Quasi-periodic frequency sweeping in electron cyclotron emission of mirror-confined plasma sustained by high-power microwaves

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Outline

• Motivation
• Dedicated experiments on ECRH-driven instabilities
• Application of quasi-linear model to many observed features of stimulated emission in ECR plasma
• Beyond the quasi-linear theory
• SMIS-37 experimental facility
• Chirping microwave emission during plasma decay stage
• Application of the Berk-Breizman model to the experimental data
• Summary
Plasma instabilities due to fast particles

Chorus emissions in the magnetosphere of the Earth
THEMIS D spacecraft (raising tone)

THEMIS E spacecraft (falling tone)


Alfven eigenmodes

S.E. Sharapov et al., Nucl. Fusion, 53, 104022, 2013
Frequency sweeping
Toroidal Alfven Eigenmodes

Tokamaks:
- ASDEX-Upgrade
- MAST
- JET
- DIII-D
- NSTX
- JT-60U

Stellarator TJ-II

Frequency range: 30-300 kHz

(a) Magnetic spectrogram of NBI-driven Alfven instabilities in JT-60U discharge E36379;
(b) Mirnov coil signal
Dedicated experiments on ECRH-driven instabilities

- Broadband oscilloscopes (up to 60 GHz / 160 GSa/s) allow direct recording of E(t)

**SMIS-37 (IAP)**
100 kW @ 37 GHz

**JYFL Ion Source (Univ Jyvaskyla)**
250 W @ 11-14 GHz

**GDT (Budker Inst)**
800 kW @ 54.5 GHz
“Zoo” in dynamical spectrum of mw emission

SMIS-37 (IAP)

JYFL Ion Source (Uni Jyvaskyla)

GDT (Budker Inst)
Two lessons that we have learned

- Lesson 1: Quasi-linear theory for maser instability
- Lesson 2: Hole & clump dynamics in ECE
Wave-particle resonance in a magnetic mirror

Action-angle formalism

\[
H_0 = I_\perp \overline{\omega}_c (I_\perp, I_\parallel) + \frac{1}{2} I_\parallel \omega_b (I_\perp, I_\parallel)
\]

\[
\begin{align*}
\frac{d \xi_\perp}{d t} &= \overline{\omega}_c, & I_\perp &= \frac{m v_\perp^2}{2B(l)} \\
\frac{d \xi_\parallel}{d t} &= \omega_b, & I_\parallel &= \int \nu_\parallel d l
\end{align*}
\]

\[
\overline{\omega}_c = \frac{1}{\omega_b} \int \frac{\omega_c}{\nu_\parallel} d l, \quad \omega_b = \int \frac{d l}{\nu_\parallel}
\]

\[
H = H_0 + \frac{e}{mc} \mathbf{A} \cdot \mathbf{p} = H_0 + \text{Re} \left[ C(t) \sum_{s,n} V_{sn} \exp(is \xi_\perp + in \xi_\parallel - i \omega_0 t) \right]
\]

\[
s \overline{\omega}_c + n \omega_b = \omega_0
\]

Cyclotron harmonics

Bounce resonances, for electrons are usually overlapped resulting in global diffusion in phase-space (quasilinear theory)
Quasi-linear diffusion in a magnetic mirror

\[
H = H_0 + \frac{e}{mc} \mathbf{A} \cdot \mathbf{p} = H_0 + \text{Re} \left[ C(t) \sum_{s,n} V_{sn} \exp(is\xi_\perp + in\xi_\parallel - i\omega_0 t) \right]
\]

1) Interaction with waves conserves \( K = mc^2(\gamma - 1) - I_\perp \omega_0 = \text{const} \)
then consider \( (I_\perp, I_\parallel) \rightarrow (K, \kappa) \), i.e. one-dimensional distribution function \( F(t, \kappa) \)

2) Self-consistent electromagnetic field (fixed mode) \( |C(t)|^2 \rightarrow E(t), \ V_{sm} \rightarrow D, \ K \)

3) Describes both ECR heating and maser instability!
Quasi-linear model describes many observed features of stimulated emission in simple terms

- X-mode emission at the start-up phase
- Z-mode emission in rarefied decaying plasma
- Whistler waves during the stationary ECR


- Excitation of plasma waves under the double-plasma-resonance


- Stochastic grouping of ECE bursts in a decaying plasma

  Shalashov et al. PPCF 54 085023 (2012)

- Fast electron losses at GDT


- Controlled transition between periodic and CW regimes

  Shalashov et al. PRL 120 155001 (2018)
  
  JYFL ECRIS
  
  Shalashov et al. EPL. 24 35001 (2018)

- Stabilization of burst activity by two-frequency ECRH


  theory is still not finished!
Beyond the quasi-linear theory

- Fast periodic frequency sweeps in ECE discovered at SMIS-37
- Observed at fixed lines in rare plasma after ECRH switch-off
- No precipitations of fast electrons
- Low power compared to other inst.

Experimental setup

- Gyrotron
- Diagnostic chamber
- Simple mirror trap
- Input window
Experimental setup

plasma

microwave pump

Unstable distributions of hot electrons

microwave emission in electron cyclotron frequency range

mirror ratio $R = \frac{4}{10}$

trap length $\sim 25$ cm

magnetic field up to 4.3 T

80 kW @ 37.5 GHz, 1 ms

80 kW @ 37.5 GHz, 1 ms

Unstable distributions of hot electrons

1) Background plasma
   $T_e \sim 300$ eV
   $N_e \sim 10^{13} - 10^{14}$ cm$^{-3}$

2) Hot electrons
   $T_h \sim 10$ keV
   $N_h \sim 10^{10} - 10^{11}$ cm$^{-3}$

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The synchronization scheme of the experimental setup

- ECRH pulse
- Gas valve current
- Magnetic field strength at the trap center

Time, ms

-8 -6 -4 -2 0 2 4 6 8
The synchronization scheme of the experimental setup

- ECRH pulse
- Magnetic field strength at the trap center
- Gas valve current
Plasma parameters

**Rarefied plasma**
- \( \omega_{pe} / \omega_{ce} \ll 1 \)
- \( N_h < 10^{10} \, \text{cm}^{-3} \)
- \( N_h \geq N_c \)
- \( E_h \sim 100 \div 300 \, \text{keV} \)

**Dense plasma**
- \( \omega_{pe} / \omega_{ce} > 1 \)
- \( N_h \sim 10^{11} \, \text{cm}^{-3} \)
- \( N_c \sim 10^{13} \, \text{cm}^{-3} \)
- \( N_h \ll N_c \)
- \( E_h \sim 10 \, \text{keV} \)
- \( T_c \sim 300 \, \text{eV} \)

**Cold plasma density**

**Rarefied (decaying) plasma**
- \( \omega_{pe} / \omega_{ce} \ll 1 \)
- \( N_h \sim 10^{11} \, \text{cm}^{-3} \)
- \( E_h \leq 10 \, \text{keV} \)
- \( N_h \sim N_c \)
- \( T_c \sim 1 \, \text{eV} \)

ECR heating switch-off

Microwave diagnostics of plasma emission

Broadband horn antenna 2-20 GHz, input aperture 104x78 mm² + low-pass filter 24.660 GHz (30dB rejection frequency)

Broadband oscilloscope KeySight DSA-Z 594A

- 4 channels, analog bandwidth 33 GHz, sampling rate 80 GSa/s (up to 25 ms)
- 2 channels, analog bandwidth 59 GHz, sampling rate 160 GSa/s (up to 12.5 ms)
- Up to 2 billions samples per channel
- Maximum temporal resolution 6.25 ps (160 GSa/s)
Overview of plasma microwave emission

Plasma microwave emission during decay stage
Fine structure of the microwave emission spectrum
Fine structure of the microwave emission spectrum (2)
Fine structure of the microwave emission spectrum (3)
Transition from chirping to steady state emission

The microwave emission frequency bands

• Open cylindrical inserts lead to efficient mode selection of the central discharge chamber.
• Only those modes for which the cutoff frequencies of the cylindrical inserts modes with the same azimuthal number are higher than the fundamental frequency of the central section remain high-Q.
Noise-like emission during plasma decay stage
Noise-like emission during plasma decay stage

discharge chamber
Noise-like emission during plasma decay stage: seeds of narrow-band mw-emission.
Return to basic wave-particle resonance in a magnetic mirror (at the fundamental ECR)

\[ \Omega(I_\perp, I_\parallel) = \bar{\omega}_c + n\omega_b = \omega_0 \]

For very fast electrons bounce resonances do not overlap,

\[ \omega_b / \bar{\omega}_c \sim 1/30 \]
\[ \omega_b >> v_{\text{eff}}, \Delta\omega \]

then we consider one separate resonance \( n \)

\[ H = H_0 + \frac{e}{mc} A \cdot p = H_0 + \Re \left[ C(t) \sum_n V_n \exp(i\xi_\perp + in\xi_\parallel - i\omega_0 t) \right] \]

\[ \frac{\partial f}{\partial t} + \Omega \frac{\partial f}{\partial \xi} - \Re \left[ iV_n C(t) e^{i\xi - i\omega_0 t} \right] \frac{\partial \Omega}{\partial I_\perp} \frac{\partial f}{\partial \Omega} = \mathcal{S} t f. \]

Effective one-dimensional non-linear problem, \( \xi \) is the interaction phase with the field, canonical momentum \( \Omega \) has the same meaning as \( \kappa \) in the quasi-linear theory.
Self-consistent description of waves and particles

\[ E = -\frac{1}{c} \frac{\partial A}{\partial t} \approx \frac{\omega_0}{c} \text{Re} [i A_0(z, r_\perp) C(t) \exp(-i \omega_0 t)] \]

\[ \frac{dC}{dt} = -\frac{2\pi i \omega_0}{c} e^{i \omega_0 t} \int A_0^\dagger \cdot j \, d^3 r - \gamma_d C. \]

\[ H = H_0 + \frac{e}{mc} A \cdot p = H_0 + \text{Re} \left[ C(t) \sum_n V_n \exp(i \xi_\perp + i n \xi_\parallel - i \omega_0 t) \right] \]

Formally it is equivalent to the electrostatic Berk-Breizman problem → saves our efforts!
Simplest Berk-Brezman problem

Inverse non-linear Landau damping changes to "hole & clump" instability when $St f$ and $\gamma_d$ become essential

$$\frac{\partial f}{\partial t} + v \frac{\partial f}{\partial x} + \frac{e}{m} E_x \cos(kx - \omega_0 t) \frac{\partial f}{\partial \nu} = St f$$

$$\frac{\partial E_x}{\partial t} = -4\pi e \int (f - f_0) \nu d\nu - \gamma_d E_x$$

Fig. 3. Particle distribution function with holes and clumps. (a) The spatially averaged distribution as a function of time and $\Omega - \omega_0$. (b) A gray-scale image of the distribution function in phase space at $\gamma_1 t = 120$. White corresponds to the smallest values of $f$ and black the largest values. The original resonance is located in the gray area at the mid-line. The islands, corresponding to holes and clumps, are also gray although they are surrounded by other shades of the ambient phase space fluid.

Berk, Breizman, Pekker, PRL 76 1256 (1996)
How to apply the model to real experiments

1) Initial unstable distribution function → quasi-linear plateau formed during ECRH

\[ F = F_0 \exp \left( -\frac{\mathcal{K}(I_\perp, I_\parallel)}{T_e} \right) \begin{cases} 1 & \text{for } \gamma < \gamma^* \\ 0 & \text{for } \gamma > \gamma^* \end{cases} \]

\[ \gamma_L = 4\pi^3 \sigma \omega_0 \int \delta(\Omega - \omega_0) |\bar{V}_n|^2 \frac{\partial F}{\partial \mathcal{K}} \, d\mathcal{K} \, dI_\parallel \]

2) Bounce-resonance harmonic

\[ \Omega(I_\perp, I_\parallel) = \omega_0 \]

\[ \mathcal{K}(I_\perp, I_\parallel) = 0 \]

\[ \varepsilon < \varepsilon^* \]

\[ \gamma_L > 0 \]

3) Wave dissipation → Q-factor of the vacuum chamber (adjusted)

4) Collision integral for fast electrons → background plasma (adjusted)

5) Initial wave amplitude → thermal equilibrium with hot electrons
Modeling of experimental spectra with BOT code

Nitrogen

Argon

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Initial guess</th>
<th>After adjustment</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma_L$</td>
<td>$3.6 \times 10^7$ s$^{-1}$</td>
<td>$7.0 \times 10^7$ s$^{-1}$</td>
</tr>
<tr>
<td>$\gamma_d$</td>
<td>$3.45 \times 10^7$ s$^{-1}$</td>
<td>$6.44 \times 10^7$ s$^{-1}$</td>
</tr>
<tr>
<td>$\nu_{\text{diff}}$</td>
<td>$5.3 \times 10^6$ s$^{-1}$</td>
<td>$7.8 \times 10^6$ s$^{-1}$</td>
</tr>
<tr>
<td>$\nu_{\text{drag}}$</td>
<td>$6.3 \times 10^4$ s$^{-1}$</td>
<td>$9.0 \times 10^6$ s$^{-1}$</td>
</tr>
</tbody>
</table>

BOT code: Lilley M K 2011, code.google.com/p/bump-on-tail

Automatic selection of chirping events
Analysis of the experimental data: the Berk-Breizman model

Frequency change in the wave packet:

\[ \delta \omega \approx \frac{16\sqrt{2}}{3\sqrt{3\pi^2}} \gamma_L \sqrt{\gamma_d t} \equiv \sqrt{At} \quad \Rightarrow \quad \gamma_L^2 \gamma_d \approx 5A \]


Instability condition:

\[ \gamma_L \geq \gamma_d \quad \Rightarrow \quad \gamma_L = \gamma_d \approx 3\sqrt{5A} \]
Analysis of the experimental data: the Berk-Breizman model

\[ \delta \omega \approx \sqrt{At} \]
Analysis of the experimental data: the Berk-Breizman model

\[ \delta\omega \approx \sqrt{At} \]

\[ A = (0.4 - 2) \times 10^{21} \text{ s}^{-3} \]

\[ \gamma_L = (1 - 2.5) \times 10^7 \text{ s}^{-1} \]
Conclusions

- The chirping frequency patterns in the plasma EC emission, which are very similar to those predicted by the Berk-Breizman model, were observed during the plasma decay stage, characterized by a high relative density of the fast electrons.

- To explain the features of plasma emission spectra we formulate a self-consistent kinetic equation for the distribution of resonant electrons and an equation for complex amplitudes of unstable modes excited under combined (cyclotron and bounce) resonance in an inhomogeneous plasma.

- We show that the problem of disturbing electromagnetic modes in an inhomogeneous plasma with a mirror magnetic field configuration can be reduced to equations describing a much simpler situation of exciting electrostatic oscillations in a homogeneous plasma under conditions of nonlinear Landau damping, which are described in the well-known Berk-Breizman model.

- Under the conditions of our experiment a case may occur where these combined resonances do not overlap.

- Similarities to nature phenomena: Earth magnetosphere, solar flares.

Thank you for your attention!

ご清聴ありがとうございました