Effects of spatial channeling on the structure of Alfvén eigenmodes

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Outline

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   and its manifestations

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1. Phenomenon of Spatial Channeling (SC) and its manifestations

Phenomenon of Spatial Channeling

SC is the energy and momentum transfer across the magnetic field by destabilized eigenmodes .

Particles driving an instability $\Rightarrow$ Eigenmodes $\Rightarrow$ Bulk plasma

Typically, the region of emission (Region 1) does not coincide with the region of absorption (Region 2).

<u>Region 1</u>
Emission of the waves

<u>Region 2</u>
Absorption of the waves

The energy and momentum received by modes in unstable Region 1 are transferred to electrons / ions in stable Region 2.

1. Ya.Kolesnichenko, Yu.Yakovenko, V. Lutsenko, Phys. Rev. Lett. **104** (2010) 075001

2. Ya.Kolesnichenko, Yu.Yakovenko, V. Lutsenko, R. White, A. Weller, Nucl. Fusion **50** (2010) 084011.

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Manifestations of SC

- ❖ **Degradation of plasma energy confinement (Outward SC)**
- ❖ **Improvement of plasma energy confinement (Inward SC)**
- ❖ **Plasma rotation and frequency chirping**
- ❖ **Modification of mode structure**
- ❖ **Curving of the wave front**

Outward SC, i.e., energy and momentum transfer by the modes from the plasma core to the periphery.

Presumably [1-3], it could deteriorate the energy confinement time in the NSTX experiment [4] where the increase of the NBI power a factor of three (from 2 MW to 6 MW) did not increase the central temperature, and high frequency instabilities (0.5–1.1 MHz) were observed.

It could play a role in the Wendelstein 7-AS experiment where an NBI-driven low frequency, 40-70 kHz, bursting instability occurred, leading to thermal crashes. Momentum transfer during SC may explain the observed frequency chirping [2].

1. Ya.Kolesnichenko, Yu.Yakovenko, V. Lutsenko, Phys. Rev. Lett. **104** (2010) 075001.
2. Ya.Kolesnichenko, Yu.Yakovenko, V. Lutsenko, R. White, A. Weller, Nucl. Fusion **50** (2010) 084011.
3. E. Belova, N. Gorelenkov, N. Crocker, et al., Phys. Plasmas **24** (2017) 042505.
4. D. Stutman et al., Phys. Rev. Lett. **102** (2009) 115002.

Inward SC, i.e., energy and momentum transfer by the modes from the plasma periphery core to the core.

Two possible mechanisms are known.

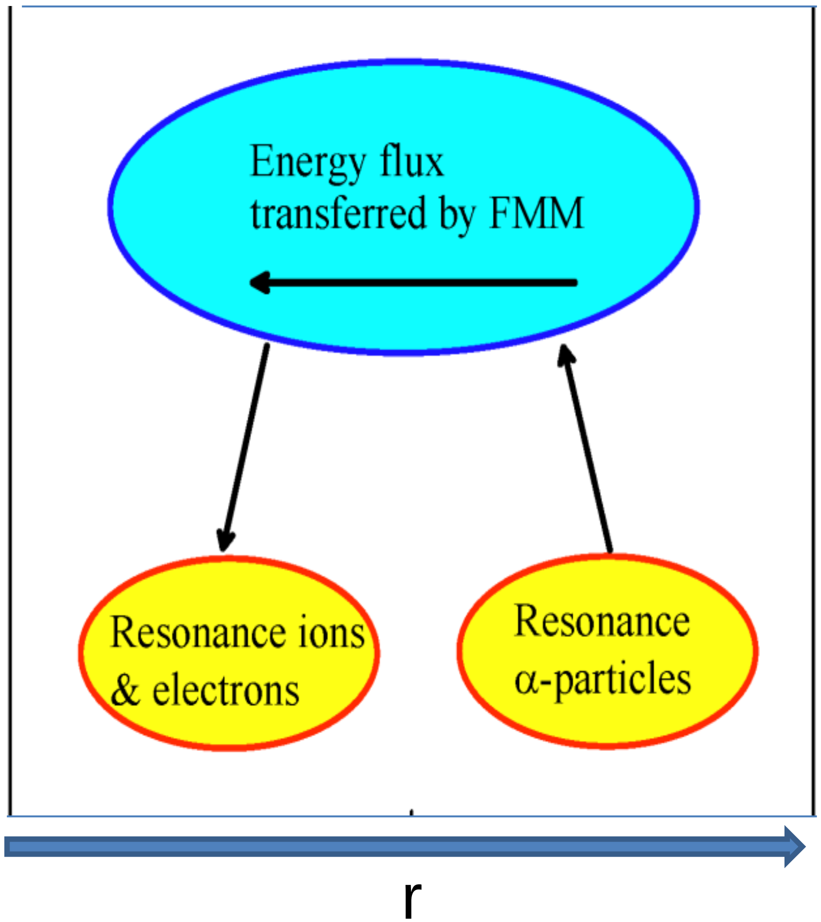
The first mechanism:
In tokamaks, an instability driven by alpha particles at the periphery can heat thermal ions in the core.
Fast Magnetoacoustic Modes (FMM) with frequencies close to harmonics of ion gyrofrequency can be in resonance with alphas at the periphery and with thermal ions in the core:

$$\frac{V_{\|i}^{res}}{V_{\|\alpha}^{res}} = \frac{k_{\|}(r_\alpha)}{k_{\|}(r_i)}\frac{x_i}{x_\alpha} \quad \text{for } B = B_0\left(1 - \frac{x}{R}\right).$$

We observe that $V_{\|i} << V_{\|\alpha}$ for $x_i << x_\alpha$.

Kolesnichenko, Lutsenko, Tyshchenko, Weisen, Yakovenko and JET Contributors, Nucl. Fusion **58** (2018) 076012.

Improvement of plasma performance in JET



In the DTE1 experiments the overall **plasma confinement time** was slightly higher **and anomalous ion heating** took place at the largest fusion power.

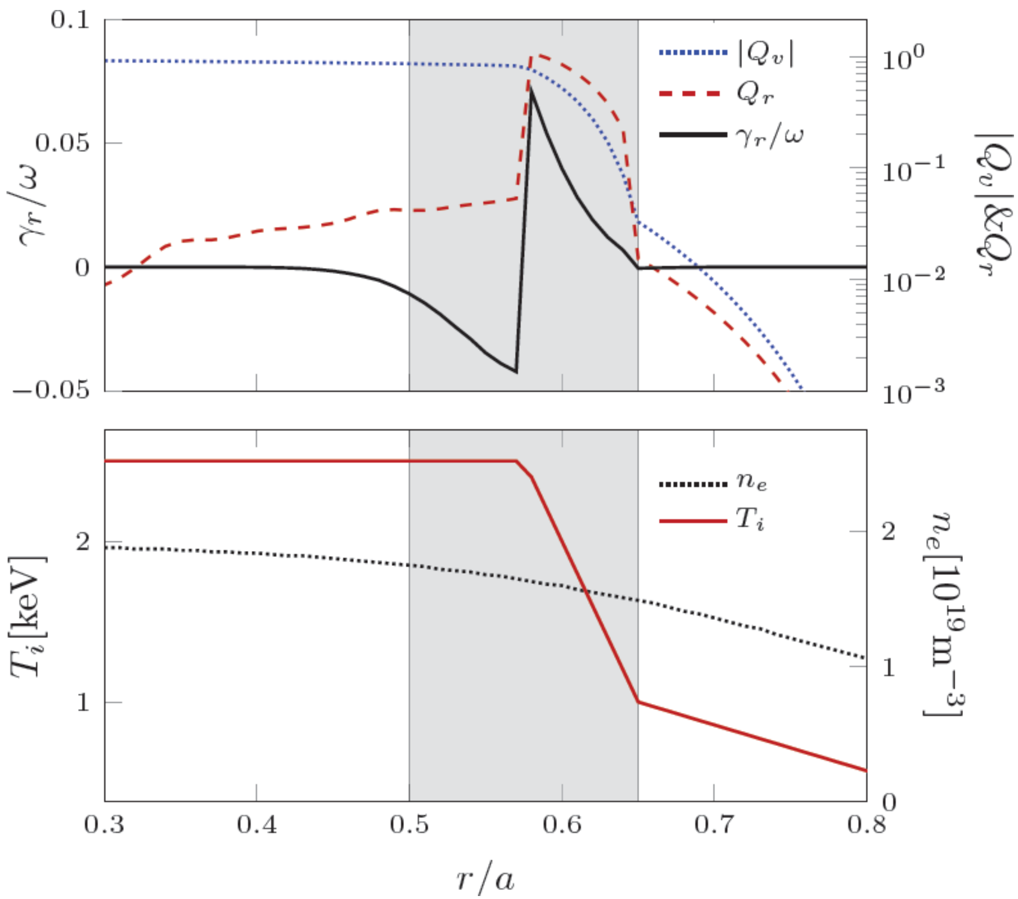
*[1] Thomas et al., PRL **80** (1998) 5548; 28th EPS Conf. CFPP, 2001.*
[2] Weisen et al., AIP Conf. Proc., 1612 (2014) 77-86.

<u>Inward SC</u>
of alpha power due to
Fast Magnetoacoustic Modes
(FMM) can explain it.

Kolesnichenko, Lutsenko, Tyshchenko, Weisen, Yakovenko and JET Contributors, Nucl. Fusion **58** (2018) 076012

Second mechanism of the inward SC: Temperature gradient driven instability

In stellarators, because of non-axisymmetric resonances, ∇T-driven Alfvén instabilities can arise.



$$\gamma = \int a^{-2} r dr \gamma_r$$

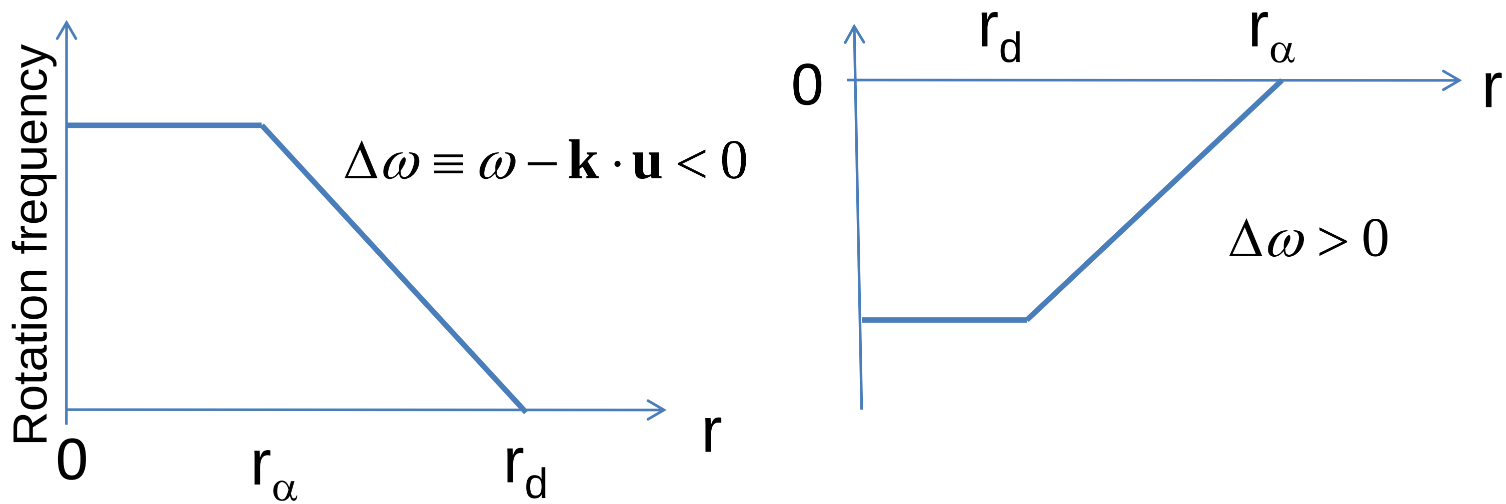
The mode is driven at r/a > 0.57 and damped at r/a < 0.57, leading to an inward energy flux [1].

This mechanism may explain long-lasting high-frequency oscillations observed in W7-X [2].

[1] Ya. Kolesnichenko, A. Tykhyy, Phys. Lett. A **382** (2018) 2689.
[2] T. Windisch et al., Plasma Phys. Contr. Fusion **59** (2017) 105002.

Plasma rotation and frequency chirping (1)

- SC leads to sheared plasma rotation due to the momentum transfer from an unstable region (located at $r \sim r_\alpha$) to a region where the mode is damped (located at $r \sim r_d$).
- Steady state can be reached for the time determined by viscosity.



- When the instability burst lasts for $\Delta t < \tau^{vis}$, frequency chirping occurs, **u**(t).
- Depending on whether $r_\alpha < r_d$ or $r_\alpha > r_d$, the frequency chirps down or up.

[1] Ya.Kolesnichenko, Yu.Yakovenko, V. Lutsenko, R. White, A. Weller, Nucl. Fusion **50** (2010) 084011.

Plasma rotation and frequency chirping (2)

Resonance particles give to / receive from the waves the momentum

$$\mathbf{k}n_k = \mathbf{k}\frac{W_k}{\omega}$$

They are affected by the force

$$\mathbf{f}_\sigma = -2\gamma_\sigma \mathbf{k}\frac{W_k}{\omega}, \text{ with } \sigma = \alpha, e \quad \gamma_\alpha > 0, \gamma_e < 0.$$

When $r_\alpha < r_d$, the maximum rotation is determined by fast ions.
Hence $\operatorname{sgn} u_j = \operatorname{sgn} f_j = -\operatorname{sgn} k_j, \quad j = \vartheta, \varphi$

and the frequency chirps down $\Delta\omega = \mathbf{k}\cdot\mathbf{u}(t) < 0 \quad \text{when } \Delta t < \tau^{vis}.$

This may explain the frequency chirping down by 23 kHz in a Wendelstein 7-AS experiment where a bursting instability occurred, with thermal crashes after each burst [1].

[1] Ya.Kolesnichenko, Yu.Yakovenko, V. Lutsenko, R. White, A. Weller, Nucl. Fusion **50** (2010) 084011.

Radial phase dependence of the mode

Eigenmodes are waves propagating along flux surfaces:

$$\tilde{\Phi} = \hat{\Phi}(\psi)\exp(i\xi - i\omega t)$$

where ψ is a flux radial coordinate, $\xi = \xi(\vartheta, \varphi)$,
ϑ and φ are the poloidal and toroidal angles.

This implies that the wave energy flux across the magnetic flux surfaces vanishes, $\mathbf{S} \cdot \nabla \psi = 0$.

However, in the presence of SC $\mathbf{S} \cdot \nabla \psi \neq 0$.

Hence, **the wave phase becomes dependent on the radius:**

$$\boxed{\xi = \xi(\vartheta, \varphi, \psi)}$$

In other words, **the SC leads to curving the wavefront**.

This may explain observations of radial spiral mode structure made in DIII-D and NSTX [1-3].

[1] M.A. Van Zeeland et al., Nucl. Fusion 49 (2009) 065003.
[2] B.J. Tobias et al., Phys. Rev. Lett. 106 (2011) 075003.
[3] N.A. Crocker et al., Plasma Phys. Contr. Fusion 53 (2011) 105001.

2. Radial energy flux during SC

Transverse energy flux of eigenmodes

An eigenmode is a wave standing in the radial direction.
It can be considered as a superposition of two traveling waves, one of them moving outwards, another one moving inwards.
Thus, the mode energy flux density is

$$S^{(m)} = S_+ + S_-, \qquad S_\pm = \pm |V_g^t| W_\pm .$$

In the absence of the energy sources and sinks (stable plasma) the eigenmode energy flux vanishes, $S_+ + S_- = 0$, i.e. $\delta W \equiv W_+ - W_- = 0$.

The mode energy flux in an unstable plasma is $S^{(m)} = \left|V_g^t\right| \delta W = V_g W^{(m)}$,

where $V_g = V_g^t \dfrac{\delta W}{W^{(m)}}$ is the mode group velocity,

and $W^{(m)} = W_+ + W_-$ is the mode energy density.

Kolesnichenko, Lutsenko, Tyshchenko, Weisen, Yakovenko and JET contributors, Nucl. Fusion **58** (2018) 076012.

Killing eigenmodes

The equation of local energy balance in the eigenmode is

$$\frac{\partial W^{(m)}}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r} r S^{(m)} = 2\gamma^{loc} W^{(m)} \qquad (1)$$

Integrating (1) over the plasma volume yields the overall mode growth rate

$$\gamma = \frac{\int d^3 r \gamma^{loc} W^{(m)}}{\int d^3 r W^{(m)}} \qquad \gamma^{loc} = \gamma_+ - \gamma_- .$$

The steady state solution of (1) is $S^{(m)}(r) = \frac{2}{r}\int_0^r dr' r' \gamma^{loc}(r') W^{(m)}(r')$.

$S^{(m)} \neq 0$, even for $\gamma = 0$, when the regions with γ_+ and γ_- do not coincide.

$S_{\max}^{(m)} \approx \gamma_+ \Delta_\alpha W^{(m)}$, where Δ_α is the width of unstable region.

Because $S^{(m)} = \left|V_g^t\right| \delta W$, the mode is weakly distorted [$\delta W << W^{(m)}$] provided that

$$\boxed{\gamma_+ \Delta_\alpha << V_g^t}$$

Otherwise the mode structure changes considerably and the mode can even disappear.

Radial energy flux when the mode survives and γ > 0

$$\frac{\partial W}{\partial t}+\frac{1}{r}\frac{\partial}{\partial r}rS+\frac{1}{2}\sum_m \mathrm{Re}\,(E_{b,m}B^*_{\parallel,m})=0 \qquad (1)$$

When mode shape persists, the solution of (1) is

$$\boxed{S(r)=-\frac{2}{r}\left(\gamma-\gamma^{(r)}\right)\mathcal{W}^{(r)}} \qquad (2)$$

where $\mathcal{W}^{(r)}=\int_0^r dr'r'W(r')$, $W(r)$ is the mode energy density,

$$\gamma^{(r)}=-\frac{1}{4\mathcal{W}^{(r)}}\sum_m\int_o^r dr'r'\,\mathrm{Re}\left(\mathbf{j}_m^{res}\cdot\mathbf{E}_m^*\right) \qquad \gamma^{(r)}\big|_{r=a}=\gamma$$

$$\boxed{S=S_{\mathrm{mode}}+S_{heat}}$$

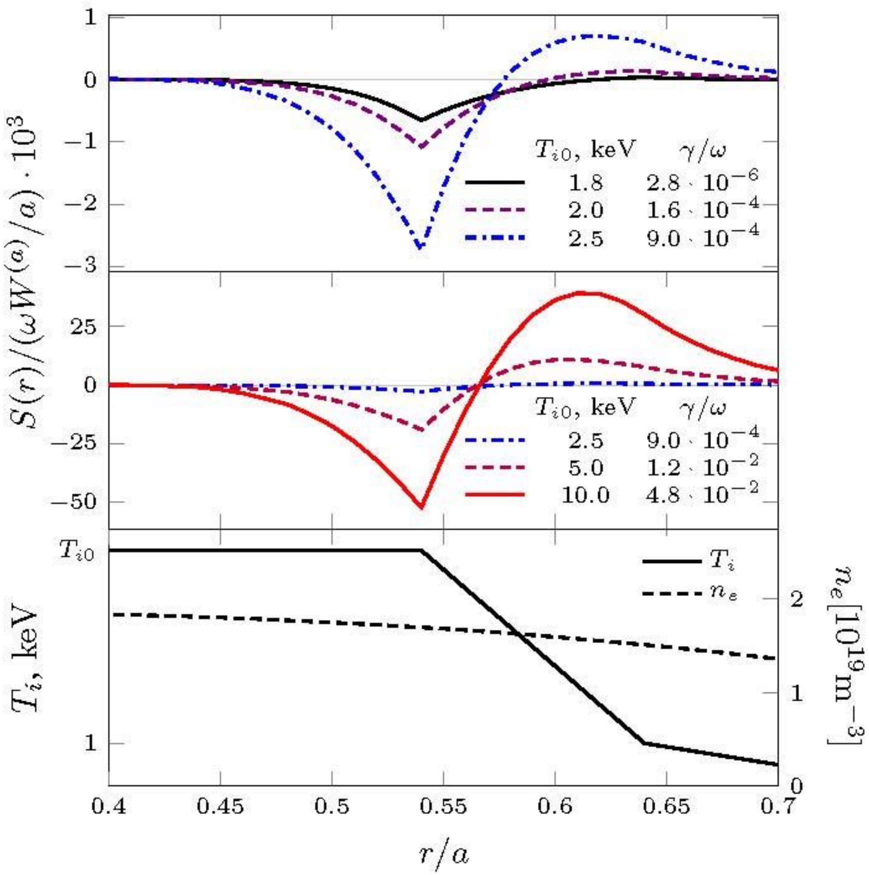
S_{mode}, the 1st term in (2), supports the mode shape, supplying energy for the growth of the mode amplitude in stable regions.

S_{heat}, the 2nd term in (2), provides plasma heating in the stable region, i.e., it is responsible for SC.

Ya.I. Kolesnichenko, A.V. Tykhyy, Phys. Plasmas **25** (2018) 102507.

Flux dependence on the growth rate of the grad-T instability in W7-X

The sign of S and the ratio of S_{mode} and S_{heat} depend on the mode growth rate.



(i) **The inward flux** (S<0) is generated at low temperature, when $\boxed{\gamma_\alpha \approx |\gamma_d|}$.

(ii) **The outward flux** (S>0) appears in the periphery at high temperature, when $\boxed{\gamma_\alpha >> |\gamma_d|}$.

Ya.I. Kolesnichenko, A.V. Tykhyy, Phys. Plasmas **25** (2018) 102507.

3. Nature of transverse energy flux of Alfvén waves

Radial energy transfer by traveling waves

The radial group velocity
of fast magnetoacoustic waves is $V_g^t = \frac{d\omega}{dk_r} = \frac{k_r}{k} V_A$.

What is the radial group velocity of Alfvén waves?

Radial energy transfer by Alfvén waves

For classical Alfvén waves $\omega = k_{\|} V_A, \quad V_g^t = \dfrac{d\omega}{dk_r} = 0, \quad \nabla \cdot \boldsymbol{\xi}_{\perp} = 0.$

Moreover, equations for Alfvén eigenmodes in toroidal plasmas are usually derived in the assumption that $\tilde{B}_{\|} = 0$, in which case the radial flux vanishes:

$$S \propto (\tilde{\mathbf{E}} \times \tilde{\mathbf{B}})_r = \tilde{E}_b \tilde{B}_{\|} = 0.$$

$$\mathbf{e}_r = \frac{\nabla r}{|\nabla r|}, \ \mathbf{e}_b = \mathbf{b} \times \mathbf{e}_r, \ \mathbf{e}_{\|} = \mathbf{b} \equiv \frac{\mathbf{B}}{B}.$$

Nevertheless, Alfvén eigenmodes in toroidal plasmas do exist, which implies that there is an energy exchange between flux surfaces, i.e.,

$$V_g^t \neq 0, \quad \nabla \cdot \boldsymbol{\xi}_{\perp} \neq 0, \quad S^t \neq 0.$$

The reason is that $\omega \neq k_{\|} V_A$, **for Alfvén gap modes and for global Alfvén eigenmodes (GAE).**

Ya.I. Kolesnichenko, A.V. Tykhyy, Phys. Plasmas **25** (2018) 102507.

Energy flux of Alfvén waves within ideal MHD

Due to $\nabla \cdot \boldsymbol{\xi}_{\perp} \neq 0,$ it can be shown that the equations for Alfvén waves and fast magnetoacoustic waves are coupled. In the WKBJ approximation,

$$\left(\frac{\omega^2}{k_{\|}^2} - V_A^2\right)(\mathbf{k}\times\boldsymbol{\xi})_{\|} = k_{\|}\hat{j}_{\|}\nabla\cdot\boldsymbol{\xi}, \quad \left(\frac{\omega^2}{k^2} - V_A^2\right)\nabla\cdot\boldsymbol{\xi}_{\perp} = k_{\|}\hat{j}_{\|}(\mathbf{k}\times\boldsymbol{\xi}_{\perp})_{\|}$$

where $\hat{j}_{\|} = 4\pi j_{\|}/(cB).$

Using these equations and $\tilde{B}_{\|} = -B\nabla\cdot\boldsymbol{\xi}_{\perp},$ we obtain **the radial energy flux of traveling Alfvén waves:**

$$\boxed{S^t = \frac{c}{8\pi}\mathrm{Re}\left(\tilde{E}_b^*\tilde{B}_{\|}\right) = \frac{\omega k_r}{8\pi k_{\perp}^2}|\tilde{B}_{\|}|^2 = \frac{c^2 k_r}{8\pi\omega}\left(k_{\|}^2 - \frac{\omega^2}{V_A^2}\right)|\tilde{\Phi}|^2}$$

The group velocity is $\boxed{V_g^t = \dfrac{S^t}{W^t} = \dfrac{k_r V_A^2}{\omega k_{\perp}^2}\left(k_{\|}^2 - \dfrac{\omega^2}{V_A^2}\right)}$

Ya.I. Kolesnichenko, Yu.V. Yakovenko, M.H. Tyshchenko, Phys. Plasmas **25** (2018) 122508.

Energy flux of Alfvén waves within ideal MHD

Thus, $|S^t| \propto \tilde{B}_{||}^2 \propto |\omega^2 - k_{||}^2 V_A^2|$.

The same result can be obtained by proceeding from a GAE equation (not using the WKBJ approximation)**, which is derived in the assumption** $\tilde{B}_{||} = 0$.

$\tilde{B}_{||} \neq 0$ **can be recovered from the solution of these equations.**

Ya.I. Kolesnichenko, Yu.V. Yakovenko, M.H. Tyshchenko, Phys. Plasmas **25** (2018) 122508.

4. Modification of the structure of Alfvén eigenmodes

Equations for a TAE mode

We consider the influence of the SC on the well-known TAE mode [1,2] with $m=1$ and 2, $n=1$ in a tokamak with $A=R/a=4$, $q=1+x^2$, $x=r/a$, and homogeneous plasma density.

Assuming $\tilde{\Phi} = \sum \Phi_m \exp(-i\omega t + im\vartheta - in\varphi)$ we proceed from

$$\frac{1}{x}\frac{d}{dx}x\left(\hat{\omega}^2 - \hat{k}_m^2\right)\frac{d\Phi_m}{dx} + \left[2\frac{\hat{k}_m \hat{k}'_m}{x} - \frac{m^2}{x^2}\left(\hat{\omega}^2 - \hat{k}_m^2\right)\right]\Phi_m + \frac{1}{x}\frac{d}{dx}x^2\frac{\hat{k}_*^2}{A_{ef}}\frac{d\Phi_{m+1}}{dx} = i\nabla\cdot\hat{\mathbf{j}}_m^{res}$$

$$\frac{1}{x}\frac{d}{dx}x\left(\hat{\omega}^2 - \hat{k}_{m+1}^2\right)\frac{d\Phi_{m+1}}{dx} + \left[2\frac{\hat{k}_{m+1} \hat{k}'_{m+1}}{x} - \frac{(m+1)^2}{x^2}\left(\hat{\omega}^2 - \hat{k}_{m+1}^2\right)\right]\Phi_{m+1} + \frac{1}{x}\frac{d}{dx}x^2\frac{\hat{k}_*^2}{A_{ef}}\frac{d\Phi_m}{dx} = i\nabla\cdot\hat{\mathbf{j}}_{m+1}^{res}$$

where $\hat{\omega} = \omega\dfrac{R}{V_A}$, $\hat{k}_m = (m/q - n)$, and $\hat{\mathbf{j}}^{res} = \dfrac{4\pi\omega R^2 a^2}{c^2}\mathbf{j}^{res}$ is a normalized

current describing the resonance mode - particle interaction.

[1] C.Z. Cheng, M.S. Chance, Phys. Fluids 29 (1986) 3695.
[2] G.Y. Fu, J.W. Van Dam, Phys. Fluids B **1** (1989) 1949.

Equations for a TAE mode: the resonance term

The perturbative solution $\gamma = \frac{1}{4\mathscr{W}} \sum_m \mathrm{Re} \int d^3 r \, \mathbf{j}_{\perp m}^{res} \cdot \nabla_\perp \Phi^*$

is not applicable in the presence of strong SC because neither mode structure nor frequency are known.

To study effects of the SC we use a model resonance term, which we construct as follows.

Following [1], we write the resonance current term as

$$\nabla \cdot \hat{\mathbf{j}}_m^{res} = \sum_\sigma \sum_{\mu=\pm1} \left(\frac{1}{x} \frac{d}{dx} + \frac{\mu m}{x^2} \right) (x\Phi'_m - \mu m \Phi_m) Q[F_\sigma]$$

where F_σ is a distribution function of resonant particles, and σ labels particle species.

Q[F] may contain both a destabilizing part and a stabilizing part.

[1] Ya. Kolesnichenko, A. Tykhyy, Plasma Phys. Contr. Fusion **60** (2018) 125004.

Equations for a TAE mode: construction of Q(F)

We use a model with $Q[F] = Q(x)$

$$Q(x) = C\,\frac{x_0^4 - x^4}{q^2(x)},$$

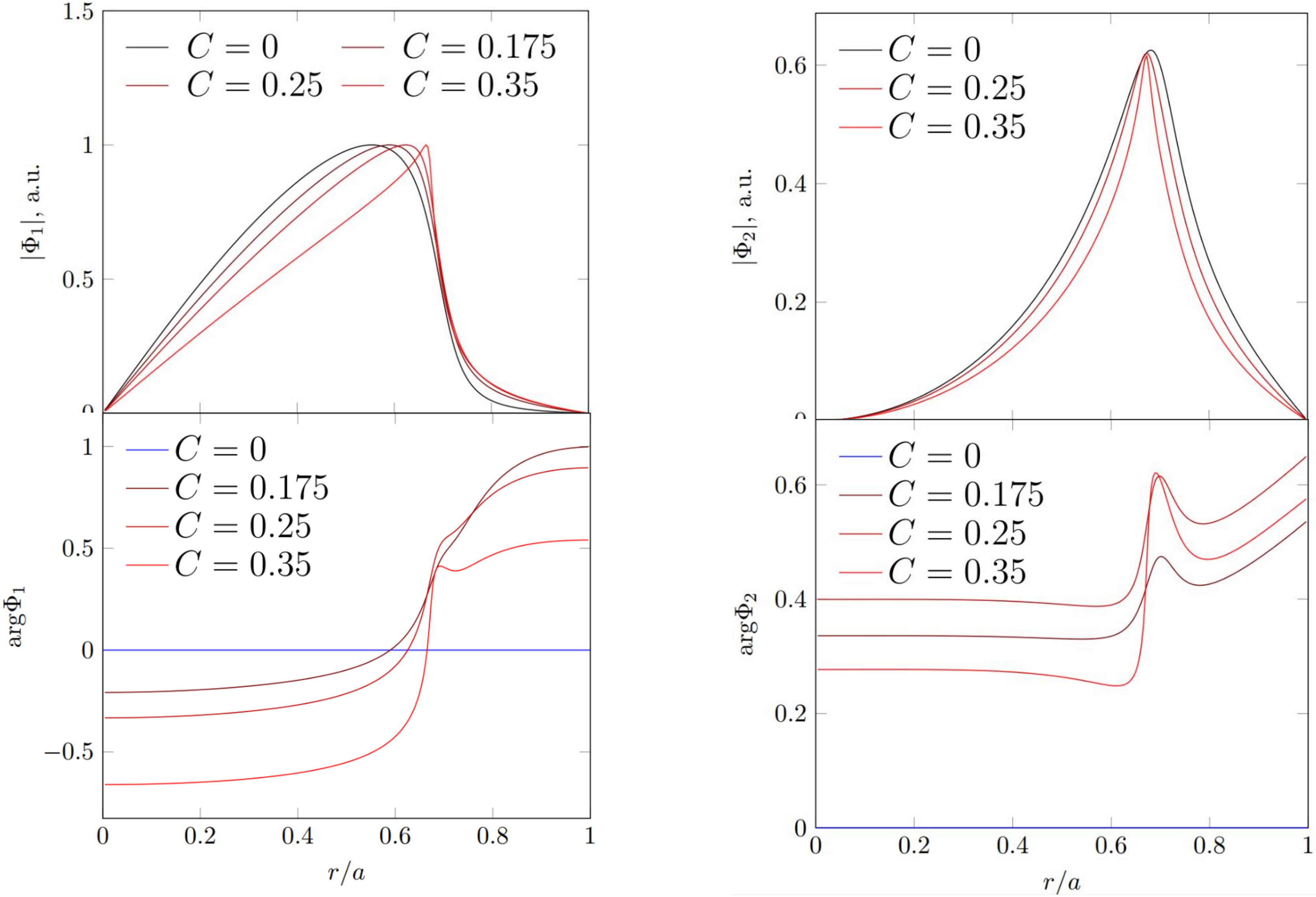
where x_0 = 2/3 is selected so that the mode is close to the margin of stability ($\gamma \approx 0$), and the parameter $C > 0$ describes the strength of drive (for $Q > 0$) and damping (for $Q < 0$).

This Q(x) can model, e.g., an instability driven by a temperature gradient, like in [1], or an instability driven by fast ions and damped by the plasma particles.

The mode is driven at $x<x_0$ and damped at $x>x_0$.

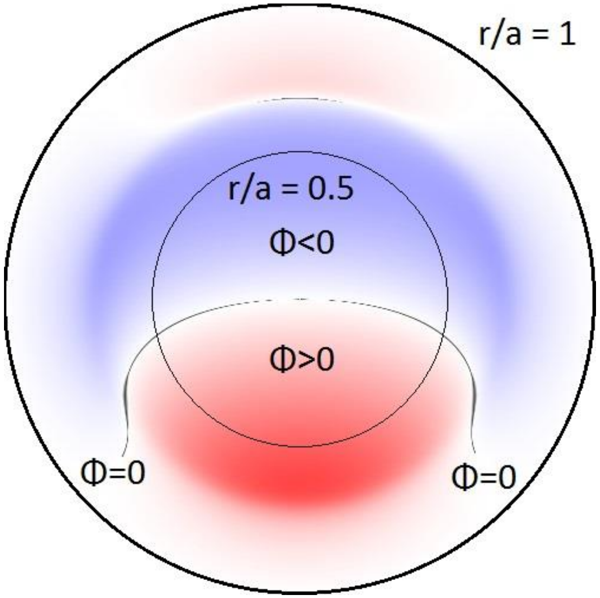
[1] Ya. Kolesnichenko, A. Tykhyy, Phys. Lett. A **382** (2018) 2689.

Influence of SC on mode structure

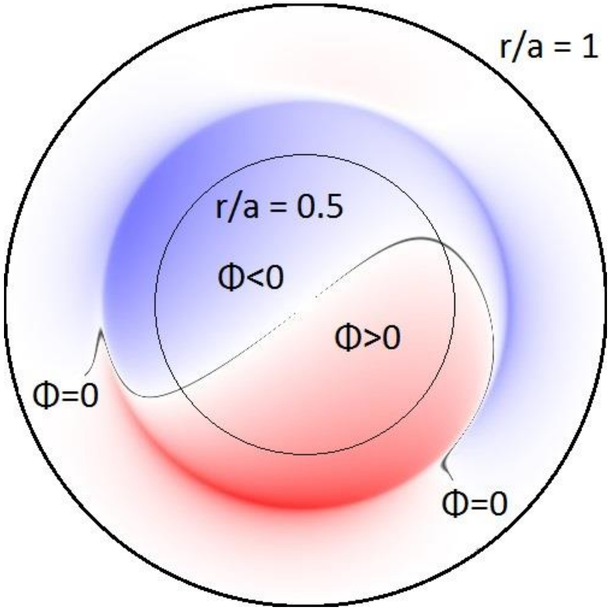


The mode becomes more peaked, and its phase becomes dependent on x.
For *C* > 0.37 the mode merges into the continuum and disappears.

The poloidal structure of the mode



$$\frac{\gamma_{max}^{loc}}{\omega} = 10^{-5}$$



$$\frac{\gamma_{max}^{loc}}{\omega} = 1.5\times 10^{-2}$$

Spiral structure of the mode is formed due to SC.
This may explain experimental observations in [1-3].

[1] M.A. Van Zeeland et al., Nucl. Fusion 49 (2009) 065003.
[2] B.J. Tobias et al., Phys. Rev. Lett. 106 (2011) 075003.
[3] N.A. Crocker et al., Plasma Phys. Contr. Fusion 53 (2011) 105001.

Analysis of the numerical results (1)

The energy flux is maximum at $x = x_0$. At this radius

$$S_{\max}^{(m)} = \frac{2a}{x_0}\int_0^{x_0} dx x \gamma^{loc}(x) W^{(m)}(x) = V_g^t \delta W\Big|_{x_0} \qquad (1)$$

where $$\frac{\gamma^{loc}}{\omega} \approx \frac{1}{\hat{\omega}^2} Q(x) = 4q_*^2 Q(x) = 9Q(x), \qquad \omega \approx \frac{V_A}{2q_* R} = \frac{V_A}{3R}.$$

It follows from (1) that $\delta W << W(x_0) \equiv W_0$, i.e. the mode does not change, when

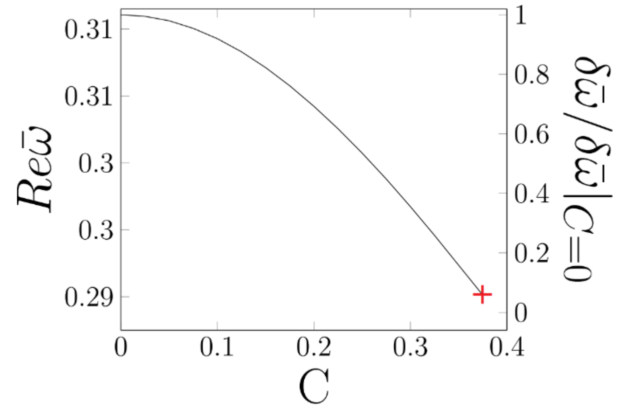
$$\boxed{\frac{S_{\max}^{(m)}}{W_0^{(m)} V_A} \equiv \frac{6}{A x_0}\int_0^{x_0} dx x Q(x) \frac{W(x)}{W_0} << \frac{V_g^t(x_0)}{V_A}}$$

The mode changes considerably when $\boxed{\frac{S_{\max}^{(m)}}{W_0^{(m)}} \;\hat{}\; V_g^t(x_0).}$

Analysis of the numerical results (2)

Evaluation of the group velocity

$$V_g^t \propto \delta\omega \equiv \left(\omega - \omega_{AC}^{low}\right),$$



where ω_{AC}^{low} is lower continuum branch frequency because the eigenmode frequency is close to it.

We find that $\delta\omega^{\gamma\neq 0} / \delta\omega^{\gamma\approx 0} \approx 1/6 << 1$,
which tends to change the mode structure.

Using [1], we assume that $|V_g^t| = 2\varepsilon_{ef} V_A \dfrac{|k_r k_{\parallel}|}{k_{\perp}^2}$, with $\varepsilon_{ef} \equiv \left|\dfrac{\omega - \omega_{AC}^{low}}{\omega_{AC}^{low}}\right|$

$$\tilde{\omega} = \frac{\omega R}{V_A} = 0.31 \text{ for } C = 0.001\,(\gamma^{loc} \approx 0),\ \tilde{\omega} = 0.2923 \text{ for } C = 0.35\ (\gamma_{\max}^{loc} / \omega \approx 1.5\times 10^{-2}),\quad \tilde{\omega}_{AC}^{low} = 0.2888.$$

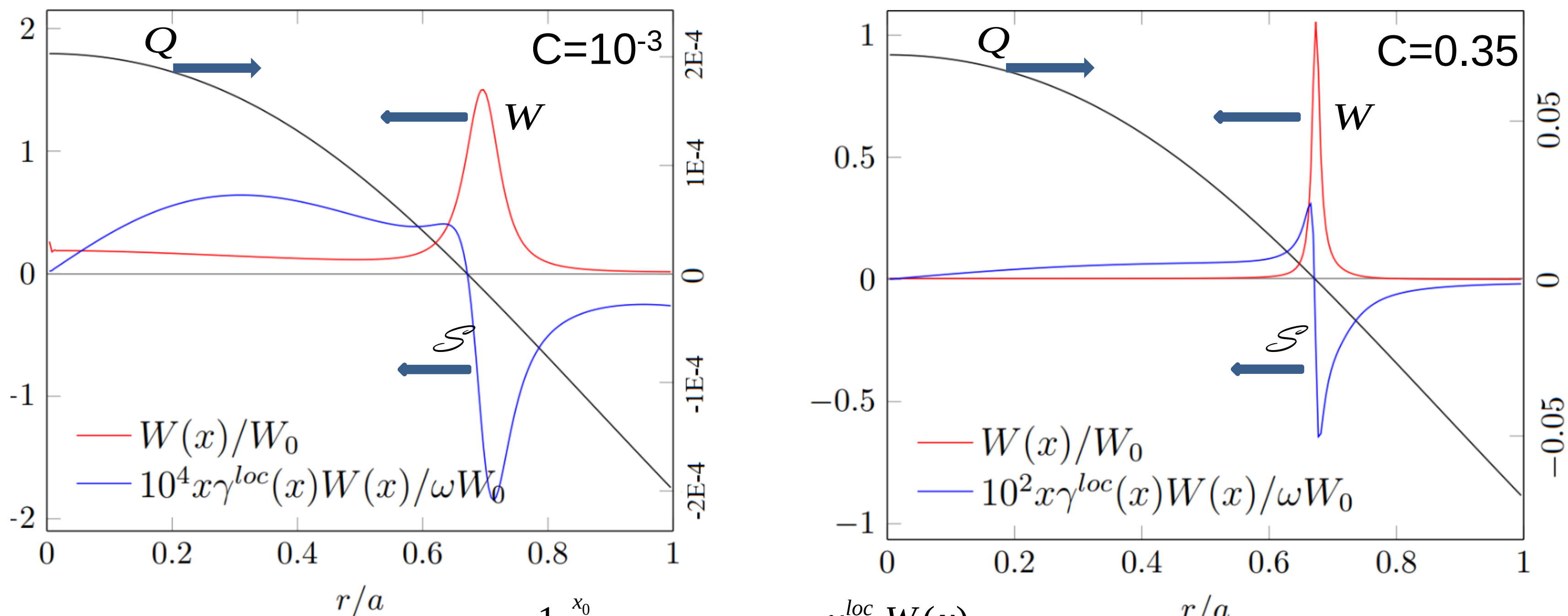
Taking $k_r=2\pi/(a\nu_r)>>k_\theta$ with $\nu_r \geq 1$, we obtain:

$$\boxed{\frac{V_g^t}{V_A}\nu_r = 1.95\times 10^{-3} \text{ for } \gamma^{loc} \approx 0 \quad \text{and} \quad 3.18\times 10^{-4} \text{ for } \gamma^{loc} \neq 0.}$$

[1] Ya.I. Kolesnichenko, Yu.V. Yakovenko, M.H. Tyshchenko, Phys. Plasmas **25** (2018) 122508.

Analysis of the numerical results (3)

Evaluation of the energy flux



$$S_{\max}^{(m)} \equiv \frac{1}{x_0}\int_0^{x_0} dx\mathscr{S}, \quad \mathscr{S} \propto x\frac{\gamma^{loc}}{\omega}\frac{W(x)}{W_0}.$$

We infer from these figures that $\frac{\gamma_{\max}^{loc}}{\omega} = 10^{-5}$ for C=0.001 and $\frac{\gamma_{\max}^{loc}}{\omega} = 1.5\times 10^{-2}$ for C=0.35.

$$\frac{S_{\max}^{(m)}}{W_0^{(m)}V_A} = 7\times 10^{-6} \quad for \quad \gamma_{\max}^{loc}/\omega = 10^{-5}. \qquad \frac{S_{\max}^{(m)}}{W_0^{(m)}V_A} = 10^{-4} \quad for \quad \gamma_{\max}^{loc}/\omega = 1.5\times 10^{-2}.$$

The energy flux increases (SC intensifies), and mode energy density becomes more localized as the drive grows larger.

Analysis of the numerical results (4)

Ratio of the energy flux to the wave group velocity

$$\frac{S_{\max}^{(m)} / W_0^{(m)}}{V_g^t} = 3.6 \times 10^{-3} \nu_r \qquad for \qquad \gamma_{\max}^{loc} / \omega = 10^{-5}.$$

$$\frac{S_{\max}^{(m)} / W_0^{(m)}}{V_g^t} = 0.31 \nu_r \qquad for \qquad \gamma_{\max}^{loc} / \omega = 1.5 \times 10^{-2}.$$

The normalized energy flux is on the order of the group velocity when the instability drive is sufficiently large. This explains the change of the mode structure in this case.

5. Summary

- **The phenomenon of SC of energy and momentum is a factor that can affect plasma performance in toroidal devices. SC is associated with energetic ions, but it can occur also in a Maxwellian plasma.**
- **Effects of SC are studied by solving a TAE mode equation with a model term describing the drive and damping characterized by a radial mismatch in the location.**
- **It is found that SC leads to a spiral mode structure. This may explain experiments on DIII-D and NSTX where radial mode phase dependence was observed.**
- **It is shown that the mode width and its frequency decrease when the drive increases; eventually the mode disappears.**
- **These numerical results are supported by an analytical consideration.**

Thank you