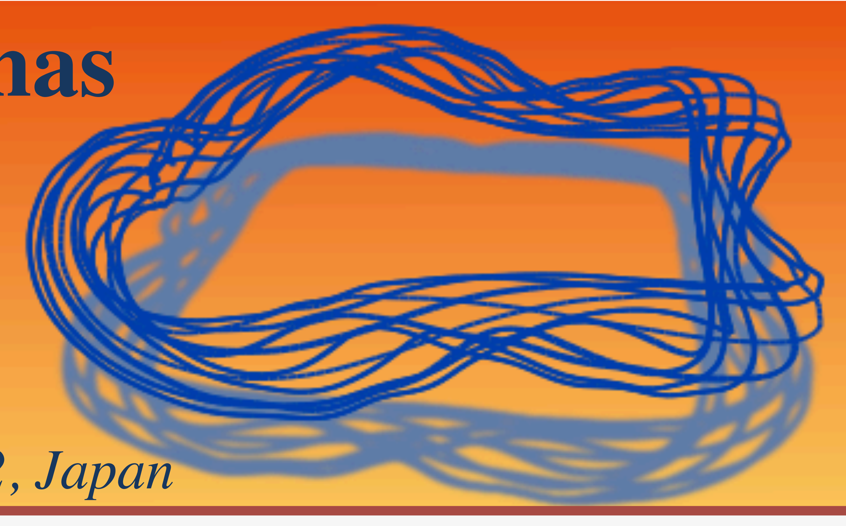
Flux-surface Averaged Radial Transport in Toroidal Plasmas with Magnetic Islands



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In toroidal magnetic confinement fusion research, one-dimensional (1D) transport models rely on one radial coordinate that labels nested toroidal flux surfaces. The presence of magnetic islands in the magnetic geometry does not impede making 1D transport calculations if the island regions are excluded and then, if necessary, treated separately. In this work we show a simple way to modify the flux-surface coordinate and corresponding metric coefficients when an island region is excluded. Comparison with the metrics obtained from Poincaré plots are shown, as well as applications to two types of plasma: Heliac (TJ-II, CIEMAT, Spain), where the geometrical effects alone cannot explain the experimental results when islands move throughout minor radius; and Heliotron (LHD, NIFS, Japan), where we estimate the effect of possible heat losses in flux-gradient relations.

MOTIVATION: Use well-developed 1D transport codes in presence of notable islands.

A radial coordinate cannot be defined in the whole domain, which obliges to:

- Perform complicated 3D calculations (not practical in many circumstances).
- Develop new codes that evolve and match different domains [1].

We propose a different, simple solution: take away the islands region and solve separately if needed → a coordinate modification is enough.

BASIC IDEA (cylindrical case): Associate island region to cylindrical annulus.

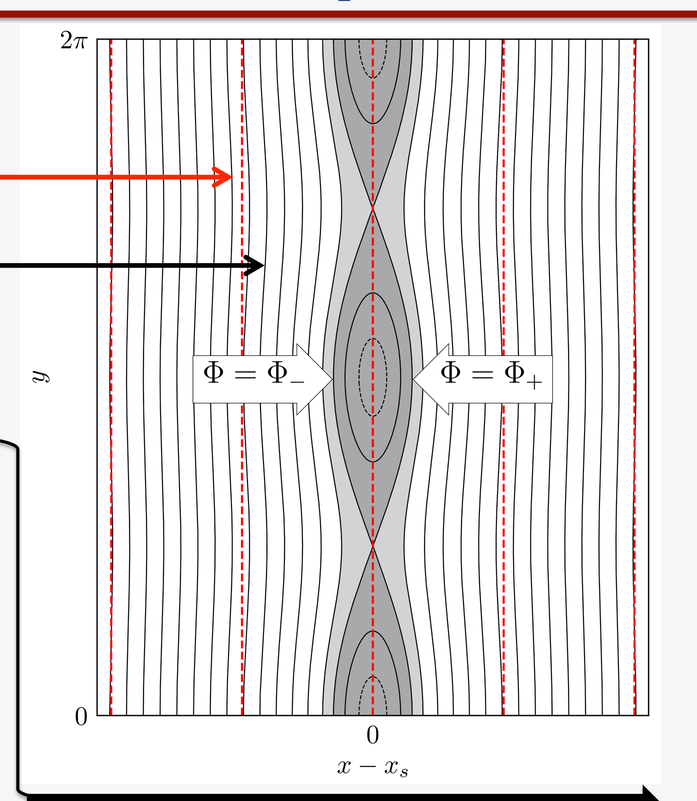
- Original coordinate: cylindrical r; or maybe normalized r/a.
- New coordinate ρ outside island region:
  - Continuous by definition, $\rho(r_I-\Delta/2) = \rho(r_I+\Delta/2)$
  - Keeps normalization $\rho(r = a) = 1$

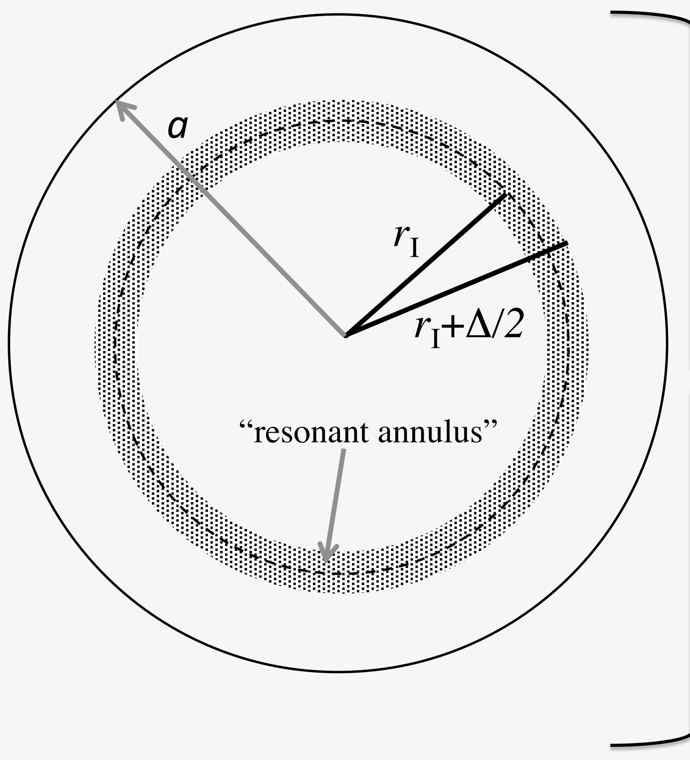
Example (out of many choices):

- **Inside part** (from the magnetic axis to the *inner* separatrix): Square root of the toroidal flux $B_0\pi r^2$ normalized to $B_0\pi a^2-\Phi_I$
- **Outside part** (from the *outer* separatrix to the edge): Square root of the flux $B_0\pi r^2-\Phi_I$ normalized to $B_0\pi a^2-\Phi_I$

$$\rho_0 = \sqrt{\Phi_0/\Phi_{0a}}$$

$$\bar\rho_0 = \sqrt{\Phi/\Phi_{0a}}$$





Generalize based on "magnetic flux annulus" [2]

$$\rho = \begin{cases} \sqrt{\dfrac{\Phi}{\Phi_{0a}-\Phi_I}}; & \Phi < \Phi_- \\ \sqrt{\dfrac{\Phi-\Phi_I}{\Phi_{0a}-\Phi_I}}; & \Phi > \Phi_+ \end{cases}$$

GENERAL TRANSFORMATION:

Typical equilibrium coordinates used in 1D transport codes can be considered as $\bar\rho_0$ an approximation of a real configuration where small magnetic islands may be present. Defining

$$\beta^2 \equiv \Phi_I/\Phi_{0a} < 1$$

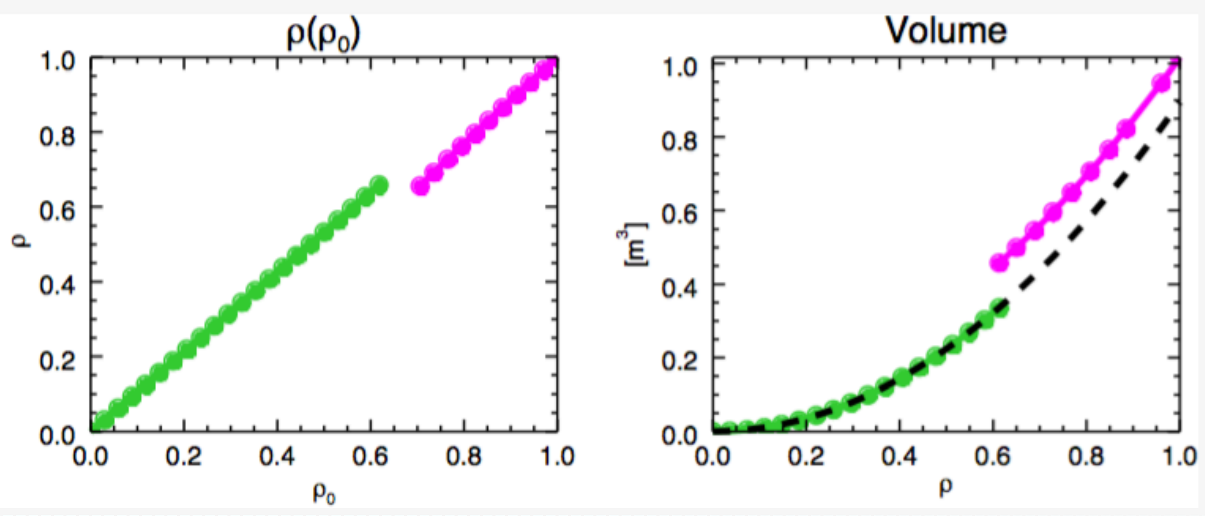
$$\gamma \equiv \sqrt{1/(1-\beta^2)} \geq 1$$

the coordinate transformation is

$$\rho = \begin{cases} \gamma\bar\rho_0; & \bar\rho_0 < \bar\rho_{0-} \\ \gamma\bar\rho_0\sqrt{1-\beta^2/\bar\rho_0^2}; & \bar\rho_0 > \bar\rho_{0+}, \end{cases}$$

As a first approximation, β^2 can be related with cylindrical prescription for island width:

$$\beta^2 = \frac{2\bar\rho_{0s}\Delta}{a}$$



OBSERVATIONS about the transformation:

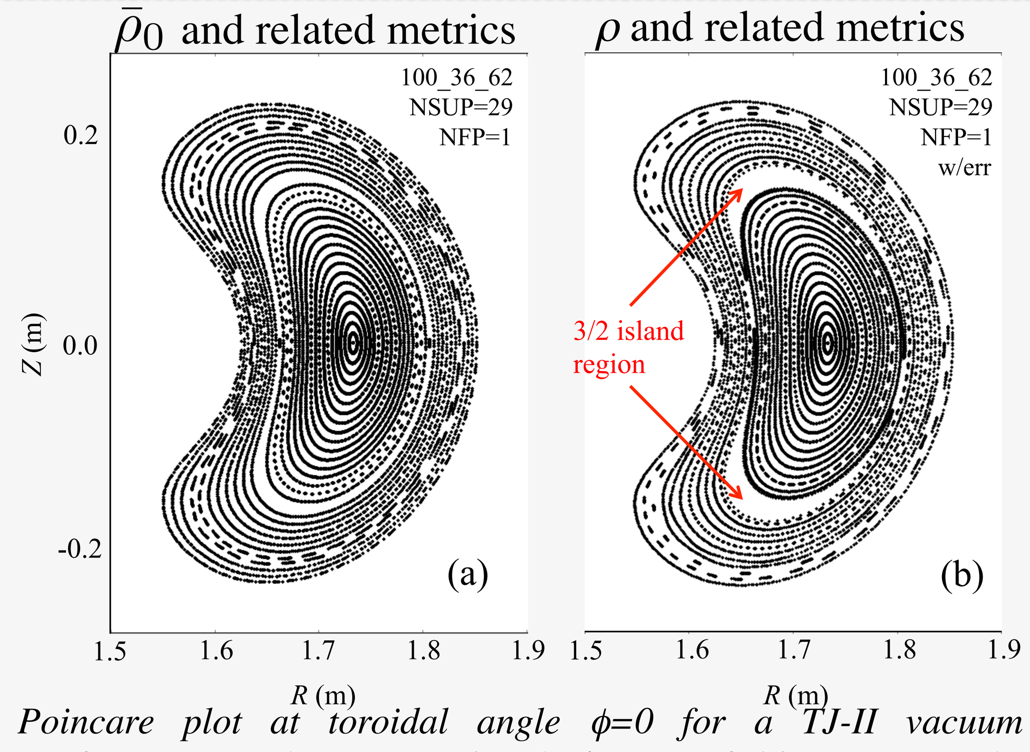
- Implies **discontinuities** in the radial fluxes ← it is a piecewise transformation.
- Excludes the separatrix (otherwise there would be bi-valuated functions). The point representing the separatrix is, however, included in the transport problem by simply imposing its condition of common boundary.
- **Would equally apply to a fully chaotic region** enclosed by the flux surfaces Φ_- and Φ_+, to which a flux Φ_I would be associated.

Transport in the island region can be solved separately and taken into account as an additional term in the discontinuities of the fluxes.

CHECKING THE ANNULAR MODEL: The metric coefficients obtained from the model agree with those obtained directly from Poincaré maps of the perturbed magnetic field. This has been done for TJ-II heliac (see figure) and LHD heliotron [3] magnetic configurations. In both cases, a suited effective width Δ is taken as input in the annular model. This width corresponds fairly well with the average island width.

OBSERVATIONS:

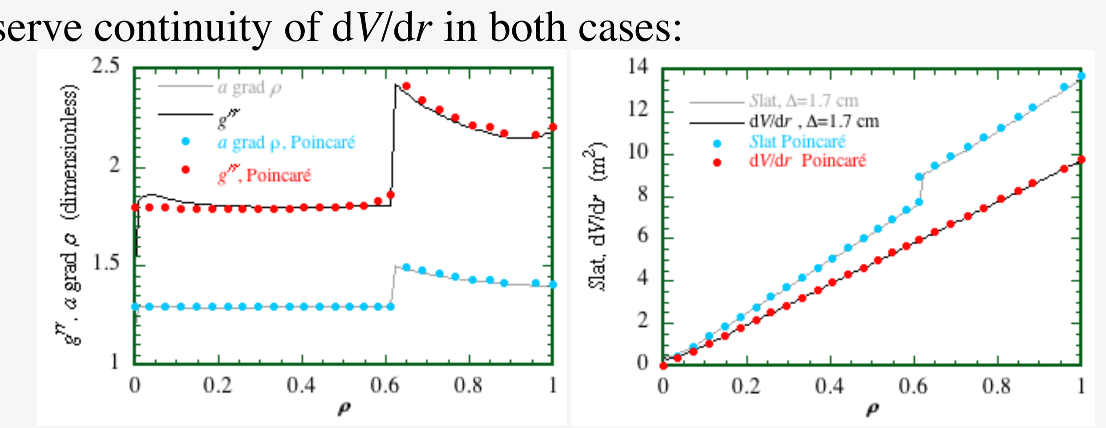
1D transport codes must deal with discontinuous metric coefficients. We have verified this possibility for the ASTRA shell [4] by comparison with the MAxS code (a numerical implementation of the "multiple domain scheme" [1]). Only precaution: set a high enough number of grid points.



Poincare plot at toroidal angle ϕ=0 for a TJ-II vacuum configuration without (a) and with (b) error field to promote the opening of m = 2 islands around the ι=3/2 magnetic resonance.

- $\bar\rho_0$ and related metrics are transformed by the annular model to obtain ρ for a given island width and location.
- ρ and related metrics are obtained numerically from Poincaré plots.

Observe continuity of dV/dr in both cases:



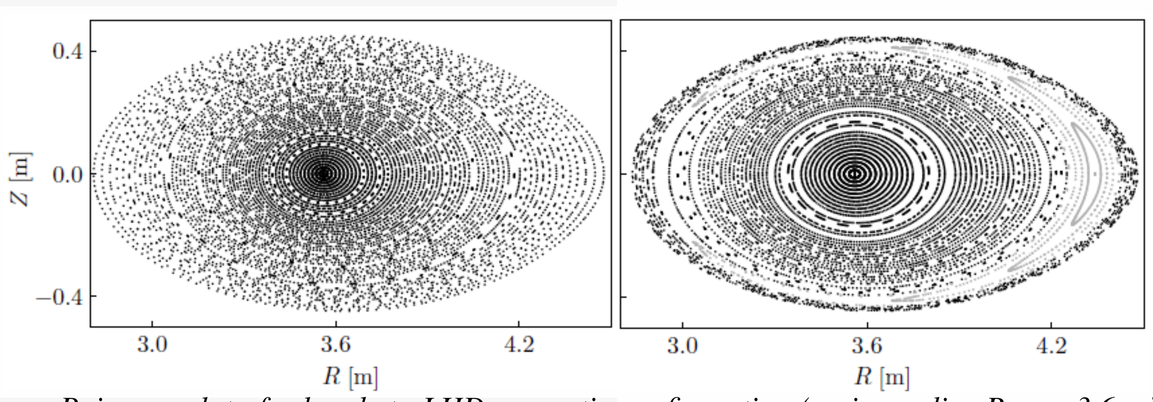
Comparison between dV/dr, S_{lat}=gradr·dV/dr (left), gradr and g^{rr} (right) computed using: the analytical annular model with island width Δ=1.7 cm (continuous lines) and from the Poincare plot with islands in Fig. (b) (dots) [3]. NB: r=aρ is chosen to ease the comparison with cylindrical values.

APPLICATION EXAMPLE: LHD

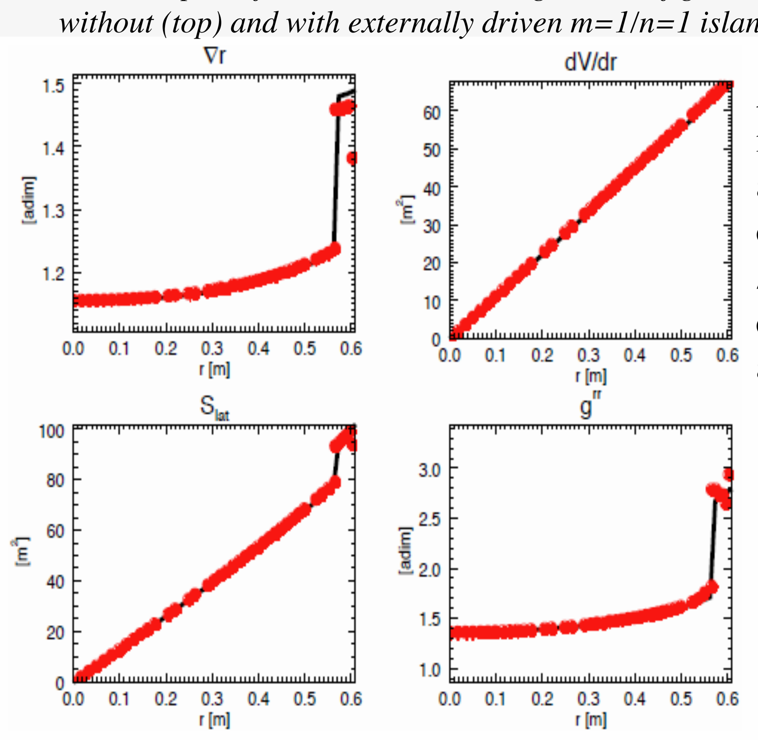
Consequences of a m=1/n=1 island in **LHD** magnetic configurations.

Metric and kinetic profiles:

- Metric profiles from the annular model. The unperturbed configuration ρ_0 is obtained from VMEC.
- Need to transform the input kinetic profiles: **take away** the data that fall in the resonant region. See Thomson Scatt. data.
- The remaining data and coordinates are **(re-)mapped** onto the radial coordinate ρ, which eliminates the island part of the profiles thus rendering them continuous and monotonic.



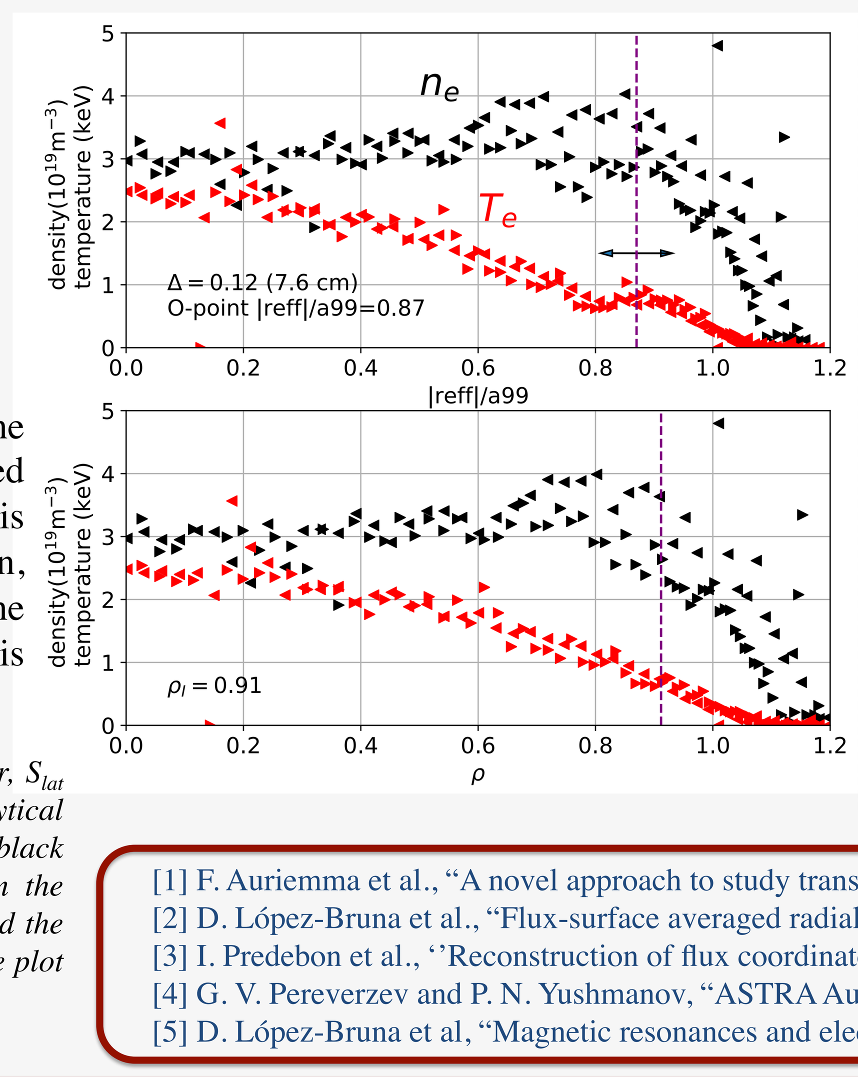
Poincare plot of a low-beta LHD magnetic configuration (major radius Rax = 3,6 m) without (top) and with externally driven m=1/n=1 island (bottom).



As for the TJ-II case, the metrics is well approximated as long as a suited width is chosen for the transformation, Δ= 7.6 cm in this case. The equatorial island width is about 12 cm.

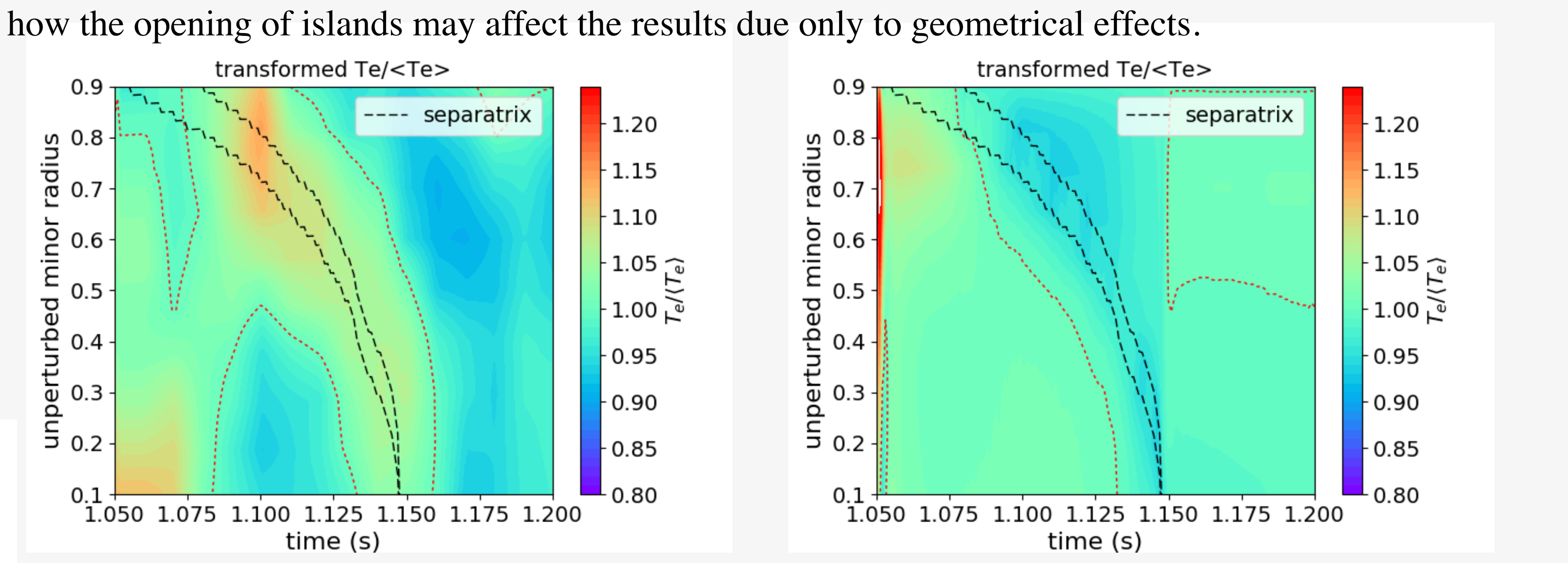
Comparison between gradr, dV/dr, S_{lat} and g^{rr} computed using the analytical annular model with Δ= 7.6 cm (black line) acting on the metrics from the Poincaré plots without island, and the metrics directly from the Poincare plot (red dots) with 1/1 island, [3]. NB: r=aρ

- LHD discharge #140534 around t = 4 s: a base plasma with NBI heating and superimposed modulated ECH (MECH). We reproduce the T_e response to MECH setting a diffusive transport model with neoclassical transport coefficients.
- The consequences of energy sink at the island regions in LHD MECH experiments are investigated [2]. A clear advantage of the annular model is that we can study the possible effect of islands in other experiments **without** the need of re-calculating the metrics from Poincare plots.



APPLICATION EXAMPLE: TJ-II

Geometrical effect of magnetic islands on transport in **TJ-II** plasmas with evolving rotational transform. Predictive transport analysis is performed, based on mostly neoclassical heat fluxes, in order to study how the opening of islands may affect the results due only to geometrical effects.



Evolution of T_e-profiles normalized to the time average. Dotted lines represent the path followed by the separatrix of a moving ι = 5/3 island chain through minor radius during the scan. Left: **experimental** data from ECE for TJ-II shot #21671. Right: **(predictive) transport analysis** results due to geometrical effects. Here the normalized minor radius corresponds to the unperturbed normalized coordinate. Values inside the separatrix are not calculated. Red dots mark $T_e = \langle T_e \rangle$.

- Experiment: low order rationals of ι leave a distinguishable trace in T_e-gradients when such "rationals" move through minor radius.
- Simulated discharge: we move the ι = 5/3 rational from edge to core as done in the experiments [5].
- We have set a transport model that mimics steady state T_e-profiles for the given electron heat source and density profiles (ECH plasma).
- Apply annular model. We impose Δ=1.5ρ/a cm, so the maximum effective width of 1.5 cm happens near the edge and decreases until collapsing at ρ = 0.

In the experiments the effect is found to be opposite: local temperatures above the mean follow the rational. Namely, the present exercise with the island-modified geometry indicates that the change in electron temperatures found experimentally should be related with an improvement of confinement, at or by the magnetic resonance, not by geometrical effects.

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