Real-time simulation of the NBI fast-ion distribution

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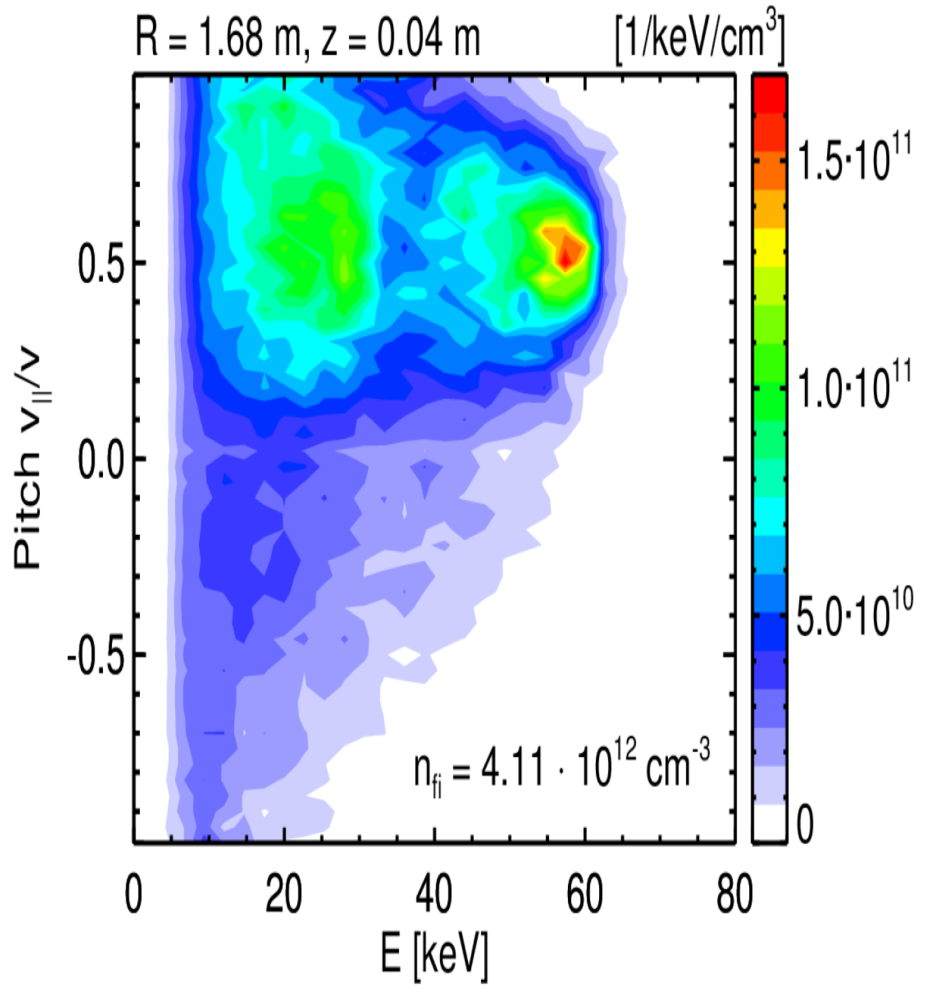
RABBIT: Real-time model for the NBI fast-ion distribution

Motivation:

- Fast ion distribution function is required for instance:
  - Heating profiles for transport calculations
  - pressure and current-drive for equilibrium reconstructions
- Sophisticated simulation codes exist (e.g. TRANSP/NUBEAM based on Monte Carlo), but long computation time (~ 30 s per time-step)
- → Too slow for real-time applications (e.g. discharge control systems, real-time transport solvers like RAPTOR)
- → Develop fast model Rapid Analytical Based Beam Injection Tool – RABBIT [M. Weiland, NF 2018] (~ 20 ms per time-step)

TRANSP fast-ion distribution function



Kinetic equation

Kinetic equation for distribution function f(x, v, t)

$$\frac{\partial f}{\partial t} + \underbrace{\boldsymbol{v} \cdot \nabla_{\boldsymbol{x}} f + \boldsymbol{a} \cdot \nabla_{\boldsymbol{v}} f}_{\text{Orbit effects}} = \nabla_{\boldsymbol{v}} \cdot \Gamma_c(f) \quad + \text{Source}$$

Orbit effects

$$\boldsymbol{a} = \frac{q}{m}\left(E + \frac{\boldsymbol{v}}{c} \times B\right)$$

Time dependence
(=0 for steady state
solution)

collisions
(e.g. slowing down,
pitch angle scattering)

Source = NBI
deposition

Kinetic equation - outline

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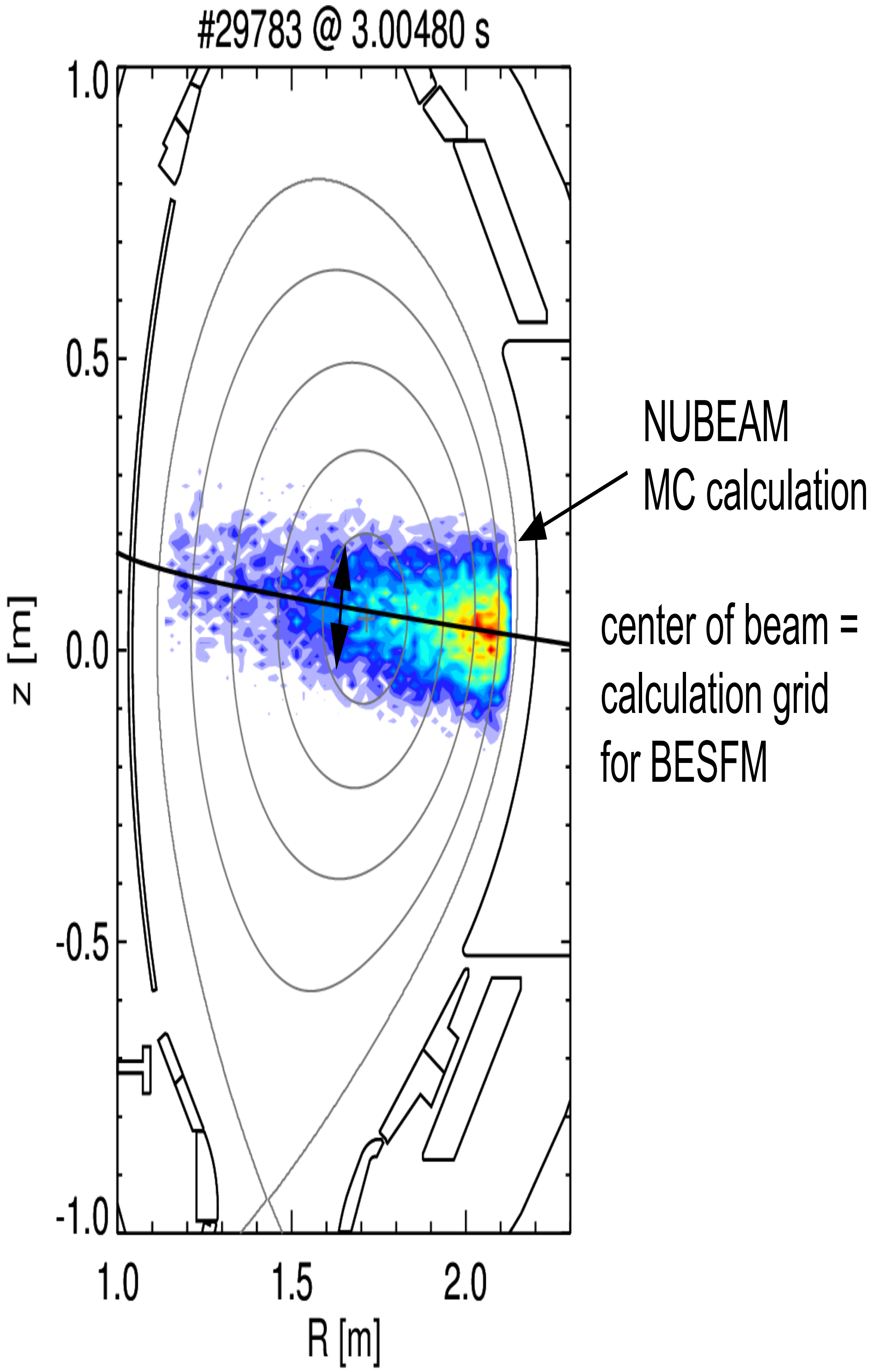
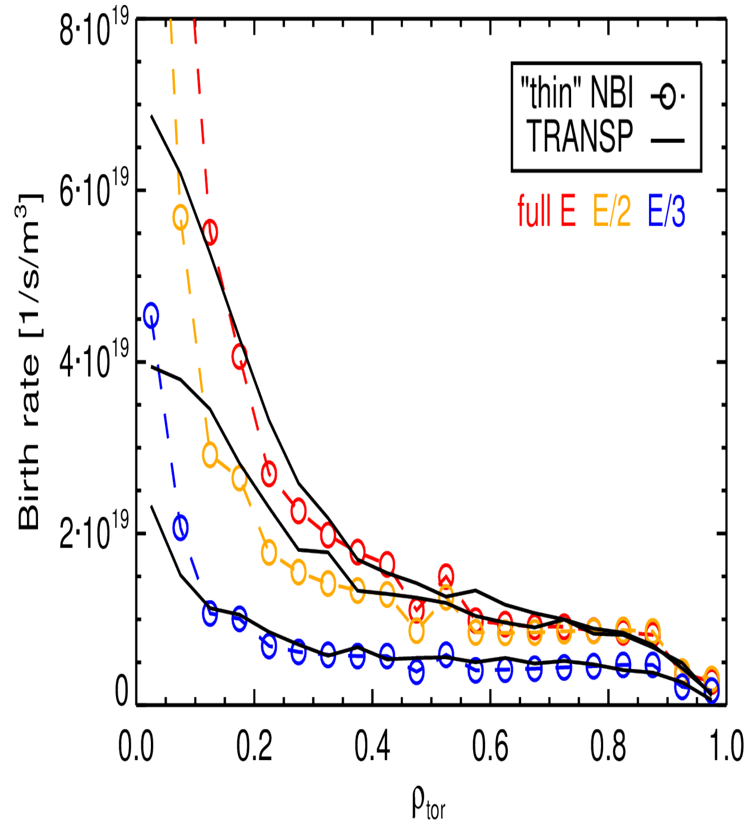
5. Applications

Beam deposition (birth profile)

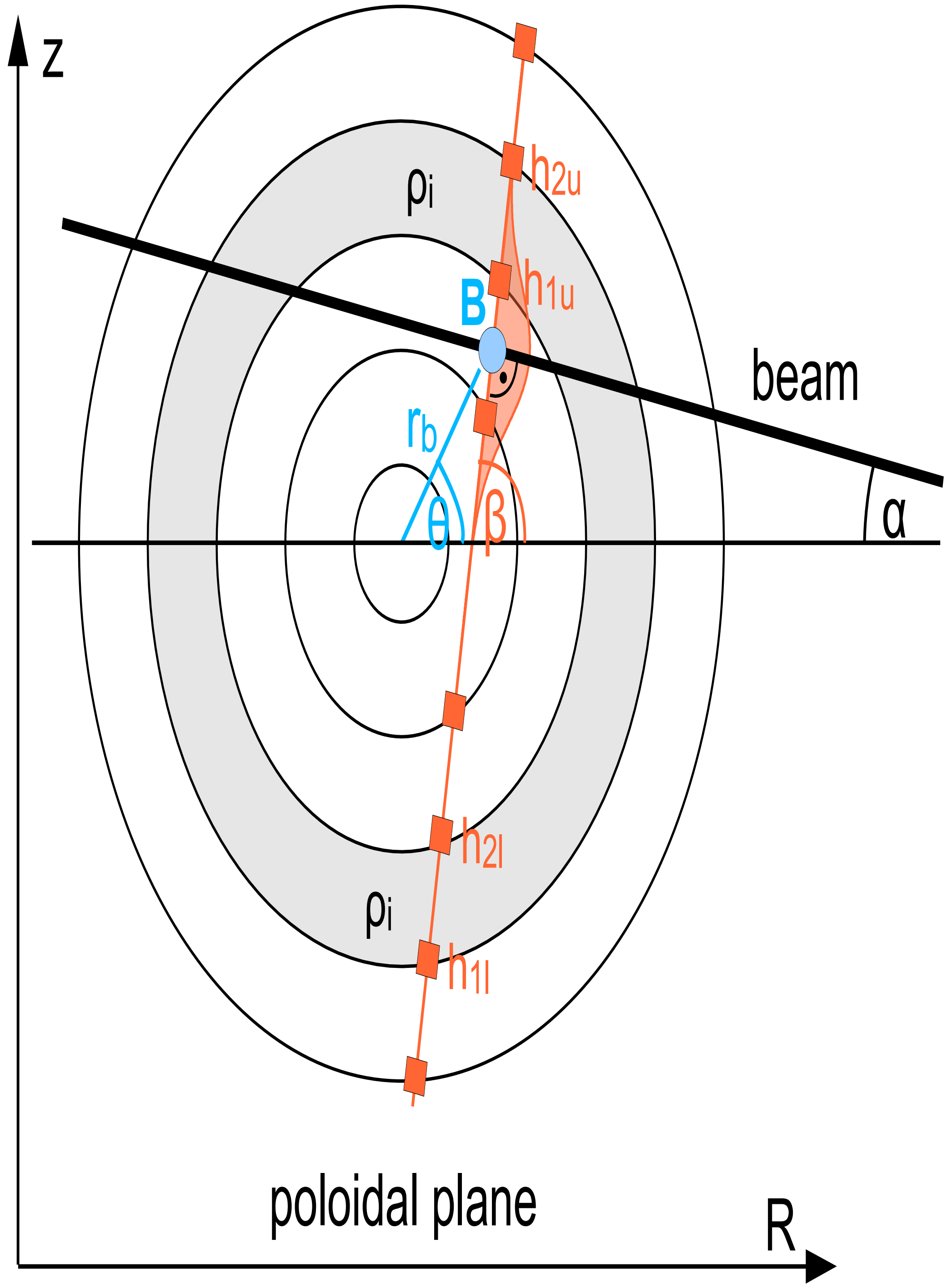


- Injection of fast neutrals into the plasma → Ionization → newly "born" fast ion
- Fast-ion birth rate = - (beam attenuation rate)
- Calculation of beam attenuation: BESFM Code by A. Lebschy, R. Dux, IPP
- We use the simplest geometry: NBI as thin line
- Good approximation for attenuation – for birth profile, we need to take into account the beam width:



- Assume a Gaussian broadening with standard deviation σ(l), l = coordinate along beam, σ(l) defined by NBI parameters (e.g. divergence)

Analytic model for the poloidal beam width



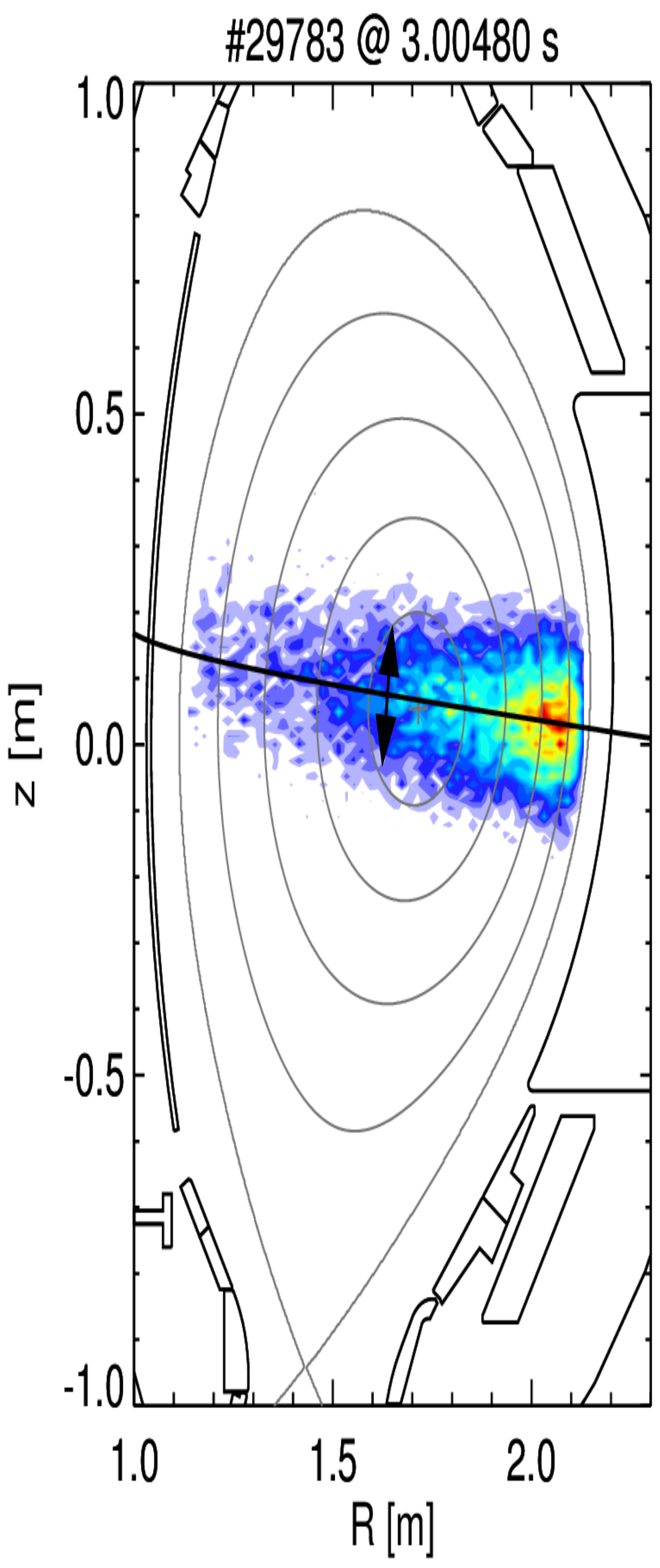
- We assume Gaussian spreading along orange line (standard deviation σ)
- Assume circular concentric flux surfaces
- Transformation between flux coordinate ρ and geometric radius r based on ratio at B:
  $r(\rho) = \rho * (r_b / \rho_b)$
- → Crossings ■ with ρ-cells can be calculated analytically
- → Contribution into i-th cell ρ_i:

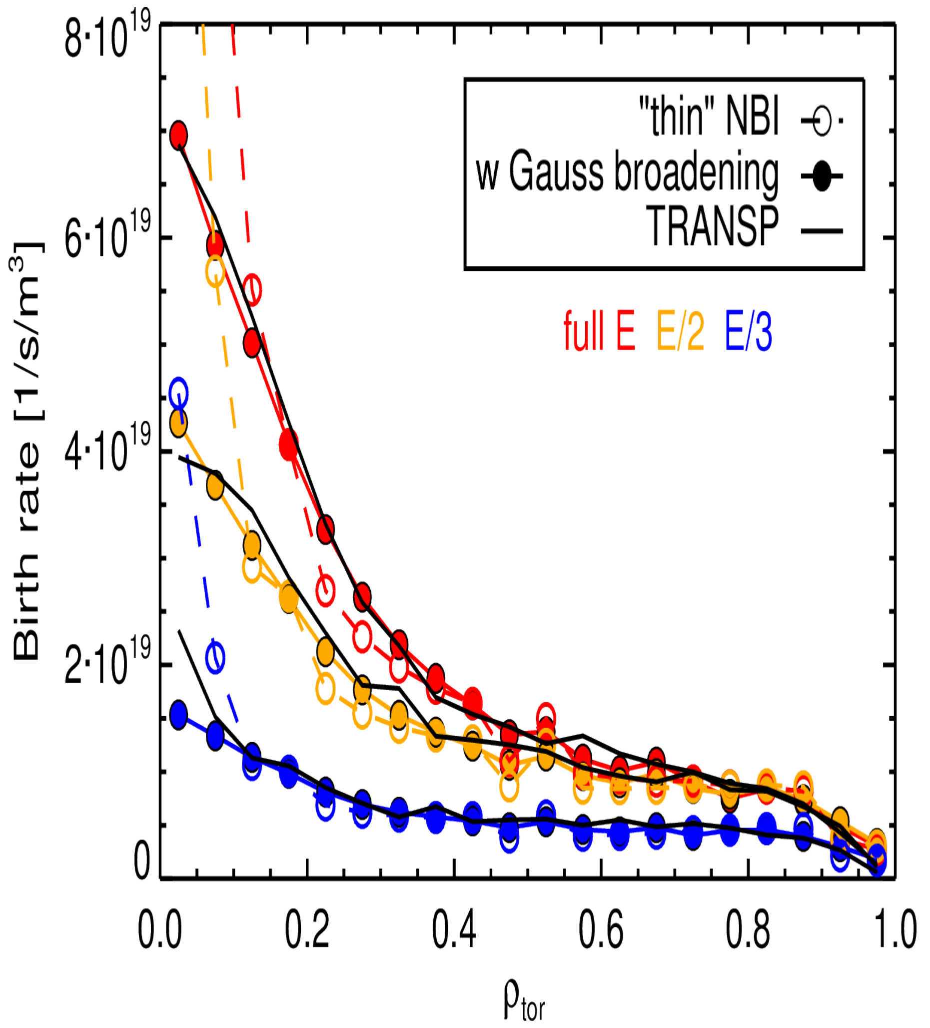
$$\frac{1}{2}\left(\operatorname{erf}\frac{h_{2u}}{\sqrt{2}\sigma} - \operatorname{erf}\frac{h_{1u}}{\sqrt{2}\sigma}\right) + \frac{1}{2}\left(\operatorname{erf}\frac{h_{2l}}{\sqrt{2}\sigma} - \operatorname{erf}\frac{h_{1l}}{\sqrt{2}\sigma}\right)$$

- Correction for plasma elongation:
  Scale beam width σ according to elongation b/a at B.

$$\sigma = \sigma_0 \cdot \sqrt{\frac{(a\cos\theta)^2 + (b\sin\theta)^2}{(a\cos\beta)^2 + (b\sin\beta)^2}}$$

Beam deposition (birth profile) with beam-width correction





- Taking into account a Gaussian broadening of the beam leads to good agreement with TRANSP/NUBEAM

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(=0 for steady state
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Analytic solution of the Fokker-Planck equation

$$\underbrace{\frac{1}{\tau_s v^2}\frac{\partial}{\partial v}[(v^3+v_c^3)f]}_{\text{slowing down}} + \underbrace{\frac{\beta\, v_c^3}{\tau_s v^3}\frac{\partial}{\partial \xi}(1-\xi^2)\frac{\partial f}{\partial \xi}}_{\text{pitch angle scattering}} + \underbrace{\cancel{\frac{1}{\tau_s v^2}\frac{\partial}{\partial v}\left[\left(\frac{T_e}{m_{fi}}v^2+\frac{T_i}{m_{fi}}\frac{v_c^3}{v}\right)\frac{\partial f}{\partial v}\right]}}_{\text{speed diffusion}} = \cancel{\frac{\partial f}{\partial t}} - \underbrace{\frac{S}{2\pi v^2}\delta(v-v_0)K(\xi)}_{\text{source term}}$$

S: deposition, v0: injection velocity (mono-energetic)
K(ξ): broad pitch distribution

- Solution [Cordey/Core 74]:

Legendre polynomials

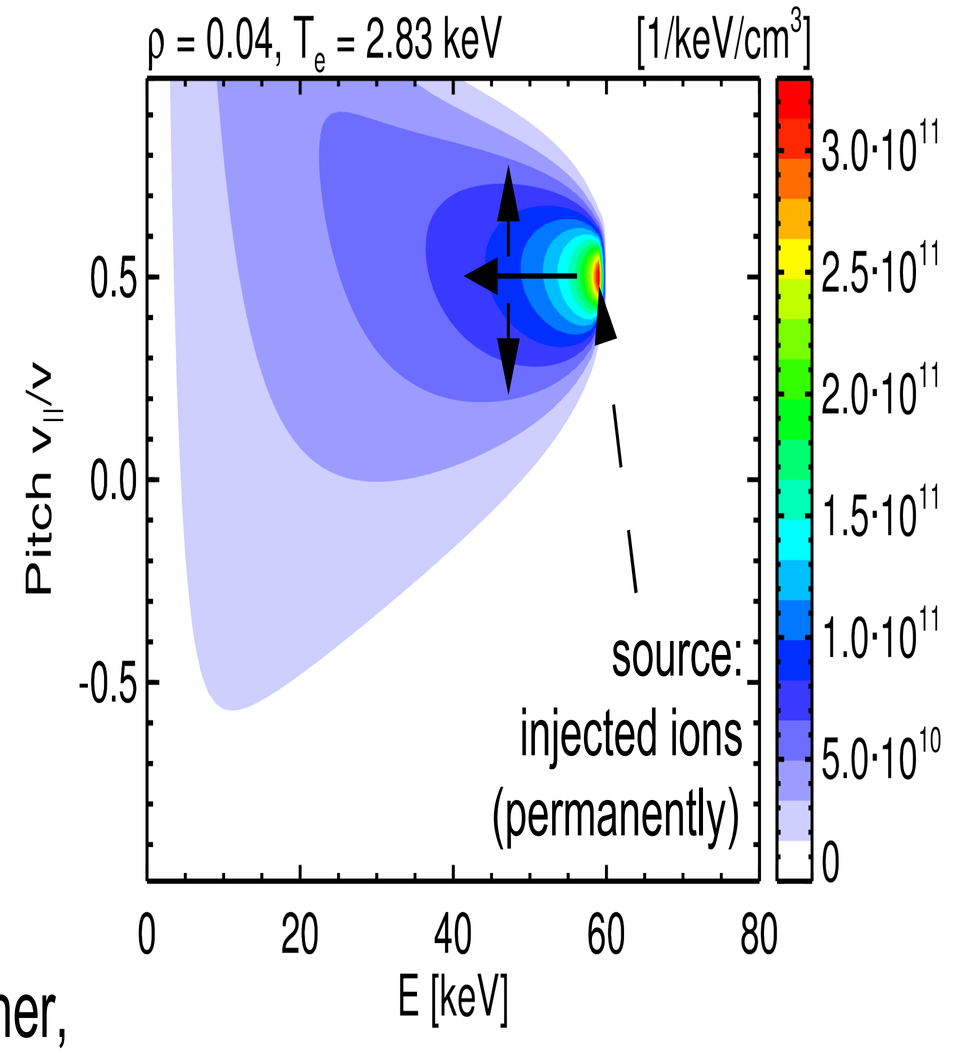
$$\boxed{f(v,\xi) = \frac{1}{2\pi}\frac{S\cdot\tau_S}{v^3+v_C^3}\cdot\sum_{l=0}^{\infty}\left(l+\tfrac{1}{2}\right)u^{l(l+1)}P_l(\xi)K_l\cdot H(v_0-v)}$$

v_C, v_0 : critical, injection velocity

$$u = \left(\frac{v_0^3+v_C^3}{v^3+v_C^3}\frac{v^3}{v_0^3}\right)^{\pm\beta/3} \qquad \beta = \frac{Z_{eff}}{2}\frac{m_i}{m_{beam}}$$

$$K_l = \int K(\xi)P_l(\xi)\,d\xi$$

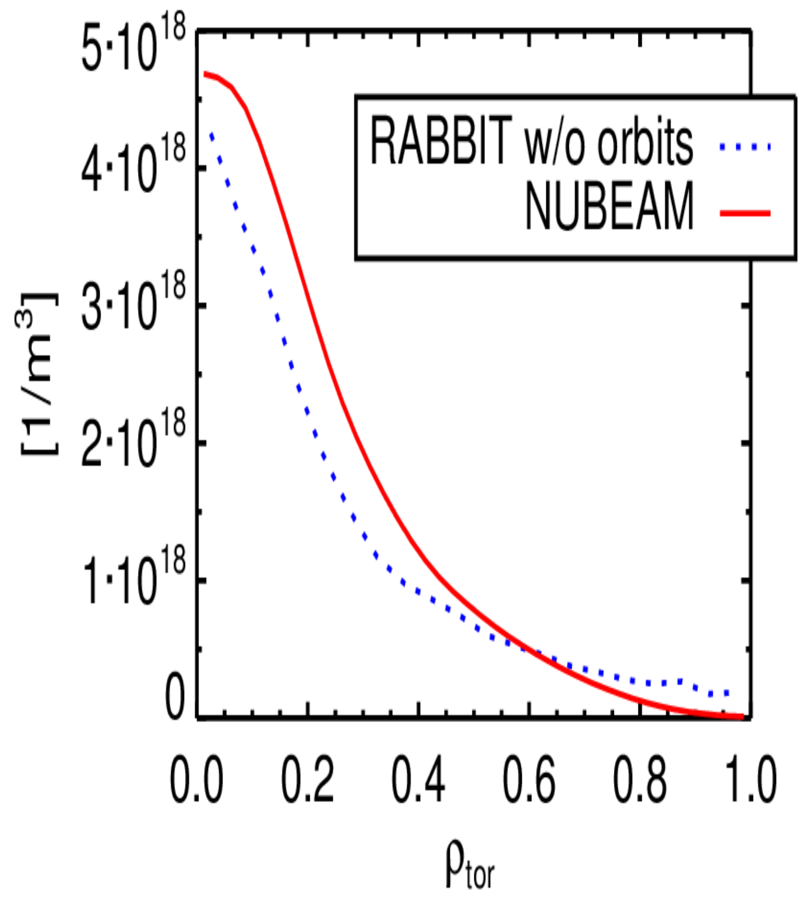
$$\tau_S = 6.27\cdot 10^8\cdot\frac{A_b}{Z_b^2\ln\Lambda_e}\cdot\frac{(T_e[\mathrm{eV}])^{3/2}}{n_e[\mathrm{cm}^{-3}]}\ \mathrm{s} \qquad v_C^3 = 1.51\cdot 10^{14}\cdot(T_e[\mathrm{eV}])^{3/2}\cdot\frac{\sum_i n_i Z_i^2\ln\Lambda_i/A_i}{n_e\ln\Lambda_e}$$

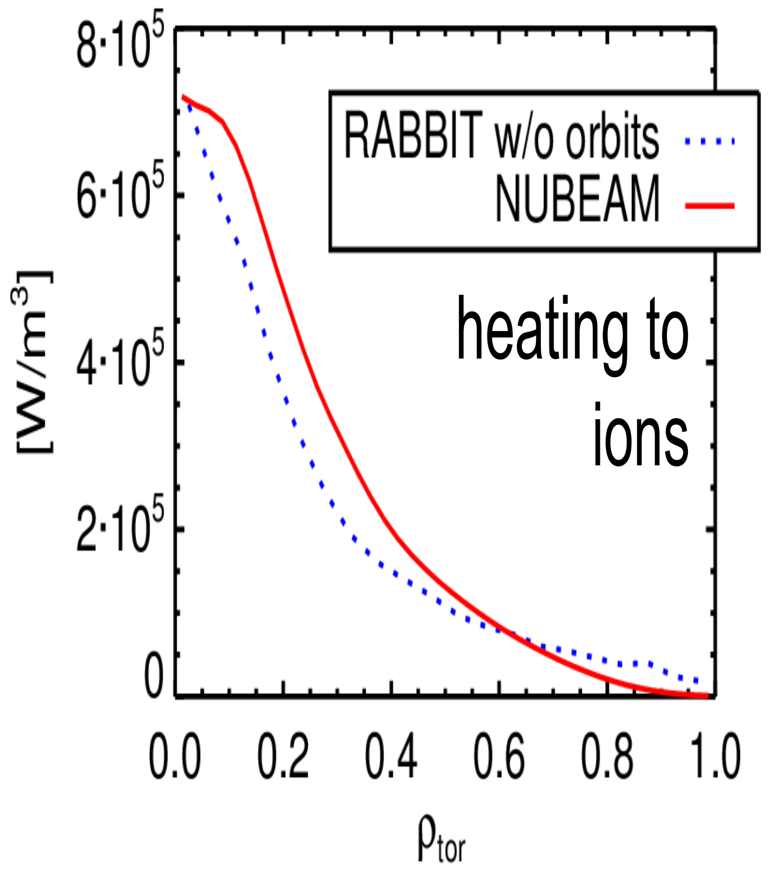


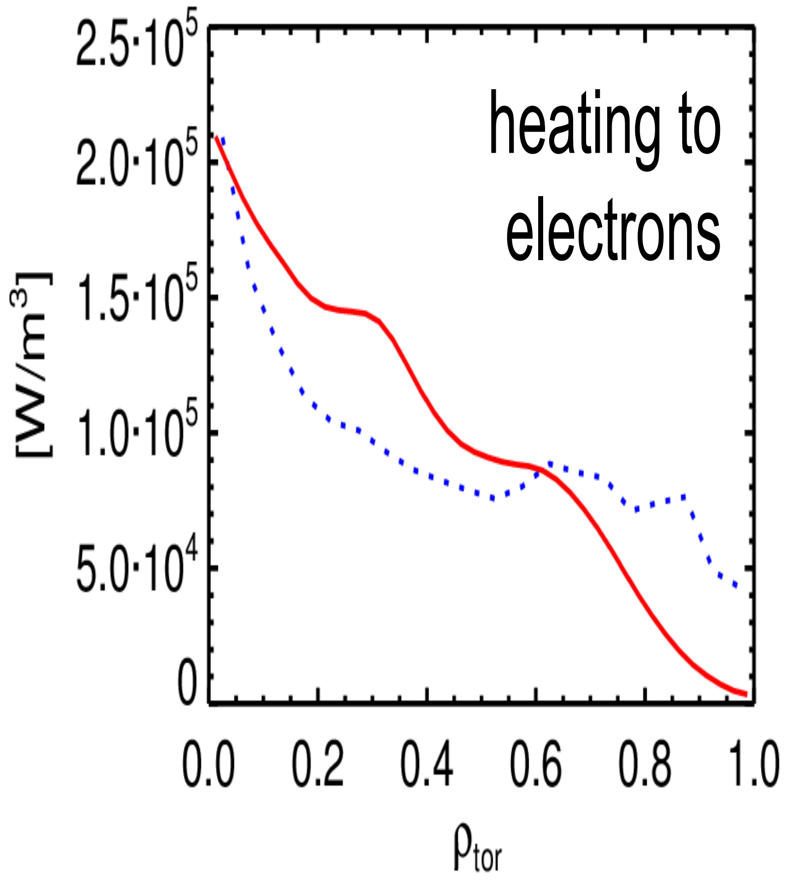
- Uniform plasma solution, i.e. each radial cell is independent of each other, no particle trapping etc.
- A correction for speed diffusion is applied above injection energy.

Density and heating profiles

- In the end we are interested in integrals of f, e.g.:
  Heating power (to electrons and ions), fast-ion pressure and current drive
- These integrals can also be solved analytically. Due to orthogonality of the Legendre polynomials, only first few moments are necessary (l=0, 1)
- E.g. fast-ion density: $n_{\mathrm{fi}} = \iint 2\pi v^2 \cdot f(v,\xi)\,\mathrm{d}v\,\mathrm{d}\xi = \frac{S\tau_S}{3} \ln\left(\frac{v_0^3 + v_C^3}{v_C^3}\right)$







- Profile shapes do not (yet) agree well, due to missing orbit-effects
- Under-estimation in the core, over-estimation at the edge

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3. Orbit effects

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4. Time dependence (=0 for steady state solution)

2. collisions (e.g. slowing down, pitch angle scattering)

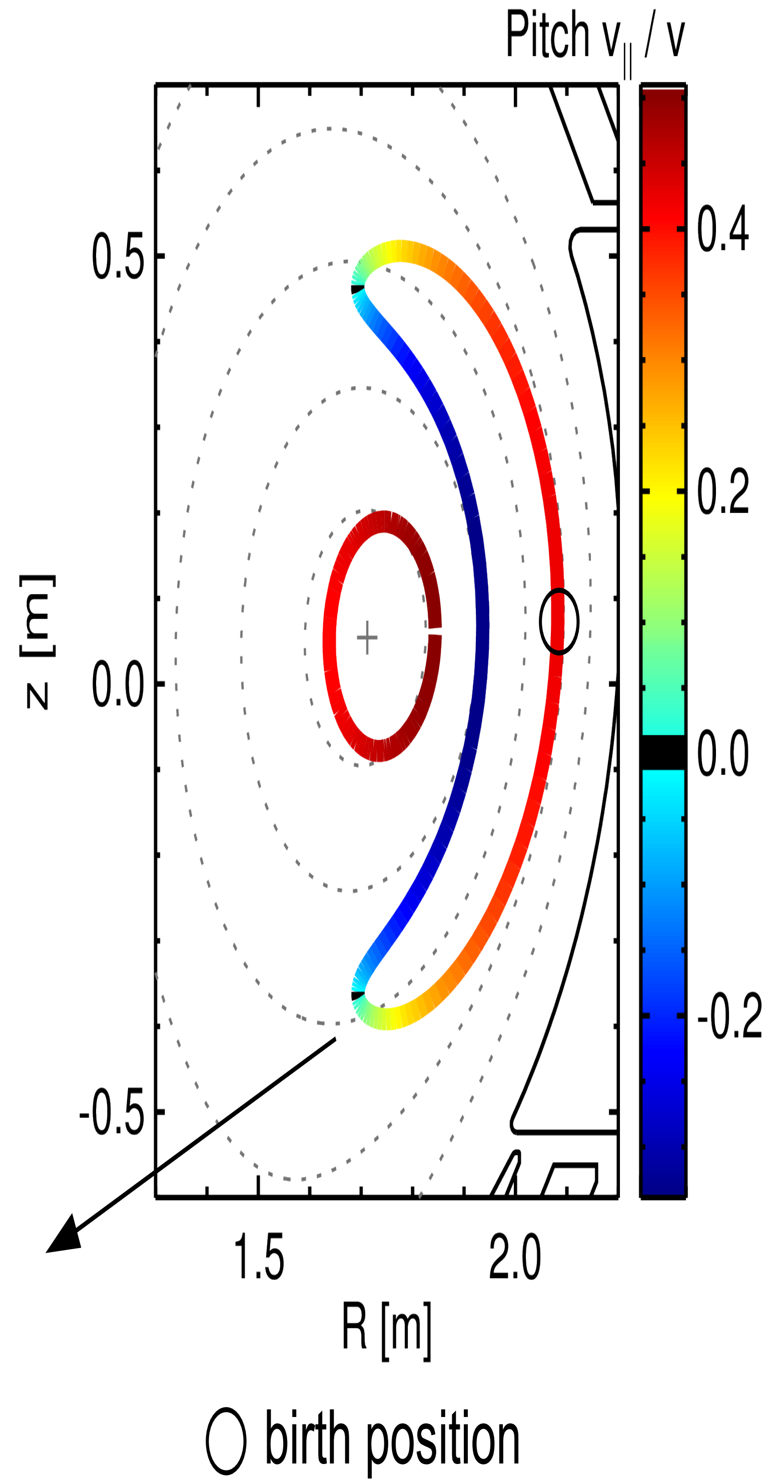
1. Source = NBI deposition

- In MC codes (e.g. NUBEAM)
  - MC representation of source
  - Calculate orbits for each MC marker
  - Apply collision operator during orbit steps
- For real-time: Only ad-hoc treatment of orbit effects possible

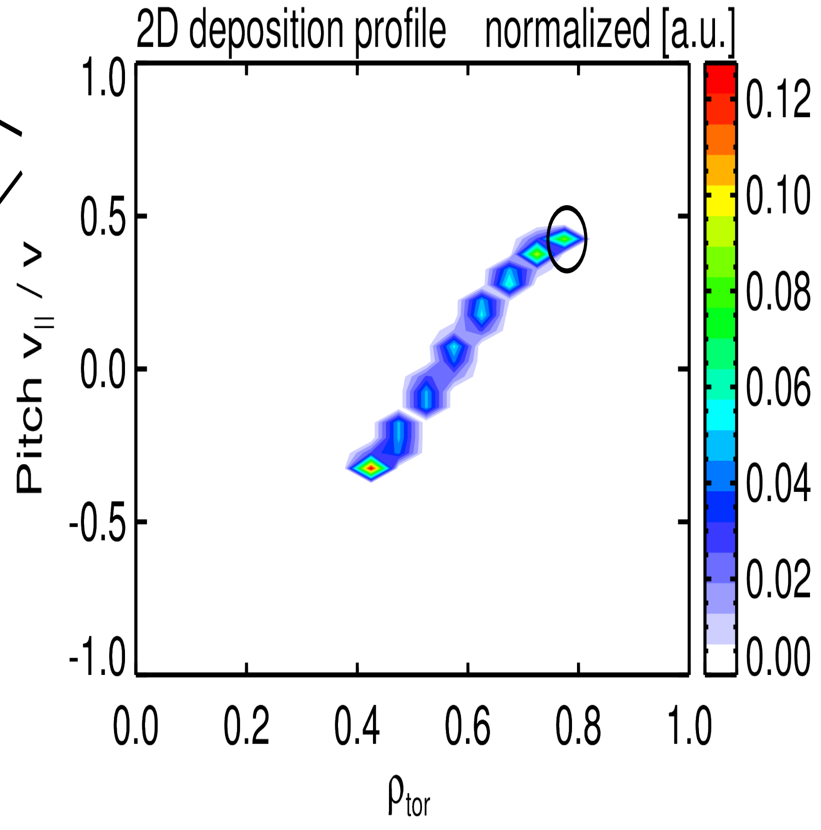
5. Applications

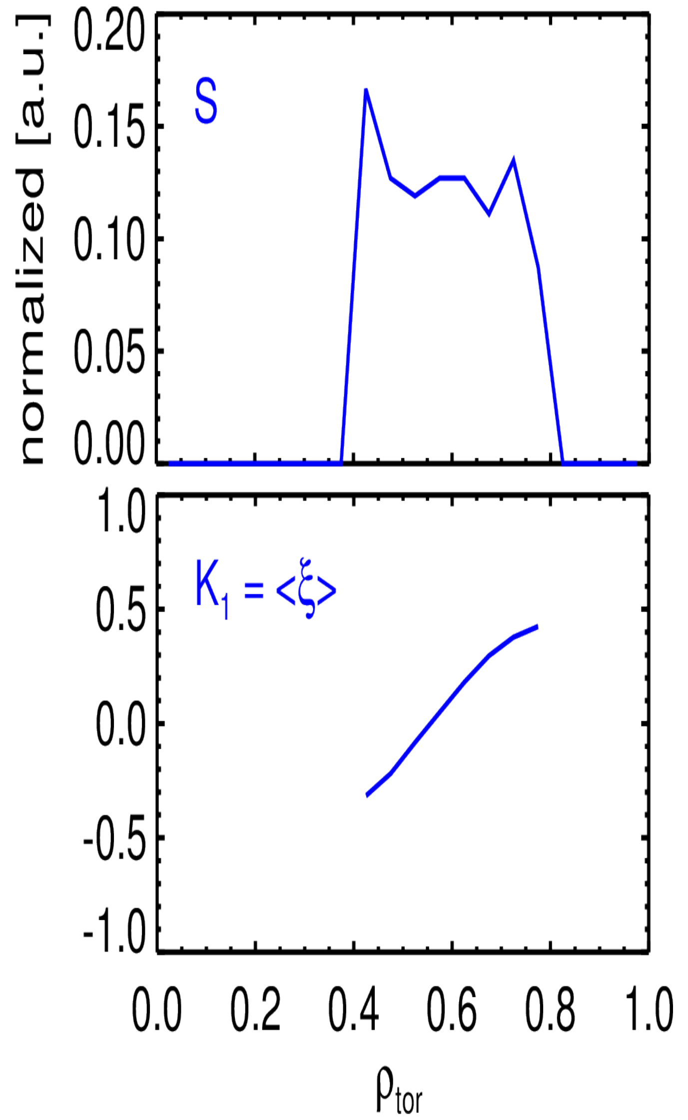
How to include the effect of first fast-ion orbit

- Orbit effects lead to a broadened deposition (towards the plasma center) and to changes of the pitch-distribution in the velocity space
- They can be taken into account, by averaging the deposition over the first orbit:
  - ➔ assume that slowing-down process starts on random position of first orbit
  - ➔ neglect orbit effects during slowing down



example for banana orbit:

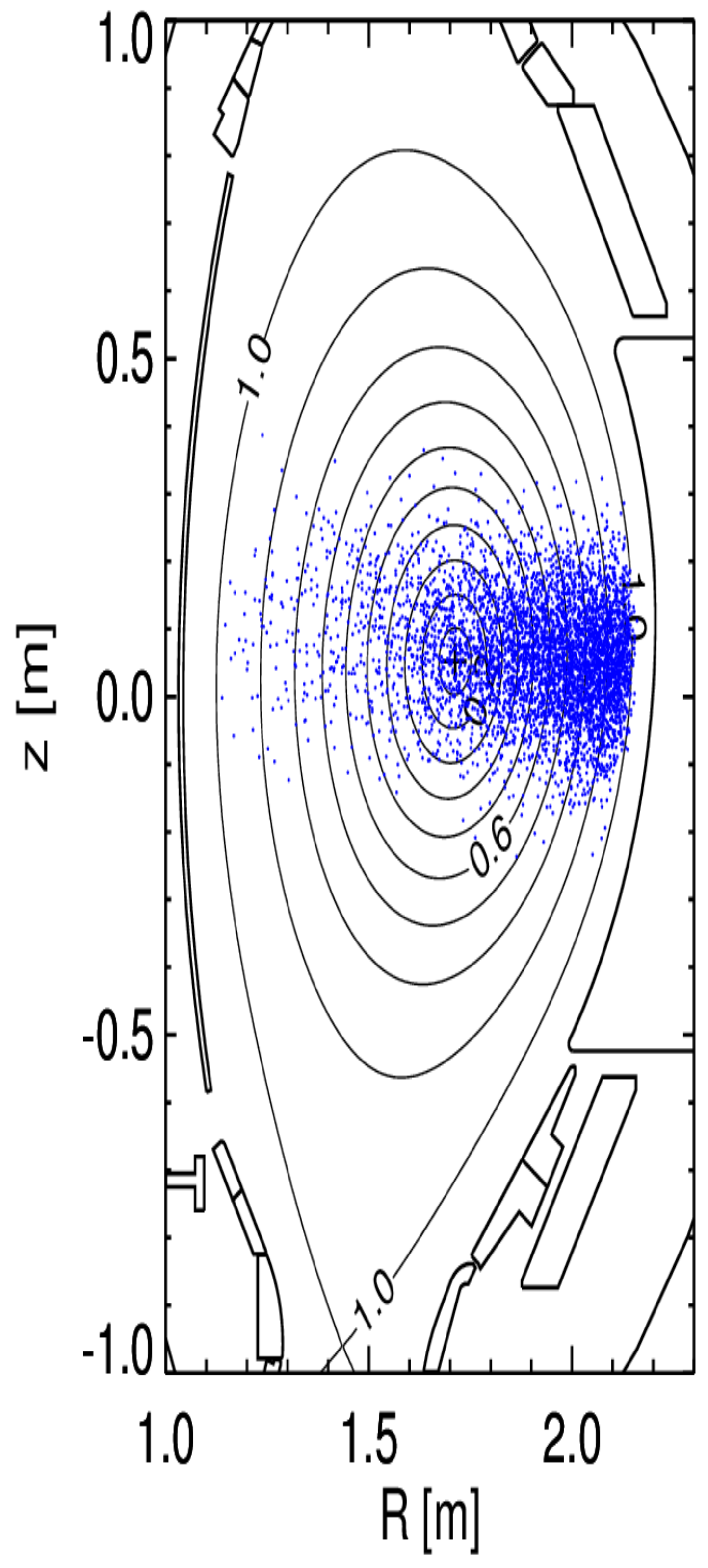




Monte-Carlo orbit-average is too slow for real-time applications

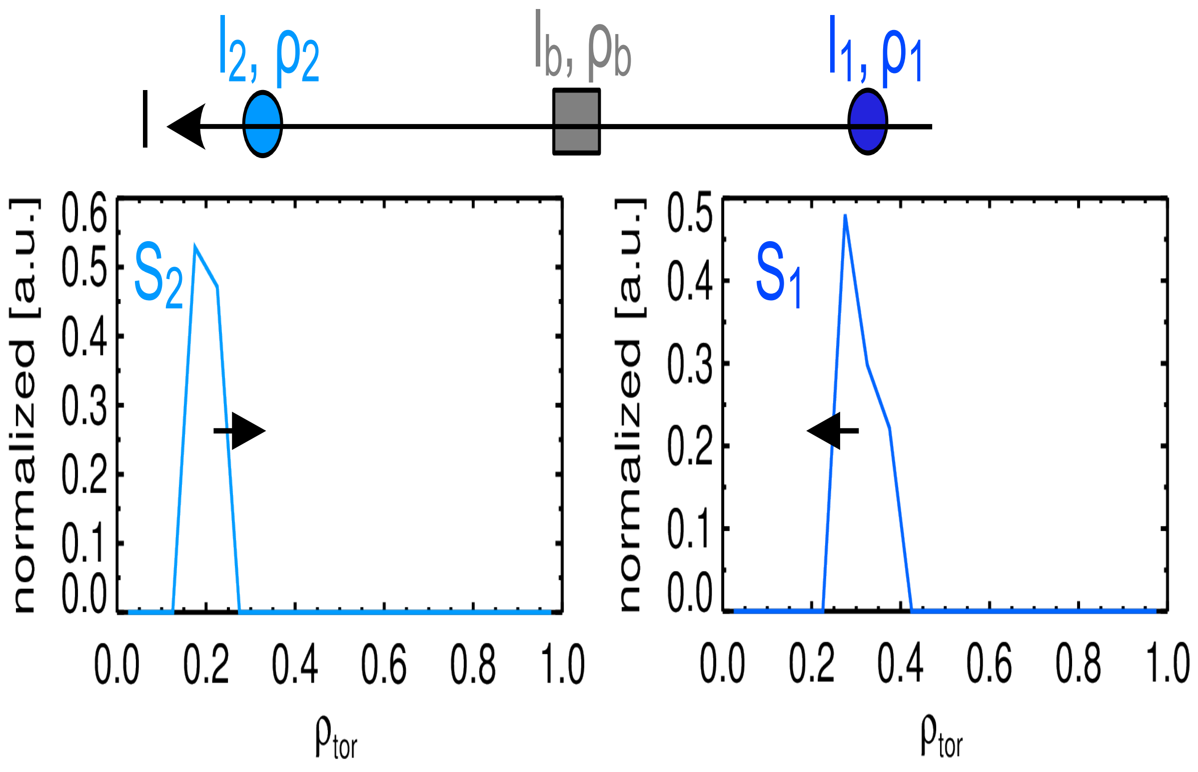
- Monte-Carlo orbit-average:
  - Take MC representation of birth distribution
  - Calculate orbit for each MC marker (e.g. ~5000)
  - → too slow for real-time purposes (takes ~1s)

- Possible solutions:
  - Either: Use approximation formulas for the orbits
  - Or: Reduce number of orbits (strongly)

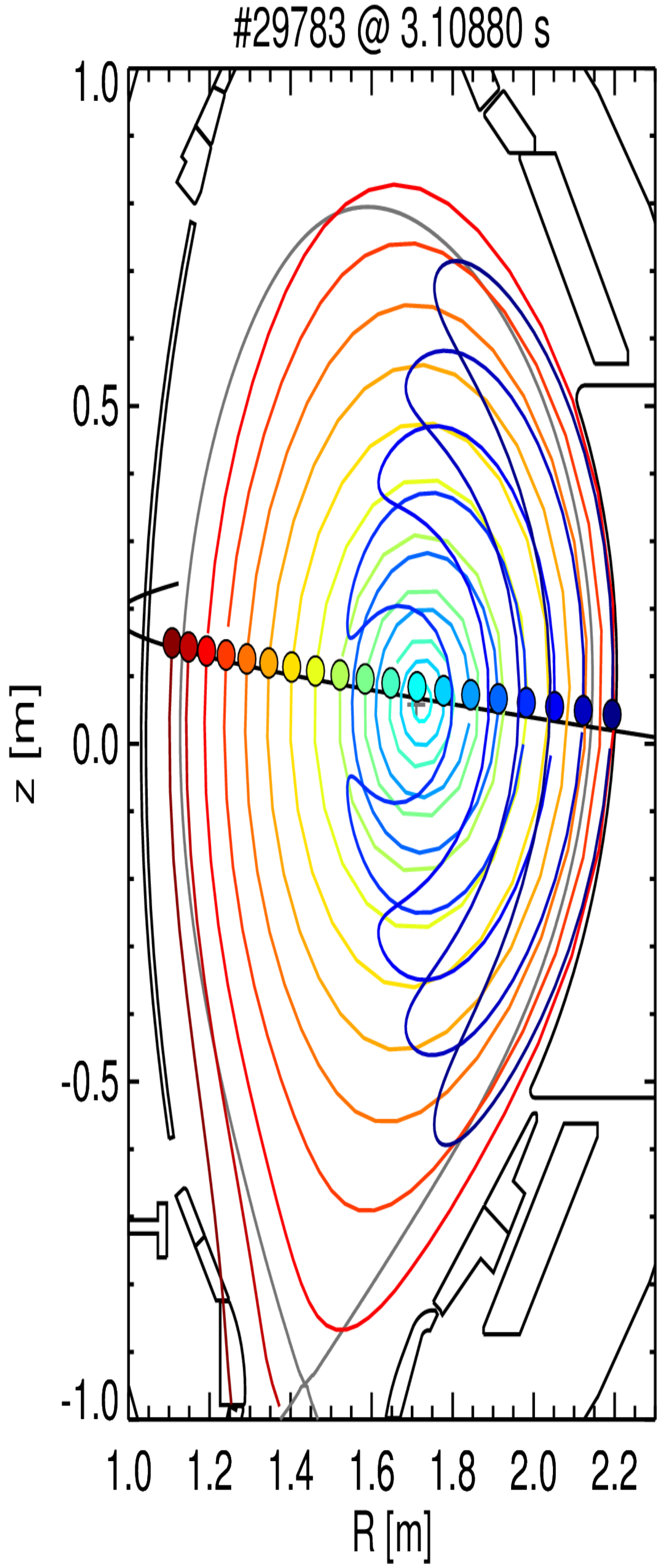


Orbit average in real-time

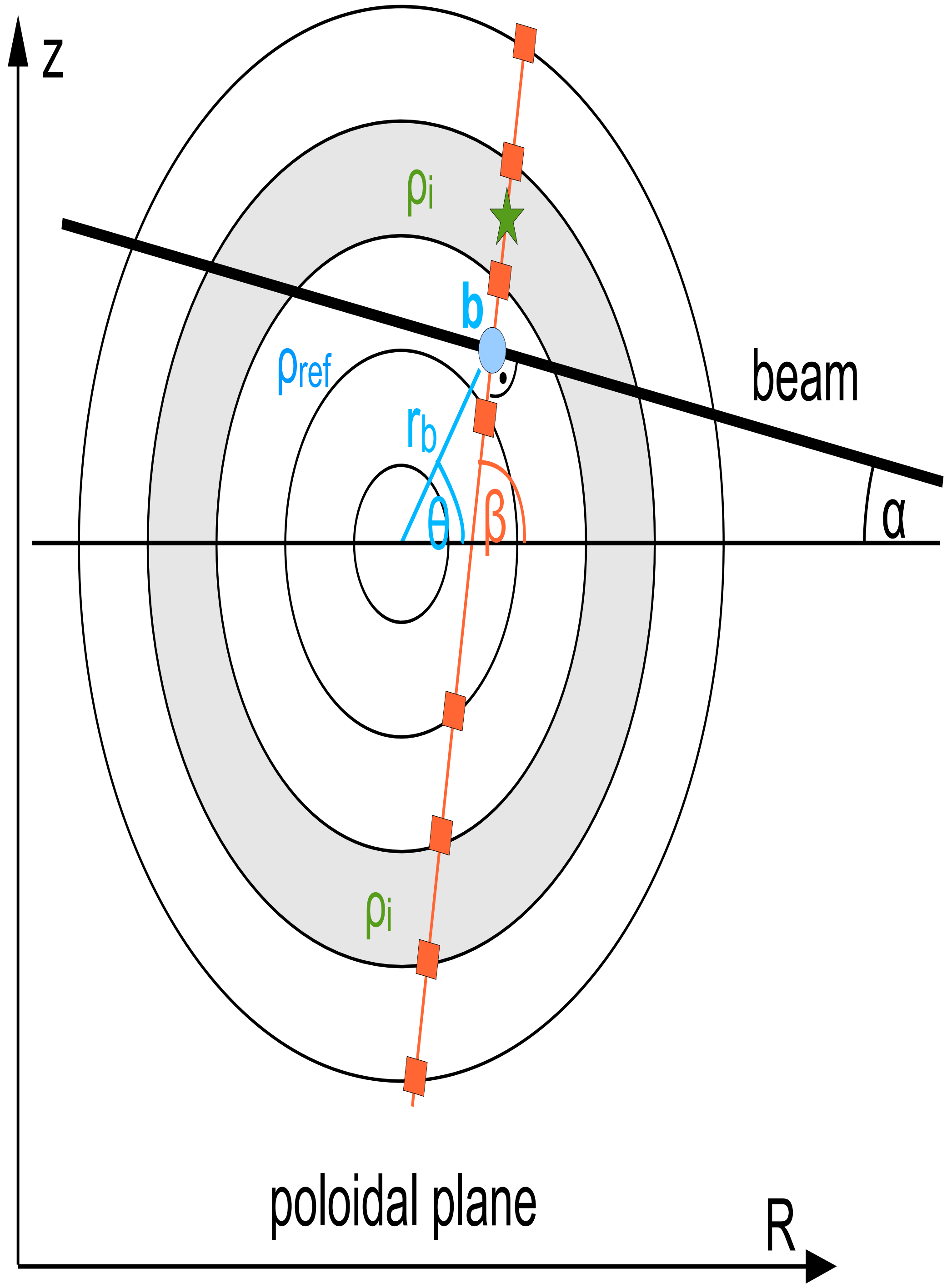
- Calculate orbit only every n-th grid point
  (4^{th} order Runge-Kutta guiding center integrator)
  Right: All calculated orbits for full energy component
- Here: 19 orbits x 3 energy components
  → possible within ~12 ms.
- In between: Shift neighboring profiles and interpolate linearly



$$S_b(\rho) = S_1(\rho - (\rho_b - \rho_1)) \cdot \frac{I_2 - I_b}{I_2 - I_1} + S_2(\rho - (\rho_b - \rho_2)) \cdot \frac{I_b - I_1}{I_2 - I_1}$$



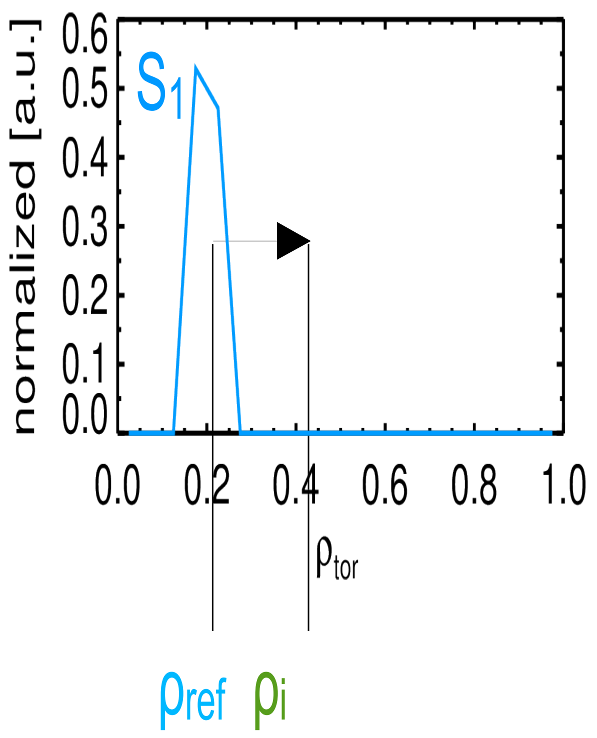
Orbit-average: compatible with beam-width model



- Up to now, we have calculated the orbit-average along the beam (at **b**).
- For the beam-width correction, we need to extrapolate from the ρ-cell containing **b** (ρ_{ref}) along the orange line to other radial cells
- E.g. from ρ_{ref} to ★(ρ_i):

$$S_i(\rho) = S_b(\rho - (\rho_i - \rho_{ref}))$$

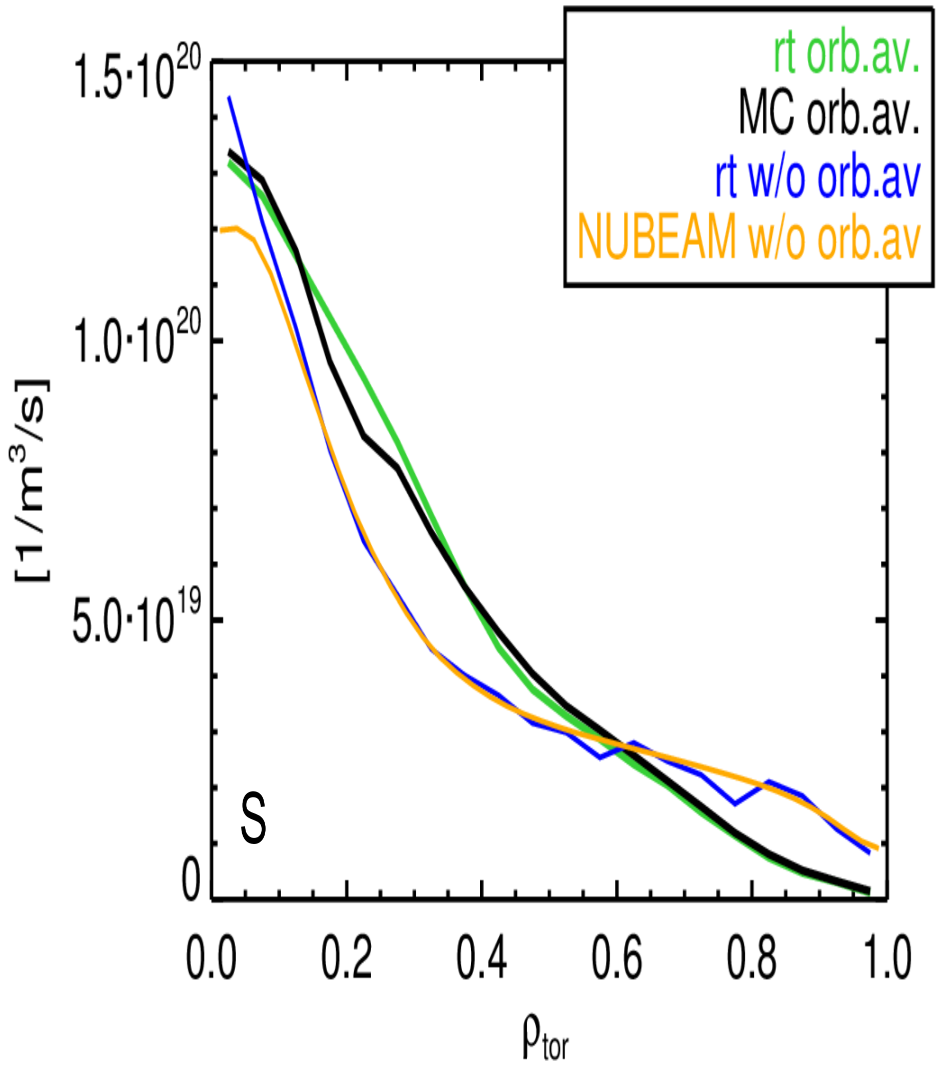
(similar to the interpolation method)



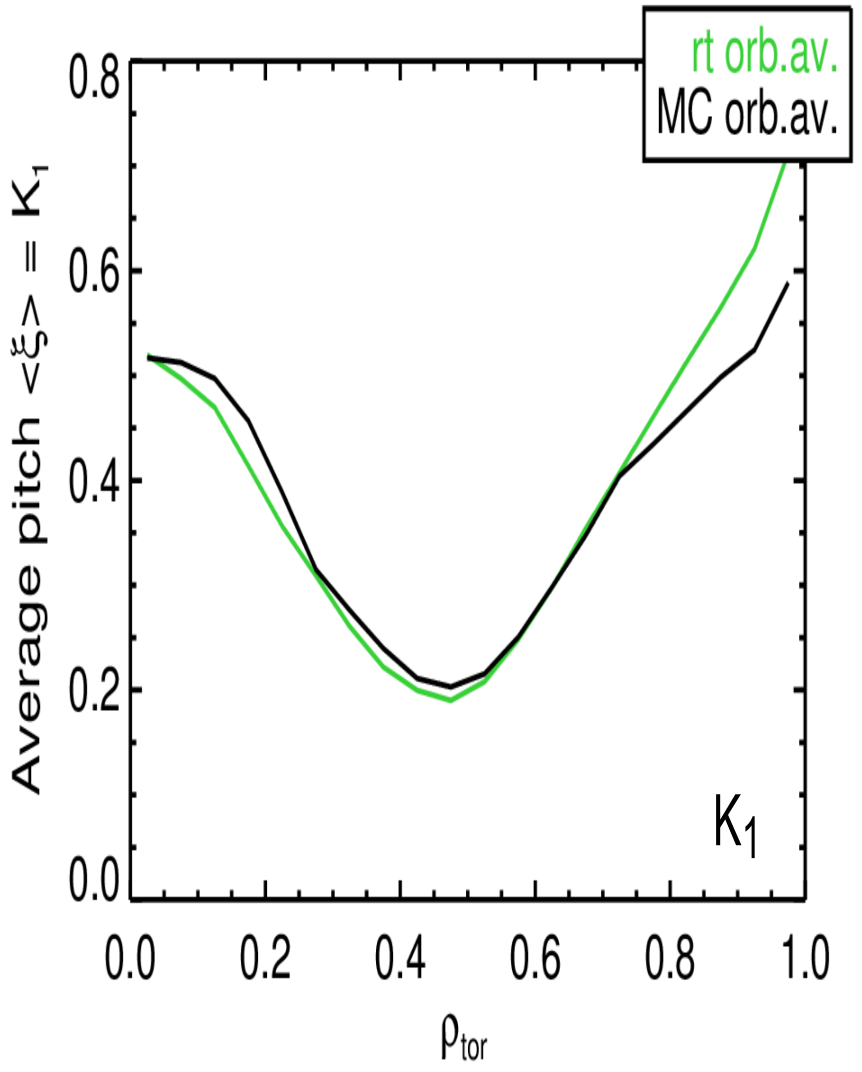
Results of „RABBIT orbit average“ in good agreement with MC orbit average

- Test accuracy of the RABBIT rt orbit average:
  Compare it to Monte-Carlo orbit average (including fully realistic NBI geometry)
- Very good agreement is found, despite orders of magnitude difference in calculation time (~5000 orbits vs. ~60 orbits)

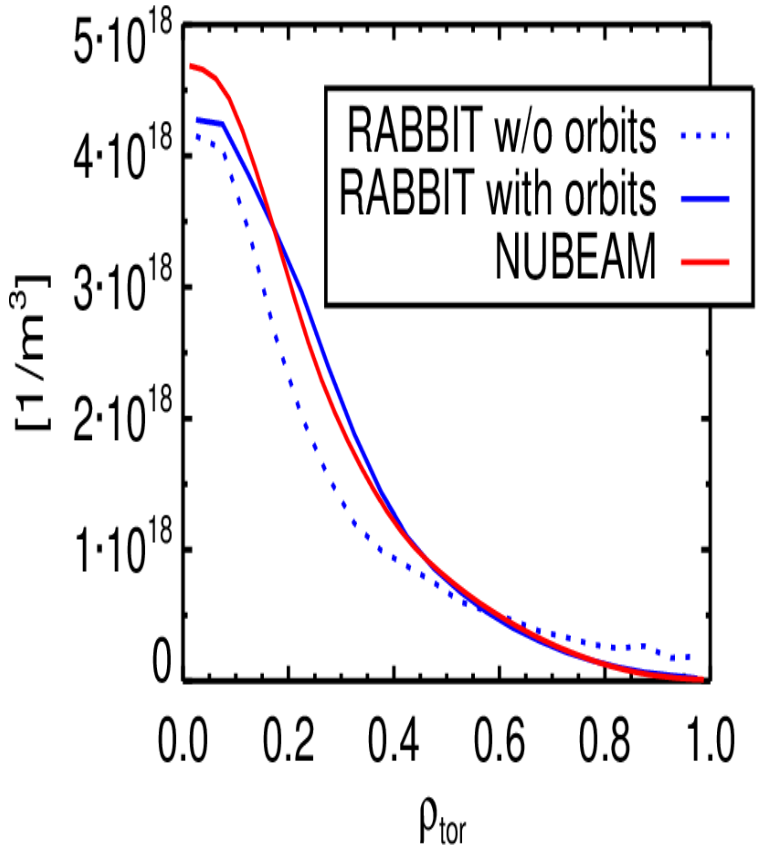
Birth profile (sum over all E components):



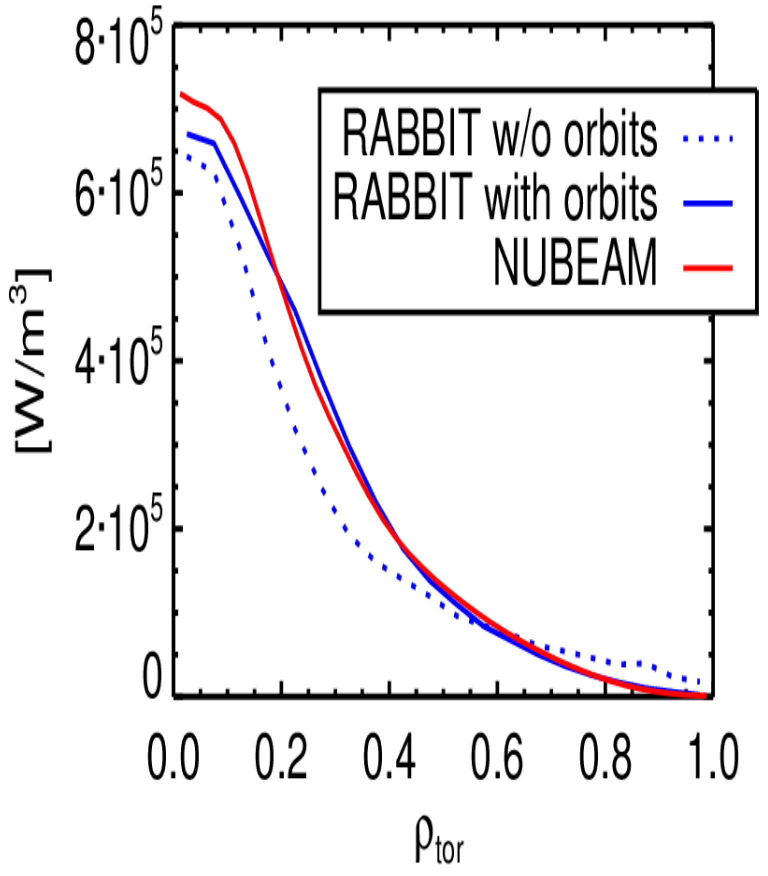
Average pitch $v_{||}/v$ of full-E component:



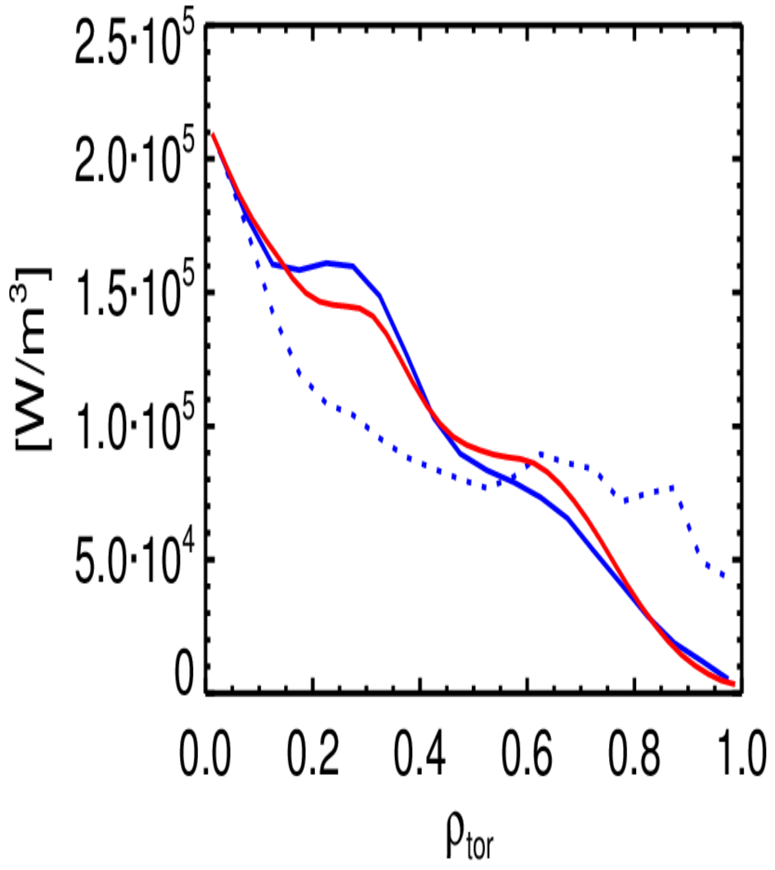
Comparison to NUBEAM



Fast-ion density

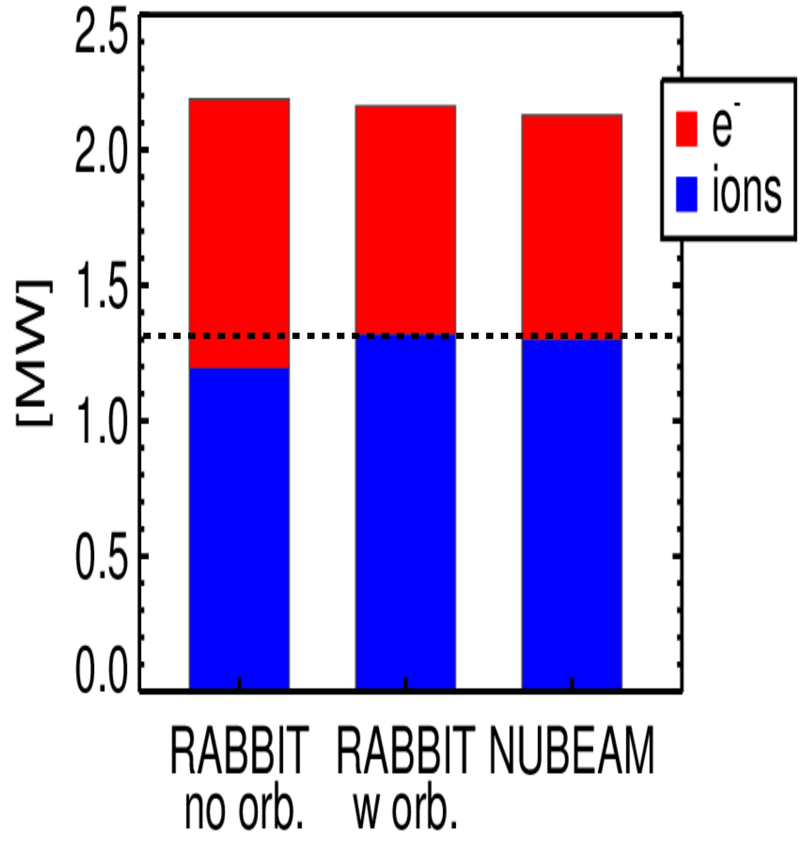


Heating to ions



Heating to electrons

- Orbit-average leads to good agreement in profile shape
- Slight deviations remain in plasma center, affecting only small fraction of plasma volume
- Orbit-average has also an impact on volume-integrated heating distribution to electrons/ions and improves agreement



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deposition

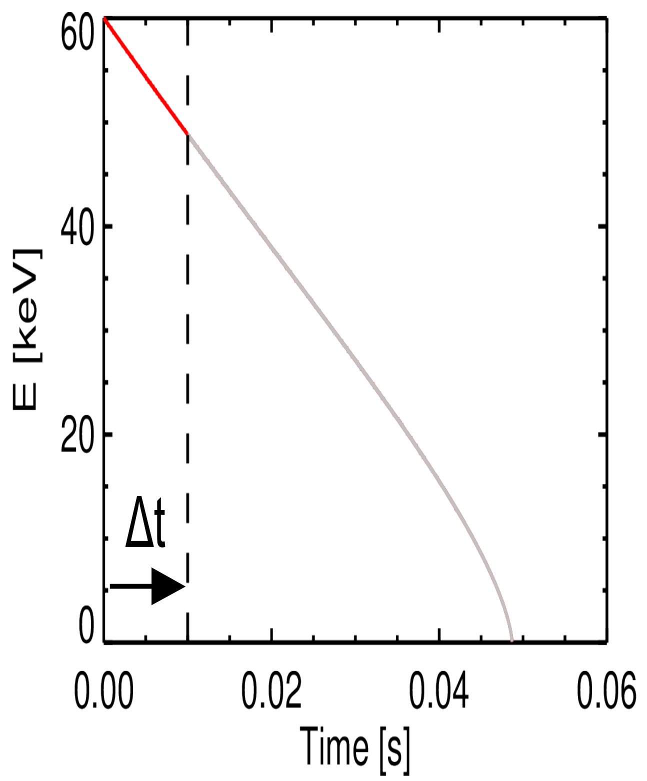
5. Applications

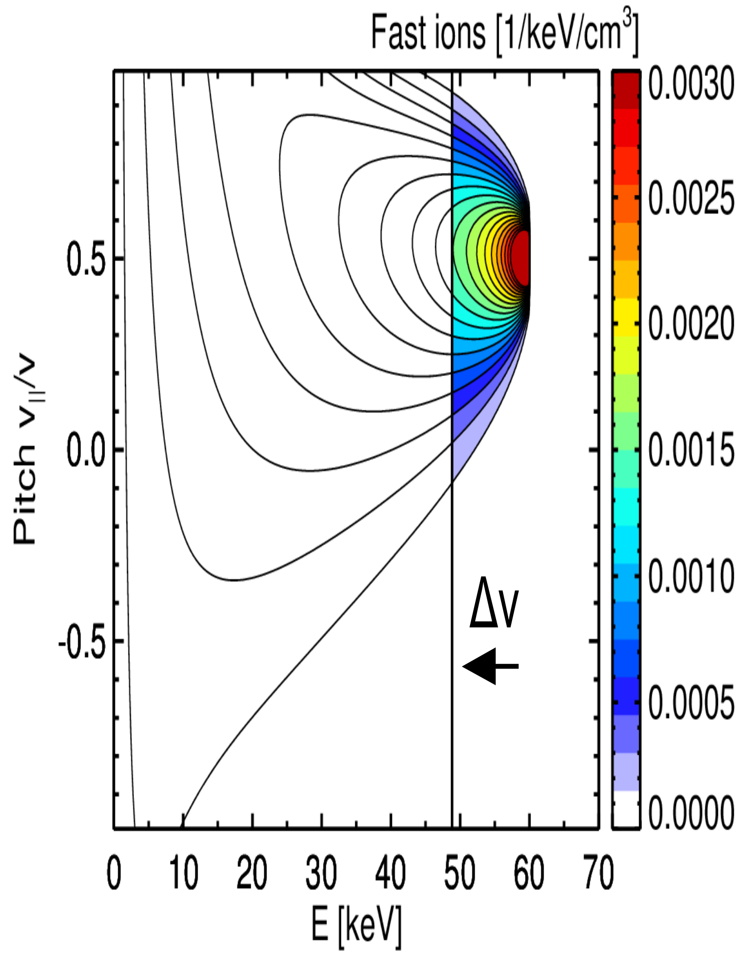
Time dependence

- For time-dependent simulation: Discrete time steps Δt
- Calculate how far the fast-ions slow down during time-step:

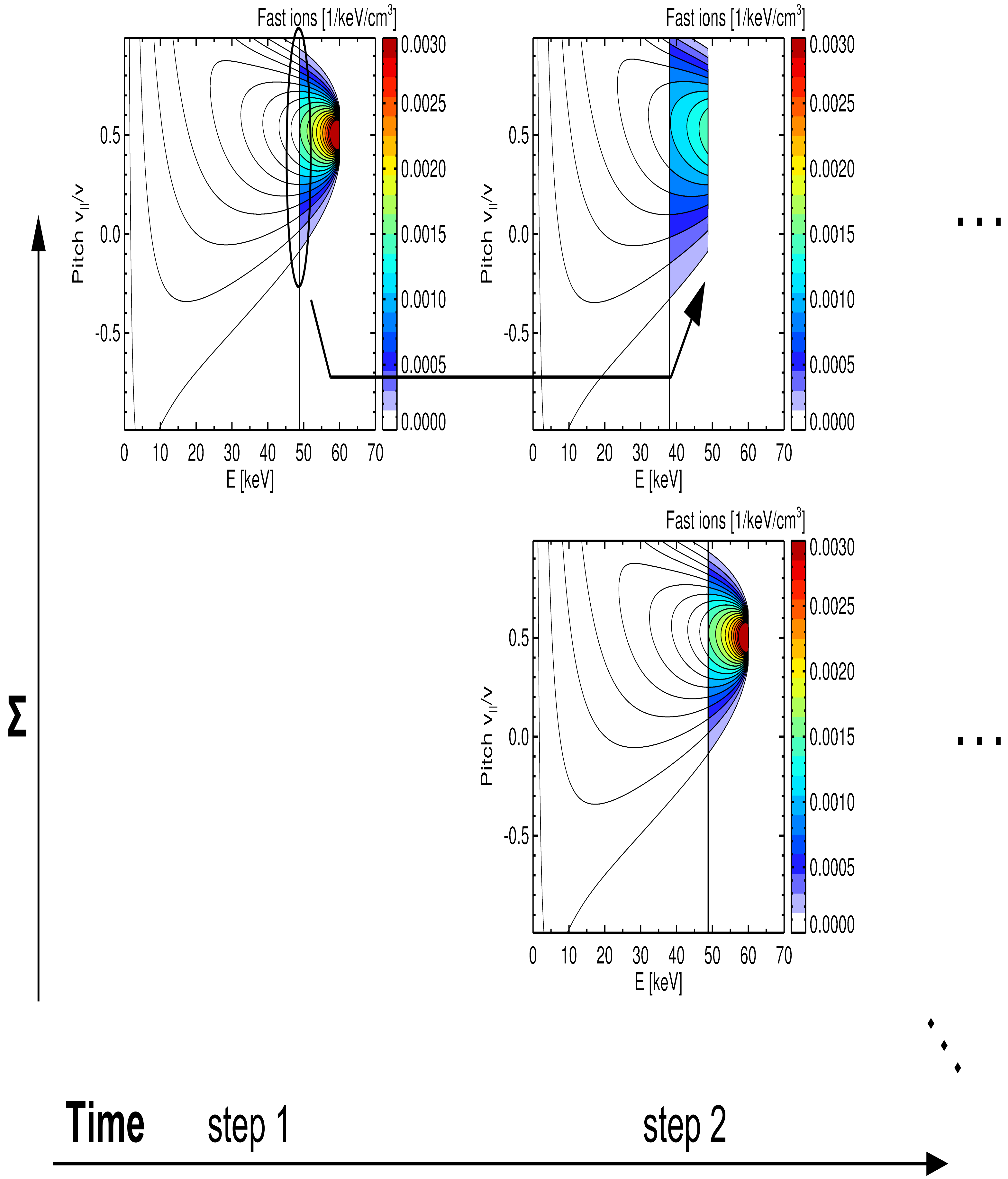
$$v_{\text{final}}^3 = \left(v_{\text{start}}^3 + v_c^3\right) \cdot \exp\frac{-3 \cdot \Delta t}{\tau_s} - v_c^3$$

→ multiply steady state solution with box function





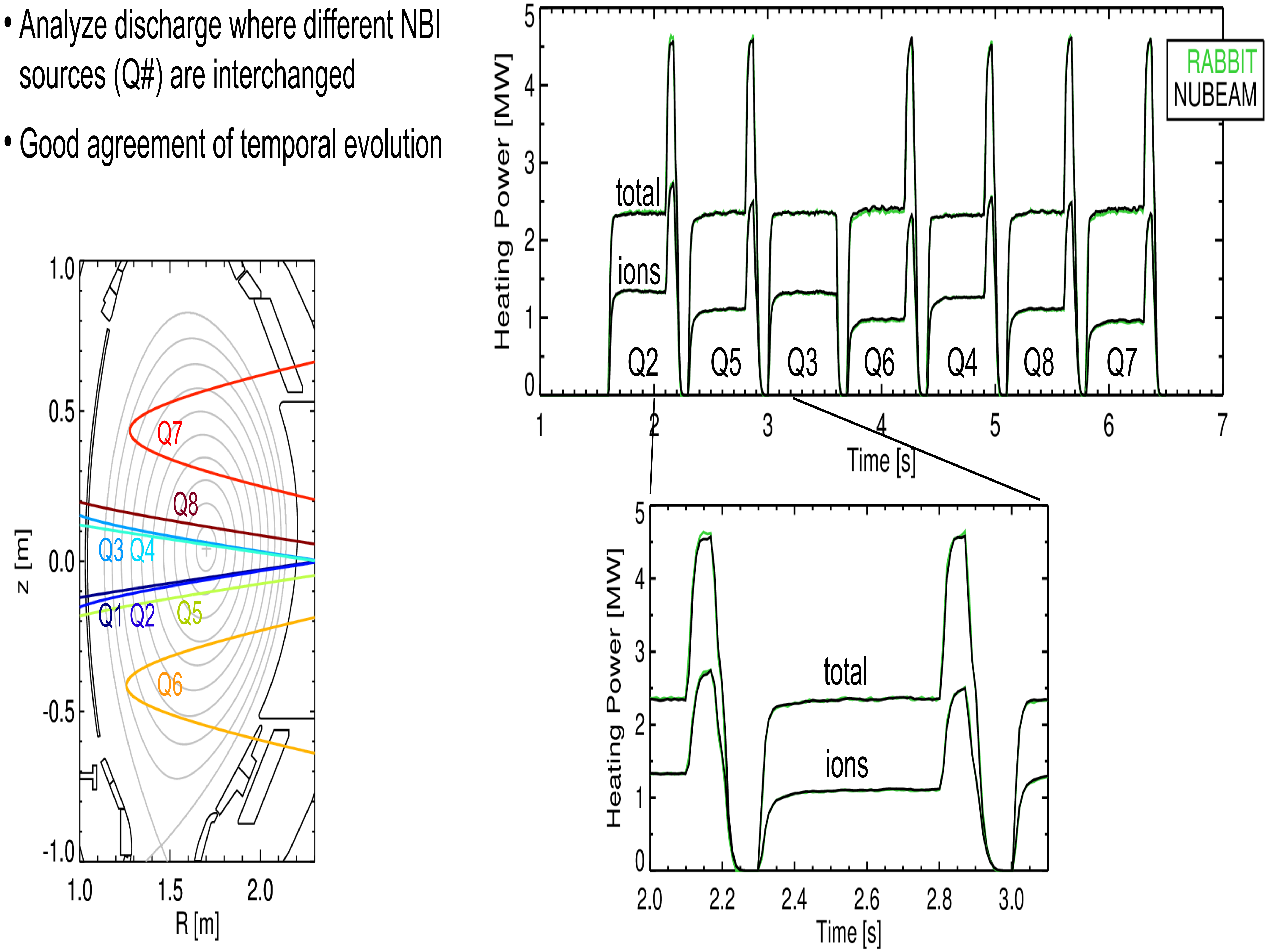
Time dependence via train of fast-ion pulses



- Final state of „step 1“ is starting point of „step 2“
- If beam is still turned on in „step 2“, add a new pulse at nominal injection energy
- continue ...
  (add new rows each time-step, sum over rows)

Comparison of time evolution with NUBEAM

- Analyze discharge where different NBI sources (Q#) are interchanged
- Good agreement of temporal evolution



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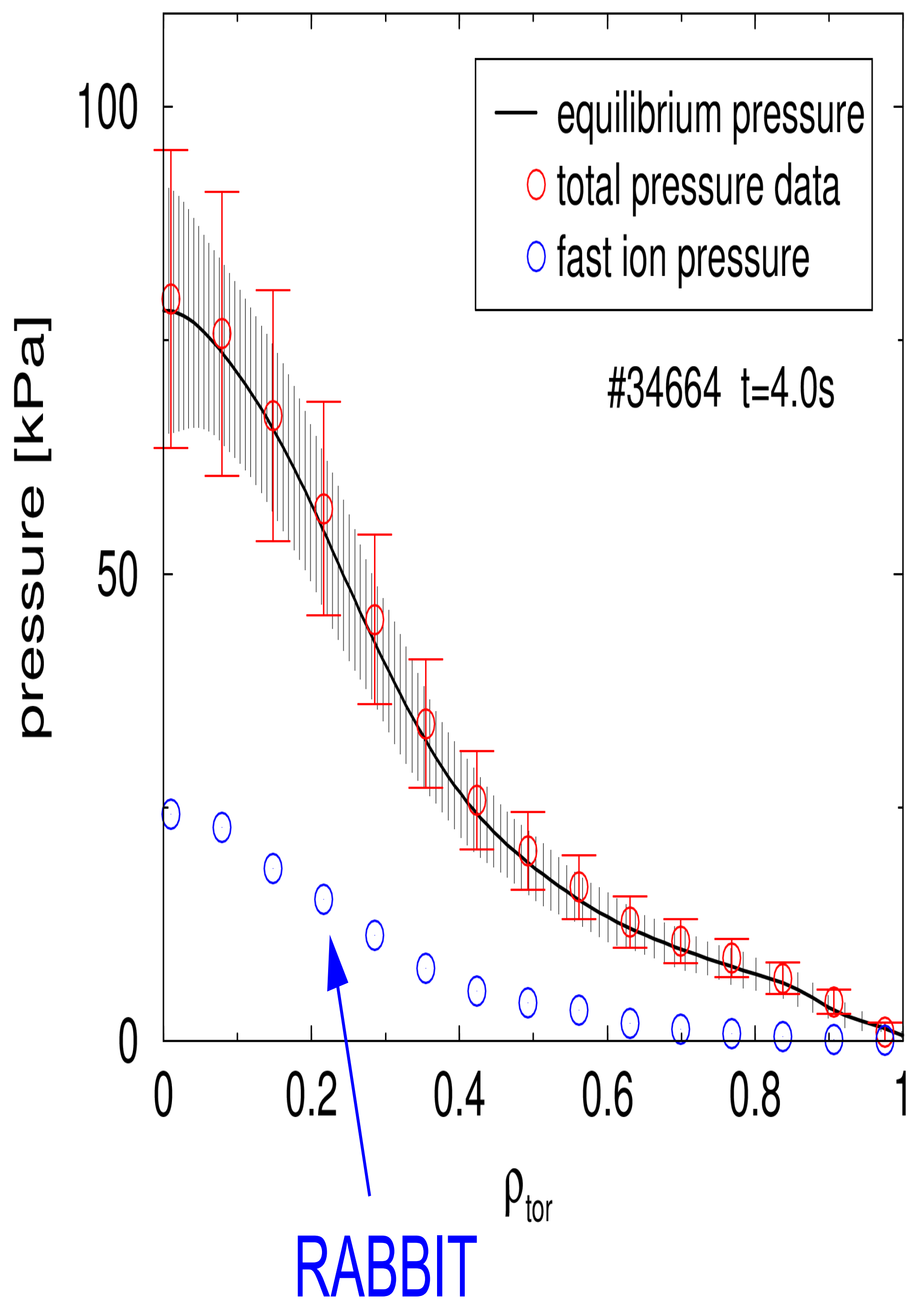
RABBIT for equilibrium reconstructions with the IDE code

Equilibrium reconstruction: Solve Grad-Shafranov-Schlüter equation for ψ:

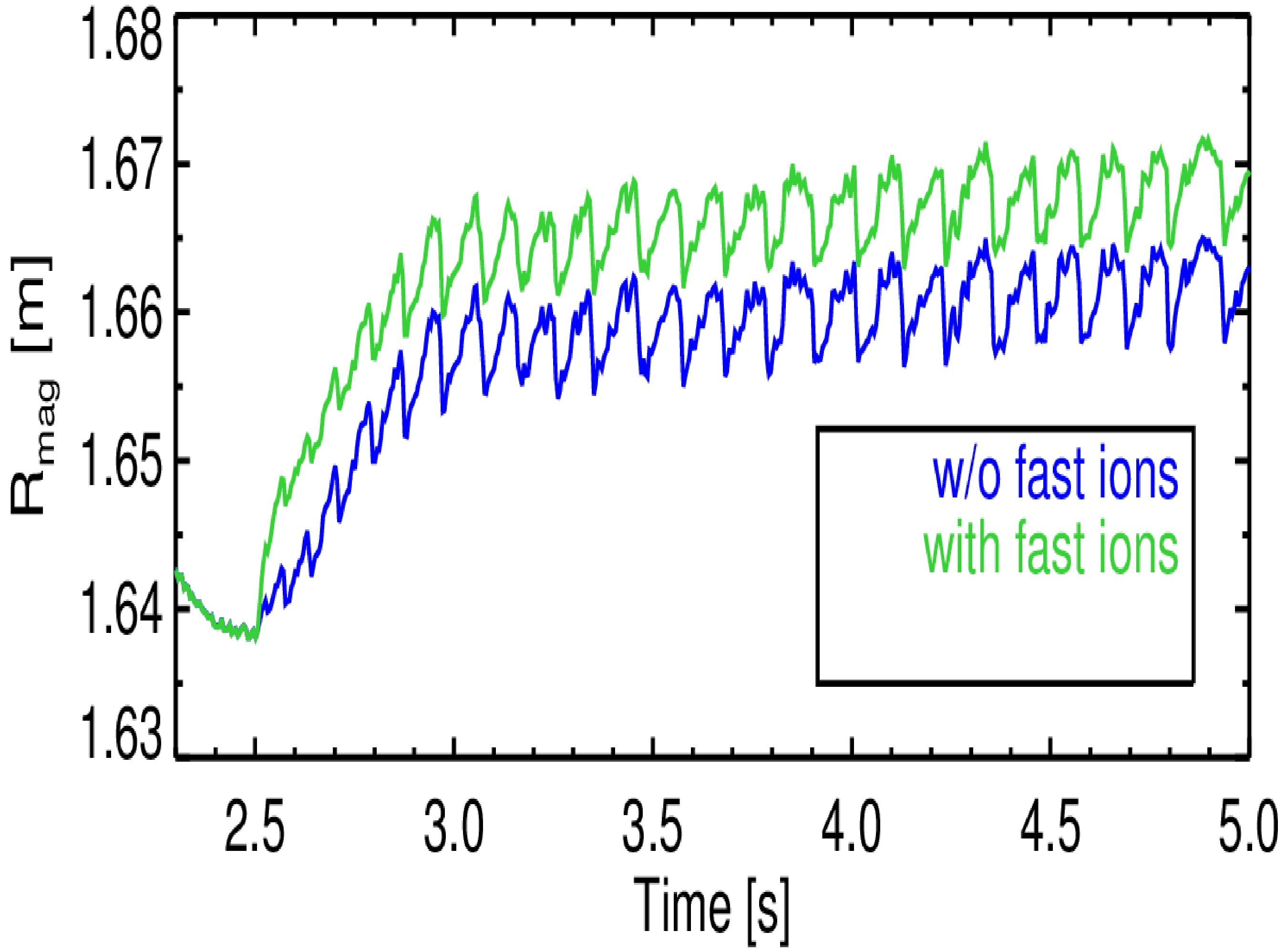
$$\Delta^* \Psi = R^2 \mu_0 \frac{\mathrm{d}p}{\mathrm{d}\Psi} + F \frac{\mathrm{d}F}{\mathrm{d}\Psi}$$

- IDE code: Equilibrium reconstruction based on integrated data analysis [R. Fischer, 2016, FST]
- Fast-ions from NBI can contribute significantly to the **pressure** p(ψ)
- Neutral beam **current** drive is relevant for current diffusion equation
- With RABBIT these profiles can be calculated routinely and also directly after the discharge



RABBIT for equilibrium reconstructions with the IDE code

- Fast-ion pressure
  → increases Grad-Shafranov shift
  → correction for magnetic axis
- Relevant for correct interpretation of diagnostics (e.g. MSE)
- In this case: Sawtoothing plasma, sawtooth-induced fast-ion redistribution is relevant, too



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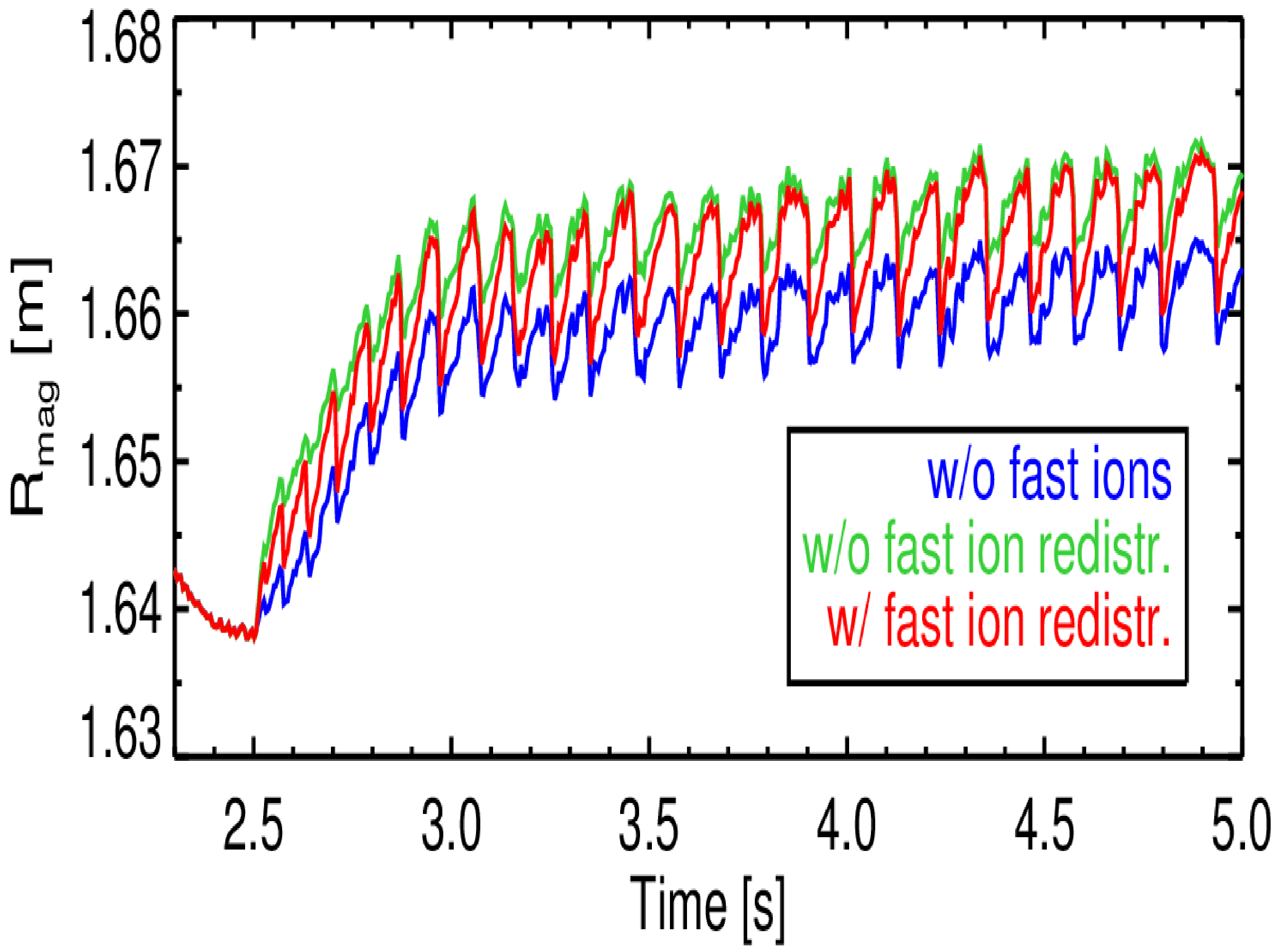
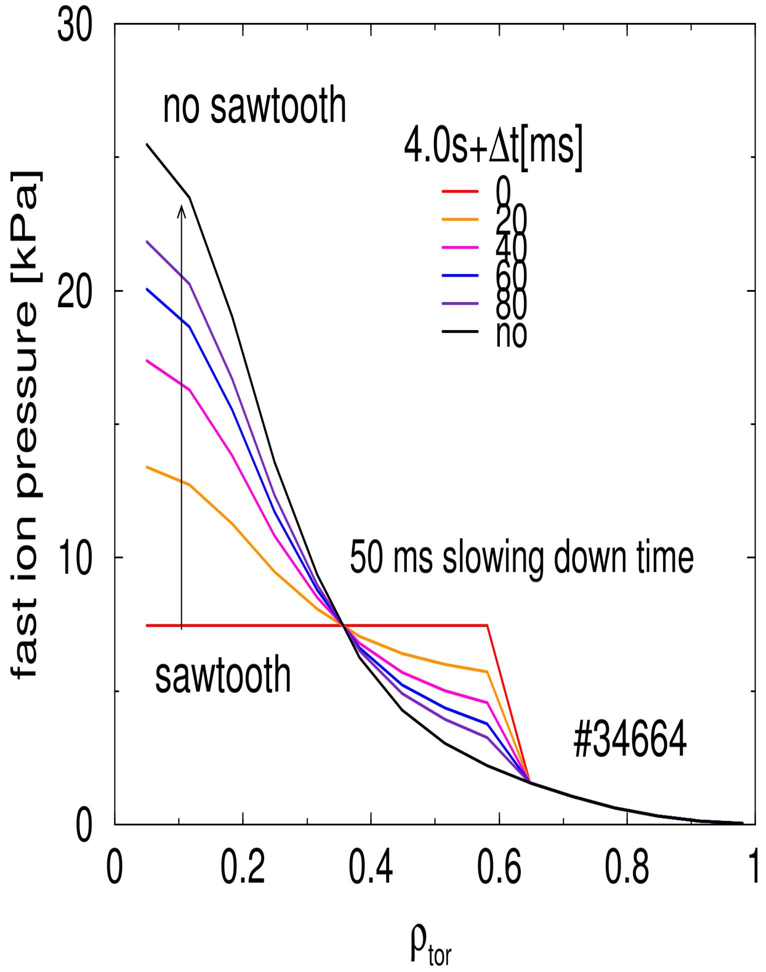
  → here modeled by post-processing RABBIT profiles

  → Outlook: implement redistribution directly in RABBIT.

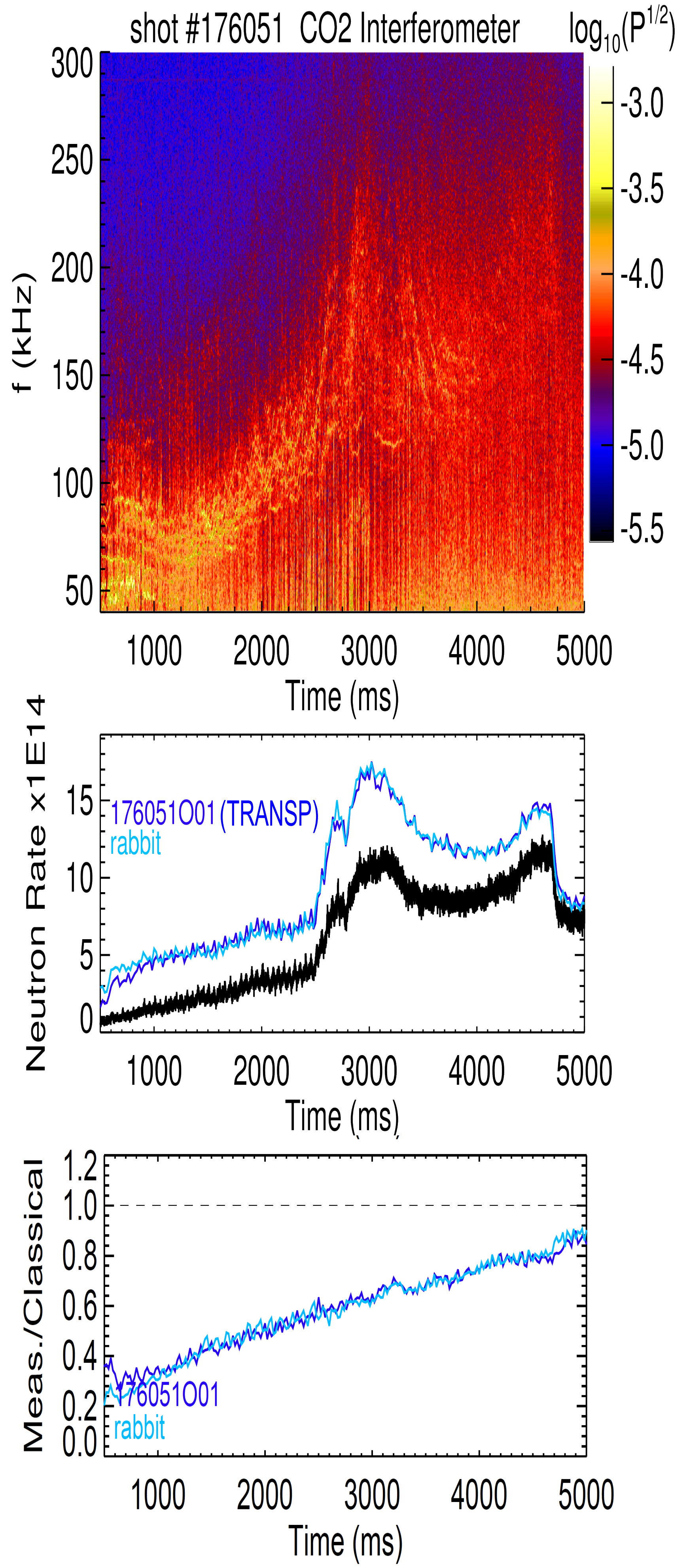




Detect AE-induced fast-ion transport by analyzing neutron rates

- Alfvén eigenmodes (AE) can be measured directly by fluctuation diagnostics
- Do they cause strong fast-ion transport?
  → can be assessed by comparing measured neutron rate with neo-classical prediction
- With RABBIT, this information is available directly after the discharge or even in real-time

- Used in scenario development for reversed shear steady state scenario experiments
  → Goal: Reduce fast-ion transport to optimize perfomance
  → RABBIT useful for decision making between discharges



Database comparison between TRANSP and RABBIT

- Comparison between RABBIT and TRANSP for all shots from the experimental session
  → Good agreement

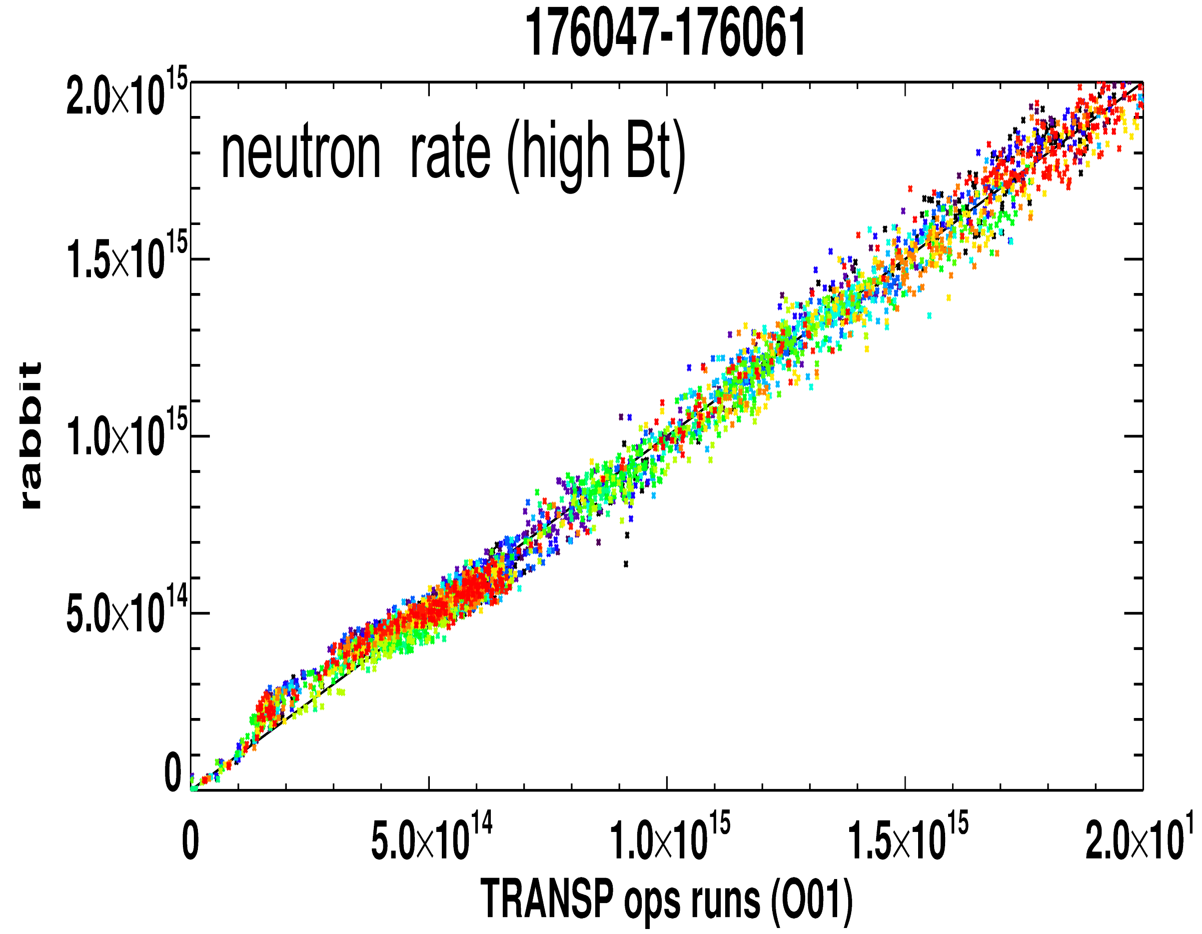
- Outlook: Use for real-time control of AE modes

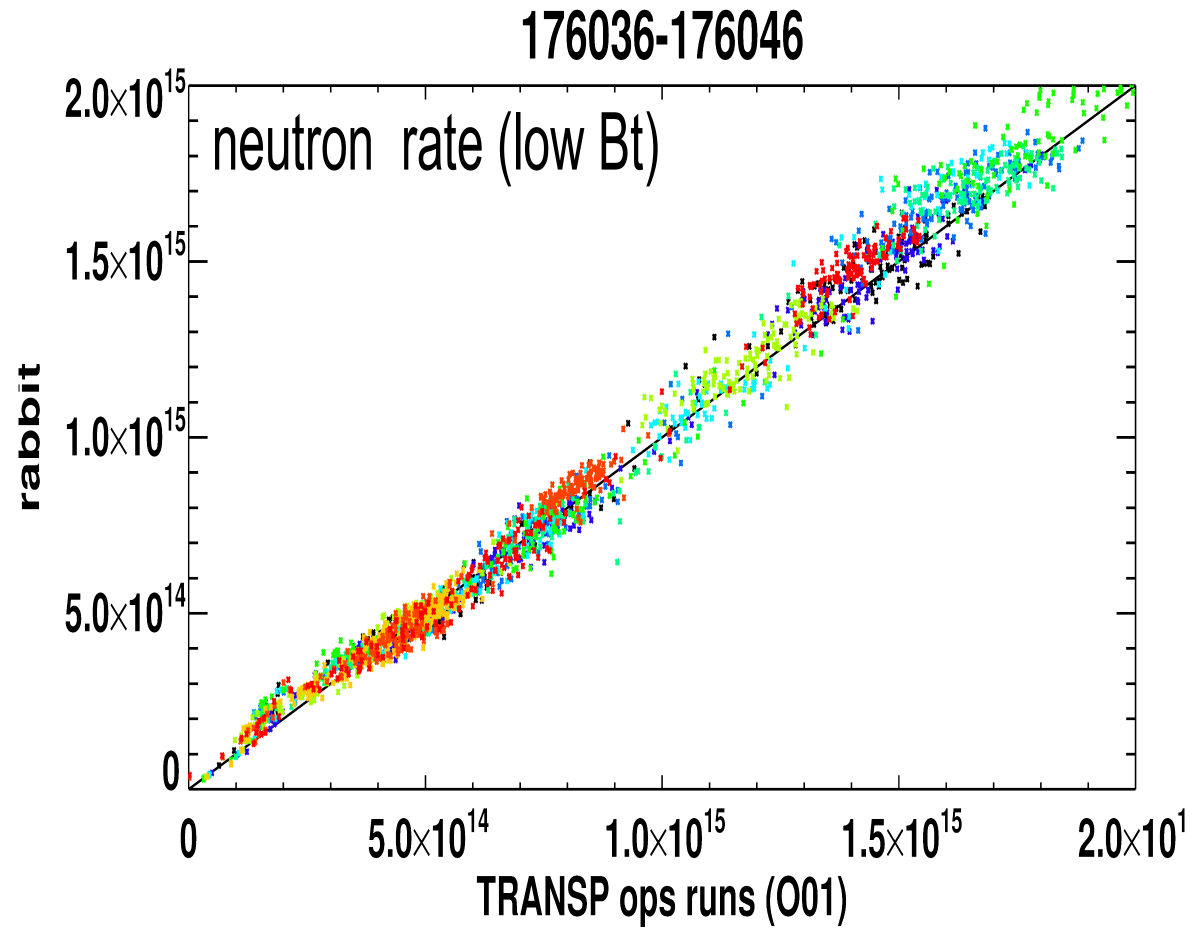
  → use ECE to detect AE amplitude

  → use RABBIT to detect detrimental fast-ion transport

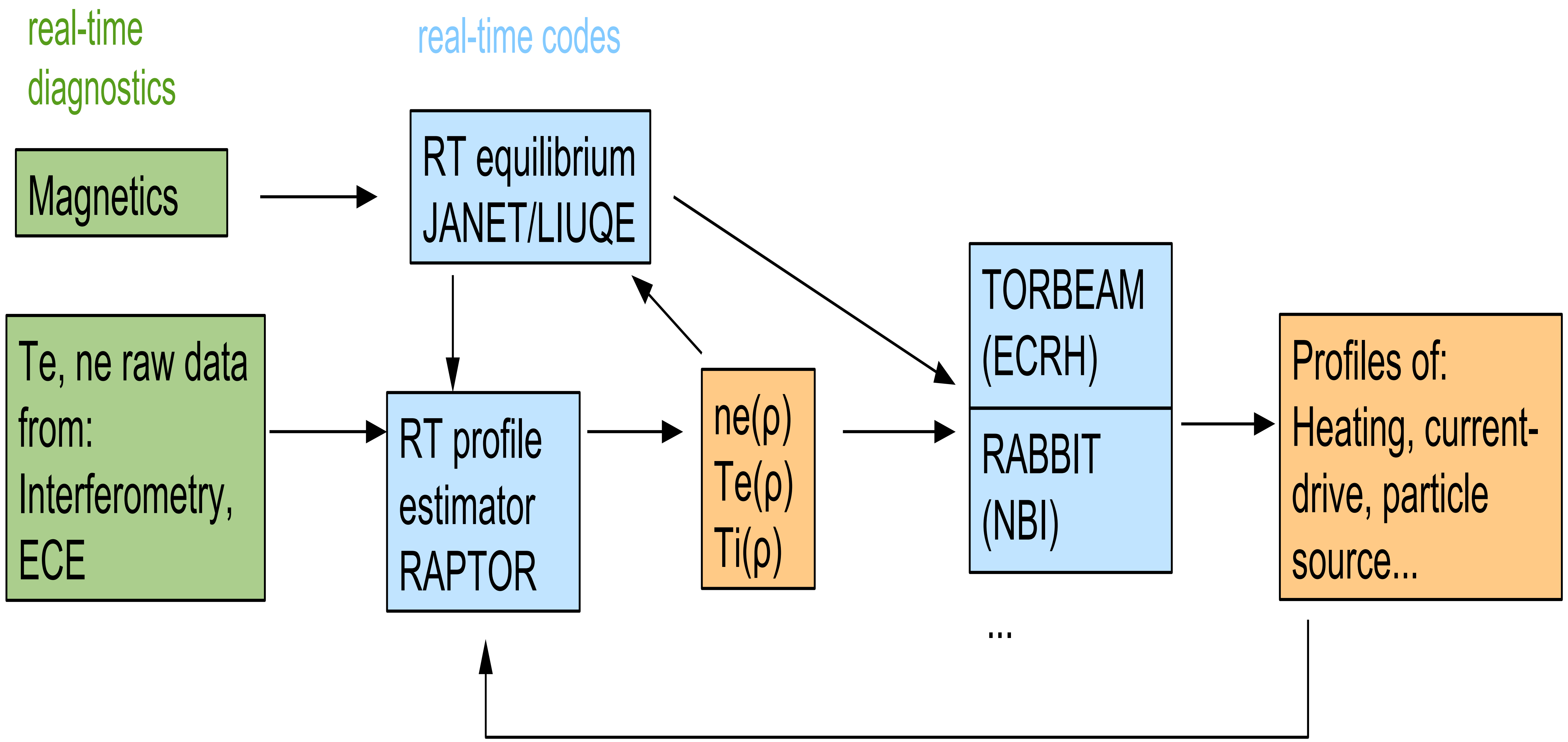
  → trigger counter-measures, e.g. reduce NBI power (=AE drive)

- Use for data-mining (plan to run Rabbit on every historic DIII-D discharge)





Real-time implementation of RABBIT on ASDEX Upgrade and TCV



- RAPTOR is 1D real-time transport solver → relies on realistic inputs of sources by auxiliary heating
- Status of RABBIT implementation:
  Hardware is installed, software is being finalized, first tests/results are expected in few weeks

Summary – RABBIT: A high-fidelity real-time NBI code

Three parts:	Calc. time:	Method:
• 1. Beam attenuation – birth profile	~ 7 ms	Simplified geometry (“thin beam”), analytic treatment of beam width
• 2. Orbit average	~ 12 ms	Conventional GC-integrator, but for very few orbits (~20 per E-comp.)
• 3. Time dep. solution of FP equation	~ 1 ms	Fully analytic

- In total ~20 ms per time-step (1 thread per beam) - faster than NUBEAM by roughly a factor of ~1000
- Good agreement with NUBEAM e.g. for heating profiles, neutron rates, fast-ion pressure
  tested so far on ASDEX Upgrade, DIII-D and JET

Applications / Outlook:

- For improved equilibrium reconstruction (e.g. IDE)
- Neutron rates allow to assess fast-ion transport (e.g. at DIII-D, intershot and real-time)
- Real-time control applications (with RAPTOR) at ASDEX Upgrade and TCV
- In ASTRA for the currently developed ASDEX Upgrade flight simulator
- In integrated modeling frameworks (IMAS, ETS, JINTRAC, OMFIT, …)

Backup

CPU hardware

Intel Xeon E5-2680 v3 (2.5GHz) CPUs,