MHD spectroscopy of pellet injected plasmas

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Outline

• Introduction and experimental motivation

• Analytical estimates of the Alfvén spectrum for plasmas with modulated plasma density

• Numerical study of the Alfvén spectrum in plasmas with modulated plasma density

• Model for the expansion of pellet material in a tokamak

• Response of Alfvén eigenmodes to expanding pellet material

• Summary
Pellet injected plasmas are complex

- Pellets of frozen deuterium are injected to fuel the plasma and mitigate dangerous instabilities.
- Varied applications require pellets with a wide range of parameters.
- Pellets usually break down within milliseconds, and assimilation of the deposited material takes longer (∼ 10 ms).
- We examine pellet injected plasmas in JET.
• 4mm fuelling pellets were injected into JET from the inboard side.

• High ICRH power, $P_{ICRH} \geq 4$ MW, excited toroidal Alfvén eigenmodes (TAEs).

TAEs are modified by pellet injection

- After a pellet is injected into the plasma, the frequency of a TAE drops by $\Delta f \approx 70 \text{ kHz}$ over $\tau \approx 15 \text{ ms}$.

3D density inhomogeneities affect TAE coupling

- 3D density inhomogeneities introduce poloidal and toroidal dependence into the Alfvén velocity:

\[ v_A(r, \theta, \zeta) \propto \frac{B(r,\theta)}{\sqrt{n_i(r,\theta,\zeta)}}. \]

- Therefore, the refractive index is periodic, coupling poloidal and toroidal harmonics.

- This coupling modifies the Alfvén wave spectrum. Can we use Alfvén waves for MHD spectroscopy of pellet injected plasmas?

- To understand how pellet injection modifies the Alfvén spectrum we introduce 3D density profiles to MHD codes:
  - Stellgap – calculates the Alfvén continuum,
  - AE3D – calculates the Alfvén eigenmodes.

- We compare this numerical work to analytical estimates.
• Introduction and experimental motivation

• **Analytical estimates of the Alfvén spectrum for plasmas with modulated plasma density**

• Numerical study of the Alfvén spectrum in plasmas with modulated plasma density

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• Summary
Obtaining the TAE coupled equations

- We solve the shear Alfvén wave equation:
  \[
  \nabla \cdot \left( \frac{\omega^2}{v_A^2} \nabla \phi \right) + (\mathbf{B} \cdot \nabla) \left[ \frac{1}{B} \nabla^2 \left( \frac{\mathbf{B}}{B} \cdot \nabla \phi \right) \right] = 0
  \]

  - **Inertia term**
  - **Line bending term**

- Introduce:
  - Metric tensor with toroidicity and Shafranov shift,
  - Poloidal modulation of the ion density:
    \[
    n_i(r, \theta) = n_{i0}(1 + \delta \cos \theta)
    \]

  \(\delta > 0\) : modulation peaking on the low field side.
  \(\delta < 0\) : modulation peaking on the high field side.
Effect of poloidal density asymmetry on TAEs

• Solve the eigenmode equations for the eigenfrequency for an even TAE:

\[

\omega = \frac{\nu_A}{2aqR_0} \left[ 1 - \frac{1}{2} \left( \varepsilon + \frac{\delta}{2} \right) \left( 1 - \frac{\pi^2S^2}{2} \right) \right]

\]

Frequency of gap centre

New coupling term due to poloidal modulation in density

• Two effects can change TAE frequency:

  ▪ Increasing flux surface averaged density decreases \( \nu_A (\propto 1/\sqrt{n_i}) \) and the frequency of the gap centre.

  ▪ Increasing size of inhomogeneity \( \delta \):
    ▪ decreases the eigenfrequency for LFS (\( \delta > 0 \)), or
    ▪ increases the eigenfrequency for HFS (\( \delta < 0 \)).
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How does density modulation affect continuum?

$n = 2$ and $n = 3$ Alfvén continua without density modulation
Toroidal density modulation creates new gaps

\[ n_i(r, \theta) = n_{i0}(1 + \frac{1}{2} \cos \zeta) \]

Alfvén continuum without density modulation
Poloidal density modulation widens existing gaps

\[ n_i(r, \theta) = n_{i0} \left( 1 + \frac{1}{2} \cos \zeta \right) \]

Alfvén continuum without density modulation

\[ n_i(r, \theta) = n_{i0} \left( 1 + \frac{1}{2} \cos \theta \right) \]
Poloidal density asymmetry reduces eigenfrequency

- All eigenvalues found by AE3D for a plasma with ion density:

\[ n_i(r, \theta) = n_{i0}(1 + \delta \cos \theta) \]

- Decrease in frequency due to modulation only, \( \langle n_i \rangle_{\zeta, \theta} \) is constant.
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Modelling the expansion of a pellet wake

- As a pellet moves through the plasma, it leaves a wake of overdense material.

- This material then expands along the magnetic field lines.
  - Perpendicular expansion is inhibited by $j \times B$ force.

- Model the 1D expansion of the wake with the fluid equations:

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 0
\]

\[
\frac{\partial (\rho \mathbf{V})}{\partial t} + \nabla \cdot (\rho \mathbf{V} \mathbf{V}) + \nabla p = 0
\]

\[
\frac{\partial}{\partial t} \left( \rho \left[ \frac{1}{2} V^2 + \epsilon \right] \right) + \nabla \cdot \left( \rho \mathbf{V} \left[ \frac{1}{2} V^2 + h \right] \right) = Q
\]
Modelling the expansion of a pellet wake

• The wake is heated by thermal electrons that slow down as they deposit energy:

\[ Q = -\frac{\partial}{\partial t} \int \frac{1}{2} m_e n_b v^2 f \, d^3v \]

where \( f(v, t) = \left( \frac{m_e}{2\pi T_b} \right)^{\frac{3}{2}} \exp\left[-\left(3 \, v \, t + \frac{v^3}{v_b^3}\right)^{\frac{2}{3}}\right] \).

• Mass density is Gaussian:

\[ \rho(L, t) \propto \sqrt{k(t)} \exp(-k(t) \, L^2). \]

• Wake expands linearly:

\[ V(L, t) \propto \Omega(t) \, L. \]

• Temperature is constant along wake:

\[ T(L, t) = T(t). \]

→ Three ODEs for \( k(t), \Omega(t), \) and \( T(t) \) to be solved.
What is the initial density profile, $n_w(r, \theta, \zeta)$?

- Previous studies suggest initial length along field line, $L_0 \approx 4$ cm.
- From injection geometry, the poloidal profile is approximately Gaussian with position $\theta_0 \approx \frac{5\pi}{4}$ and width $\Delta\theta \approx \frac{\pi}{8}$.
- Fit $D_\alpha$ measurements to obtain radial profile.

The pellet wake expands around the tokamak

$t = 0.6 \text{ ms}$

$t = 5.3 \text{ ms}$

$t = 14.6 \text{ ms}$

$t = 25.9 \text{ ms}$
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TAE frequency drops as the wake expands

- As the wake expands around the tokamak, the TAE frequency drops.

- The change in frequency depends on the initial density of the wake ($n_*$).

- By matching the numerical frequency drop of the TAE to the experimental data, we find:

  \[ n_* = 6.8 \times 10^{22} \text{ m}^{-3} \]

  at the TAE location ($r/a \approx 0.75$).
Frequency sweeping ends with density homogenisation

- At $\tau_h \approx 15 \, ms$, the wake density is homogeneous in poloidal angle to within 10%.

where:

$$\langle n_w \rangle_\zeta = \frac{1}{2\pi} \int_0^{2\pi} n_w \, d\zeta$$
Coupling term slows rapid frequency drop

\[ f_{TAE}(t) = \frac{\nu_A(t)}{4\pi q R_0} \left[ 1 - \frac{1}{2} \left( \epsilon + \frac{\delta(t)}{2} \right) \left( 1 - \frac{\pi^2 S^2}{2} \right) \right] \]

- The rapid (~ ms) frequency drop due to \( \nu_A(t) \propto \langle n_i(t) \rangle_{\zeta, \theta}^{-1/2} \) is balanced by an increase in frequency due to \( \delta(t) < 0 \).
- The coupling effect disappears as the wake density homogenises \( (\delta \to 0) \), so \( f_{TAE}(t) \) decreases over \( \tau_h \approx 15 \text{ ms} \).
Higher frequency AEs are also affected by pellets

- The frequency of elliptical Alfvén eigenmodes (EAEs) also sweep in response to the expansion of pellet material.

- Repeat the process used for TAEs to find:
  - Initial wake density, $n_\ast = 4.8 \times 10^{22} \, m^{-3}$.
  - Poloidal homogenisation time, $\tau_h \approx 50 \, ms$.

at the EAE location ($r/a \approx 0.90$).
Summary

• 3D density inhomogeneities couple Alfvén eigenmode harmonics.

• A model for the expansion of a pellet wake was derived.

• As a pellet wake expands, the Alfvén eigenfrequencies sweep.

• The initial density of a pellet wake was estimated from the drop in eigenfrequency.
  ▪ \( n_\ast = 6.8 \times 10^{22} \, m^{-3} \) at the TAE location \( (r/a \approx 0.75) \).
  ▪ \( n_\ast = 4.8 \times 10^{22} \, m^{-3} \) at the EAE location \( (r/a \approx 0.90) \).

• Frequency sweeping stops when the plasma density is homogeneous in poloidal angle.
  ▪ \( \tau_h \approx 15 \, ms \) at the TAE location \( (r/a \approx 0.75) \).
  ▪ \( \tau_h \approx 50 \, ms \) at the EAE location \( (r/a \approx 0.90) \).

More details can be found in our paper: H J C Oliver et al 2019 Nuclear Fusion
Additional slides
Poloidal inhomogeneity couples poloidal harmonics

- We obtain two coupled differential equations for the two poloidal harmonics $m, m - 1$:

$$
\frac{d}{dr} \left[ \left( \frac{\omega^2}{v_A^2} - k_{\|m}^2 \right) \frac{d\phi_m}{dr} \right] - \frac{m^2}{r^2} \left( \frac{\omega^2}{v_A^2} - k_{\|m}^2 \right) \phi_m + \frac{(2\varepsilon + \delta)}{8q^2R_0^2} \frac{d^2\phi_{m-1}}{dr^2} = 0
$$

$$
\frac{d}{dr} \left[ \left( \frac{\omega^2}{v_A^2} - k_{\|m-1}^2 \right) \frac{d\phi_{m-1}}{dr} \right] - \frac{(m - 1)^2}{r^2} \left( \frac{\omega^2}{v_A^2} - k_{\|m-1}^2 \right) \phi_{m-1} + \frac{(2\varepsilon + \delta)}{8q^2R_0^2} \frac{d^2\phi_m}{dr^2} = 0
$$

- From which we obtain the normalised size of the TAE gap:

$$
\frac{\omega_+^2 - \omega_-^2}{\omega_+^2 + \omega_-^2} \approx \varepsilon + \frac{\delta}{2}
$$
Effect of poloidal density modulation on gap size

- How does the normalised gap size vary with the size of density modulation?

\[ n_i(r, \theta) = n_{i0}(1 + \delta \cos \theta) \]
Why are these parameters different for EAEs?

- The initial wake density at the EAE location is 30% lower than the initial density at the TAE location.
  - The background temperature is lower.
- Poloidal homogenisation is 2.7 times slower at the EAE location.
  - EAEs exist nearer the plasma edge, where $q$ is higher.
  - A wake must expand further at high $q$. 

James Oliver | IAEA-EPPI meeting | 3rd September 2019 | Slide 29
Fueling is much more efficient with inboard injection

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Injection track</th>
<th>TAE ($\frac{r}{a} \approx 0.75$)</th>
<th>EAE ($\frac{r}{a} \approx 0.90$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau_h$ ($ms$)</td>
<td>Inboard</td>
<td>18 ± 4</td>
<td>48 ± 12</td>
</tr>
<tr>
<td></td>
<td>Outboard</td>
<td>26 ± 2</td>
<td>69 ± 12</td>
</tr>
<tr>
<td>$\frac{\Delta f}{f_1}$</td>
<td>Inboard</td>
<td>0.29 ± 0.06</td>
<td>0.11 ± 0.05</td>
</tr>
<tr>
<td></td>
<td>Outboard</td>
<td>0.10 ± 0.01</td>
<td>0.10 ± 0.02</td>
</tr>
</tbody>
</table>

- Poloidal homogenisation of density is 2.7 times faster at the position of TAEs ($\frac{r}{a} \approx 0.75$) versus position of EAEs ($\frac{r}{a} \approx 0.90$).
- Initial wake density is a factor of 4 higher for inboard pellet injection versus outboard injection at $\frac{r}{a} \approx 0.75$.
- There is little difference in initial wake density at $\frac{r}{a} \approx 0.90$. 
Why is inboard injection more efficient?

- $\nabla B$ drift causes polarisation of the ablatant.

- The resultant electric field produces an $E \times B$ drift towards the low field side of the tokamak.

- For pellets already on the low field side, material is easily lost.

T.C. Jernigan