Analytical estimation of drift-orbit island-width for passing ions in static magnetic perturbation

ID: P2-22

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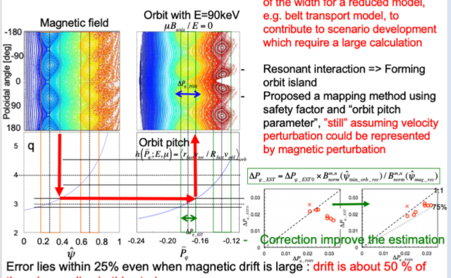
Motivation (Previous work)

K. Shinohara et al NF 2018 082026



- Width of orbit island can be a scale length of transport
- Useful if we have estimation method of the width for a reduced model, e.g. belt transport model, to contribute to scenario development which require a large calculation

Resonant interaction => Forming orbit island

Proposed a mapping method using safety factor and "orbit pitch parameter", "still" assuming velocity perturbation could be represented by magnetic perturbation

Error lies within 25% even when magnetic drift is large : drift is about 50 % of the minor radius in this study

- Deficits in previous mapping method:
  - Assumption would be broken in large drift orbit
  - Need calculations to make magnetic Poincare plot and some calculations for the mapping

Overcome

- In this work,
  - Direct estimation of $w(\hat{P}_{\varphi 0\,res})$, using newly derived analytic expression
  - Developed approximate(fast) calculation method to evaluate the analytic expression

Resonant condition for static magnetic perturbation $\omega_{tor}/\omega_{pol}$ represent orbit pitch of particle motion

Resonant condition: $\omega = n\omega_{tor} - m_{orb}\omega_{pol}=0$ for static field

=> $\omega_{tor}/\omega_{pol} = m_{orb}/n$.

$\omega_{tor}(C) = \oint_{orb}\frac{v_{tor}}{Rv_{pol}}dl_{pol}/\tau_{pol}$ and $\omega_{pol}(C) = 2\pi/\tau_{pol}$, where $C = (P_{\varphi 0}, E, \mu, \mathrm{sign}(v_{//}))$

Orbit pitch — Field pitch

$$h(C) \equiv \omega_{tor}/\omega_{pol} = \oint_{orb}\frac{v_{tor\,0}}{Rv_{pol\,0}}\frac{dl_{pol}}{2\pi} \quad \Longleftrightarrow \quad q(\psi_{p0}) = \oint_{\psi_p}\frac{B_{\varphi 0}}{RB_{\theta mag\,0}}\frac{dl_{\theta mag}}{2\pi}$$

(θ_{mag} is simply θ below)

- Resonance can be discussed by the analogy with MHD, using h
  - $h = m_{orb}/n \Leftrightarrow q = m/n$
- q is function of only ψ_p, but h is function of four phase space parameters, $P_\varphi, E, \mu, \mathrm{sign}(v_{//})$

Note:
- Hereafter, $\hat{P}_\varphi = -P_\varphi/Ze = \psi_p - \frac{M}{Ze}Rv_{tor}$

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Want to have approximate expression for orbit equivalent to that for magnetic island

Familiar expression for magnetic field island (in text books)

$$w(\psi_{p\,res}) = 4\sqrt{\frac{q(\psi_{p\,res})}{q'(\psi_{p\,res})}\delta\psi_p^{m,n}}$$

$$w(\rho_{res}) \approx 4\sqrt{\frac{\rho_{res}q(\rho_{res})}{mdq/d\rho|_{\rho_{res}}}\frac{\delta B_{norm}}{B_{0\theta}}}$$

Starting point is

$$\boldsymbol{B} = \nabla\psi_{t\,0}\times\nabla\theta_{mag} + \nabla\varphi\times\nabla\psi_p(\psi_{t\,0}, \theta_{mag}, \varphi)$$

$$\frac{d\psi_{t\,0}}{d\varphi} = \frac{\boldsymbol{B}\cdot\nabla\psi_{t\,0}}{\boldsymbol{B}\cdot\nabla\varphi} = -\frac{\partial\psi_p}{\partial\theta_{mag}}$$

$$\frac{d\theta_{mag}}{d\varphi} = \frac{\boldsymbol{B}\cdot\nabla\theta_{mag}}{\boldsymbol{B}\cdot\nabla\varphi} = \frac{\partial\psi_p}{\partial\psi_{t\,0}} = \frac{1}{q}$$

We have found equivalent forms as below

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Orbit coordinate

A.H. Boozer, PF(1984)2441 tells guiding-center velocity vector is expressed:

with $\boldsymbol{v} = \alpha\boldsymbol{H}$

$\alpha \equiv \frac{\boldsymbol{v}\cdot\boldsymbol{b}}{\boldsymbol{H}\cdot\boldsymbol{b}}$, $\boldsymbol{H} \equiv \nabla\times\boldsymbol{A}^*$, $\boldsymbol{A}^* \equiv \boldsymbol{A} + \rho_{//}\boldsymbol{B}$, $\rho_{//} \equiv \frac{Mv_{//}}{ZeB}$,

$\boldsymbol{b}$: unit vector of magnetic field $\boldsymbol{B}$, B: amplitude of $\boldsymbol{B}$, $\boldsymbol{A}$: vector potential which satisfy $\boldsymbol{B} \equiv \nabla\times\boldsymbol{A}$

In axisymmetric system, particles with a same canonical coordinate, C, lie on a surface, where $P_{\varphi 0}$ = const.

- ⇒ Can set a coordinate on the surface using $\nabla\varphi$ and $\nabla\theta_{orb}$, where φ and θ_{orb} are a toroidal angle and poloidal angle on the surface
- ⇒ Can take $hd\theta_{orb} = d\varphi$ like PEST-I $\theta_{orb} = \frac{1}{h}\int\frac{v_{tor\,0}}{Rv_{pol\,0}}dl_{pol}$
- ⇒ Can describe a particle position using a coordinate $(s, \theta_{orb}, \varphi)$, where s is surface label
- ⇒ Then $\boldsymbol{H} = T'(s)\nabla s\times\nabla\theta_{orb} + P'(s)\nabla\varphi\times\nabla s$, "$\square'$" indicates derivative w.r.t s
- ⇒ Vector potential corresponding to this $\boldsymbol{H}$ is $\boldsymbol{A}^* = T\nabla\theta_{orb} - P\nabla\varphi$
- ⇒ Realize P is $\hat{P}_\varphi$ and T can be $\hat{P}_{\theta_{orb}}$
- ⇒ Finally, $\boldsymbol{H}_0 = \nabla\hat{P}_{\theta_{orb}\,0}\times\nabla\theta_{orb} + \nabla\varphi\times\nabla\hat{P}_{\varphi\,0}(\hat{P}_{\theta_{orb}\,0}, \theta_{orb}, \varphi)$

In same form with $\boldsymbol{B}_0 = \nabla\psi_{t\,0}\times\nabla\theta_{mag} + \nabla\varphi\times\nabla\psi_{p\,0}(\psi_{t\,0}, \theta_{mag}, \varphi)$

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Derivation of width of drift-orbit island

- The form for $\boldsymbol{B}$ can be held for perturbation when div$\boldsymbol{B}$= 0 [A.H. Boozer, PF (1983) 1288] => The $\boldsymbol{H}$ is same since div$\boldsymbol{H}$=0.

$$\boldsymbol{H} = \nabla\hat{P}_{\theta_{orb}\,0}\times\nabla\theta_{orb} + \nabla\varphi\times\nabla\hat{P}_\varphi(\hat{P}_{\theta_{orb}\,0}, \theta_{orb}, \varphi)$$

- Drift orbit is described by

$$\frac{d\hat{P}_{\theta_{orb}0}}{d\varphi} = \frac{\boldsymbol{H}\cdot\nabla\hat{P}_{\theta_{orb}\,0}}{\boldsymbol{H}\cdot\nabla\varphi} = -\frac{\partial\hat{P}_\varphi}{\partial\theta_{orb}},$$

$$\frac{d\theta_{orb}}{d\varphi} = \frac{\boldsymbol{H}\cdot\nabla\theta_{orb}}{\boldsymbol{H}\cdot\nabla\varphi} = \frac{\partial\hat{P}_\varphi}{\partial\hat{P}_{\theta_{orb}\,0}} = \frac{1}{h}$$

- For small perturbed orbit $\hat{P}_\varphi = \int d\hat{P}_{\theta_{orb}\,0}\frac{1}{h} + \delta\hat{P}_\varphi^{\,m_{orb},n}\cos(n\varphi - m_{orb}\theta_{orb})$,

$$\frac{d\hat{P}_{\theta_{orb}}}{d\varphi} = -\delta\hat{P}_\varphi^{\,m_{orb},n}m_{orb}\sin(n\varphi - m_{orb}\theta_{orb})$$

- Then, we have

$$\frac{nh(\hat{P}_{\varphi 0}) - m_{orb}}{\delta\hat{P}_\varphi^{\,m_{orb},n}}d\hat{P}_{\varphi 0} = -m_{orb}\sin(Q)dQ,$$

where $Q \equiv n\varphi - m_{orb}\theta_{orb}$:phase in a island

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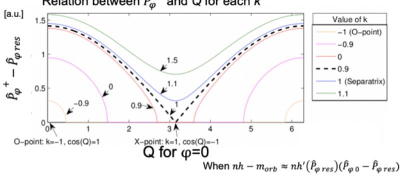
Derivation of width of drift-orbit island

- Integrating this around resonance position of $\hat{P}_{\varphi\,res}$ where $h(\hat{P}_{\varphi\,res}) = m_{orb}/n$,

we have $\int_{\hat{P}_{\varphi\,res}}^{\hat{P}_\varphi^{\pm}}\frac{nh(\hat{P}_{\varphi 0}) - m_{orb}}{m_{orb}\delta\hat{P}_\varphi^{\,m_{orb},n}}d\hat{P}_{\varphi 0} = \cos(Q) + k$

Relation between $\hat{P}_\varphi^{\pm}$ and Q for each k



- Width of the island, $w(\hat{P}_{\varphi\,res}) = \hat{P}_\varphi^{+M} - \hat{P}_\varphi^{-M}$, is obtained by setting k = 1(separatrix) and $\cos(Q) = 1$(Q=0,2π for O-point), namely solving equations

$$2 = \int_{\hat{P}_{\varphi\,res}}^{\hat{P}_\varphi^{\pm M}}\frac{nh - m_{orb}}{m_{orb}\delta\hat{P}_\varphi^{\,m_{orb},n}}d\hat{P}_{\varphi 0}$$

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Practical expression of $\delta\hat{P}_\varphi^{\,m_{orb},n}$

The perturbation amplitude $\delta\hat{P}_\varphi^{\,m_{orb},n}$ is related to the magnitude of the component of the drift velocity normal to the nonperturbed orbit surface as follows:

$$H_{norm} = \frac{\boldsymbol{H}\cdot\nabla\hat{P}_{\theta_{orb}\,0}}{|\nabla\hat{P}_{\theta_{orb}\,0}|} = \frac{\frac{\partial\hat{P}_\varphi}{\partial\theta}}{\mathcal{J}_{orb}|\nabla\hat{P}_{\theta_{orb}\,0}|}$$, where $1/\mathcal{J}_{orb} = \nabla\hat{P}_{\theta_{orb}\,0}\cdot(\nabla\theta_{orb}\times\nabla\varphi) = \boldsymbol{H}_0\cdot\nabla\varphi$ and $H_0 = H_{0\theta_{orb}} + H_{0\varphi}\frac{\nabla\varphi}{|\nabla\varphi|}$, $H_{0\theta_{orb}} = \frac{1}{R}\nabla\hat{P}_{\varphi\,0}\times\frac{\nabla\varphi}{|\nabla\varphi|}$.

$$\frac{\partial\hat{P}_\varphi}{\partial\theta} = \frac{|\nabla\hat{P}_{\theta_{orb}0}|H_{norm}}{\boldsymbol{H}_0\cdot\nabla\varphi} = \frac{hR^2H_{0\theta_{orb}}H_{norm}}{H_{0\varphi}}$$

$$\delta\hat{P}_\varphi^{\,m_{orb},n} = \frac{D^{m_{orb},n}}{m_{orb}}$$

$D^{m_{orb},n}$ is a Fourier transformed component of $D(\theta_{orb},\varphi) \equiv \frac{hR^2H_{0\theta_{orb}}H_{norm}}{H_{0\varphi}}$

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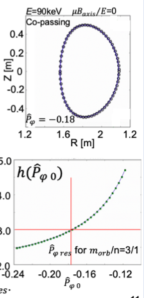
Implementation

Our purpose is to evaluate orbit island width with small calculation



- Approximate $\delta\hat{P}_\varphi^{\,m_{orb},n}$ is constant around resonant surface

$$2 \approx \frac{1}{\delta\hat{P}_\varphi^{\,m_{orb},n}}\int_{\hat{P}_{\varphi\,res}}^{\hat{P}_\varphi^{\pm M}}\frac{nh - m_{orb}}{m_{orb}}d\hat{P}_{\varphi 0}$$

- Avoid orbit calculation to define coordinate: Orbit surface is well approximated by

$$\psi_{p\,0}^{\pm} = \hat{P}_{\varphi\,0} \pm \sqrt{2MER(R_0 - R_{tip})}/Ze$$

with $R_{tip} \equiv R_{axis}\mu B_{0\,axis}/E$
[K. Shinohara et al NF 2018 082026]

- Set 100 points on 2-D contour line for a $\hat{P}_{\varphi\,0}$. On each point, evaluate $v_{tor\,0}$ and $v_{pol\,0}$ (in the axisymmetric system) using $\boldsymbol{v}_0 = \alpha\boldsymbol{H}_0$ with $v_{//\,0} = \pm 2E/M\sqrt{1 - R_{tip}/R_0}$
- Calculate orbit pitch h for the $\hat{P}_{\varphi\,0}$. repeat such calculations for different $\hat{P}_{\varphi\,0}$ and obtain profile of $h(\hat{P}_{\varphi\,0})$ => Can evaluate position of resonance, $\hat{P}_{\varphi\,res}$.

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Implementation (cont.)

- On resonant surface with the $\hat{P}_{\varphi\,res}$, we establish poloidal angle coordinate by $\theta_{orb} = \frac{1}{h}\int\frac{v_{tor\,0}}{Rv_{pol\,0}}dl_{pol}$
- On coordinate grids (360x360), we can have $H_{norm}(\theta_{orb},\varphi) = \boldsymbol{H}\cdot\nabla\hat{P}_{\varphi\,0}/|\nabla\hat{P}_{\varphi\,0}|$, where $\boldsymbol{H}(\equiv\nabla\times\boldsymbol{A}^*)$ is perturbed by magnetic perturbation through $\boldsymbol{A}^* \equiv \boldsymbol{A} + \rho_{//}\boldsymbol{B}$.
- We can have

$$\delta\hat{P}_\varphi^{\,m_{orb},n} = \frac{D^{m_{orb},n}}{m_{orb}}$$

($D^{m_{orb},n}$: a Fourier transformed component of $D(\theta_{orb},\varphi) \equiv \frac{hR^2H_{0\theta_{orb}}H_{norm}}{H_{0\varphi}}$) to solve

$$2 \approx \frac{1}{\delta\hat{P}_\varphi^{\,m_{orb},n}}\int_{\hat{P}_{\varphi\,res}}^{\hat{P}_\varphi^{\pm M}}\frac{nh - m_{orb}}{m_{orb}}d\hat{P}_{\varphi 0}$$

In the case of B

$\delta\psi_p^{\,m,n} = \frac{D_B^{\,m,n}}{m}$ for a Fourier transformed component of $D_B(\theta,\varphi) \equiv \frac{qR^2B_{0\theta}B_{norm}}{B_{0\varphi}}$

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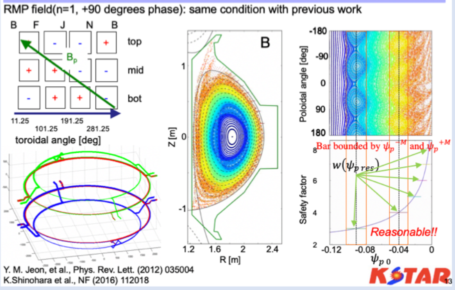
Application
Magnetic island width in vacuum field approximation

RMP field(n=1, +90 degrees phase): same condition with previous work



Y. M. Jeon, et al., Phys. Rev. Lett. (2012) 035004
K.Shinohara et al., NF (2016) 112018

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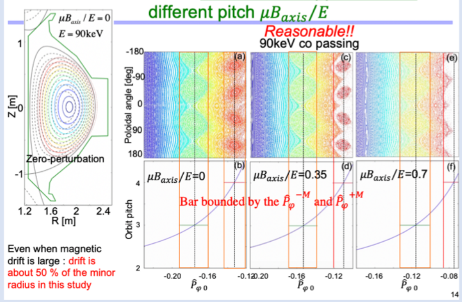
Application
Orbit island width in vacuum field approximation for different pitch $\mu B_{axis}/E$



Even when magnetic drift is large : drift is about 50 % of the minor radius in this study

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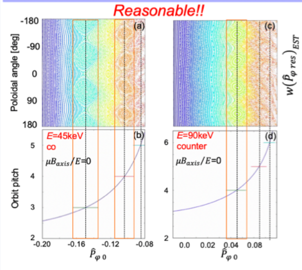
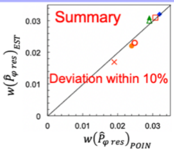
Application
Orbit island width in vacuum field approximation for different Energy or Traveling direction



Summary



Symbol	Direction of $v_{//}$	E [keV]	$\frac{\mu B_{axis}}{E}$	m_{orb}
▲	co	90	0	3
●	co	90	0	4
□	co	90	0.35	3
◆	co	90	0.7	3
△	co	45	0	3
○	co	45	0	4
×	ctr	90	0	4

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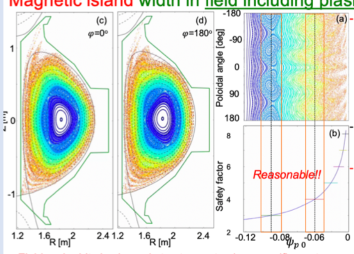
Application
Magnetic island width in field including plasma response



- Field is calculated by HINT code [Y. Suzuki, PPCF (2017) 054008]: Solve equilibrium with helical current flow at resonant surfaces Plasma shape and island shape are skewed
- Skewed structure has helical characteristic and rotate in poloidal direction when we move in toroidal direction (same for orbit)
- Field and orbits in skewed structures stay in a specific contour ("well-defined") though skewed structures move in and out in the $\hat{P}_{\varphi 0}$ and $\psi_{p\,0}$ coordinate defined in the axisymmetric system.
- Particles are not transported practically. => Might ignore the structure in the transport

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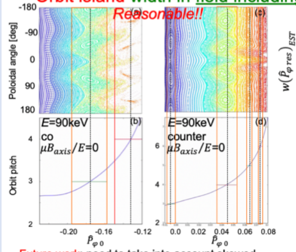
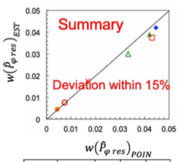
Application
Orbit island width in field including plasma response



Summary



Symbol	Direction of $v_{//}$	E [keV]	$\frac{\mu B_{axis}}{E}$	m_{orb}
▲	co	90	0	3
□	co	90	0.35	3
◆	co	90	0.7	3
△	co	45	0	3
○	ctr	90	0	3
×	ctr	90	0	4
●	ctr	90	0	5

- Future work: need to take into account skewed structure when we evaluate heat, current, particle and torque drive in the axisymmetric plasma we stand on as a base of our analysis

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Summary

- Newly derived analytic expression for direct estimation of drift-orbit island width, $w(\hat{P}_{\varphi\,res})$
- Developed approximate(fast) calculation method to evaluate the analytic expression — Overcome deficits in previous work
- Results:
  - Improved error: 10% for a field of vaccum approximation (for large magnetic drift) <= (25% previous method)
  - For newly applied self-consistent 3D field by HINT: 15%
  - Can provide an indicator to evaluate an overlapping threshold for chaotic orbits in the same way with the magnetic perturbation

Orbit pitch parameter can be used to know position of resonance for finite ω and for banana particles

- Resonant condition for finite ω $\quad \omega = n\omega_{tor} - m_{orb}\omega_{pol}$

=> $(\omega_{tor} - \omega/n)/\omega_{pol} = m_{orb}/n$

=> $\frac{m_{orb}}{n} = h - \omega/n\omega_{pol} \equiv h_\omega$

- We can have h_ω with a small calculation for a given frequency and n by our procedure.

- In the case of banana particles, ω_{tor} is toroidal precession frequency. Then, $2\pi h$ is toroidal precession angle for bounce time,$\tau_{pol} = 2\pi/\omega_{pol}$

- Useful to avoide large beam transport by choosing beams depositing the resonance retion. (Change experimental plan in a short time or Apply to real time control)

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