EUROfusion

MHD spectroscopy of pellet injected plasmas

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Outline

- Introduction and experimental motivation
- Analytical estimates of the Alfvén spectrum for plasmas with modulated plasma density
- Numerical study of the Alfvén spectrum in plasmas with modulated plasma density
- Model for the expansion of pellet material in a tokamak
- Response of Alfvén eigenmodes to expanding pellet material
- Summary

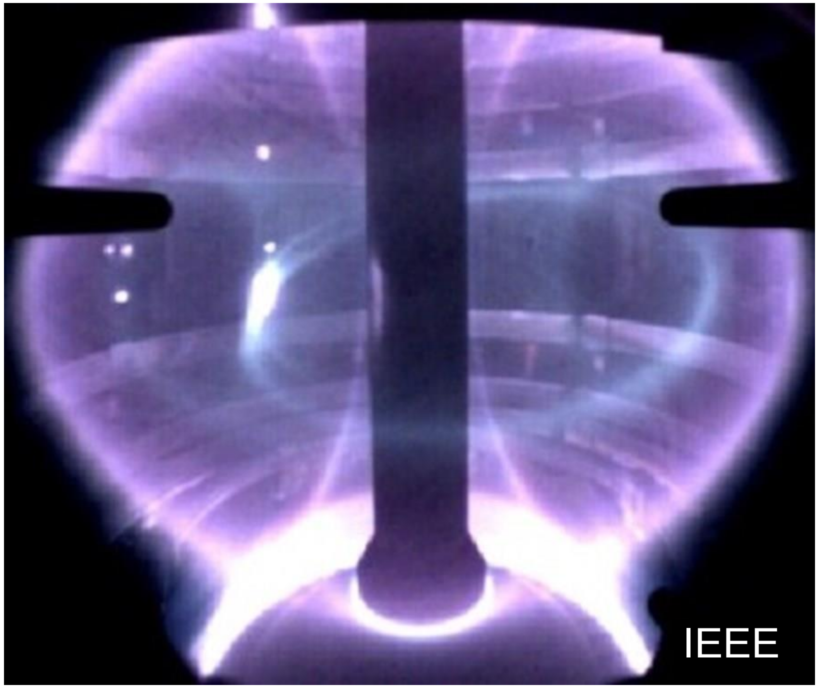
Pellet injected plasmas are complex

- Pellets of frozen deuterium are injected to fuel the plasma and mitigate dangerous instabilities.

- Varied applications require pellets with a wide range of parameters.

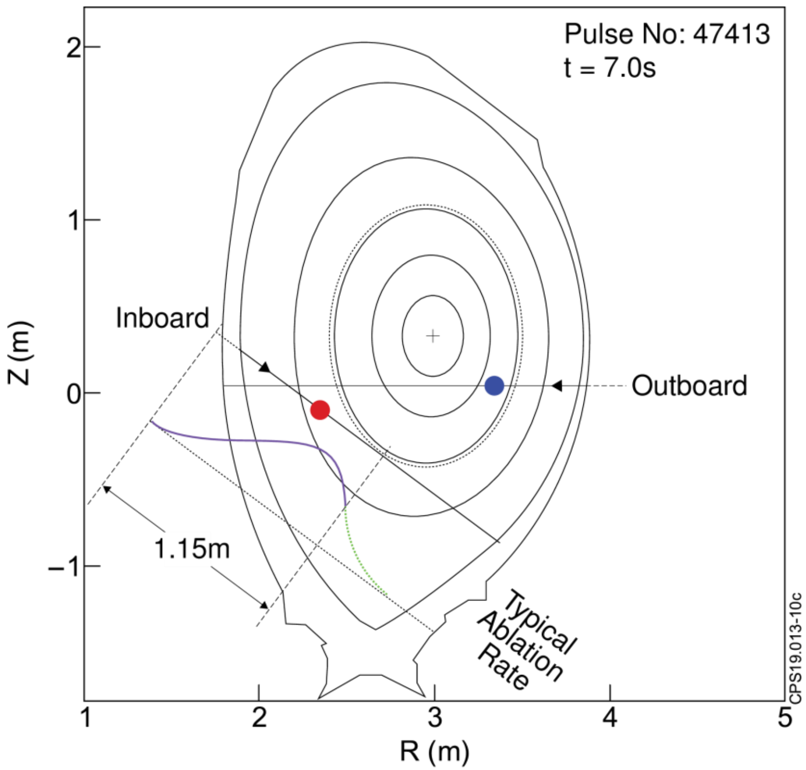
- Pellets usually break down within milliseconds, and assimilation of the deposited material takes longer ($\sim 10\ ms$).

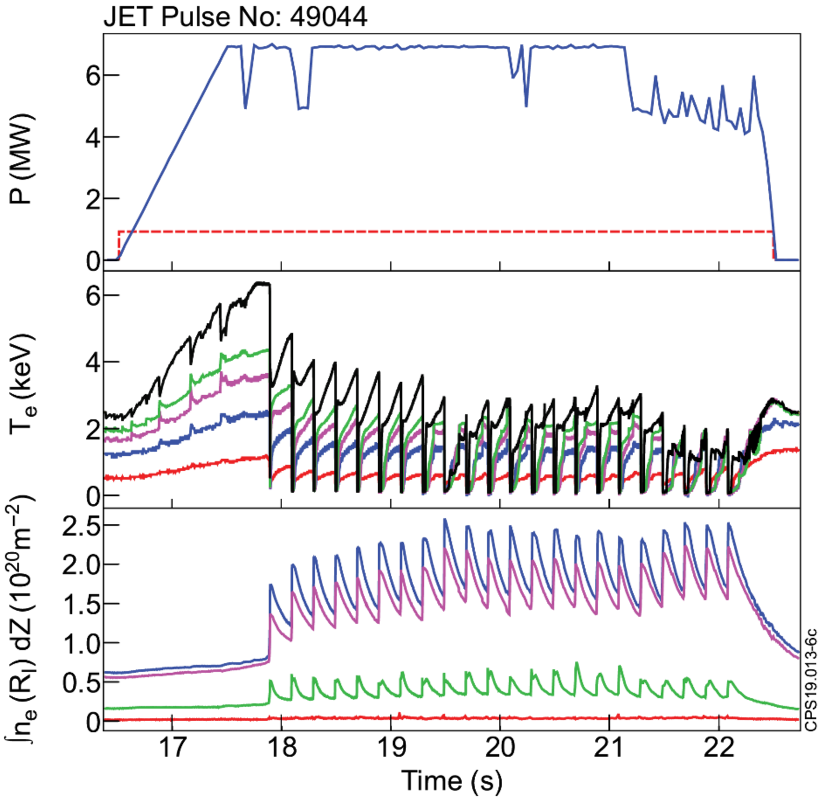
- We examine pellet injected plasmas in JET.



Fuelling pellets injected into ICRH heated plasmas

- 4mm fuelling pellets were injected into JET from the inboard side.
- High ICRH power, $P_{ICRH} \geq 4$ MW, excited toroidal Alfvén eigenmodes (TAEs).

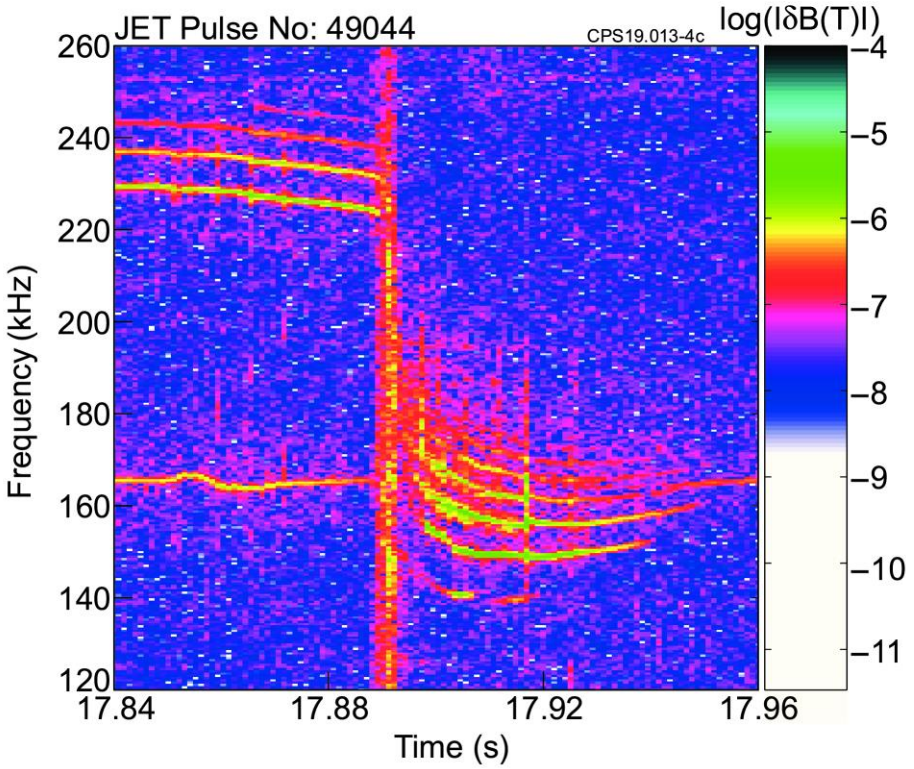


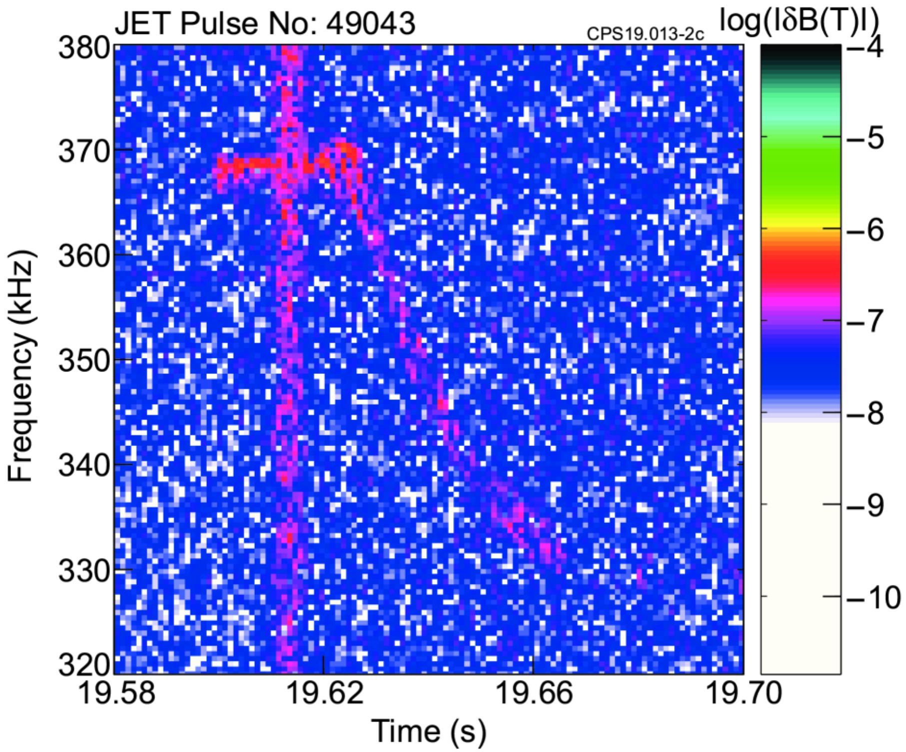


[1] T T C Jones et al, EPS Proceedings, **24B of ECA**, 13-16 (2000)

TAEs are modified by pellet injection

- After a pellet is injected into the plasma, the frequency of a TAE drops by $\Delta f \approx 70\ kHz$ over $\tau \approx 15\ ms$.





[2] S E Sharapov, H J C Oliver, B N Breizman, M Fitzgerald, L Garzotti, and JET contributors, *Nuclear Fusion* **58** 082008 (2018)

3D density inhomogeneities affect TAE coupling

- 3D density inhomogeneities introduce poloidal and toroidal dependence into the Alfvén velocity:

$$v_A(r,\theta,\zeta) \propto \frac{B(r,\theta)}{\sqrt{n_i(r,\theta,\zeta)}}.$$

- Therefore, the refractive index is periodic, coupling poloidal and toroidal harmonics.
- This coupling modifies the Alfvén wave spectrum. Can we use Alfvén waves for MHD spectroscopy of pellet injected plasmas?
- To understand how pellet injection modifies the Alfvén spectrum we introduce 3D density profiles to MHD codes:
  - Stellgap – calculates the Alfvén continuum,
  - AE3D – calculates the Alfvén eigenmodes.
- We compare this numerical work to analytical estimates.

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Obtaining the TAE coupled equations

- We solve the shear Alfvén wave equation:

$$\boldsymbol{\nabla} \cdot \left(\frac{\omega^2}{v_A^2} \nabla \phi\right) + (\boldsymbol{B} \cdot \boldsymbol{\nabla}) \left[\frac{1}{B} \nabla^2 \left(\frac{\boldsymbol{B}}{B} \cdot \boldsymbol{\nabla} \phi\right)\right] = 0$$

Inertia term — Line bending term

- Introduce:
  - Metric tensor with toroidicity and Shafranov shift,
  - Poloidal modulation of the ion density:

$$n_i(r, \theta) = n_{i0}(1 + \delta \cos \theta)$$

$\delta > 0$: modulation peaking on the *low* field side.

$\delta < 0$: modulation peaking on the *high* field side.

Effect of poloidal density asymmetry on TAEs

- Solve the eigenmode equations for the eigenfrequency for an even TAE:

$$\omega = \frac{v_A}{2\,q\,R_0}\left[1 - \frac{1}{2}\left(\varepsilon + \frac{\delta}{2}\right)\left(1 - \frac{\pi^2 S^2}{2}\right)\right]$$

Frequency of gap centre

New coupling term due to poloidal modulation in density

- Two effects can change TAE frequency:
  - Increasing flux surface averaged density decreases $v_A (\propto 1/\sqrt{n_i})$ and the frequency of the gap centre.
  - Increasing size of inhomogeneity δ:
    - decreases the eigenfrequency for LFS ($\delta > 0$), or
    - increases the eigenfrequency for HFS ($\delta < 0$).

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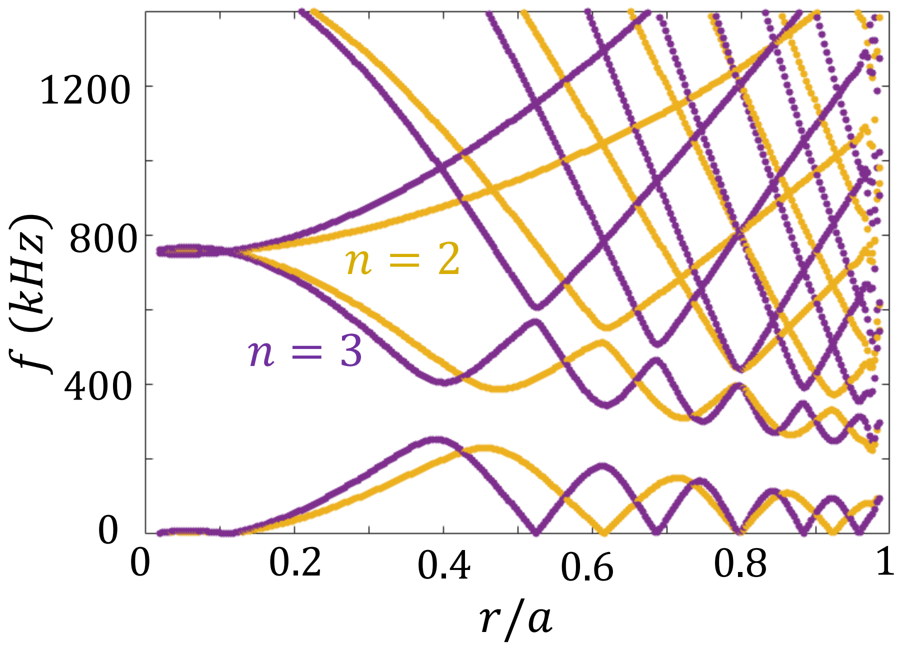
How does density modulation affect continuum?

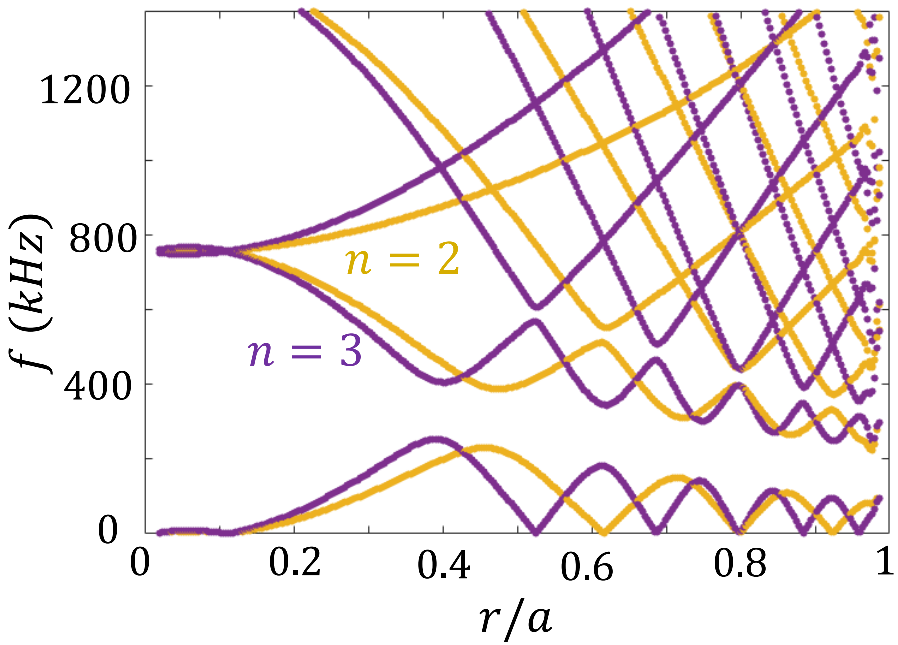
$n = 2$ and $n = 3$ Alfvén continua without density modulation

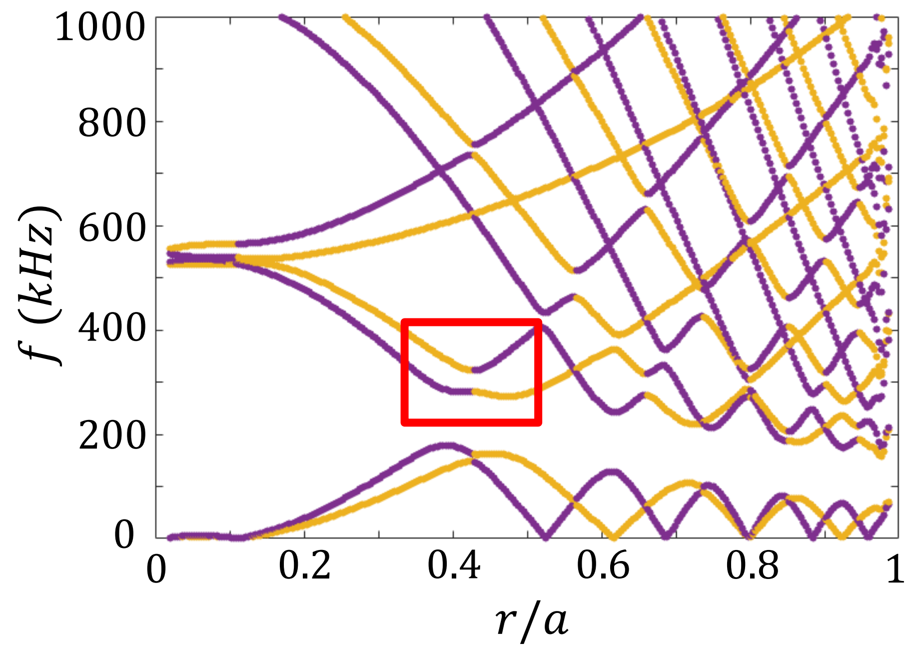


Toroidal density modulation creates new gaps

$$n_i(r,\theta) = n_{i0}\left(1 + {}^{1}\!/_{2}\ \cos\zeta\right)$$

Alfvén continuum without density modulation

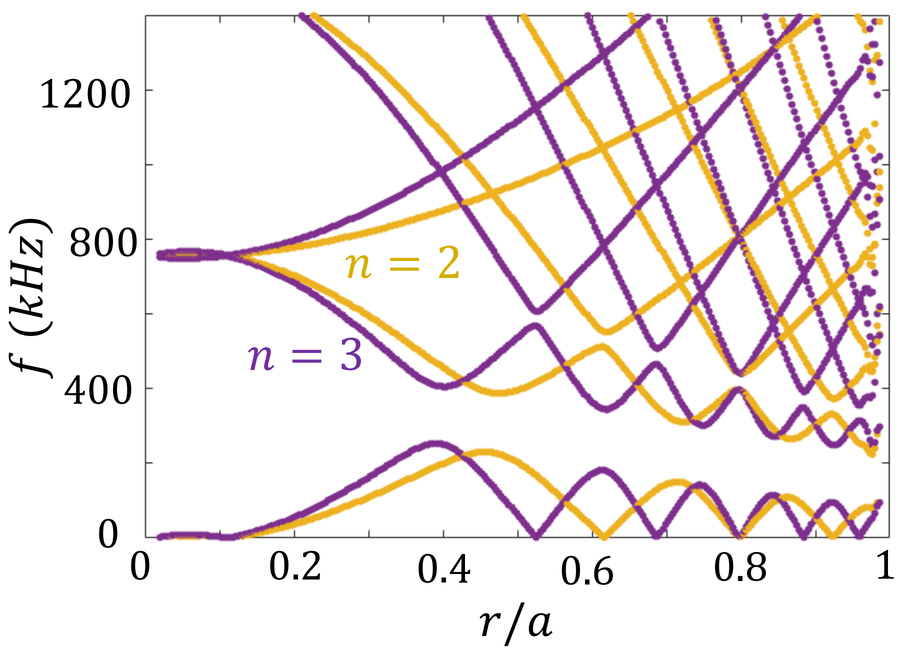


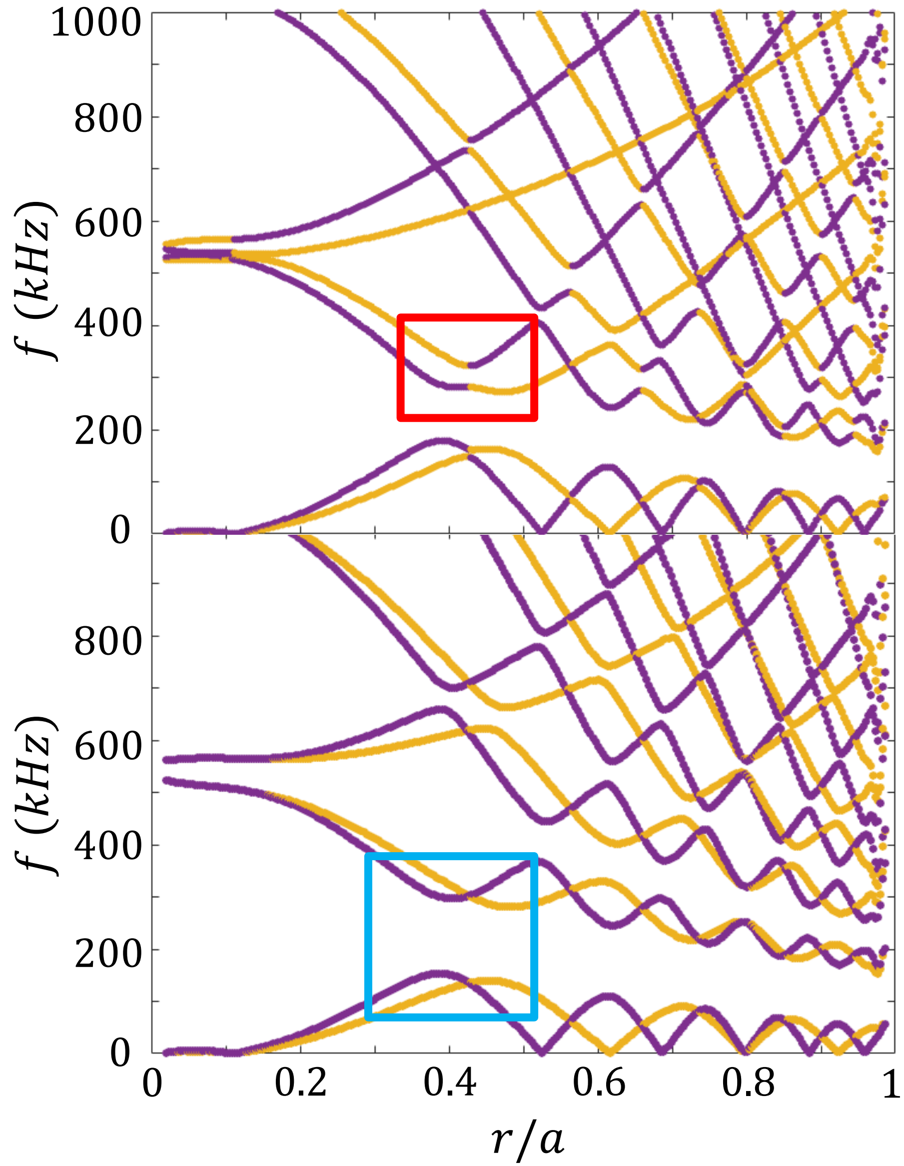


Poloidal density modulation widens existing gaps

$$n_i(r,\theta) = n_{i0}\left(1 + {}^{1}\!/_{2}\ \cos\zeta\right)$$

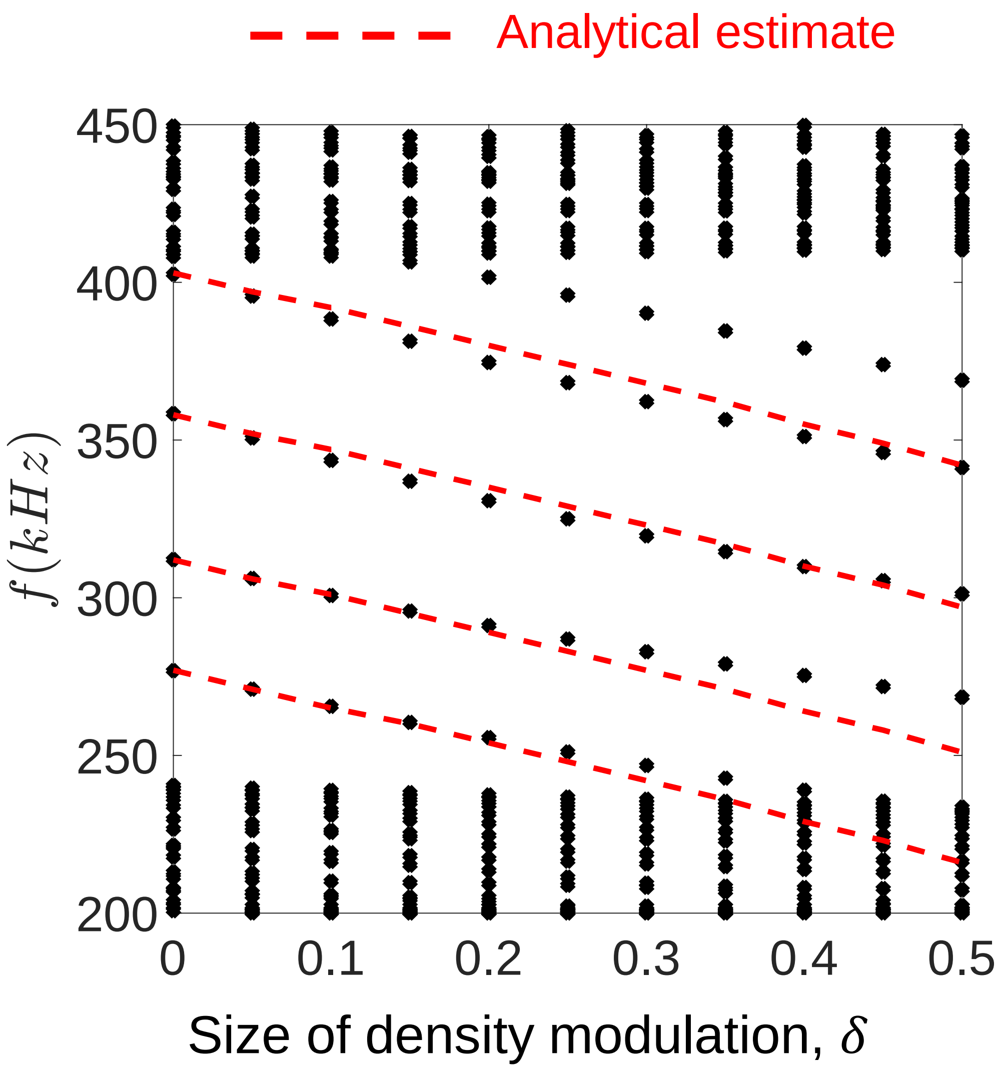
Alfvén continuum without density modulation





$$n_i(r,\theta) = n_{i0}\left(1 + {}^{1}\!/_{2}\ \cos\theta\right)$$

Poloidal density asymmetry reduces eigenfrequency



- All eigenvalues found by AE3D for a plasma with ion density:

$$n_i(r,\theta) = n_{i0}(1 + \delta\cos\theta)$$

- Decrease in frequency due to modulation only, $\langle n_i \rangle_{\zeta,\theta}$ is constant.

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Modelling the expansion of a pellet wake

- As a pellet moves through the plasma, it leaves a wake of overdense material.
- This material then expands along the magnetic field lines.
  - Perpendicular expansion is inhibited by $\boldsymbol{j} \times \boldsymbol{B}$ force.
- Model the 1D expansion of the wake with the fluid equations:

$$\frac{\partial \varrho}{\partial t} + \boldsymbol{\nabla} \cdot (\varrho \boldsymbol{V}) = 0$$

$$\frac{\partial (\varrho \boldsymbol{V})}{\partial t} + \boldsymbol{\nabla} \cdot (\varrho \boldsymbol{V} \boldsymbol{V}) + \nabla p = 0$$

$$\frac{\partial}{\partial t}\left(\varrho \left[\frac{1}{2} V^2 + \epsilon\right]\right) + \boldsymbol{\nabla} \cdot \left(\varrho \boldsymbol{V} \left[\frac{1}{2} V^2 + h\right]\right) = Q$$

Modelling the expansion of a pellet wake

- The wake is heated by thermal electrons that slow down as they deposit energy:

$$Q = -\frac{\partial}{\partial t}\int \frac{1}{2} m_e n_b v^2 f d^3 v$$

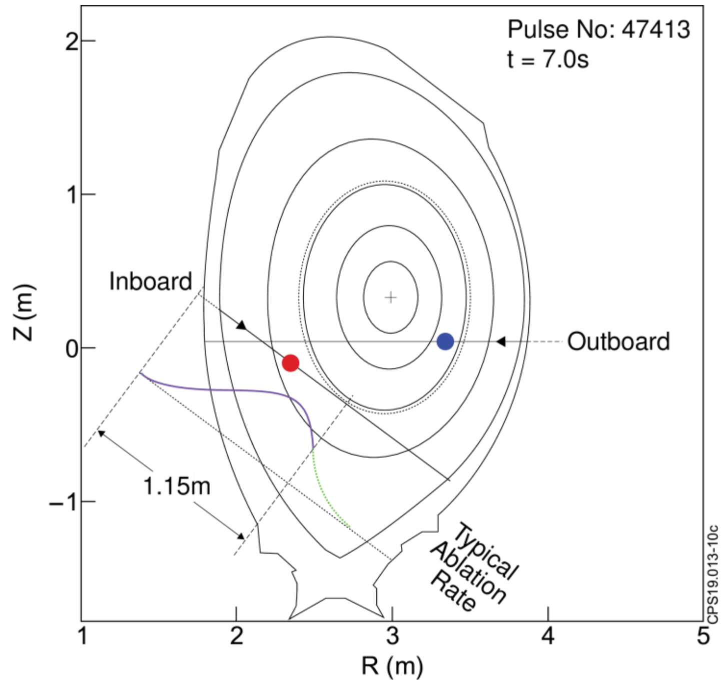
where $f(v,t) = \left(\frac{m_e}{2\pi T_b}\right)^{\frac{3}{2}} \exp\left[-\left(3\,\nu\,t + \frac{v^3}{v_b^3}\right)^{\frac{2}{3}}\right]$.

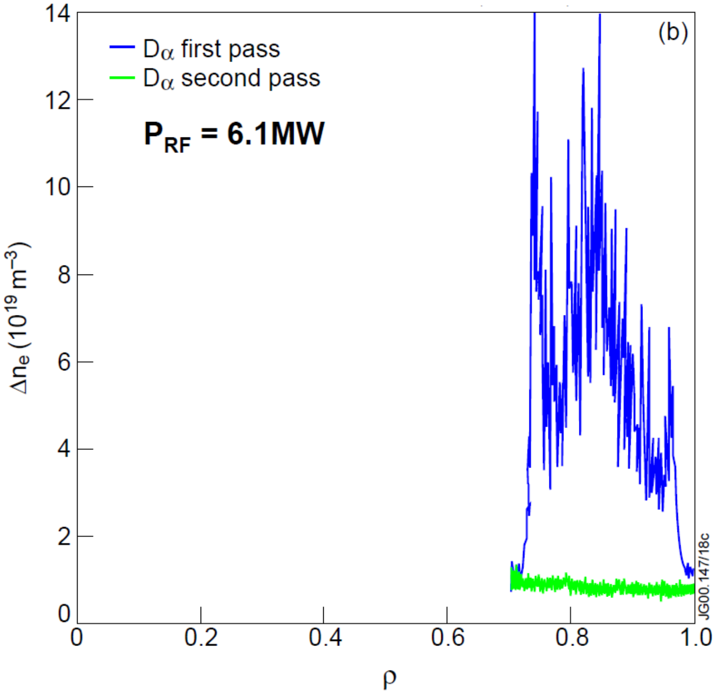
- Mass density is Gaussian: $\varrho(L,t) \propto \sqrt{k(t)}\,\exp(-k(t)\,L^2)$.
- Wake expands linearly: $V(L,t) \propto \Omega(t)\,L$.
- Temperature is constant along wake: $T(L,t) = T(t)$.

→ Three ODEs for $k(t), \Omega(t)$, and $T(t)$ to be solved.

What is the initial density profile, $n_w(r,\theta,\zeta)$?

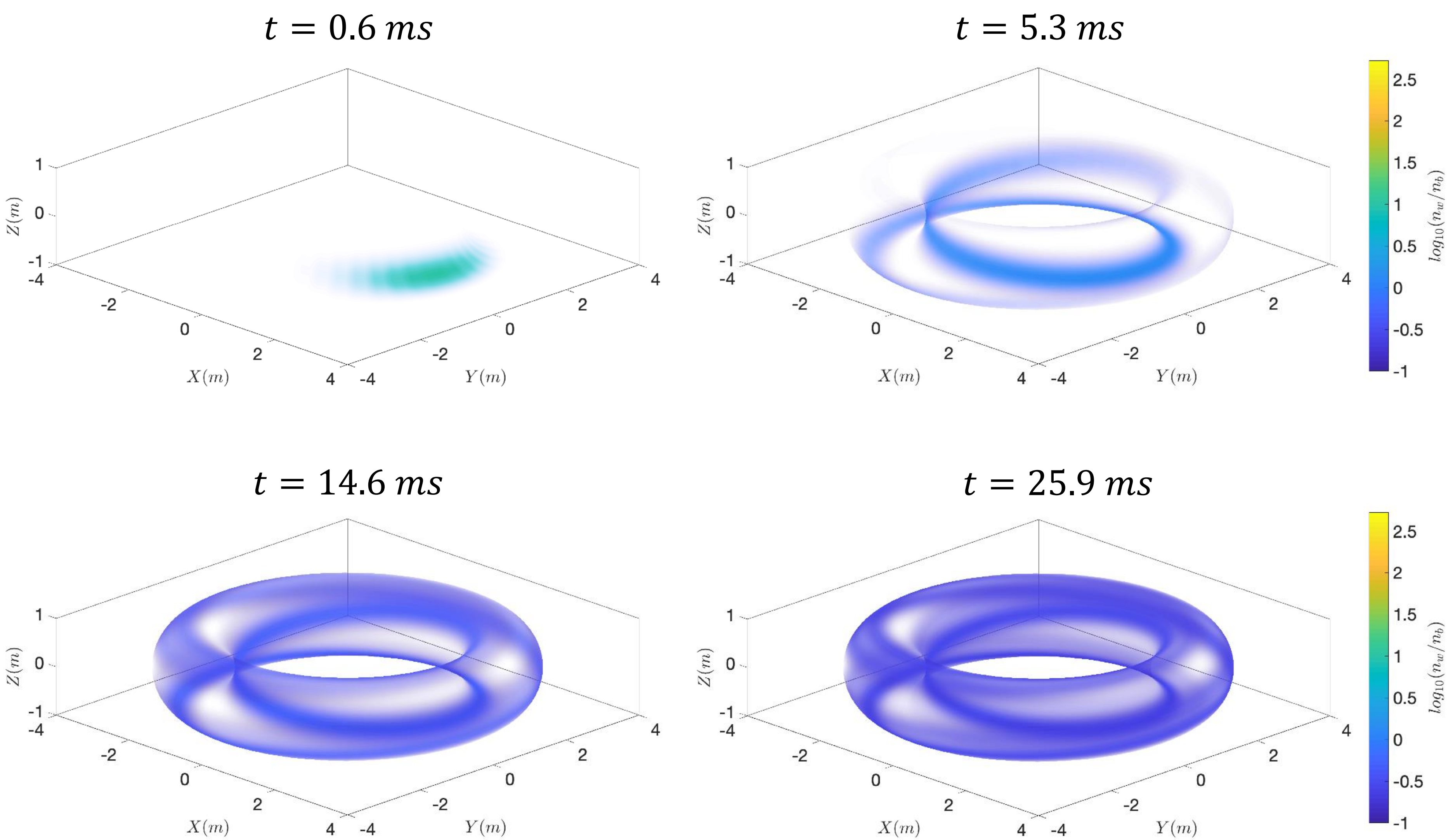
- Previous studies suggest initial length along field line, $L_0 \approx 4$ cm.
- From injection geometry, the poloidal profile is approximately Gaussian with position $\theta_0 \approx {}^{5\pi}/_{4}$ and width $\Delta\theta \approx {}^{\pi}/_{8}$.
- Fit D_α measurements to obtain radial profile.





[1] T T C Jones et al, EPS Proceedings, **24B of ECA**, 13-16 (2000)

The pellet wake expands around the tokamak



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TAE frequency drops as the wake expands

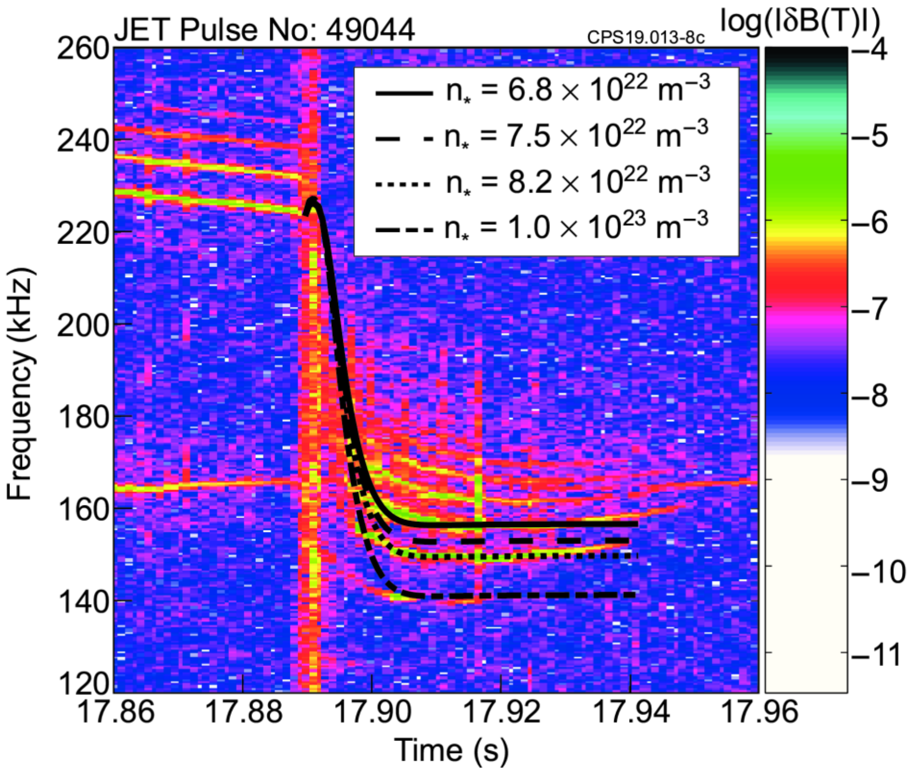
- As the wake expands around the tokamak, the TAE frequency drops.

- The change in frequency depends on the initial density of the wake (n_*).

- By matching the numerical frequency drop of the TAE to the experimental data, we find:

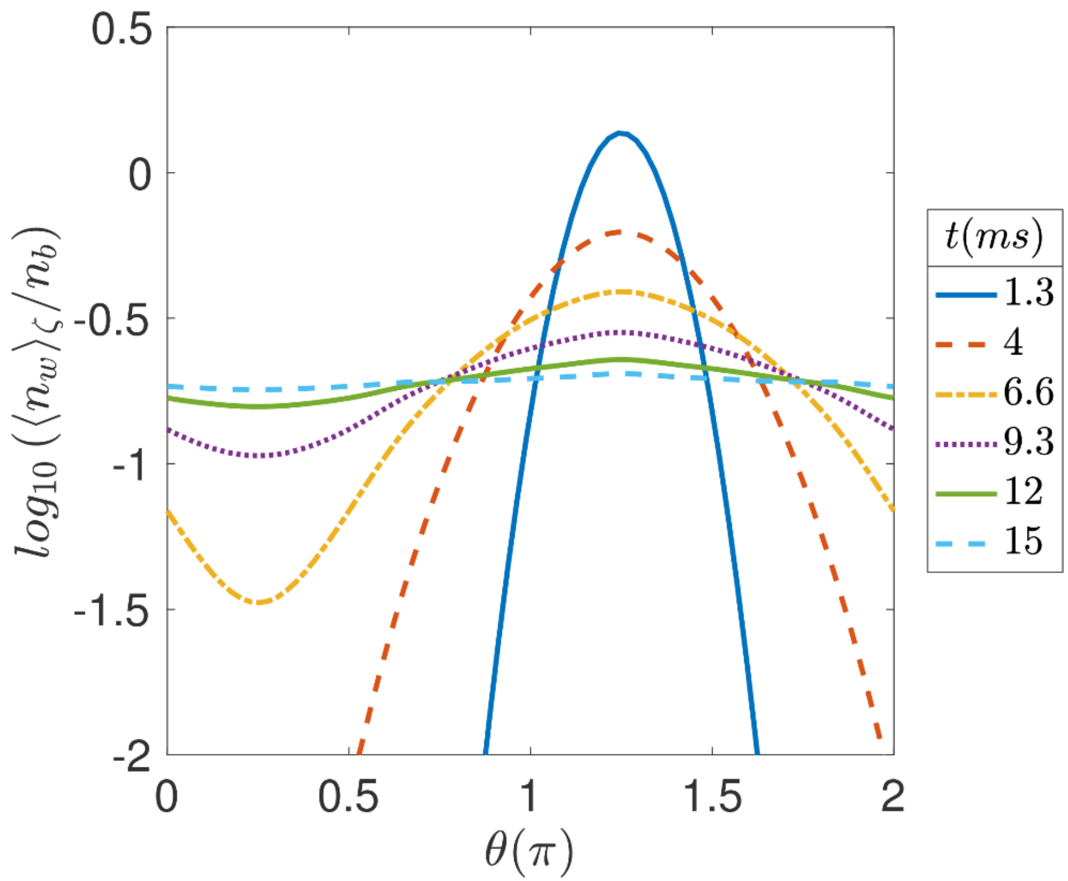
$$n_* = 6.8 \times 10^{22}\ m^{-3}$$

at the TAE location ($r/a \approx 0.75$).



Frequency sweeping ends with density homogenisation

- At $\tau_h \approx 15\, ms$, the wake density is homogeneous in poloidal angle to within 10%.



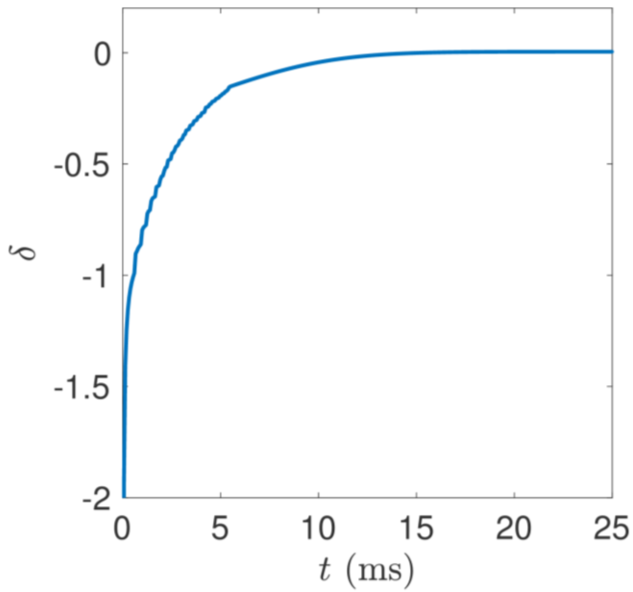
where:

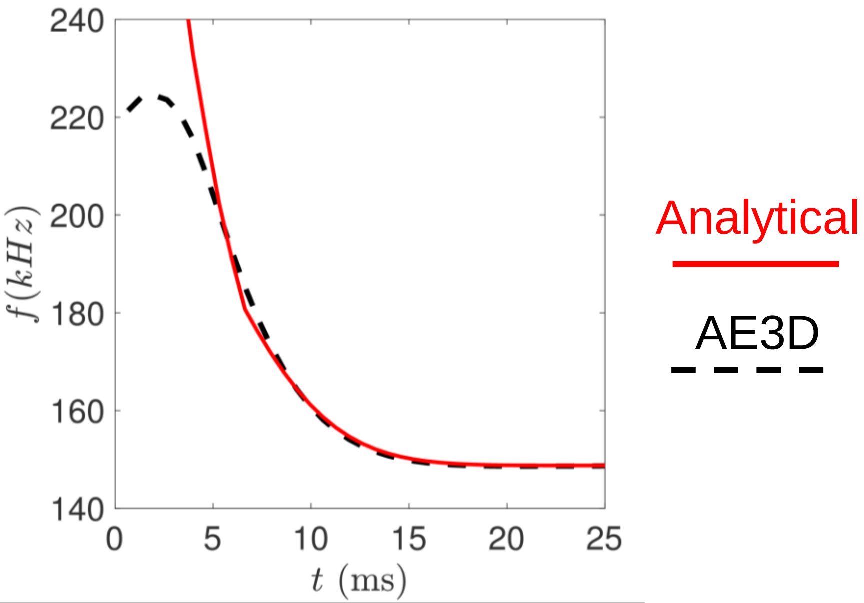
$$\langle n_w \rangle_\zeta = \frac{1}{2\pi} \int_0^{2\,\pi} n_w \, \mathrm{d}\zeta$$

Coupling term slows rapid frequency drop

$$f_{TAE}(t) = \frac{v_A(t)}{4\pi q R_0}\left[1 - \frac{1}{2}\left(\varepsilon + \frac{\delta(t)}{2}\right)\left(1 - \frac{\pi^2 S^2}{2}\right)\right]$$

- The rapid ($\sim ms$) frequency drop due to $v_A(t) \propto \langle n_i(t)\rangle_{\zeta,\theta}^{-1/2}$ is balanced by an increase in frequency due to $\delta(t) < 0$.
- The coupling effect disappears as the wake density homogenises ($\delta \to 0$), so $f_{TAE}(t)$ decreases over $\tau_h \approx 15\, ms$.





Higher frequency AEs are also affected by pellets

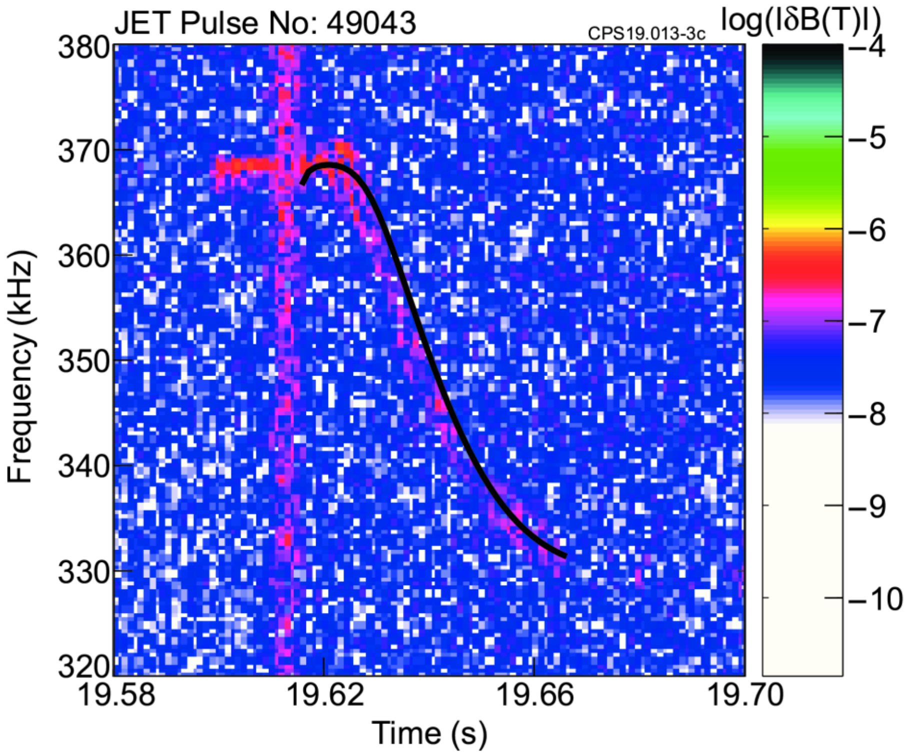
- The frequency of elliptical Alfvén eigenmodes (EAEs) also sweep in response to the expansion of pellet material.

- Repeat the process used for TAEs to find:

  - Initial wake density, $n_* = 4.8 \times 10^{22}\ m^{-3}$.

  - Poloidal homogenisation time, $\tau_h \approx 50\ ms$.

at the EAE location ($r/a \approx 0.90$).



Summary

- 3D density inhomogeneities couple Alfvén eigenmode harmonics.
- A model for the expansion of a pellet wake was derived.
- As a pellet wake expands, the Alfvén eigenfrequencies sweep.
- The initial density of a pellet wake was estimated from the drop in eigenfrequency.
  - $n_* = 6.8 \times 10^{22}\ m^{-3}$ at the TAE location ($r/a \approx 0.75$).
  - $n_* = 4.8 \times 10^{22}\ m^{-3}$ at the EAE location ($r/a \approx 0.90$).
- Frequency sweeping stops when the plasma density is homogeneous in poloidal angle.
  - $\tau_h \approx 15\ ms$ at the TAE location ($r/a \approx 0.75$).
  - $\tau_h \approx 50\ ms$ at the EAE location ($r/a \approx 0.90$).

More details can be found in our paper: H J C Oliver et al 2019 *Nuclear Fusion*

Additional slides

Poloidal inhomogeneity couples poloidal harmonics

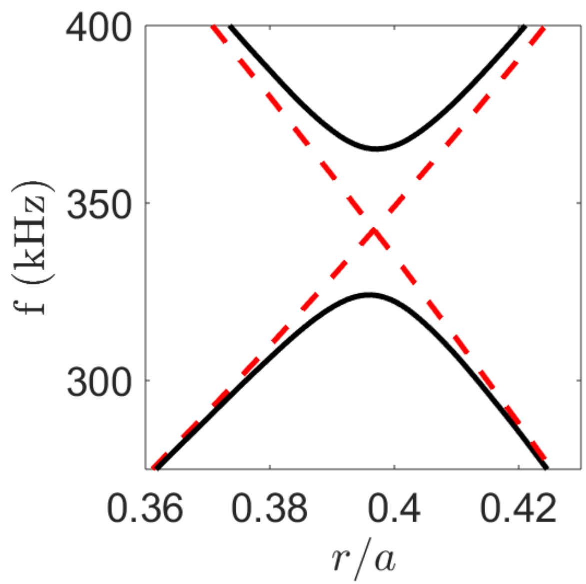
- We obtain two coupled differential equations for the two poloidal harmonics $m, m-1$:

$$\frac{d}{dr}\left[\left(\frac{\omega^2}{v_A^2} - k_{\parallel m}^2\right)\frac{d\phi_m}{dr}\right] - \frac{m^2}{r^2}\left(\frac{\omega^2}{v_A^2} - k_{\parallel m}^2\right)\phi_m + \frac{(2\varepsilon+\delta)}{8q^2R_0^2}\frac{d^2\phi_{m-1}}{dr^2} = 0$$

$$\frac{d}{dr}\left[\left(\frac{\omega^2}{v_A^2} - k_{\parallel m-1}^2\right)\frac{d\phi_{m-1}}{dr}\right] - \frac{(m-1)^2}{r^2}\left(\frac{\omega^2}{v_A^2} - k_{\parallel m-1}^2\right)\phi_{m-1} + \frac{(2\varepsilon+\delta)}{8q^2R_0^2}\frac{d^2\phi_m}{dr^2} = 0$$

- From which we obtain the normalised size of the TAE gap:

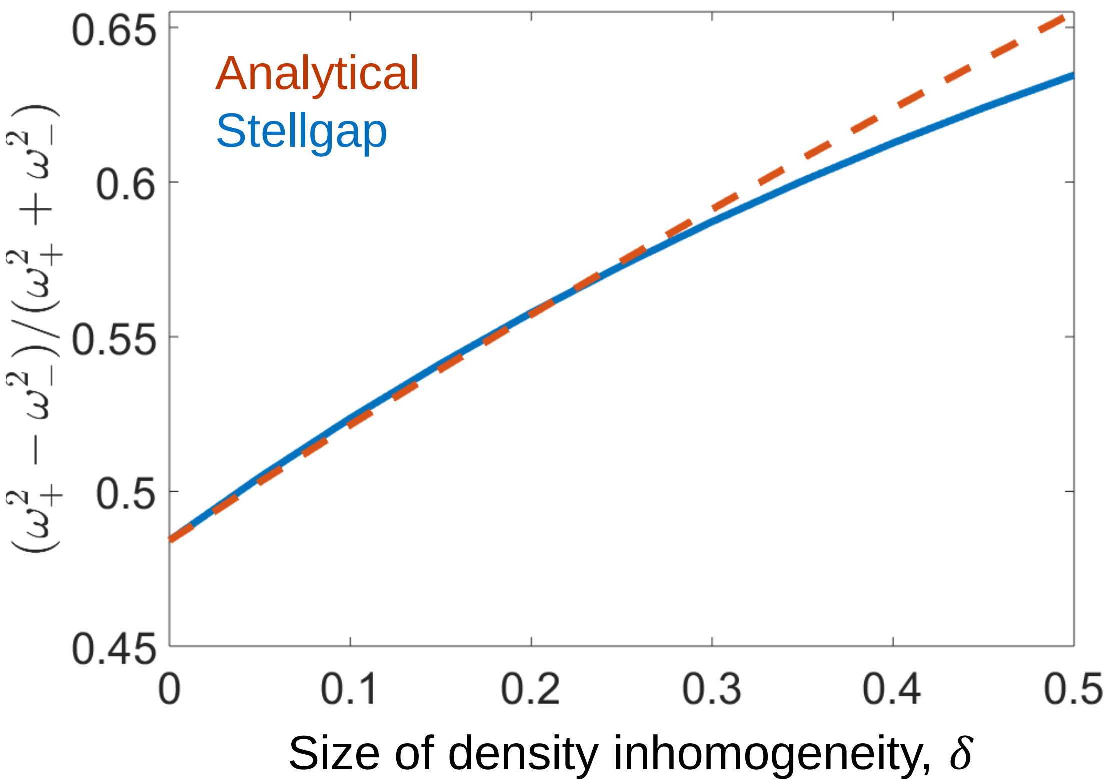
$$\frac{\omega_+^2 - \omega_-^2}{\omega_+^2 + \omega_-^2} \approx \varepsilon + \frac{\delta}{2}$$



Effect of poloidal density modulation on gap size

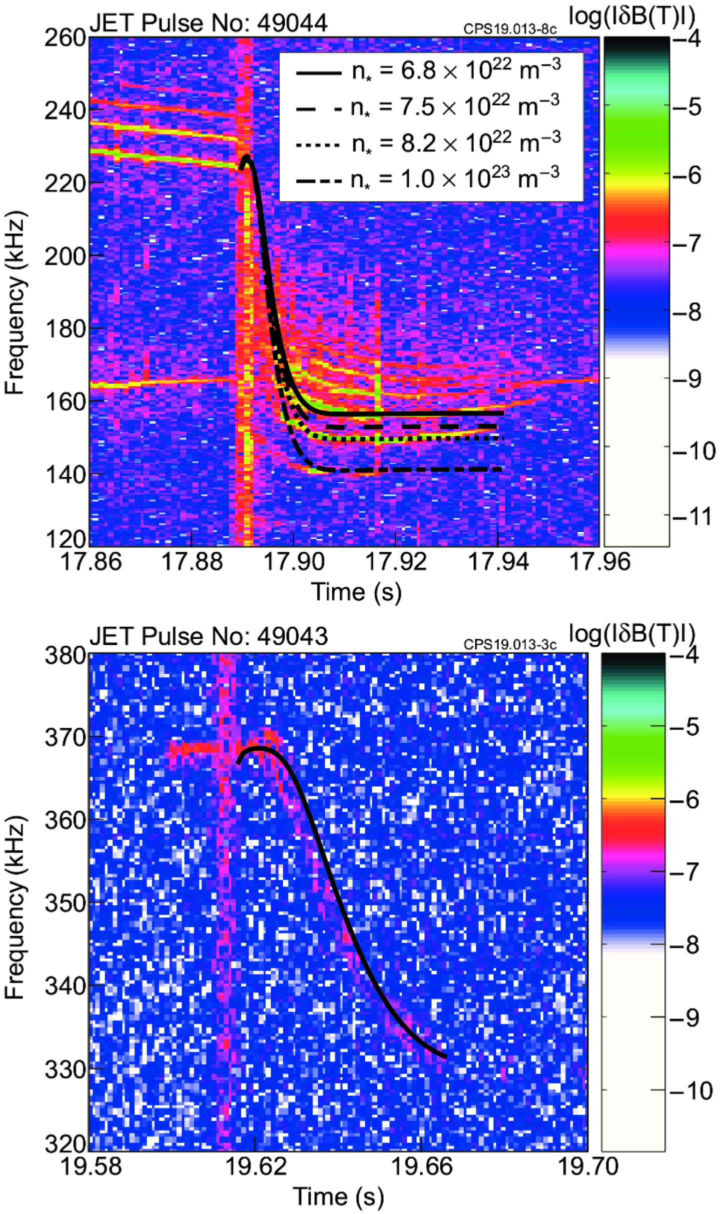
- How does the normalised gap size vary with the size of density modulation?

$$n_i(r,\theta) = n_{i0}(1 + \delta\cos\theta)$$



Why are these parameters different for EAEs?

- The initial wake density at the EAE location is 30% lower than the initial density at the TAE location.
  - The background temperature is lower.
- Poloidal homogenisation is 2.7 times slower at the EAE location.
  - EAEs exist nearer the plasma edge, where q is higher.
  - A wake must expand further at high q.



Fueling is much more efficient with inboard injection

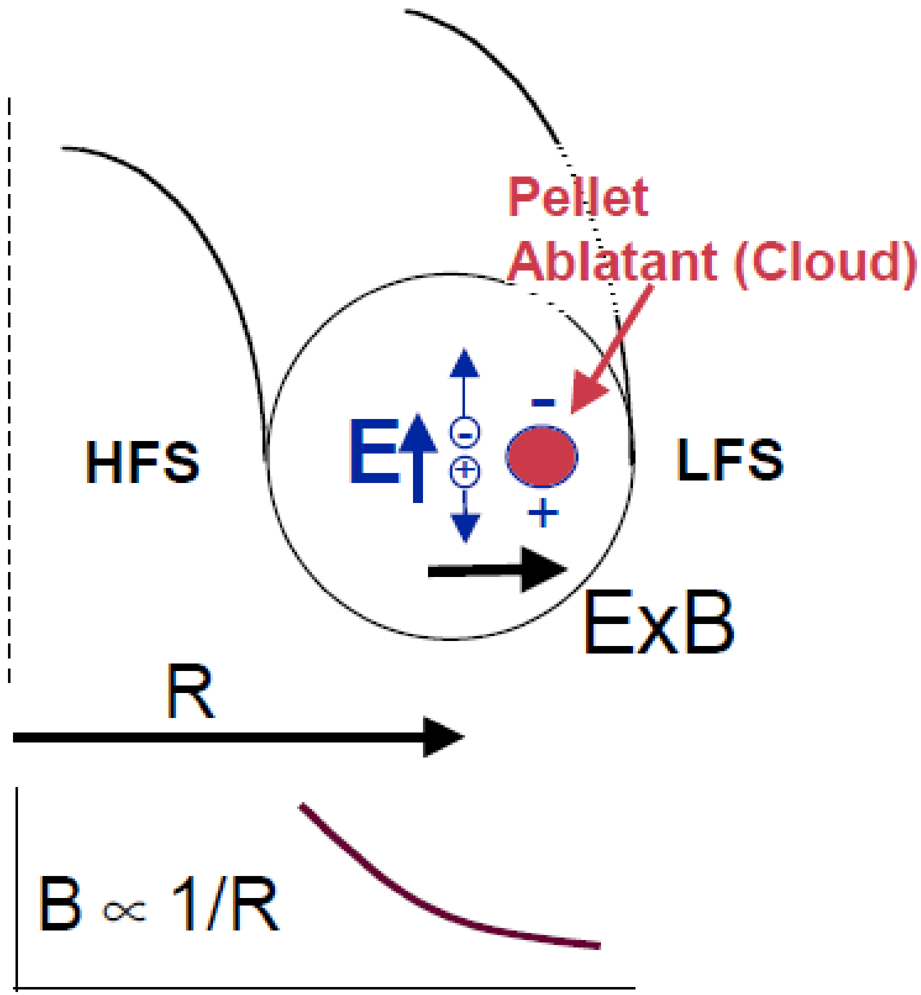
Parameter	Injection track	TAE $\left(\frac{r}{a} \approx 0.75\right)$	EAE $\left(\frac{r}{a} \approx 0.90\right)$
$\tau_h\ (ms)$	Inboard	18 ± 4	48 ± 12
	Outboard	26 ± 2	69 ± 12
$\frac{\Delta f}{f_1}$	Inboard	0.29 ± 0.06	0.11 ± 0.05
	Outboard	0.10 ± 0.01	0.10 ± 0.02

- Poloidal homogenisation of density is 2.7 times faster at the position of TAEs ($^r/_a \approx 0.75$) versus position of EAEs ($^r/_a \approx 0.90$).

- Initial wake density is a factor of 4 higher for inboard pellet injection versus outboard injection at $^r/_a \approx 0.75$.

- There is little difference in initial wake density at $^r/_a \approx 0.90$.

Why is inboard injection more efficient?



T.C. Jernigan

- ∇B drift causes polarisation of the ablatant.
- The resultant electric field produces an $E \times B$ drift towards the low field side of the tokamak.
- For pellets already on the low field side, material is easily lost.