Efficient MHD equilibrium solver for Control Oriented Transport models

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Outline

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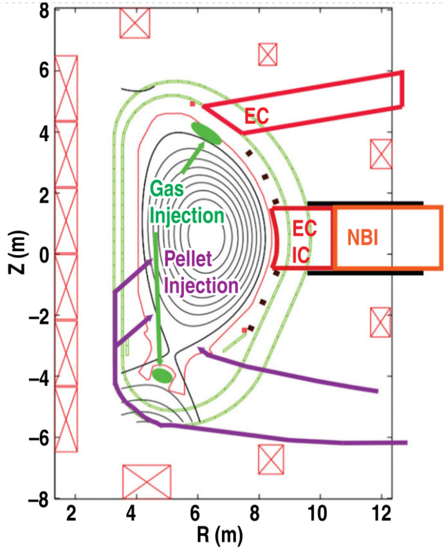
3. Equilibrium constrain: the Grad-Shafranov equation
4. A q-solver algorithm
5. Convergence & Performance
6. Summary and perspective

Plasma control: Control categories and physical actuators in ITER



Control categories

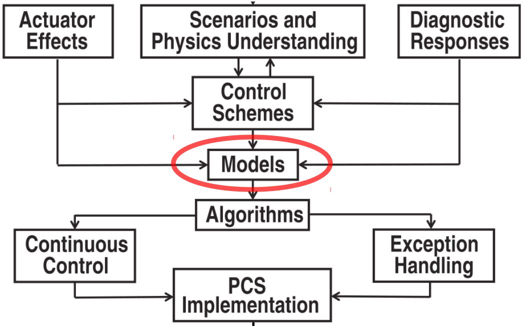
- Plasma equilibrium
- Plasma current
- Vertical stability
- Burn state
- Divertor
- Current profile
- MHD instabilities
- Fast particles
- Error field
- Disruption mitigation

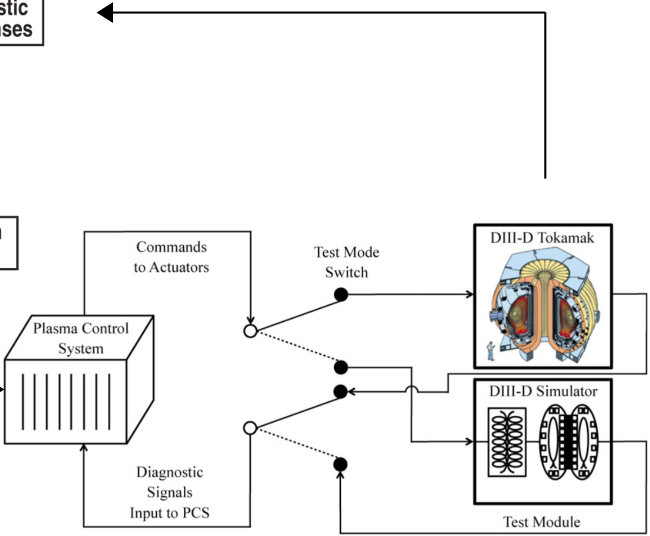
Actuators

- PF coils
- CS coils
- ECCD
- ECRH, ICRH
- NBI
- VS3 coils
- RMP coils

Elements of control process

[Humphreys PoP **22** 21806 (2015)]





Motivation

Physics-based-models for current plasma profile control

Magnetic Diffusion Equation (MDE)

$$\frac{\partial \psi}{\partial t} = \frac{\eta(T_e)}{\mu_0 \rho_b^2 \hat{F}^2} \frac{1}{\hat{\rho}} \frac{\partial}{\partial \hat{\rho}} \left(\hat{\rho} \hat{F} \hat{G} \hat{H} \frac{\partial \psi}{\partial \hat{\rho}} \right) + R_0 \hat{H} \eta(T_e) \frac{\langle \bar{j}_{NI} \cdot \bar{B} \rangle}{B_{\phi,0}},$$

$$\left.\frac{\partial \psi}{\partial \hat{\rho}}\right|_{\hat{\rho}=0} = 0, \qquad \left.\frac{\partial \psi}{\partial \hat{\rho}}\right|_{\hat{\rho}=1} = -\frac{\mu_0}{2\pi} \frac{R_0}{\hat{G}|_{\hat{\rho}=1} \hat{H}|_{\hat{\rho}=1}} I(t),$$

where

$$\hat{F}(\hat{\rho}) = \frac{R_0 B_{\phi,0}}{R B_\phi}, \quad \hat{G}(\hat{\rho}) = \left\langle \frac{R_0^2}{R^2} |\nabla \rho|^2 \right\rangle, \quad \hat{H}(\hat{\rho}) = \frac{\hat{F}}{\langle R_0^2 / R^2 \rangle},$$

are "magnetic geometric" factors determined by plasma equilibrium

Ψ : poloidal flux function

Norm. effective minor radius:

$$\hat{\rho} = \sqrt{\frac{\Phi}{\Phi_b}}$$

Φ : toroidal flux

Safety factor:

$$q = -\frac{d\Phi}{d\Psi} = -\frac{\Phi_b \hat{\rho}}{\pi \partial \psi / \partial \hat{\rho}}$$

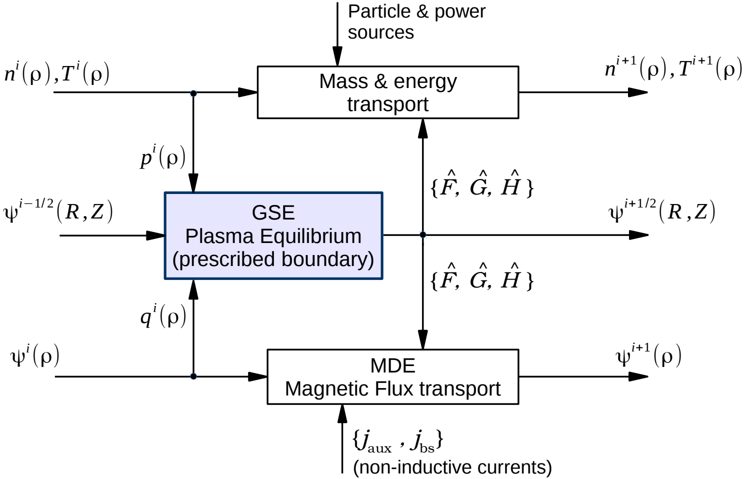
In current implementations, these profiles are externally imposed, and typically left invariant.

q profile control and good discharge reproducibility have been achieved with this simplification [e.g. Schuster NF **57** 116026 (2017), Felici NF **58** 096006 (2018)].

However, a self-consistent description would allow more general and robust control design.

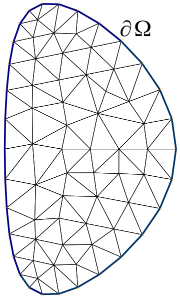
Coupling MDE with the Grad-Shafranov equation (GSE)

Simplified staggered scheme for Control Oriented Transport models



GSE as a non-linear eigenvalue problem

$$\Delta^*\Psi = -\mu_0 R^2 \frac{dP}{d\Psi} - \frac{1}{2}\frac{dF^2}{d\Psi}, \qquad F = RB_\phi$$



Using the normalization, $\psi = \Psi/\Psi_0$, Ψ_0 poloidal flux function on axis, and defining $f(\psi) = F(\Psi)$, $p(\psi) = P(\Psi)$

the (non-linear) eigenvalue nature of the equation is revealed [LoDestro PoP **1** 90 (1994)]

$$\begin{cases} -\Delta^*\psi = \dfrac{1}{\Psi_0^2}\left(R^2 \dfrac{dp}{d\psi} + \frac{1}{2}\dfrac{df^2}{d\psi}\right) & \text{in } \Omega \\ \psi|_{\partial\Omega} = 0 & 0 \leq \psi \leq 1 \end{cases}$$

Methods to solve this problem are available e.g. [Pataki JCP **243** 28 (2013)]

However, they require specification of $p(\psi), f(\psi)$ instead of $p(\rho), q(\rho)$!!!

q-solver algorithm (eulerian description)

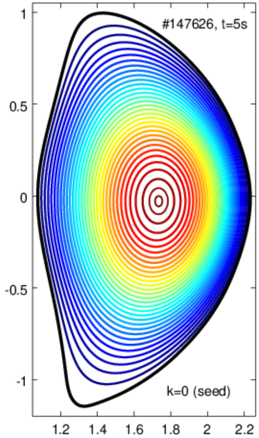
(0) A good seed: the linear eigenvalue solution

$$\begin{cases} -\Delta^* \psi = \left(R^2 \mathcal{L}_p + \mathcal{L}_f\right) \psi & \text{in } \Omega \\ \psi|_{\partial\Omega} = 0 \end{cases}$$

$(\mathcal{L}_p, \mathcal{L}_f)$ are chosen to match prescribed $(I_p, \beta) \rightarrow \Psi^{k=0}(R, Z)$

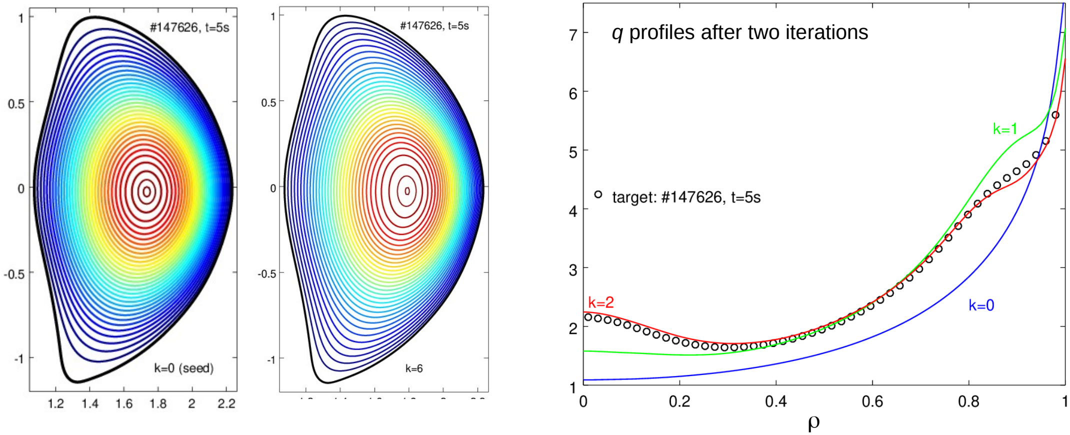
(1) Estimation of RHS$\left(\Psi_0, \frac{dp}{d\psi}, \frac{df^2}{d\psi}\right)^{k+1}$ from $\left(\Psi^k, q(\rho), \frac{dp}{d\rho}\right)$

(2) A 'standard' non-linear GSE solver,
to update the equilibrium: RHS$^{k+1} \longrightarrow \Psi^{k+1}(R, Z)$
(involves "internal" Newton iterations)

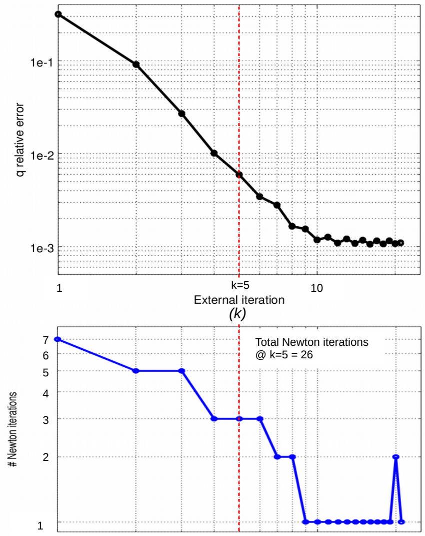


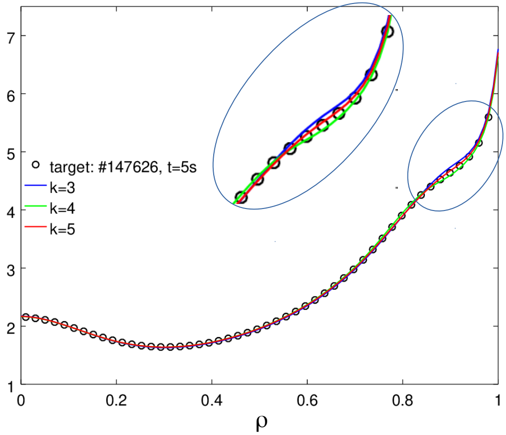
Iterate (1)-(2) over k until target q is reached ("external" iteration)

External iterations, an example



Convergence and performance I





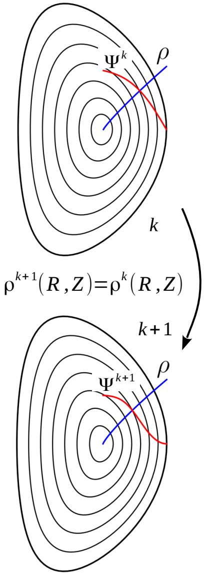
RHS$\left(\Psi_0, \frac{dp}{d\psi}, \frac{df^2}{d\psi}\right)^{k+1}$ estimation from $\Psi^k(R,Z)$, $q(\rho)$ and $p(\rho)$

$$\Psi_0 = \int_0^1 \frac{\Phi_b^2 \rho}{\pi q}\, d\rho, \quad \frac{\partial \psi}{\partial \rho} = \frac{\Phi_b^2 \rho}{\pi \Psi_0 q}, \quad \frac{dp}{d\rho} = \frac{dp}{d\psi}\frac{\partial \psi}{\partial \rho}, \quad V_\rho = \frac{\partial V}{\partial \rho}$$

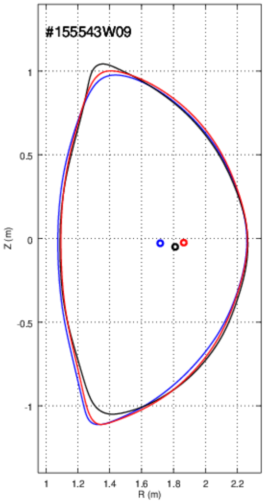
$$\langle GSE \rangle_k \quad \rightarrow \quad \frac{\Psi_0^2}{V_\rho}\frac{\partial}{\partial \rho}\left[\left\langle \frac{|\nabla \rho|^2}{R^2} \right\rangle V_\rho \frac{\partial \psi}{\partial \rho}\right] = -\frac{dp}{d\psi} - \frac{\langle R^{-2} \rangle}{2}\frac{df^2}{d\psi},$$

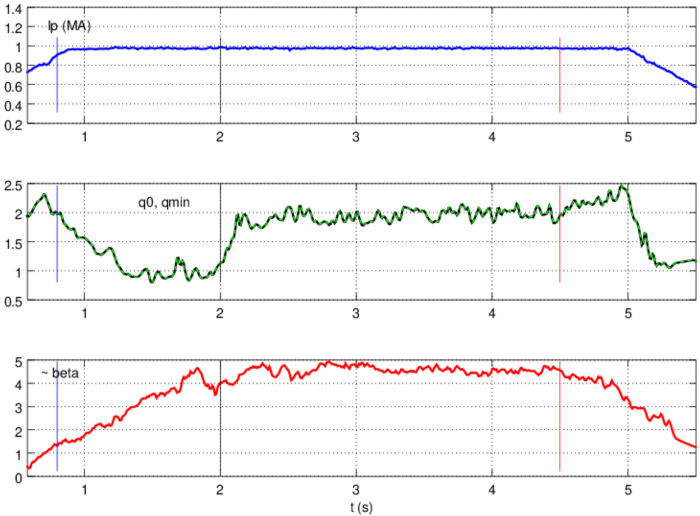
$$\langle GSE \rangle_{k+1} \quad \rightarrow \quad \frac{{\Psi_0}^2}{V_\rho}\frac{\partial}{\partial \rho}\left[\left\langle \frac{|\nabla \rho|^2}{R^2} \right\rangle V_\rho \frac{\partial \psi}{\partial \rho}\right] = -\frac{dp}{d\psi} - \frac{\langle R^{-2} \rangle}{2}\frac{df^2}{d\psi},$$

$$\frac{df^2}{d\psi} = \alpha \frac{df^2}{d\psi} - \mathcal{C}\frac{R_0}{\pi}\frac{I(\rho)}{\rho}\Psi_0 \frac{\partial \alpha}{\partial \rho} - \frac{2}{\langle R^{-2} \rangle}\left(\frac{dp}{d\psi} - \alpha \frac{dp}{d\psi}\right), \quad \alpha = \frac{\Psi_0 q}{\Psi_0 q}$$

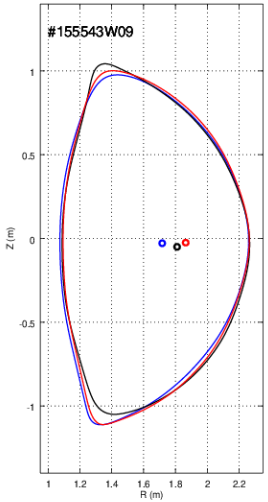


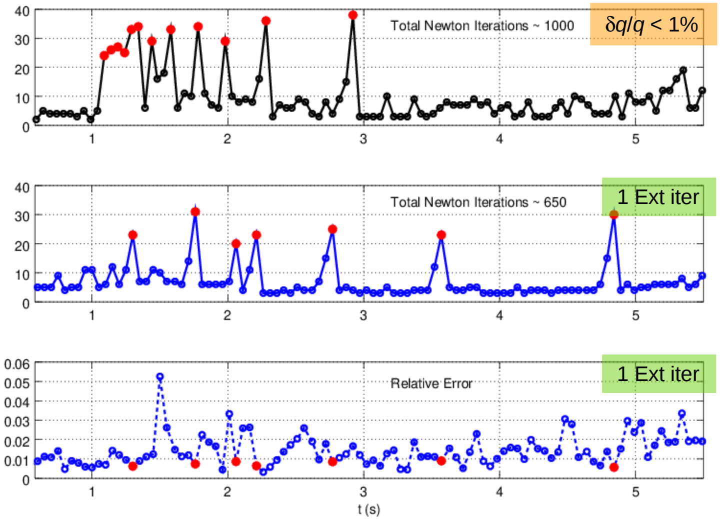
Convergence and performance II





Convergence and performance II





Summary and perspective

- A new algorithm to include the equilibrium condition in control oriented transport simulations was developed, which is robust and reasonable efficient.
- The iterative algorithm has a physically intuitive basis and could be applied straightforwardly to existing eulerian GSE solvers and other applications.
- Moreover, the same principle can be extended to use different input data such as the radial dependence of the pitch-angle.

- Improvement of the efficiency in cases with significant changes in magnetic geometry must be addressed.
- Inclusion of the effect of the coils on the shape of the plasma (free boundary problem) in an appropriate manner is being studied.