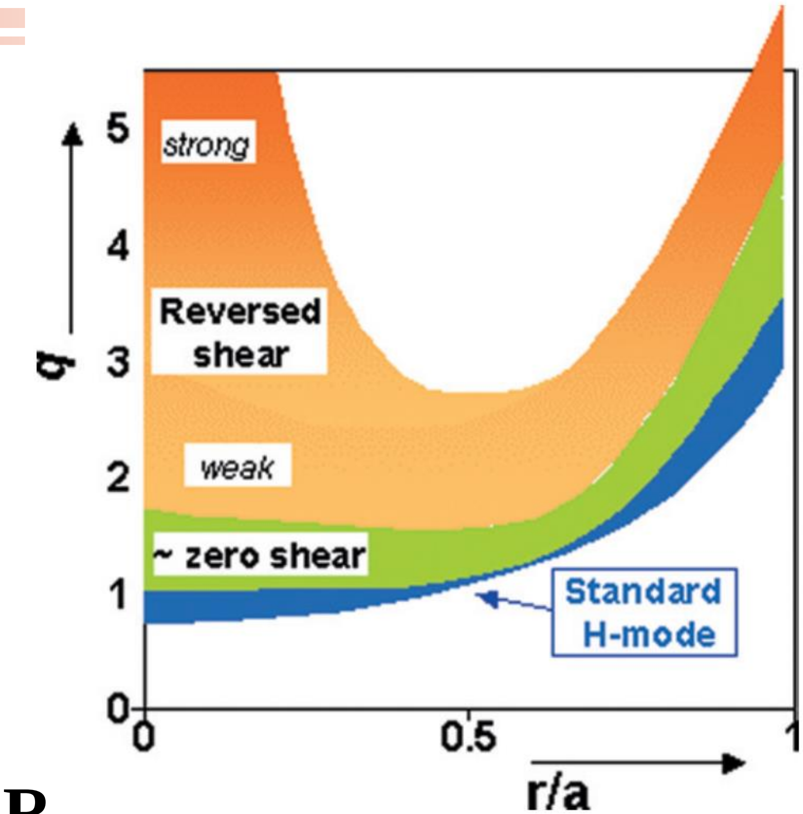
Outline

- Background and motivation
- M3D-K model
- Effects of trapped energetic ions on double tearing modes
- Summary and discussion

Motivation



- **Types of scenarios in ITER.**
  - Standard ITER scenario
  - Steady-state advanced scenario （reversed shear）
  - Hybrid scenario（weak shear）
  - ……

- **Some critical problems in ITER.**
  - MHD instabilities（Double tearing mode(DTM) could be excited with reversed shear，NTM，……）
  - Fast particles（Interaction with instabilities, ……）
  - ……

Background of double tearing modes

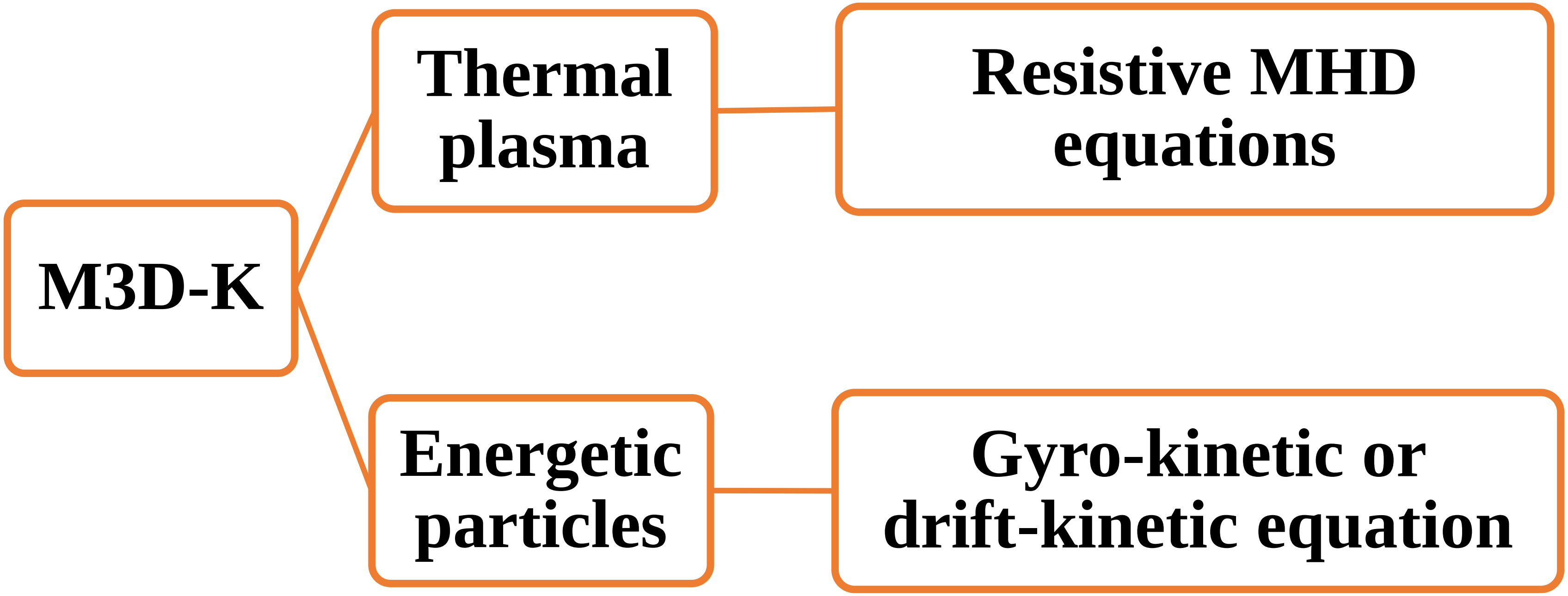
- **Influences of standard tearing mode:**
  - Reconnecting magnetic field to form magnetic islands.
  - Enhancing local radial transport.
  - Degrade plasma confinement.
  - Even triggering plasma disruptions if the island becomes large.

- **Features of double tearing mode:**
  - Two rational surfaces with same q value in reversed shear configuration.
  - Much stronger instability than standard tearing mode due to driving each other between two magnetic islands.

Background of energetic particles

- **Source of energetic particles:**
    - **Alpha particles generated in fusion reaction.**
    - **Energetic particles generated by auxiliary heating.**
- **Influences of energetic particles:**
    - **The confinement and transport of energetic particles affect**
        - **the heating efficiency**
        - **sustainment of high confinement capability of ‘burning’ plasmas**
        - **the output of high fusion energy**
    - **The loss of energetic particles leads to**
        - **excessive heat loading**
        - **damage to first wall and divertor**
    - **The interaction between energetic particles and plasma waves (e. g. Alfvén waves) and instabilities (e. g. kink mode) will**
        - **lead to the redistribution and loss of energetic particles**
        - **bring about some new plasma phenomena (e. g. fishbone).**
    - ……

M3D-K model


M3D-K
Thermal plasma
Resistive MHD equations
Energetic particles
Gyro-kinetic or drift-kinetic equation

M3D-K model

Resistive MHD equations:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$$

$$\rho \frac{d\mathbf{v}}{dt} = \mathbf{J} \times \mathbf{B} - \nabla p + \mu \nabla^2 \mathbf{v}$$

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}$$

$$\frac{dp}{dt} = -\gamma p \nabla \cdot \mathbf{v} + \rho \nabla \cdot \boldsymbol{\kappa} \cdot \nabla \frac{p}{\rho}$$

$$\mathbf{J} = \nabla \times \mathbf{B}$$

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} = \eta \mathbf{J}$$

$$\nabla \cdot \mathbf{B} = 0$$

Momentum equation with particle stress tensor*:

$$\rho \frac{d\mathbf{v}}{dt} = \mathbf{J} \times \mathbf{B} - \nabla P - \nabla \cdot \mathbf{P}_h$$

$$\mathbf{P}_h = P_\perp \mathbf{I} + (P_\parallel - P_\perp)\mathbf{bb}$$

$$P_\parallel(\mathbf{x}) = \int M v_\parallel^2 \delta(\mathbf{x} - \mathbf{X} - \rho_h) F(\mathbf{X}, v_\parallel, \mu) d^3\mathbf{X} dv_\parallel d\mu d\theta$$

$$\mathrm{P}_\perp(\mathbf{x}) = \int \frac{1}{2} M v_\perp^2 \delta(\mathbf{x} - \mathbf{X} - \rho_h) F(\mathbf{X}, v_\parallel, \mu) d^3\mathbf{X} dv_\parallel d\mu d\theta$$

$$\frac{d\mathbf{X}}{dt} = \frac{1}{B^{**}}\left[v_\parallel\left(\mathbf{B}^* - \mathbf{b}_0 \times \left(\langle \mathbf{E} \rangle - \frac{1}{e}\mu \nabla (B_0 + \langle \delta B \rangle)\right)\right)\right]$$

$$m \frac{dv_\parallel}{dt} = \frac{e}{B^{**}} \mathbf{B}^* \cdot \left(\langle \mathbf{E} \rangle - \frac{1}{e}\mu \nabla (B_0 + \langle \delta B \rangle)\right)$$

**G. Fu, et al, 2006*

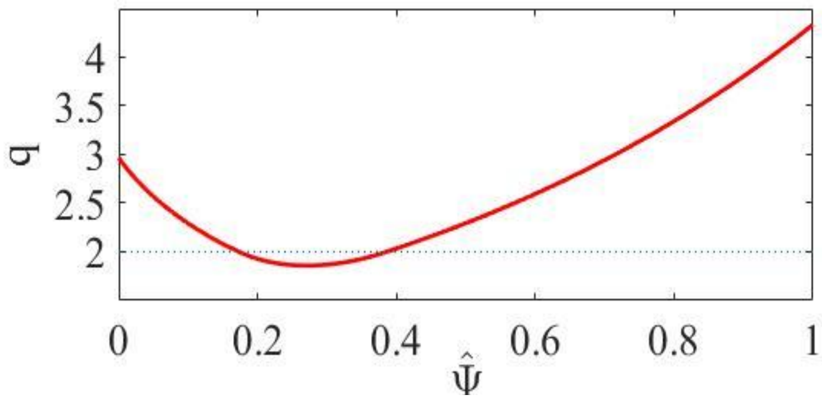
DTM without energetic ions

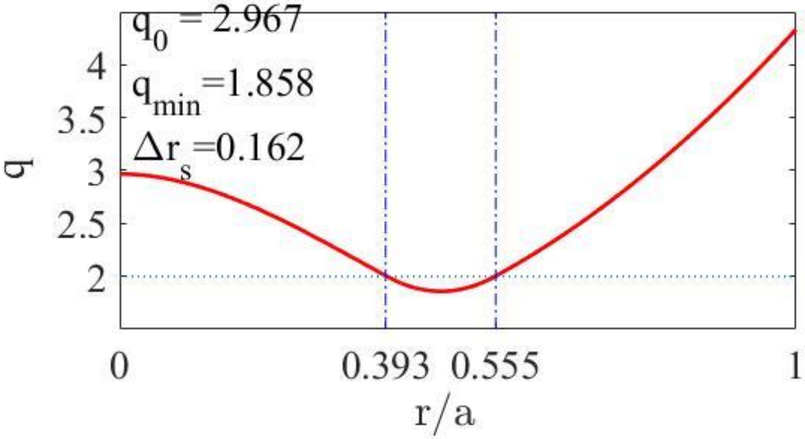
Distance between two rational surfaces (q=2)

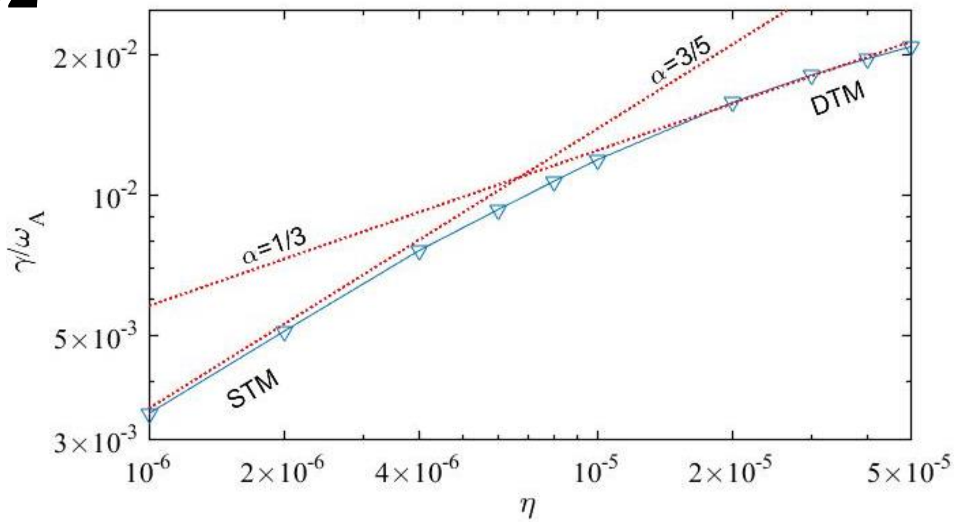
$$\Delta r_s/a = 0.162$$

Aspect ratio

$$R_0/a = 4.5$$



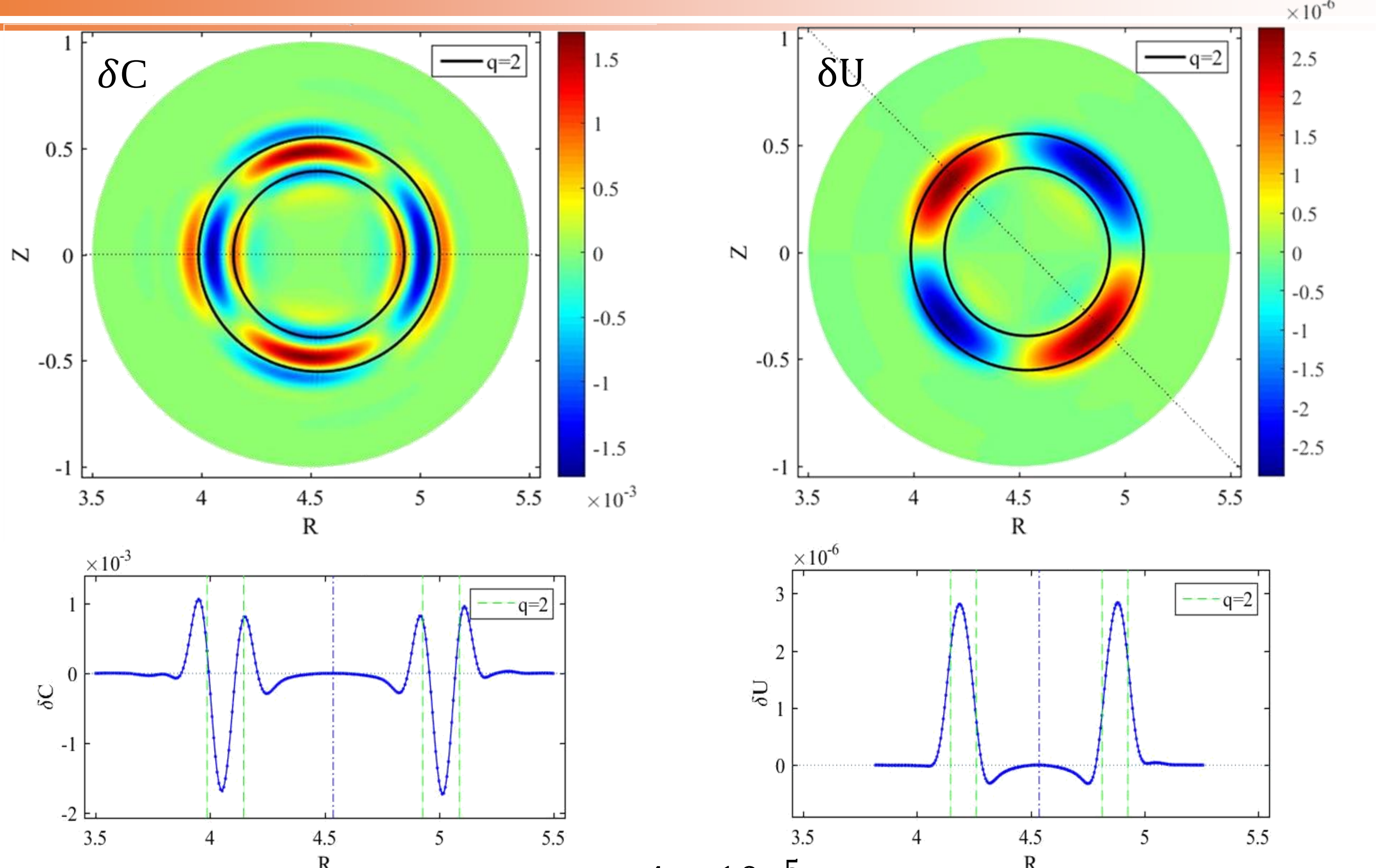




- **Large η, double tearing mode.**
- **Small η, standard tearing mode.**
- **Smaller Δr_s is needed for forming DTMs in the condition of lower η.***

**P. L Pritchett et al, 1980*

DTM without energetic ions



$$\eta = 4 \times 10^{-5}$$

The perturbed stream function is mainly located between the rational surfaces.

Distribution function of energetic ions

Distribution function of energetic ions

$$f_0 = \frac{n_{h0}}{\sqrt{\pi}(v^3+v_c^3)} exp\left[-\left(P_\varphi - \langle\frac{v_\parallel I}{\Omega_h}\rangle\right)/\Delta\psi\right]$$

$$\times \frac{1}{\Delta\Lambda}\left[erf\left(\frac{1-\Lambda_0}{\Delta\Lambda}\right) + erf\left(\frac{\Lambda_0}{\Delta\Lambda}\right)\right] exp\left[\frac{-(\Lambda-\Lambda_0)^2}{\Delta\Lambda^2}\right]$$

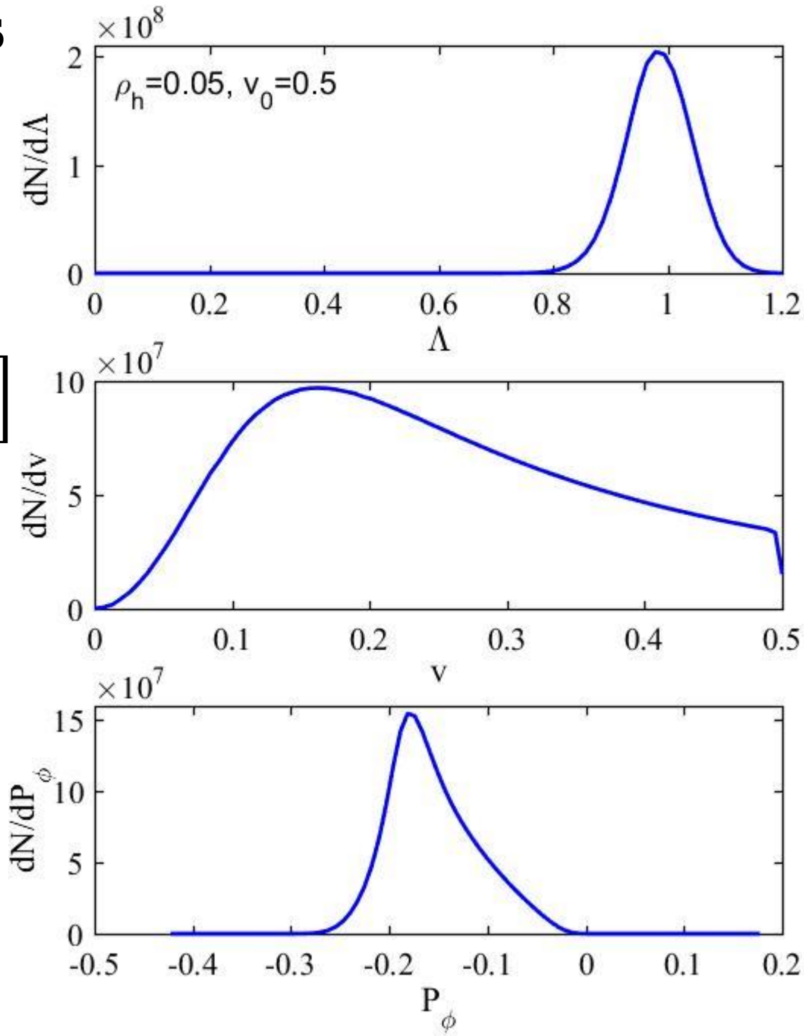
where， $v_c = 0.58 v_0$，

$\Delta\psi = 0.4(\psi_{max} - \psi_{min})$，

$\Lambda_0 = 1$，

$\Delta\Lambda = 0.1$，

$\Lambda = \mu/E$.



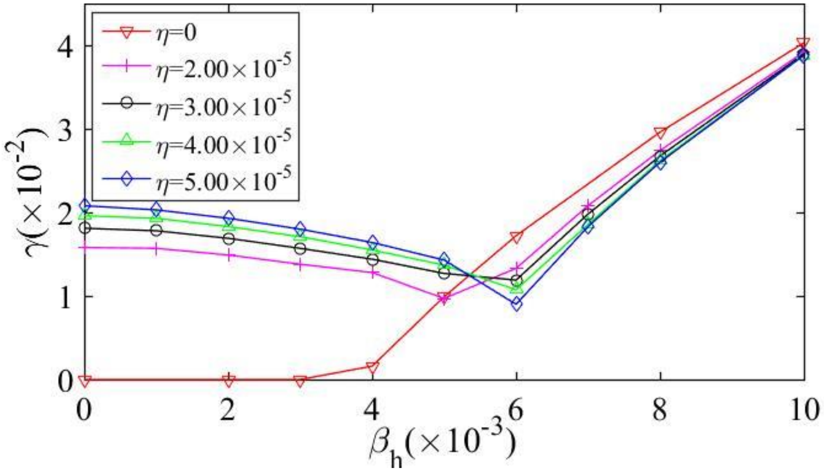
Effects of trapped energetic ions on DTMs

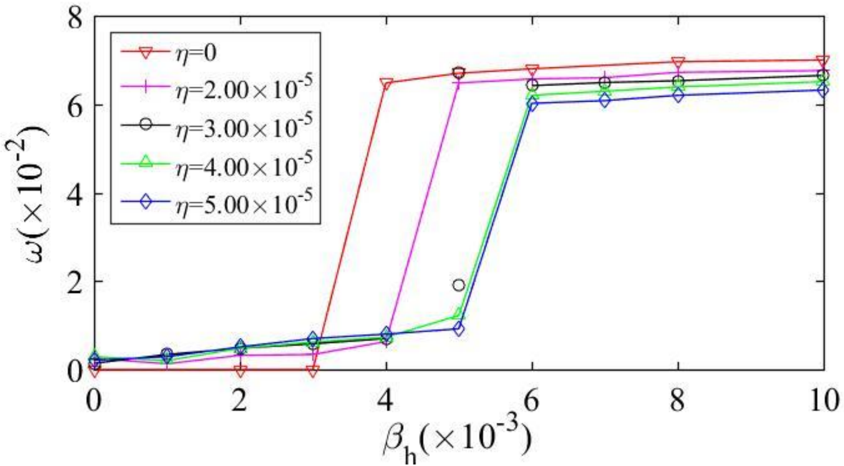
With resistivity: **For low-β_h， DTMs are stabilized by trapped energetic ions.**

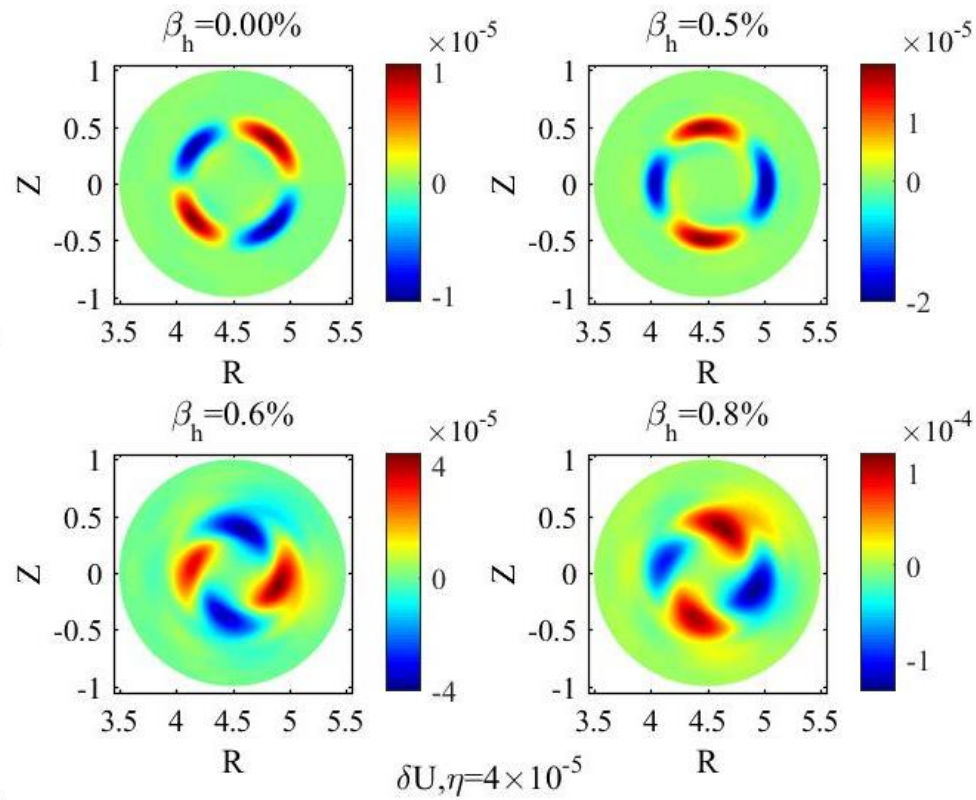
If β_h is above a threshold， a mode with higher frequency is triggered.

The threshold is related to resistivity.

Without resistivity: **low-β_h， stable; $\beta_h > 4 \times 10^{-3}$, unstable.**

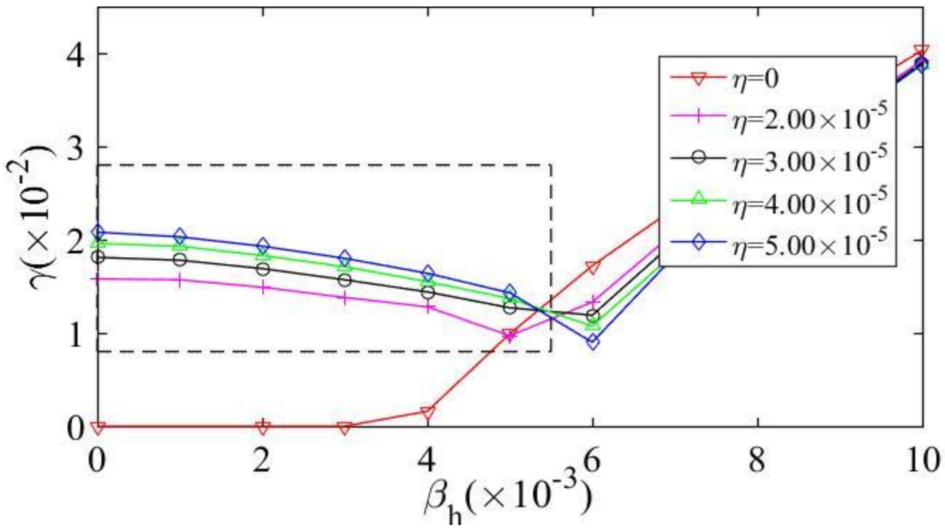


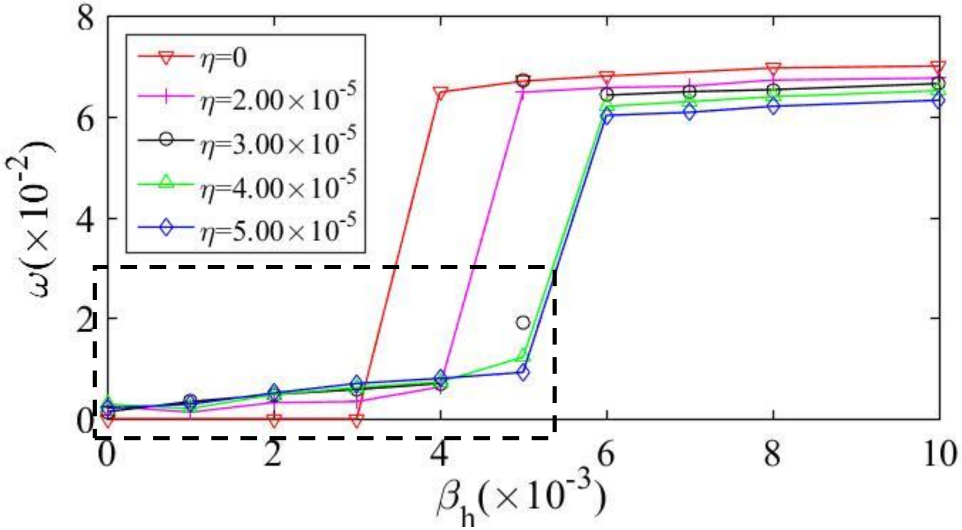




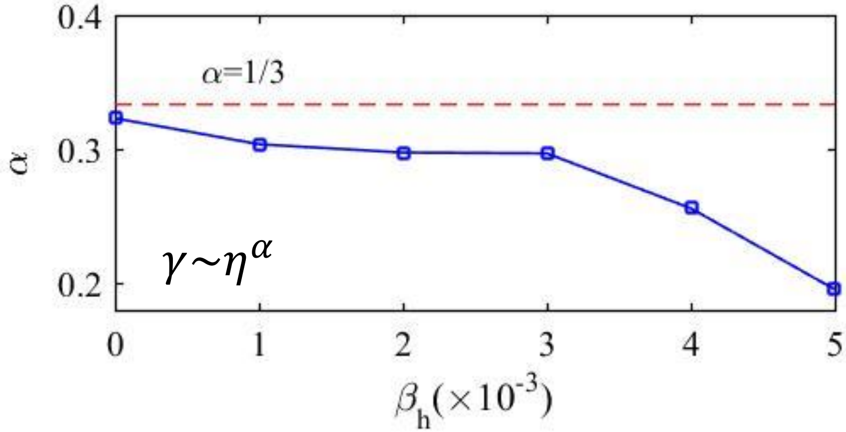
DTMs are stabilized by trapped energetic ions

Dependence of energetic ion effects on β_h (low-β_h):



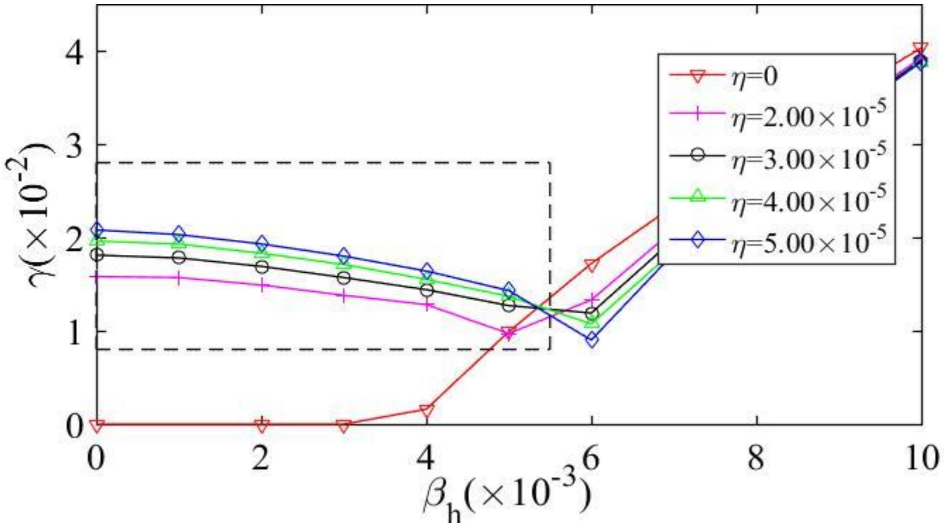


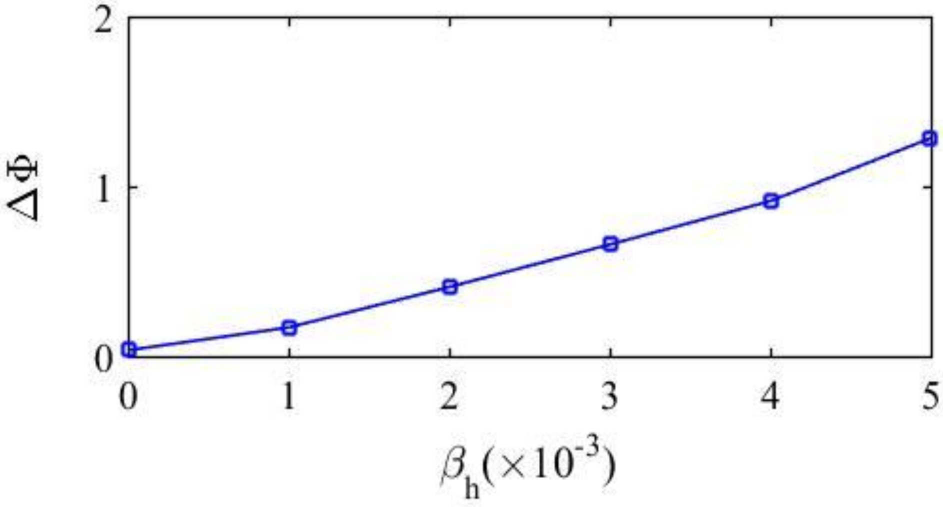
- As β_h increases, the growth rates of DTMs decrease.
- As β_h increases, the mode frequencies increase but are still very low.
- The resistivity scaling of DTMs, $\gamma \sim \eta^{\alpha}$, changes a little when β_h is low.



DTMs are stabilized by trapped energetic ions

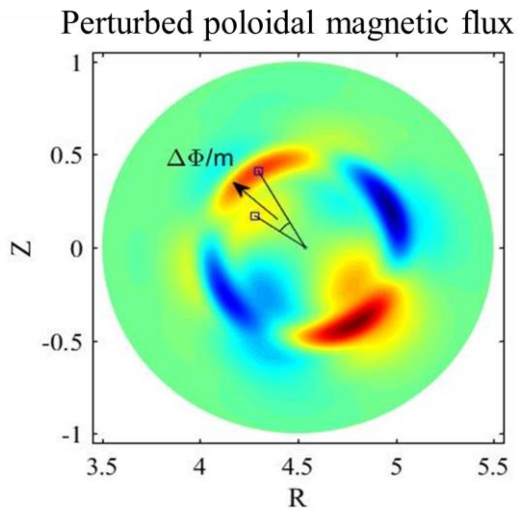
Physics explanation 1:





- **When $\beta_h = 0$, $\Delta\Phi = 0$, the DTMs are antisymmetric.**
- **As β_h increases, $\Delta\Phi$ increases and the growth rate decreases.**

- **Antisymmetric DTMs tend to transit to symmetric DTMs.**
- **Antisymmetric DTMs are more unstable than symmetric DTMs.***



**A. Otto et al. 1992*

DTMs are stabilized by trapped energetic ions

Physics explanation 2:

Assumption:

Deeply trapped energetic ions, $\boldsymbol{v}_{\parallel} \ll \boldsymbol{v}_{\perp}$

Small orbit width, $\boldsymbol{\Delta_b} \ll \boldsymbol{\Delta_m} \ll \boldsymbol{r}$

The effects of trapped energetic ions depend on the sign of

$$\left(1+\frac{n\omega_d}{\omega-n\omega_d}\right)\int_0^1 \frac{\partial f_0}{\partial r}[2E(k)-K(k)]dk^2$$

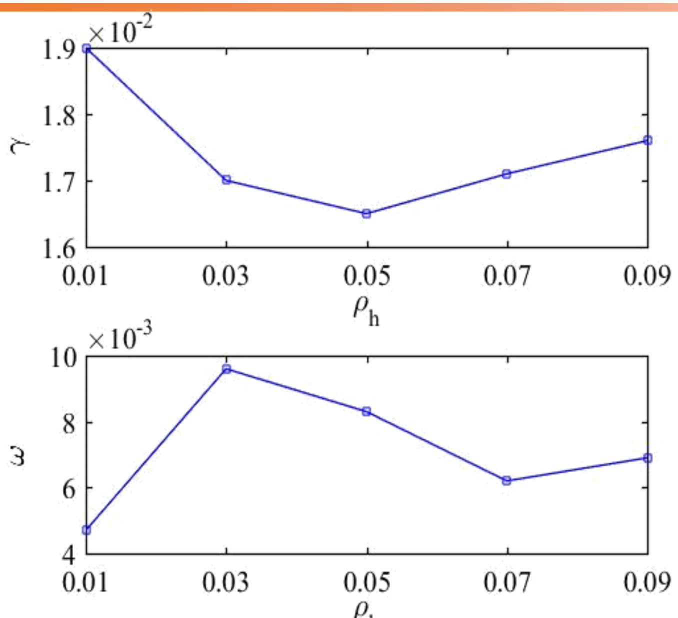
$$\omega \ll \omega_d \Rightarrow 1+\frac{n\omega_d}{\omega-n\omega_d} \sim -\frac{\omega}{n\omega_d} < 0$$

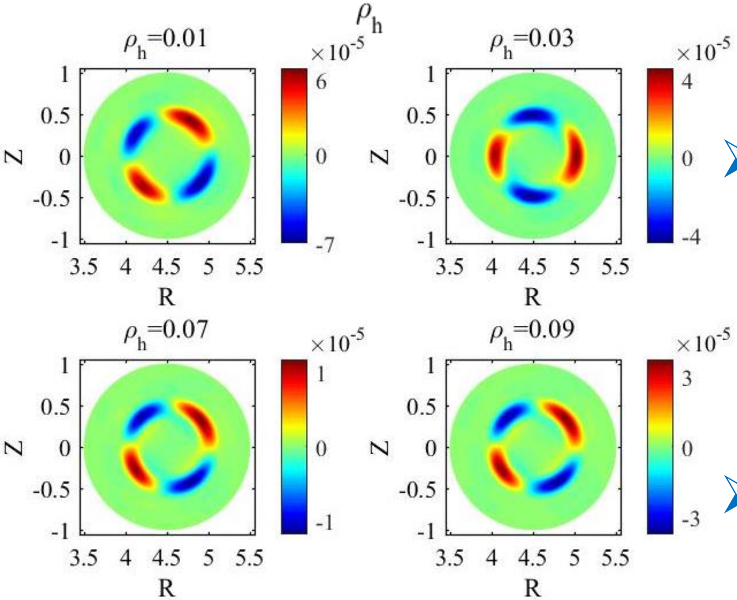
$$\frac{\partial f_0}{\partial r} < 0,\ \Lambda_0 = 1 \Rightarrow \int_0^1 \frac{\partial f_0}{\partial r}[2\mathrm{E}(k)-\mathrm{K}(k)]d\mathrm{k}^2 < 0$$

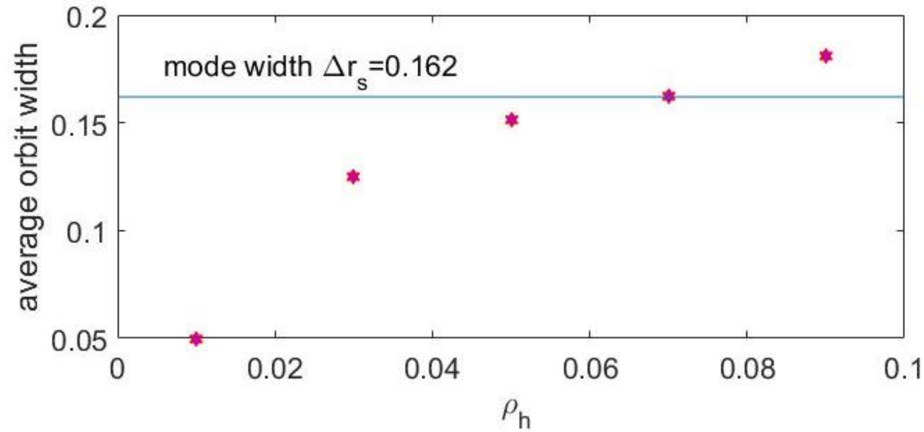
$$\Rightarrow \boxed{\delta W_h > 0}$$

- **If $\boldsymbol{\Delta_b} \ll \boldsymbol{\Delta_m} \ll \boldsymbol{r}$, DTMs can be stabilized by trapped particles.**
- **If $\boldsymbol{\Delta_b} \sim \boldsymbol{\Delta_m} \ll$ r, energetic particles spend only a fraction of their time in the region of the mode. So, the effects of energetic ions with large orbit width are weaker than small orbit width.**

Dependence of energetic ion effects on ρ_h





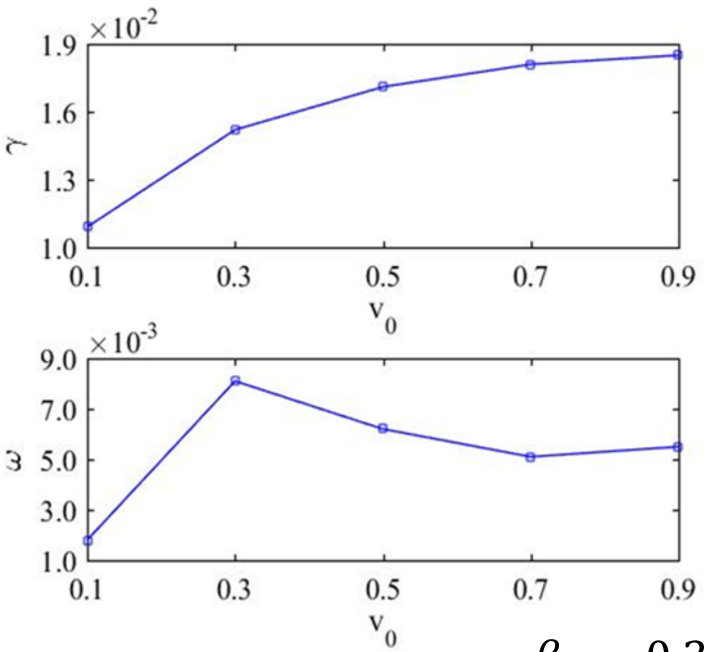


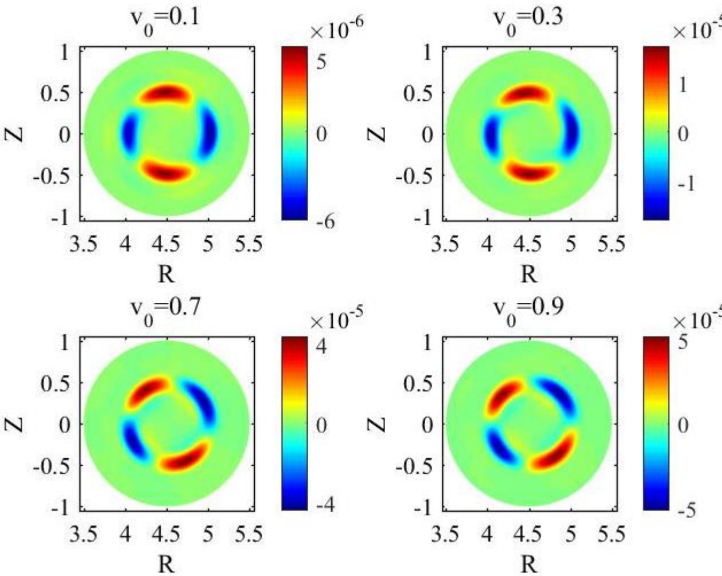
$\beta_h = 0.3\%, v_0 = 0.5$

- **For small ρ_h, $\Delta_b \ll \Delta_m$, the effects of trapped energetic ions are <u>reduced as ρ_h decreases</u>, which is similar to that on standard tearing modes.***
- **For larger ρ_h, $\Delta_b \sim \Delta_m$, the effects of trapped energetic ions are <u>reduced as ρ_h increases</u>, which can be explained by that trapped energetic ions spend less time in the region of the mode as ρ_h increases.**
- **The mode frequency and mode structure are almost unchanged.**

**Huishan Cai et al. 2012*

Dependence of energetic ion effects on v_0



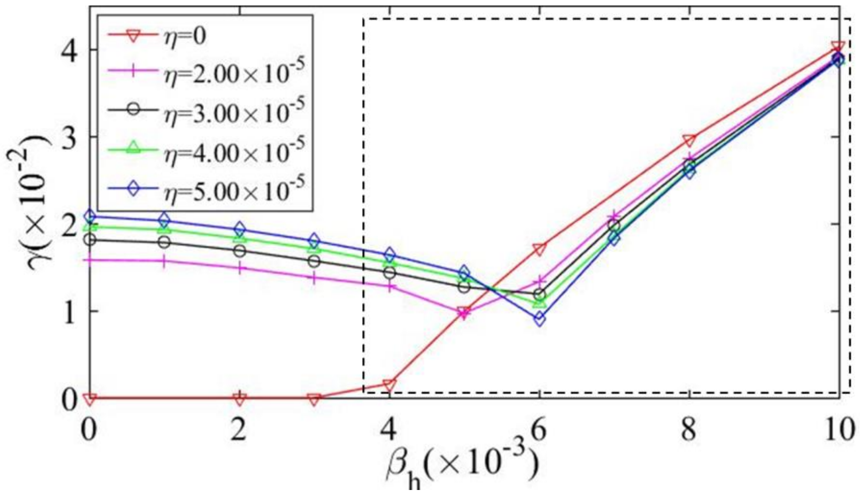


$\beta_h = 0.3\%, \rho_h = 0.07$

- **The effects of trapped energetic ions decrease as v_0 increases.**
- **The mode frequency and mode structure are not sensitive to injection velocity v_0.**

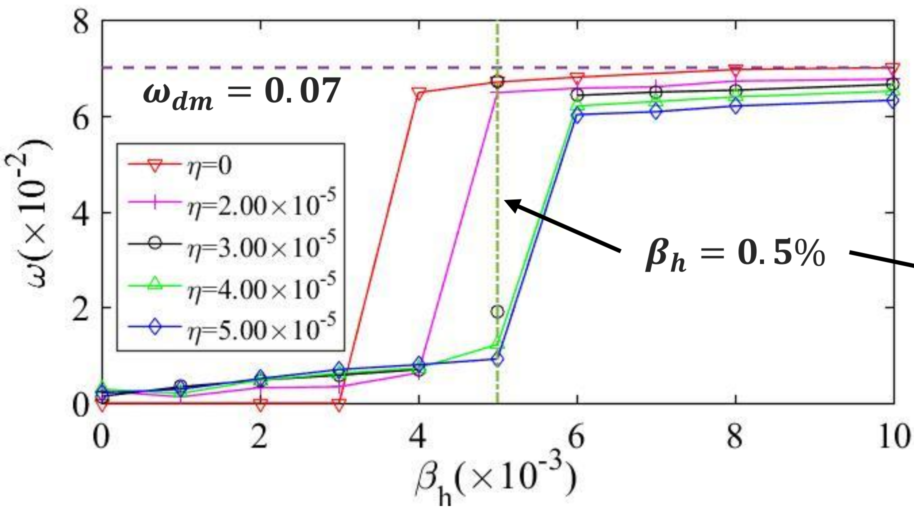
Fishbone-like excited by energetic ions

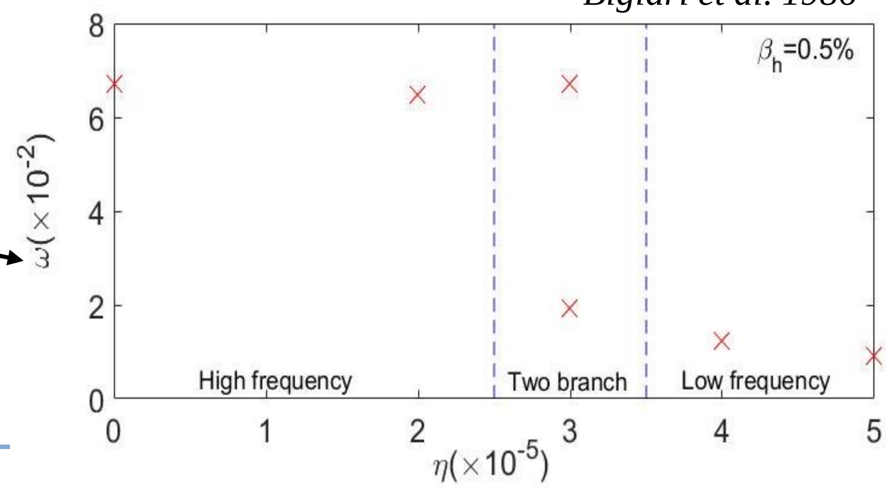
- When β_h is above a threshold, fishbone-like modes are excited, their growth rates strongly increase as β_h increases and their mode frequencies are almost constants near particle processional frequency ω_{dm}.



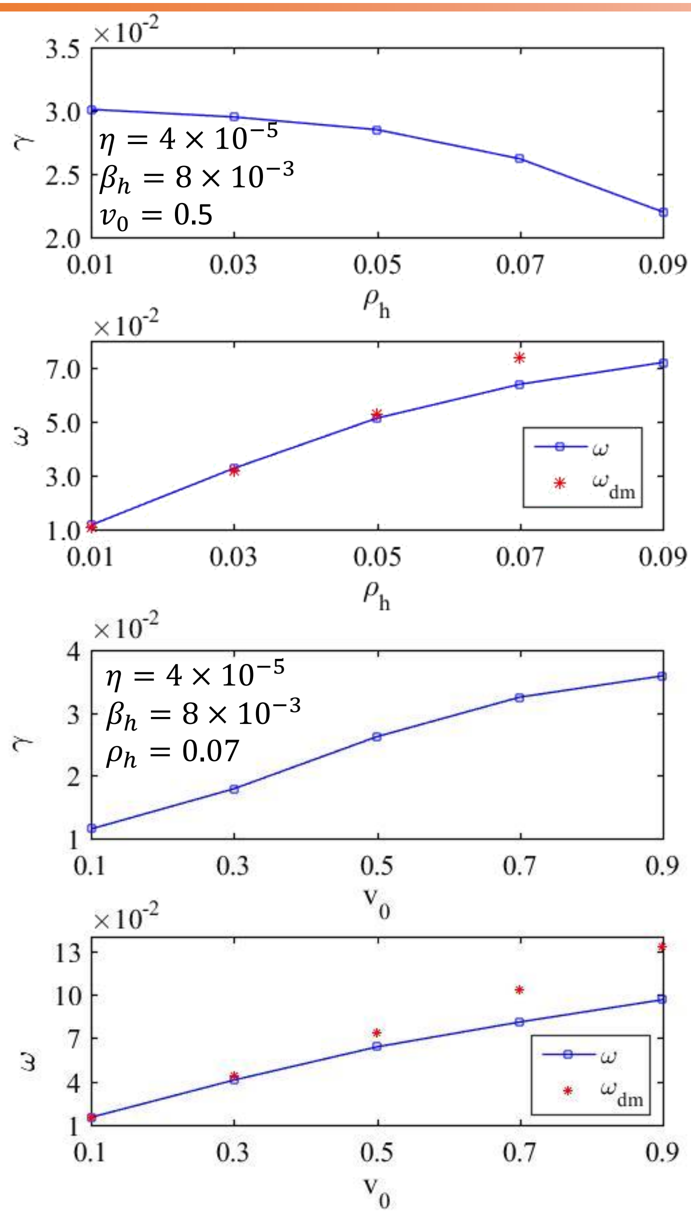
- Resistivity enhances the onset threshold of fishbone-like modes.*
- Resistivity can reduce the growth rates and mode frequencies of fishbone-like modes.
- If β_h is sufficiently large, the effects of resistivity can be neglected.

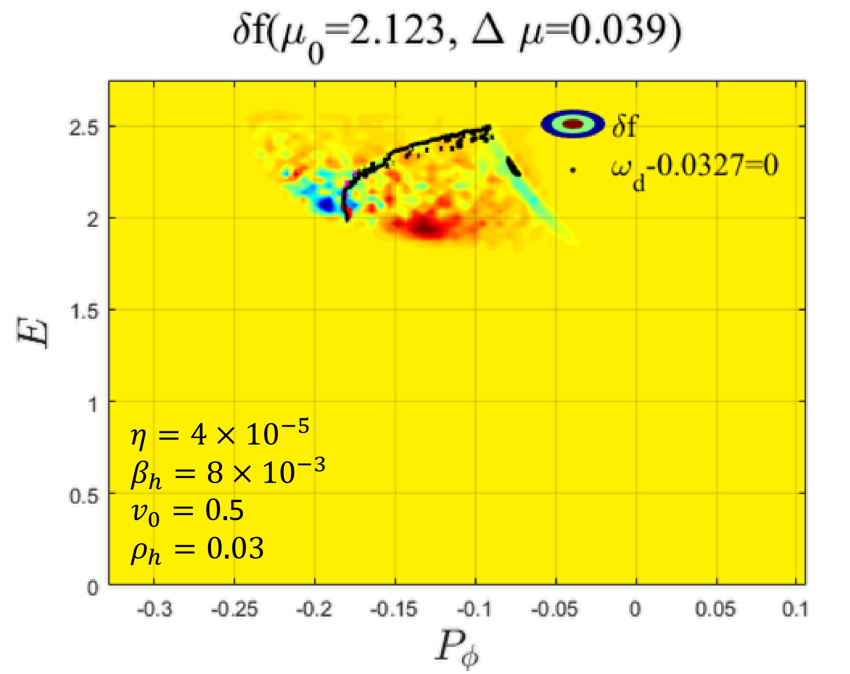
Biglari et al. 1986





Fishbone-like excited by energetic ions



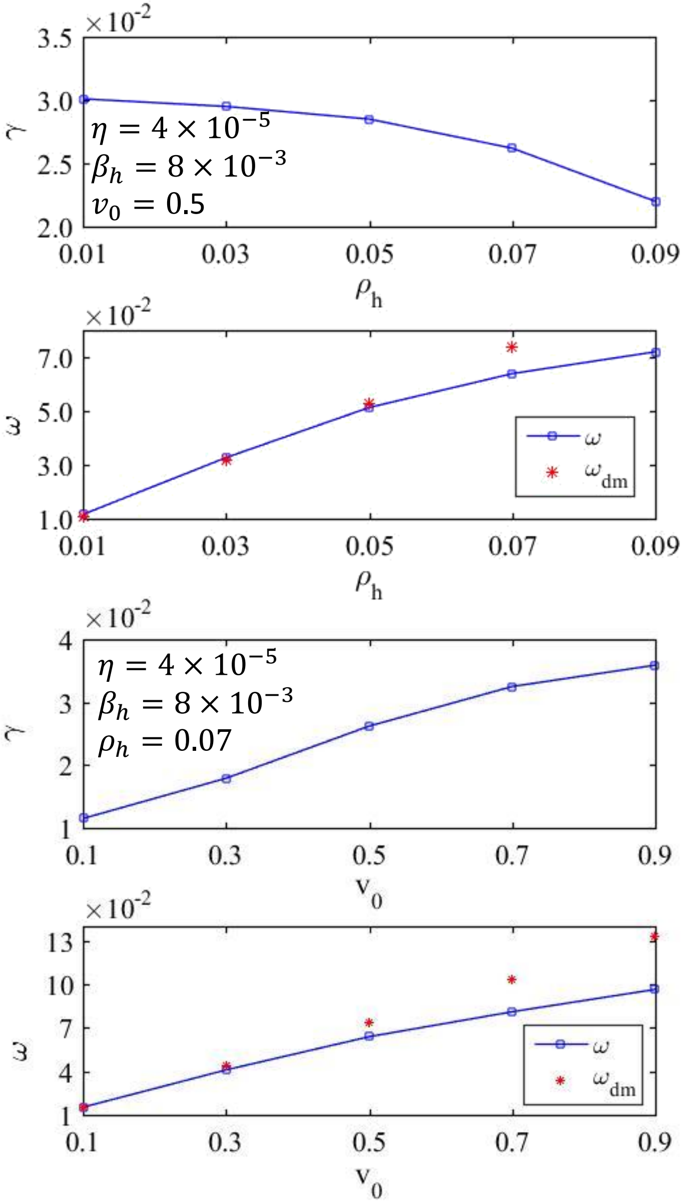


- **Mode frequency is near the processional frequency.**

$$\omega \lesssim \omega_{dm}$$

- **The resonant locations are where perturbation of distribution function is significant.**

Dependence of fishbone-like on ρ_h and v_0



- **As ρ_h increases，growth rate decreases due to increasing orbit width.**

- **As v_0 increases，fishbone-like becomes more unstable.**

Summary

1. **For low β_h, trapped energetic ions have stabilizing effects on DTMs, and these effects are related to gyro-radius ρ_h and injection velocity v_0 of energetic ions.**
   a) **As β_h increases, the growth rates of DTMs decrease while the mode frequencies increase but are still very low.**
   b) **As injection velocity v_0 increases, the effects of trapped energetic ions decrease.**
   c) **As gyro-radius ρ_h increases, the effects of trapped energetic ions increase for small ρ_h but will decrease if ρ_h is sufficiently large.**
2. **If β_h is above a threshold value, fishbone-like modes will be excited.**
   a) **As β_h increases, the growth rates strongly increase and mode frequencies are almost constants near particle processional frequency ω_{dm}.**
   b) **The threshold of DTM - FLM transition can be enhanced by resistivity. The growth rates and mode frequencies of fishbone-like are reduced by resistivity.**
   c) **If β_h is sufficiently large, the effect of resistivity can be neglect.**
   d) **The effect of trapped energetic ions is increasing with injection velocity v_0 increasing and decreasing with gyro-radius ρ_h increasing.**

Discussion

1. Trapped energetic ions make antisymmetric DTM have the tendency to transit to symmetric DTM. The regime of this transition need to be studied in detail.

2. The effects of diamagnetic frequency are neglected here, but are important. A second type of fishbone is introduced by interaction between energetic ions and kinks when the mode frequency is comparable to the diamagnetic frequency of background plasma. The second type of fishbone-like modes can also be probably excited with DTMs since the physics regime of the linear effects of energetic ions on ordinary kinks, double kinks and DTMs is the same.

3. Only the trapped energetic ions effects on DTMs are studied. Kolesnichenko et al. found double-kink fishbone can be excited by circulating energetic ions with ideal MHD. In the presence of resistivity, similar phenomenon may also exist.

Thank you !