Simulation of the internal kink mode in visco-resistive regimes

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Abstract:

We present numerical simulation results of the nonlinear evolution of the (1,1) internal kink mode in the presence of various kinds of equilibrium plasma flows. The present studies are carried out in the framework of a two fluid model and extend our past investigations done with a RMHD model [1]. Two-fluid effects are found to significantly influence the mode dynamics in a number of different ways. In the linear regime diamagnetic effects in combination with flows provide a synergistic stabilizing influence that also carries over to the nonlinear regime. In addition one observes novel symmetry breaking phenomena in the linear growth rates as well as in the nonlinear saturated states of the mode. Our study also explores the influence of strong viscosity on the mode evolution.

1 Introduction

The (m=1, n=1) internal kink instability is significant in the context of tokamaks for various reasons. The (1,1) mode develops inside the q=1 rational surface (q being the safety factor), when the q at the axis is smaller than 1. It is involved in triggering sawtooth oscillations which can influence plasma quality and confinement. In advanced tokamaks, plasma rotation strongly influences sawtooth dynamics[1-5]. However, there still is not a full understanding of the effect of flows on the kink mode. There are some conflicting results in the literature regarding the nature of stabilisation due to flows depending on the parameter regime of the studies [6, 7]. It may be noted that most of the past flow studies have been done in the low viscosity regime, although viscosity can be high in tokamak operations, particularly due to enhancements from turbulent effects and could therefore significantly influence the effect of flow shear on the internal kink mode [8]. The diamagnetic drift is present in the two fluid regime, between the ion and electron fluid whose velocity is denoted by $v_d$ and frequency is denoted by $\omega^*$. The effect of diamagnetic drift is proportional to the electron density gradient which is depending on $\alpha$ in our model.
where we have used a density profile of the form \( n = n_0 \exp(-\alpha \frac{r^2}{a^2}) \), where \( n \) stands for density and \( r \) is the radial coordinate.

In this work, we have addressed this issue and investigated the stability of the (1,1) mode in the presence of sheared flows over a range of viscosity regimes using the CUTIE[9] code in the two fluid regime. CUTIE is a nonlinear, global, electromagnetic, quasi neutral, two fluid, which allows interaction between plasma property profiles and electromagnetic turbulence. CUTIE is based on a periodic cylinder model of the tokamak geometry in which the magnetic flux surfaces are concentric circles, and which is an appropriate approximation for large aspect ratio devices in which the aspect ratio, \( R/a \), is significantly greater than unity, or equivalently the inverse aspect ratio is small. In our earlier study using RMHD, we had obtained a number of interesting results [2]. We had observed that viscosity may play a role in changing the effect of flows on the (1,1) mode. There are additional effects in the two fluid case, and we see that our results are more complex than those obtained in the RMHD case. We have begun with linear studies which we have carried out using the Resolvent method, a method of finding eigenvalues, explained in our earlier paper[1]. In our linear studies, we observe that the symmetry of growth rate and frequency curves as a function of flow on reversal of direction is broken. We also note the change in growth rate over a range of viscosity regimes and various values of drift frequencies. In the nonlinear case, we observe that the results are different from those in the RMHD case qualitatively and quantitatively. These can be attributed to the presence of an intrinsic poloidal flow in the two fluid case, due to the \( k_\theta v_d \) effect which alters the nature of the kink instability considerably. The intrinsic poloidal flow and imposed flows can interact to produce more complicated scenarios than possible with purely axial or poloidal flows.

2 Linear Studies

2.1 Axial Flows

We have used an imposed axial flow profile of the form

\[
V_{0z}/V_A = M_z \tanh(\rho - \rho_s)
\]

where, \( V_{0z} \) is the imposed axial flow velocity, \( V_A \) is the Alfvén velocity, \( M_z \) is the axial Mach Number, \( \rho = \frac{r}{a} \), is the normalised radial coordinate \( r \) being the radial coordinate and \( a \) being the minor radius, and \( \rho_s = \rho \) when \( r = r_s \) is the resonant radius.

We also define the Prandtl Number, \( Pr \), which is a measure of viscosity in the system and Lundquist number, \( S \), which is a measure of resistivity in the system, as follows,

\[
Pr = \frac{\tau}{\tau_A} \quad \text{and} \quad S = \frac{\tau_A}{\tau},
\]

where \( \tau_A = a/v_A \) is the Alfvén time. Here, \( a \) is the minor radius and \( v_A \) is the the Alfvén velocity and, \( \tau_\eta = (4\pi a^2/c^2\eta) \) is the resistive diffusion time, \( \eta \) being the resistivity present in the system. In our studies, we have used \( S = 10^6 \) throughout, and \( Pr \) values used are indicated locally. The value of \( \alpha \), denoted by dndr in the figures, is fixed at \( \alpha = 1.5 \) for the figures indicated with fixed \( \alpha \).
We have studied the growth rate and frequency variation of the internal kink mode in two ways. We have plotted for different values of $\alpha$, denoted by $d\text{ndr}$, which is proportional to electron density gradient in one case Fig. 1 where we study the growth rate of the mode as a function of axial flow for different $\alpha$, Fig. 2 is plotted likewise with mode frequency vs flow. In Fig. 3 we plot mode growth rate as a function of flow for different Prandtl numbers, denoted by $Pr$, for a fixed $\alpha$, and Fig. 4 is the corresponding figure with mode frequency plotted as function of $Pr$ likewise. We observe that the symmetry of growth rate curve about $M_z = 0$ observed in the zero density gradient i.e. RMHD case, disappears as we increase density gradient becoming more asymmetrical as we continue to increase density gradient. This is because a self consistent poloidal flow is generated by the $\omega\times$ effect, there is a net helical flow generated in the system, which leads to the observed asymmetry, in agreement with our earlier results on helical flows in an RMHD system. We then have the case of varying $Pr$ for a fixed, finite density gradient. In this case, we observe that while low $Pr$ increases the growth rate, high $Pr$ reduces it, consistent with our observations in the RMHD case. The frequency shows interesting trends, and it seems to be the case that frequency and viscosity are not directly proportional, contrary to the expectation that viscosity being dissipative in nature should gradually reduce the frequency.

### 2.2 Poloidal flow

We have used the following purely poloidal imposed flow profile,

\[
\frac{V_{0\theta}}{v_A} = M_\theta(\rho) \tag{2}
\]

where the poloidal Mach number,

\[
M_\theta(\rho) = \Omega \tau_A \rho (1 + k\rho)
\]
Here, $V_{0\theta}$ is the equilibrium poloidal flow, and $\Omega$ is the poloidal angular frequency and $k$ measures the shear in the flow.

In the following, we have Figs. 5 where we have plotted mode growth rate vs poloidal flow for different density gradients at a fixed $Pr$, Fig. 6 where we have plotted mode frequency vs poloidal flow for different density gradients at a fixed $Pr$, Fig. 7 where we have plotted growth rates vs poloidal flow for different $Pr$ at a fixed $\alpha$, Fig. 8 where we have plotted frequencies vs poloidal flow for different $Pr$ at a fixed $\alpha$, are the 2fluid results using imposed poloidal flow. The growth rates in the first case Fig. 5 and Fig. 7 display opposite trends, that is the slopes of the growth rate curves are flat in Fig. 7 for finite density gradient, and the growth rate curve of the zero density gradient case has a finite slope as function of flow. We observe the opposite trend for Fig. 5 with regard to the slopes of the growth rate curves for finite density showing a finite slope and the curve for the zero density gradient showing zero slope as a function of flow. There is interesting behaviour in the frequency curves in Fig. 6 showing curved trends while Fig. 8 showing straight line trends for the frequencies as a function of flow.

2.3 Helical Flow

In this section we discuss helical flow results. Helical flows are a combined effect of axial and poloidal flows. The two fluid has an intrinsic poloidal flow present, therefore the case with an imposed axial flow is similar to the RMHD helical flow case. The RMHD helical flow results seem to agree with axial flow two fluid results as we expect, but due to the presence of intrinsic poloidal flow, while the 2fluid helical flow results behave very differently. These display a behaviour at variance from helical flow RMHD results. In the RMHD case [1], we have an asymmetry in the effect of the helical flows, and the results we obtain are qualitatively similar to those obtained for axial flow here in the two fluid case. Here, we have both imposed axial and poloidal flows, in addition to the intrinsic poloidal flows. In Fig. 9 we observe the variation of mode growth rate with increasing axial flow while keeping poloidal flow constant, for different density gradients at a fixed
Pr, the difference between the growth rates trends is quite stark. It is similarly seen for frequencies in Fig. 10, we observe the variation of mode frequency with increasing axial flow while keeping poloidal flow constant, for different density gradients at a fixed Pr. The results in Fig. 11 we observe the variation of mode growth rate with increasing axial flow while keeping poloidal flow constant, for different Pr at a fixed $\alpha$, seem to conform to our expectation about viscosity changing its nature. The following figure Fig. 12 we observe the variation of mode frequency with increasing axial flow while keeping poloidal flow constant, for different Pr at a fixed $\alpha$, denoted by $dndr$ in the figures, shows a different trend again.

3 Nonlinear Studies

We present the nonlinear results in the following part. The nonlinear results are distinctly different from those in the RMHD case. The nonlinear runs presented in this section were
carried out with a $\alpha = 2.0$ and for $Pr = 100$ and $Pr = 30$, being the high and low viscosity regimes for our nonlinear studies respectively.

In the following we examine the nonlinear evolution of the kink mode in the two fluid regime with an imposed axial flow. In Fig. 13 we observe the nonlinear evolution of $|\psi|$, for the case with axial flow with a fixed Mach number but opposite direction in comparison with the no flow case for $Pr=30$. We have studied separately the $Pr=100$ case where we observe stabilisation. Broadly, the trends seen in RMHD for helical flow are replicated here, in accordance with our expectations. High viscosity is stabilising the flow cases relative to the no flow cases. Low viscosity on the other destabilises the mode in one case, but stabilises it in another case.

In the next figure, we have, Fig. 14 we observe the nonlinear evolution of $|\psi|$, for the case with imposed poloidal flow with a fixed Mach number but opposite direction in comparison with the no flow case for $Pr=30$, we examine the cases with an imposed poloidal flow. For the $Pr=100$ case, we observe no difference. This is in addition to the
intrinsic poloidal flow. We notice that with high viscosity there is no significant effect on evolution, except a slight destabilisation in the linear case for positive imposed poloidal flow.

In the case of low viscosity, we notice a more marked difference in saturation levels with a similar trend as for the case of high viscosity.

Finally, we turn to the cases with combined imposed flows Fig. 15, where we observe the nonlinear evolution of $|\psi|$, for the case with helical flow with a fixed Mach number but opposite direction in comparison with the no flow case for Pr=100, Fig. 16, where we observe the nonlinear evolution of $|\psi|$, for the case with imposed poloidal flow with a fixed Mach number but opposite direction in comparison with the no flow case for Pr=30. Here, we thus have a total of three flows in the system, and it behaves quite differently as compared to the RMHD helical flow evolutionary runs we have studied earlier. However, they seem to behave like poloidal flow runs mentioned above.

\begin{figure}[h]
\centering
\includegraphics[width=0.4\textwidth]{fig13}
\caption{Nonlinear Axial Flow results for Pr=30}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.4\textwidth]{fig14}
\caption{Nonlinear Poloidal Flow results for Pr=30}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.4\textwidth]{fig15}
\caption{Nonlinear Helical Flow results for Pr=100}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.4\textwidth]{fig16}
\caption{Nonlinear Helical Flow results for Pr=30}
\end{figure}
4 Summary and Discussion

In summary, we have studied the effect of flow and viscosity on the internal kink mode in a two fluid regime using the CUTIE code. We have studied both linear and nonlinear aspects of the same. We have studied the trends of growth rate and frequency variation in the linear regime using imposed axial flow, imposed poloidal flow and a helical flow which is a combination of both. In the axial flow case, we have observed that symmetry of the growth rate curves under reversal of direction of flow breaks down with finite density gradient. Increasing viscosity in the runs tends to lower growth rate of the mode. In the poloidal flow case, the slopes of the frequency curves in the Fig. 6 and Fig. 8 behave differently. In Fig. 6 the zero density frequency has a different slope from the cases involving finite density gradient. However, in the Fig. 8 we have all the cases having the same slope. These phenomenon are under investigation, dealing with the interaction of the density gradient and the viscosity present in the system in presence of flow, here, the poloidal flow. It is of note that there is already a intrinsic poloidal flow present in the system due to $\omega^*$ effects in the two fluid regime, absent in the single fluid study we have carried out earlier [1]. Next, we examine combined axial and poloidal flows, which produce helical flows. In particular, we notice a variation in both slope and nature of growth rate and frequency curves. In the nonlinear case, we have observed that the results for the axial flow case are similar to those of RMHD helical flow runs for similar parameters. The poloidal flow seems to have a stronger effect at lower viscosities and negligible effect at higher viscosities. The helical flow cases are similar qualitatively to the poloidal flow cases.

References

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