Kinetic Aspects of High-Z Pellet Modeling for Disruption Mitigation

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Characteristic features of the problem

- Pellets offer more controllable penetration of the injected material than MGI

- Hot plasma electrons heat the pellet surface. The ablated gas throttles the electron heat flux to the surface until 3D expansion makes the gas shield semi-transparent

- Collisional scattering and slowing down of the hot electrons in the pellet material need a kinetic description with the electron gyromotion included

- Strong backscattering of the hot electrons reduces the electrostatic sheath potential

- Elastic scattering reduces the hot electron penetration depth and the resulting ablation rate
Schematic of high-Z pellet ablation

- Hot electron velocity distribution is nearly isotropic due to strong elastic scattering.
- The electrostatic sheath potential scales as $Z^{-1/3}$.
- The ablation cloud is semi-transparent for hot electrons.
- Unmagnetized hot electrons diffuse radially.
- Magnetized hot electrons diffuse along the field lines.
- Penetration depth of the hot electrons is $\delta_p = \sqrt{D\tau_{drag}}$.
- The solid pellet radius decreases slowly due to ablation.
- Outgoing mass flux is constant within the ablation cloud: $G = 4\pi Mr^2 NV$.
- Magnetic field.
Strong backscattering reduces electrostatic sheath

- Hot electron flux into the pellet scales as \( j_{\text{hot}} \sim \frac{1}{\sqrt{Z}} n_\infty \sqrt{\frac{T_\infty}{m}} \)

- Return flux of the emitted cold electrons satisfies the Child-Langmuir law:
  \[ j_{\text{cold}} \sim \left( \frac{e \phi}{m} \right)^{3/2} \frac{m}{e^2 d^2} \]

- The sheath width is roughly the Debye length:
  \[ d^2 \sim \frac{T_\infty}{n_\infty e^2} \]

- The return cold flux balanced the hot particle flux, which gives
  \[ e \phi \sim \frac{T_\infty}{Z^{1/3}} \ll T_\infty \]
Ablation scenario revision for high-Z pellets

- Hot electron diffuse into the pellet until they slow down due to electron drag. Their penetration depth is \( \delta_p \sim \frac{1}{Z\sqrt{Z}} \frac{1}{N_p \sigma_{ee}} \sqrt{\ln \Lambda_{ee} / \ln \Lambda_{ei}} \). Strong elastic scattering reduces the penetration depth and the resulting ablation rate.

- The heated surface layer of the pellet expands radially and becomes semi-transparent when it broadens to pellet radius. The flow is roughly sonic at this point \((MV^2_\ast \sim T_\ast)\).

- When the flow becomes sonic, the semi-transparent cloud is heated to a temperature

  \[
  T_\ast \sim ZT_\infty n_\infty \sqrt{\frac{T_\infty}{m}} \sigma_{ee} \frac{R_p}{V_\ast}
  \Rightarrow
  \quad V_\ast^3 \sim \frac{R_p}{M} \frac{Zn_\infty \ln \Lambda_{ee}}{T_\infty^{1/2}}
  \]

- Half of the incoming heat flux is absorbed in the cloud: \( NR_p \sim N_p \delta_p \)
Flow velocity, cloud density and ablation rate

- Flow velocity for sonic expansion:

\[ V_\ast \sim \left( \frac{ZT_\infty n_\infty}{m} \sqrt{\frac{T_\infty}{\sigma_{ee}}} \frac{R_p}{M} \right)^{1/3} \]

- Density of semi-transparent ablation cloud:

\[ N_\ast \sim N_p \frac{\delta_p}{R_p} \sim N_p \frac{\delta_p}{R_p} \sim \frac{1}{Z \sqrt{Z}} \left( \frac{\ln \Lambda_{ee}}{\ln \Lambda_{ei}} \right)^{1/2} \frac{1}{R_p \sigma_{ee}} \]

- Ablation rate estimate:

\[ G \sim 4\pi MR_p^2 N_\ast V_\ast \sim 4\pi MR_p \frac{1}{Z \sqrt{Z}} \left( \frac{\ln \Lambda_{ee}}{\ln \Lambda_{ei}} \right)^{1/2} \frac{1}{\sigma_{ee}} \left( \frac{R_p}{M} \right) ZT_\infty n_\infty \sigma_{ee} \sqrt{\frac{T_\infty}{m}} \]

\[ G \sim \frac{4\pi M}{(\pi e^4)^{2/3}} \left( \frac{1}{m^{1/2} M} \right)^{1/3} R_p^{4/3} \frac{n_\infty^{1/3} T_\infty^{1/6}}{Z^{7/6} (\ln \Lambda_{ei})^{1/2} (\ln \Lambda_{ee})^{1/6}} \]
Kinetic heating calculation

- Kinetic equation for hot electrons:

\[
div(uf) - \frac{1}{u^2} \frac{\partial}{\partial u} u^3 v_{ee} f + \omega c \frac{\partial f}{\partial \psi} = \frac{\nu_{ei}}{2} \sin \theta \frac{\partial}{\partial \theta} \sin \theta \frac{\partial f}{\partial \theta} + \frac{\nu_{ei}}{2} \sin^2 \theta \frac{\partial^2 f}{\partial \psi^2}
\]

- The hot electron distribution is nearly isotropic in the high-Z case, i.e. \( f(r,u) = F(r,u) + O(1/Z) \).

- The kinetic equation reduces to an axisymmetric diffusion-type equation for the isotropic distribution:

\[
-\frac{1}{3} \frac{\partial}{\partial z} u^2 \frac{\partial F}{\partial z} - \frac{1}{3\rho} \frac{\partial}{\partial \rho} \rho \left( \nu_{ei} \frac{\partial F}{\partial \rho} + \nu_{ ei} \frac{\partial F}{\partial \rho} \right) - \frac{1}{u^2} \frac{\partial}{\partial u} u^3 v_{ee} F = 0
\]

- Power deposition by hot electrons per unit volume:

\[
Q = \frac{16\pi^2 e^4 ZN \ln \Lambda_{ee}}{m} \int F \, du^2
\]
Two reductions of the kinetic equation

- **Strongly magnetized hot electrons:** $\omega_c >> v_{ei}$
  \[
  \frac{\partial F}{\partial \tau} + \frac{\partial^2 F}{\partial \xi^2} = 0
  \]

- **Unmagnetized hot electrons:** $\omega_c << v_{ei}$
  \[
  \frac{\partial F}{\partial \tau} + \frac{1}{r^2} \frac{\partial}{\partial \xi} r^2 \frac{\partial F}{\partial \xi} = 0
  \]

- **Normalized line-integrated density:**
  \[
  \xi = \frac{8\pi Ze^4}{T_\infty^2} \left( 3Z \ln \Lambda_{ee} \ln \Lambda_{ei} \right)^{1/2} \int_Z^\infty N(z')dz'
  \]

- **Normalized time-like velocity variable:**
  \[
  \tau = \frac{1}{8} \left( u \frac{m}{T_\infty} \right)^8
  \]
Comparison of power deposition profiles for unmagnetized and magnetized hot electrons

We use an asymptotic expression for the cloud density to express $r$ in terms of $\xi$ in the diffusion equation:

$$N \sim r^{-7/3} \Rightarrow r^2 \sim \xi^{-3/2}$$
Gas flow modeling

- We use the fluid model from [Parks and Turnbull 1978], but with a kinetic calculation of the power deposition $Q$.
- Flow velocity at subsonic-supersonic transition ($\mu = 1$): $V_*^2 = 2\pi R_*^3 \frac{(\gamma - 1)Q_*}{G}$

\[
G = 4\pi MR^2NV = \text{const} \\
MNV \frac{dV}{dR} + \frac{d}{dR} NT = 0 \\
\frac{G}{4\pi R^2} \frac{d}{dR} \left( \frac{\gamma T}{M} \frac{V}{\gamma - 1} + \frac{V^2}{2} \right) = Q
\]

\[
N = \frac{G}{(4\pi R^2 V)} \\
\left( \mu^2 - 1 \right) \frac{dV}{dR} = \frac{2V}{R} \left[ 1 - 2\pi R^3 \frac{(\gamma - 1)\mu^2}{2V^2 G} Q \right] \\
\left( \mu^2 - 1 \right) \frac{d\mu}{dR} = \frac{\mu(\gamma \mu^2 - 1)}{R} \left[ 1 - 2\pi R^3 \frac{(\gamma - 1)\mu^2}{V^2 G} Q \right] - \frac{\mu(\mu^2 - 1)}{R}
\]

- Spherical expansion
- Cloud remains neutral with a constant adiabatic index $\gamma$
- Surface boundary condition implies negligible sublimation energy
- Radiative losses not accounted for
- Ignore the electrostatic sheath which scales as $Z^{-1/3}$
Reduction of ablation rate due to elastic scattering

- Predictions from [Sergeev et al. 2006] are too high.
- Difference between our results and [Parks & Samulyak] shows significant sensitivity of the heat deposition to elastic scattering.
- All ablation rates agree well in pellet radius scaling.

\[ G = 1205.45 R_p^{1.44} \]
\[ G = 64.4 \]
\[ G \approx 350 R_p^{4/3} \]

\[ n_b = 10^{14} \text{ cm}^{-3}, \quad T_b = 2 \text{ keV} \]
The first principle kinetic calculation of the heat deposition gives a noticeably lower ablation rate for the high-Z pellets than the pre-existing estimates.

Strong elastic scattering of the incident electrons reduces the role of electrostatic shielding.

Magnetization of the incident electrons can modify the heat deposition geometry significantly.

Kinetic calculations of the heat deposition provide an updated input for fluid simulations of the pellet ablation process.