NONLINEAR GYROKINETIC ANALYSIS OF LINEAR OHMIC CONFINEMENT TO SATURATED OHMIC CONFINEMENT TRANSITION

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Abstract

One of the long lived conundrums in ohmically heated plasmas is that the energy confinement time $\tau_E$ shows a transition from a linear regime proportional to the density (LOC) to a saturation regime (SOC) weakly dependent on the density. In the viewpoint of the first principle nonlinear global gyrokinetic simulations, we here present an investigation of LOC to SOC transition for the first time. In this study, by varying a single parameter plasma density, the confinement time estimated by $\tau \propto 1/\lambda_{\text{eff}}$ shows a transition from a linearly increasing regime to a saturation regime as the plasma density increases. The effective transport diffusivity is defined as $\lambda_{\text{eff}} \equiv \frac{n_e T_e}{n_i T_i + n_e T_i}$, where $n_e$, $T_e$, and $T_i$ are density, temperature and heat diffusivity for electron (e) and ion (i). The above nonlinear result follows the trend from the mixing length quasilinear estimation for the heat transport. A transition of trapped electron dominant heat transport from TEM to ion dominant heat transport from ITG is observed when the LOC to SOC transition occurs. In the simulations, the Coulomb collision operator for ion-ion collision and the pitch-angle scattering operator for electron-ion collision are included. Trapped electrons are more scattered into passing regime for a higher density as electron-ion collision becomes more frequent.

1. INTRODUCTION

The transition of energy confinement time $\tau_E$ from the linear Ohmic confinement (LOC) regime to saturated Ohmic confinement (SOC) regime in Ohmic plasmas is a long-lived conundrum in tokamak fusion plasmas. It was found in early experiments in the Alcator C tokamak[1] that the energy confinement time $\tau_E$ is linearly proportional to the line-averaged electron density $\tilde{n}_e$, which is referred to as the LOC regime. The confinement time saturates in the SOC regime as the density increases, showing weak dependency on the density. Similar observations were also made in later experiments in Texas Experimental Tokamak (TEXT)[2], DIII-D[3] and ASDEX Upgrade[4]. Recently, the intrinsic rotation reversal observed in Alcator C-Mod ohmic L-mode plasmas was found to have a strong correlation with the LOC to SOC transition[5, 6]. Furthermore, $E \times B$ staircase was recently found to exist in SOC regime and disappear in the low density LOC regime[7].

Though LOC-SOC transition is studied widely in experiments and many devices, it remains as a mystery and the understanding of the underlying physics is still poor. Interpretation of early Ohmic plasmas is on the basis of a theoretical consideration[8] that trapped electron modes at low density and ITG modes at high density, presumably assuming TEM is stabilized by collision and ITG becomes dominant. Experiments in ASDEX Upgrade obtained a consistent result[4] and a transition from TEM to ITG was observed as collisionality increases[9]. The study of Alcator C-Mod tokamak Ohmic plasmas and comparisons with local gyrokinetic simulations address the importance of collision in stabilizing TEM by scattering trapped particles with justifications in a linear limit[10]. As far as we know, a robust and concrete theoretical understanding of the mechanism governing the LOC regime and the transition to SOC is still lacking.

Recently, direct measurements[11] of long wavelength ($k_B \rho < 0.3$) electron temperature fluctuations in Alcator C-Mod showed a significant reduction (~40%) crossing from LOC to SOC, which is consistent with the hypothesis that a microturbulence transition of ITG to TEM occurs alongside with LOC-SOC transition. However in that paper, the linear microinstability analysis presented a strong local dependence that ITG can be still dominant in the LOC regime deeper in the core. A further investigation in C-Mod[12] reported that with nearly indistinguishable density and temperature profiles, LOC-SOC transition occurs as density increases, though linear gyrokinetic analysis demonstrated the microinstability remains as ITG in both LOC and SOC regimes. Understanding of the physics underlying LOC-SOC transition and the relation to ITG-TEM microturbulence transition still remains controversial and requires further efforts. We also note that almost all...
the existing theoretical study are from linear gyrokinetics, and there exist rare nonlinear gyrokinetic study on this issue.

Fortunately, with a recent development of gyrokinetic modeling scheme, especially with the implementation of bounce-averaged gyrokinetic model[13] in the ITG-TEM microinstability[14] and microturbulence[15] simulations, we are able to perform nonlinear gyrokinetic study of ITG-TEM simulations with significantly reduced computational costs. In this paper, we present for the first time an investigation of LOC-SOC transition from the viewpoint of the first principle nonlinear global gyrokinetic simulations. The rest of the paper is organized as the following. In Sec. 2, setup of numerical simulations will be presented. Observation of LOC-SOC transition from the nonlinear simulations and detailed analysis and discussions of the underlying physics will be given in Sec. 3. A quasilinear estimation will also be discussed based on the linear microinstability simulations and scattering of trapped electrons by collisions. Finally, a conclusion and discussion along with future work will be presented in Sec. 4.

2. SIMULATION SETUP

In this work, we use the gyrokinetic PIC code gKPSP, which numerically solves the gyrokinetic equations[16, 17] and the bounce-averaged kinetic equations for ions and trapped electrons, respectively. Passing electrons are modeled as passively following the evolution of turbulent electrostatic field and zonal flow. A zonal flow conserving Krook operator[18] is employed to control discrete particle noises and to provide a source to maintain initially given temperature and density profiles.

In order to investigate the LOC-SOC transition physics, the following parameters are adopted. The major and minor radius are set as \( R_0 = 1.86m \) and \( a = 0.67m \) with the inverse aspect ratio \( a/R_0 = 0.36m \). The central magnetic field is given as \( B_0 = 1.91T \). The safety factor \( q \) satisfies \( q = 0.58 + 3.04\varepsilon + 8.5\varepsilon^2 \) where \( \varepsilon \) denotes the local inverse aspect ratio. The ions are set as hydrogen with realistic ion to electron mass ratio \( m_i/m_e = 1836 \). Radial profiles of the main physical quantities are plotted in Fig. 1. In this figure, the gradients of ion temperature \( (T_i) \), electron temperature \( (T_e) \), and density \( (n) \) are chosen as \( R_0/L_{Ti} = 4.0 \), \( R_0/L_{Te} = 6.0 \), and \( R_0/L_n = 2.2 \), respectively. The central ion temperature is set to satisfy \( a/\rho_i = 250 \) and \( T_i/T_e = 0.58 \) at the mid-radius \( r = 0.5a \). Here \( \rho_i = T_i/m_i/\Omega_i \) denotes the ion gyroradius and \( \Omega_i = eB/(m_i\varepsilon) \) represents the ion gyro frequency. An axisymmetric concentric circular geometry is used in this series of simulations. The toroidal simulation domain is set as 1/4 wedge with the maximum toroidal mode number \( n_B = 116 \), such that the maximum poloidal wave number \( k_p\rho_i = 1.22 \) at mid-radius. The number of ion and trapped electron marker particles are 200 and 40 per cell, respectively. The radial grid size is set as \( dr \sim \rho_i \).

In this series of numerical experiments, only density is varied, while all other parameters are fixed. Note that in Ref. [12], LOC-SOC transition is observed with nearly unchanged profiles, our simulation setup is reasonable. Our selections of density and temperature gradients are also in accordance with those Ohmic or ECRH heating plasma experiments of LOC-SOC studies. The theory investigation with gyrokinetic simulations can provide more advantages as we can fix all other parameters and study the physical effects from a single parameter variation. In experiments, it is usually impossible to vary a single parameter, and many effects from other parameters’ variation are involved. What’s more, the Coulomb collision operator for ion-ion collision and the pitch-angle scattering operator for electron-ion collision are included in this series of simulations. Here we define the effective ion-ion collisionality \( \nu_{ii} \equiv \nu_{ii}/\omega_{bi} \), here \( \nu_{ii} \) is ion-ion collisionality, \( \omega_{bi} \) is ion bounce frequency \( \omega_{bi} = V_{Ti}/(qR_0\varepsilon^{-3/2}) \) with \( V_{Ti} = T_i/m_i \) the ion thermal velocity. The electron-ion effective collisionality \( \nu_{ei} \equiv \nu_{ei}/\omega_{be} \), here \( \nu_{ei} \) is electron-ion collisionality, \( \omega_{be} \) is electron bounce frequency \( \omega_{be} = V_{Te}/(qR_0\varepsilon^{-3/2}) \) with \( V_{Te} = T_e/m_e \) the electron thermal velocity. Neoclassical transport is not involved in this study.

3. OBSERVATION OF LOC-SOC TRANSITION: NONLINEARGYROKINETIC RESULT

In this section, we will present the observation of LOC-SOC transition from a series of nonlinear gyrokinetic simulations by varying plasma density. The underlying physics including the background nonlinear microturbulence will also be discussed.

3.1. Showing the LOC-SOC transition
Transport diffusivities are presented in Fig. 2 as a function of density. There includes electron heat diffusivity $\chi_e$ (black), ion heat diffusivity $\chi_i$ (red) and particle diffusivity $D$ (blue), which are normalized to gyroBohm diffusivity $\chi_{GB} \equiv \rho_i^2 V_{Ti}/a$. The effective electron-ion collisionality is also indicated on the top $x$-axis. As the density ($\nu_{ei}$) increases, we can identify the following interesting characteristics of the transport. Electron heat diffusivity $\chi_e$ decreases fast and finally vanishes to the noise level. Ion heat diffusivity $\chi_i$ decreases first at low density and then gradually goes up. The particle diffusivity $D$ gradually evolves from an out transport to inward transport. As $\chi_e$ goes to noise level, $D$ becomes nearly unvaried. We will not explain the physics underlying these observations now, but revisit this figure later. We now proceed to estimate the confinement time based on these diffusivities.

**FIG. 1.** Initial setup of radial profiles for safety factor $q$, magnetic shear $S_q$, density gradient $R_0/L_n$, ion temperature gradient $R_0/L_{Ti}$, electron temperature gradient $R_0/L_{Te}$ and ion (electron) temperature $T_i$ ($T_e$).

**FIG. 2.** Electron heat diffusivity $\chi_e$ (black), ion heat diffusivity $\chi_i$ (red) and particle diffusivity $D$ (blue) as a function of plasma density and effective electron-ion collisionality ($\nu_{ei}^*$).

We here define an effective heat transport diffusivity $\chi_{eff} = \frac{n_e \chi_e \nabla T_e + n_i \chi_i \nabla T_i}{n_e \nabla T_e + n_i \nabla T_i}$, where $n_e(i), T_e(i)$ and $\chi_{e(i)}$ are density, temperature and heat diffusivity for electron (e) and ion (i). The effective heat diffusivity is computed and plotted in Fig. 3a as a function of density ($\nu_{ei}$). We further estimate the confinement time by $\tau_{eff} \propto 1/\chi_{eff}$, as plotted in Fig. 3b. From the figure, we can clearly identify a transition from a linearly increasing regime to a saturated regime where the confinement time saturates and slightly decreases, as the density ($\nu_{ei}$) increases.
Here, as presented, we show a transition from linear confinement to saturated confinement by increasing the plasma density ($\nu_{ei}^*$) while keeping all other parameters fixed. The red arrow in Fig. 3b indicates approximately the nonlinear transition density $n_{NT} = 5.0 \times 10^{19}$ for this series of simulations’ parameters. Note that as density increases, electron-ion collisionality ($\nu_{ei}^*$) increases accordingly. Suppression of trapped electron turbulence by the electron-ion collision is observed and responsible for the transition from linear confinement to saturated confinement.

![Image](image_url)

**FIG. 3.** (a) effective heat diffusivity $\chi_{\text{eff}}$, (b) the confinement time $\tau_{\text{eff}}$ estimated from $\chi_{\text{eff}}$ as a function of the density ($\nu_{ei}^*$). The red arrow shows the nonlinear transition density $n_{NT}$, where the LOC – SOC transition occurs.

### 3.2. Microturbulence characteristics in LOC and SOC regimes

In this subsection, we will investigate the characteristics of background microturbulence for varying densities.

![Image](image_url)

**FIG. 4.** Electrostatic potential spectra in $(\omega, k_\theta)$ space for increasing densities. Red arrows show approximately the moving trend of the strength of the spectrum.

In Fig. 4, it shows the electrostatic potential spectra at mid-radius ($r = 0.5a$) in $(\omega, k_\theta)$ space for densities $n_0 = 1.5 \times 10^{19}, 2.5 \times 10^{19}, 3.5 \times 10^{19}$ and $8.0 \times 10^{19}$. The red dashed lines divide the spectrum range of ITG
(bottom half) and TEM (top half) in each sub figures. It can be seen from the figure that as density increase, the spectra gradually moves from TEM regime to ITG regime, as indicated by red arrows. To quantify the strength of TEM spectra, we here define

\[ S_{\text{TEM}} = \frac{\sum_A \varphi(\omega, k_\theta)}{\sum_B \varphi(\omega, k_\theta)} \]  

(1).

Here the summation condition A is that if \( k_\theta > 0, \omega > 0 \) and \( \varphi(\omega, k_\theta) > 0.5 \times \text{maximum of } \varphi(\omega, k_\theta) \). Summation condition B is that if \( k_\theta > 0 \) and \( \varphi(\omega, k_\theta) > 0.5 \times \text{maximum of } \varphi(\omega, k_\theta) \).

The strength of ITG spectra is therefore computed by the definition of \( S_{\text{ITG}} = 1 - S_{\text{TEM}} \). The computed strength of TEM and ITG spectra is plotted in Fig. 5 as a function of density \( (\nu_{ei}^*) \). It can be seen from the figure that TEM strength is dominant at lower density, while ITG gets dominant as density increases. The green arrow shows the spectrum strength flip density \( n_{\text{ST}} = 3.2 \times 10^{19} \) approximately. Therefore, the suppression of trapped electron turbulence by increasing density (electron-ion collision) is confirmed in this spectra analysis. Turbulence transits from TEM dominance to ITG dominance along with the LOC-SOC transition in this series of simulations.

4. INTERPRETATION OF THE NONLINEAR RESULT

4.1. Linear and quasilinear analysis

In this subsection, we present the linear and quasilinear analysis and comparison with the nonlinear result. The maximum linear growth rate \( (\gamma) \) is shown in Fig. 6a as a function of density \( (\nu_{ei}^*) \). It is seen from the figure that the maximum linear growth rate is reduced sharply at lower densities, while nearly not varied at higher densities, as density increases. Accordingly, by using the mixing length estimation of the heat diffusivity \( \chi_{\text{MLE}} = \gamma / k_\theta^2 \), and following the previous method to estimate the confinement time \( \tau_{\text{MLE}} = 1 / \chi_{\text{MLE}} \). We obtain the quasilinear estimated confinement time as a function of density \( (\nu_{ei}^*) \), as plotted in Fig. 6b. It can be seen from the figure that as density increases, the mixing length estimated confinement time shows a transition from a linear regime to a saturated regime along with a transition density \( (n_{\text{LT}} = 3.5 \times 10^{19}) \).

The quasilinear estimation is consistent with the nonlinear result, and shows that the confinement time transits from a linear regime to a saturated regime along with a transition of the background microturbulence from TEM to ITG. We note that there is an important observation that the linear transition density \( (n_{\text{LT}} = 3.5 \times 10^{19}) \) is different (and usually smaller than) from the nonlinear transition density \( (n_{\text{NT}} = 5.0 \times 10^{19}) \). This might be able to explain some experiments[12] that only ITG can be confirmed all through LOC-SOC transition from the linear gyrokinetic analysis. The reason why linear gyrokinetic analysis is not enough to recognize the
background turbulence is that the TEM-ITG transition from linear analysis is typically a bifurcation process, while the suppression of trapped electron heat flux is not but a gradually evolving process, as one can tell from Fig. 2. We also would like to note that the linear transition density is close to the spectrum strength flip density \( n_{ST} = 3.2 \times 10^{19} \), which is in agreement.

Revisiting Fig. 2, there are comprehensive nonlinear phenomena to discuss. Firstly, the trapped electron heat flux is mitigated and suppressed eventually by higher density due to the suppression of TEM microturbulence. Secondly, the ion heat flux shows an initial drop at lower density but gradually increases at higher density. The initial drop of ion heat flux is mainly due to the reduction of linear growth rate, which follows the quasilinear process. At higher density, due to higher effective ion-ion collisionality \( \nu_{ii}^* \), zonal flow are damped[19] and therefore the ion heat flux increases. This mechanism is responsible for the degradation of confinement in the saturated regime, as one can see in Fig. 3. Finally, the particle flux shows a flip from outward flux induced by TEM to inward flux induced by ITG. This flip of particle flux introduces a density range with nearly vanishing particle transport, which should be interesting to experimental operations.

4.2. Suppression of TEM by breaking precession resonance

In order to understand how density (collision) suppress trapped electron turbulence, we here make efforts to study the effects of collision on the trapped electrons, and consequently the trapped electron precession resonance. Collisionless trapped electron mode is destabilized by the magnetic curvature drift resonance[20]. In our study, small collisionality is found to stabilize the trapped electron mode. Thus the impacts of electron-ion collision on the trapped electron population is critical for this stabilization mechanism. Fig. 6 shows the loss rate of trapped electrons in a unit time of \( R_0/V_{Ti} \) as a function of density \( \nu_{ei}^* \). It is seen from the figure trapped electrons are lost faster at higher densities. In other words, electron-ion collision scatters trapped electrons into passing electrons much faster at higher collisionality. Though those lost trapped electron population can be mostly compensated by scattering passing electrons into trapped regime, the environment of trapped electron precession resonance can be destroyed. Thus small electron-ion collision can stabilize TEM by destroying the destabilization mechanism, i.e., precession resonance.
5. CONCLUSION AND FUTURE WORK

In this study we have performed a series of nonlinear global gyrokinetic simulations to show that the estimated confinement time transits from a linear regime to a saturated regime with a set of parameters in accordance with typical LOC-SOC experiments. This LOC-SOC transition in our numerical experiments is induced by the only variable in the simulations, i.e., density or more specifically density induced collision. Collision scatters trapped electrons and destroy the trapped electron precession resonance to stabilize trapped electron turbulence. Background microturbulence is found to transit from TEM to ITG alongside with the LOC-SOC transition. Quasilinear analysis shows nearly consistent result with the nonlinear gyrokinetic simulations. The nonlinear transition density is found larger than the linear transition density. Our study in this paper is consistent with typical LOC-SOC experimental observations, and can provide a theoretical base to understand and explain those LOC-SOC experiments.

Alongside LOC-SOC transition, experiments report many other phenomena, such as intrinsic rotation reversal, $E \times B$ staircase, etc.. Therefore, for the future work, gyrokinetic simulation in relating our theoretical studies to those phenomena is interesting. We are also interested to adopt the experimental profiles directly in our gyrokinetic simulations to understand the experiments. If possible, a direct insight into trapped electron toroidal precession resonance and the collision effects will be also interesting for future study.

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