Nonlinear excitations of zonal structures by Toroidal Alfvén Eigenmodes

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Zonal structures and Alfvén Waves

- Zonal structures have important self-regulatory roles in the dynamics of microscopic drift-wave type turbulences including drift Alfvén waves

  - they are linearly stable (symmetry reasons, without velocity-space free energy) and predominantly only radially varying on mesoscales

  - they have a unique role in the cross-scale coupling of disparate spatiotemporal scales in burning plasmas as complex systems

  - self-regulation is essentially achieved via spontaneous excitations of modulational instabilities above a critical threshold in the driving fluctuation intensity

- Zonal structures act as non-local spectral transfer of energy and scatter driving instabilities into the short-radial wavelength stable domain.

- Zonal structures include zonal flows [Hasegawa etal 79], zonal magnetic fields (zonal currents) [Chen etal 01, Guzdar etal 01, Gruzinov etal 02] and radial corrugations of (energetic particle) profiles [Zonca etal 00].
- Zonal magnetic fields have been observed in CHS [Fujisawa etal 07].
- Recent numerical simulation results have shown that low frequency forced driven zonal flows may have a role in the nonlinear TAE saturation [Todo etal 10], but have not observed spontaneous excitation of zonal structures.
- Here, focus on spontaneous excitation of zonal structures (zonal flows and currents) by TAEs $\Rightarrow$ more easily induced including proper trapped ion responses $\Rightarrow$ zonal structure dominated by zonal current, not zonal flow.
- Theoretical analyses and numerical simulations must rely on kinetic descriptions in realistic equilibrium geometries for realistic predictions.
- Same conclusions apply for radial modifications of energetic particle profiles: see this afternoon posters by Vlad (TH/P6-03; fishbones), Di Troia (TH/P6-21; TAE/EPM) X. Wang (TH/P6-23; BAE).

Theoretical model

- The field variables are $\delta\phi$ and $\delta A_{\parallel}$ and are used to investigate the nonlinear couplings among the pump TAE, $(\omega_0, \boldsymbol{k}_0)$, the upper and lower TAE sidebands, $(\omega_\pm, \boldsymbol{k}_\pm)$, and the zonal mode $(\omega_z, \boldsymbol{k}_z)$.
- Indicating TAE and zonal mode with the subscripts A and z, respectively, one then has, for example, $\delta\phi = \delta\phi_A + \delta\phi_z$ and $\delta\phi_A = \delta\phi_0 + \delta\phi_+ + \delta\phi_-$.
- Assume for simplicity high toroidal mode numbers TAE and adopt the ballooning-mode decomposition in (r, θ, ϕ) field-aligned flux coordinates

$$\delta\phi_0 = A_0 e^{i(n\phi - m_0\theta - \omega_0 t)} \sum_j e^{-ij\theta} \Phi_0(x-j) + \text{c.c.}, \qquad \text{Defs.} \begin{cases} m = m_0 + j, \\ x = nq - m_0, \\ \int |\Phi_0|^2 dx = 1. \end{cases}$$

$$\delta\phi_\pm = A_\pm e^{\pm i(n\phi - m_0\theta - \omega_0 t)} e^{i(\int^r k_z dr - \omega_z t)} \sum_j e^{\mp ij\theta} \begin{bmatrix} \Phi_0(x-j) \\ \Phi_0^*(x-j) \end{bmatrix} + \text{c.c.},$$

$$\delta\phi_z = A_z \exp\left[i\left(\int^r k_z dr - \omega_z t \right) \right] + \text{c.c.}.$$

Zonal mode equations

- Consider long wavelengths, typical for TAE excitation by energetic particles $|k_\perp \rho_i|^2 \sim |k_z \rho_i|^2 < \epsilon = r_0/R_0 < 1$.
- Vorticity equation for the zonal mode [Chen etal 01] ($k_\parallel = x/qR_0$ and $\langle \ldots \rangle_x \equiv \int dx |\Phi_0|^2 (\ldots)$)

$$i\omega_z \chi_{iz} \delta\phi_z = \frac{c}{B_0} k_z k_\theta k_z^2 \rho_i^2 \left\langle \left(1 - \frac{k_{0\parallel}^2 v_A^2}{\omega_0^2} \right) \right\rangle_x (A_0^* A_+ - A_0 A_-) \ .$$

- Zonal flow polarizability $\chi_{iz} \simeq 1.6 q^2 \epsilon^{-1/2} k_z^2 \rho_i^2$ [Rosenbluth and Hinton 98] depends on both kinetic response and equilibrium geometry ($\omega_A = v_A/(qR_0)$)

$$i\omega_z \chi_{iz} \delta\phi_z = \frac{c}{B_0} k_z k_\theta k_z^2 \rho_i^2 \left(1 - \frac{\omega_A^2}{4\omega_0^2} \right) (A_0^* A_+ - A_0 A_-).$$

- Parallel Ampère's law for the zonal mode yields $\delta A_{\parallel z}$ or equivalently $\delta\psi_z \equiv \omega_0 \delta A_{\parallel z}/ck_{0\parallel}$.

- Strong electron current screening effect on scale lengths that are longer than the collisionless skin depth $\delta_e = c/\omega_{pe}$, with ω_{pe} the electron plasma frequency. Furthermore, $\delta_e \ll \rho_i$ for $m_e/m_i \ll \beta \ll 1$.

- Parallel Ampère's law reduces then to $\delta j_{z\parallel e} \simeq 0$, i.e.

$$\delta\psi_z = i\frac{c}{B_0}\frac{k_z k_\theta}{\omega_0}(A_0^* A_+ + A_0 A_-) \quad .$$

- This equation can also be readily derived from massless electron force balance along $\boldsymbol{B}_0$.

TAE sideband equations

- Zonal modes scatter TAE pump onto shorter wavelength sidebands.
- TAE sideband equations, for a fixed TAE pump, close the zonal mode equations and allow computing the zonal mode dispersion relation and onset condition for the modulational instability (spontaneous excitation).
- Use the theoretical framework of the general fishbone like dispersion relation (radial envelope evolution equation) to write

$$A_{\pm}\epsilon_{A\pm}b_{\pm} = -2i\frac{c}{B_0}k_\theta k_z \omega_0 b_0 \begin{pmatrix} A_0 \\ A_0^* \end{pmatrix} (\delta\phi - \delta\psi)_z \quad ,$$

- Definitions: $b_0 = \rho_i^2 \langle |\nabla_0 \Phi_0|^2 \rangle_x$, $b_{\pm} = \rho_i^2 \langle |\nabla_{\pm} \Phi_0|^2 \rangle_x = b_0 + b_z$, $b_z = k_z^2 \rho_i^2$.
- Nonlinearity includes Reynolds and Maxwell stresses plus the nonlinear correction to the ideal Ohm's law.

- TAE sideband linear dielectric response

$$\epsilon_{A\pm} = \left(\frac{\omega_A^4}{\epsilon_0 \omega^2} \Lambda_T(\omega) D(\omega, k_z) \right)_{\omega=\omega_\pm} ,$$

$$D(\omega, k_z) = \left(\Lambda_T(\omega) - \delta \hat{W}(\omega, k_z) \right) ,$$

- Defs: $\Lambda_T = \sqrt{-\Gamma_- \Gamma_+}$, $\Gamma_\pm = (\omega^2/\omega_A^2 - 1/4) \pm \epsilon_0 \omega^2/\omega_A^2$, $\epsilon_0 = 2(r/R_0 + \Delta')$.

- $\delta \hat{W}(k_z, \omega)$ plays the role of a normalized potential energy.

- Solutions of $D(\omega, k_z) = 0$ are $\omega = \pm \omega_T(k_z)$, with the pump TAE frequency given by $\omega_0 = \omega_T(k_z = 0)$.

- Defining $-i\omega_z = \gamma_z$ (zonal mode growth rate) and $\Delta_T \equiv \omega_T(k_z) - \omega_0$ (TAE sideband frequency shift)

$$D(\omega_\pm, k_z) = \pm \frac{\partial D}{\partial \omega_0} (i\gamma_z \mp \Delta_T) .$$

Zonal mode dispersion relation

- After using TAE sideband equations, zonal mode equations become

$$\delta\phi_z = 2\left(\frac{c}{B_0}k_\theta k_z|A_0|\right)^2\left(\frac{\omega_0^2}{\omega_A^2}-\frac{1}{4}\right)\left(\frac{b_0 b_z}{b_+\chi_{iz}}\right)\frac{\epsilon_0}{\Lambda_T(\omega_0)}\frac{2\omega_0/\omega_A^2}{\partial D/\partial\omega_0}\frac{(\delta\phi-\delta\psi)_z}{\gamma_z^2+\Delta_T^2}\equiv -\alpha_{\phi T}\frac{(\delta\phi-\delta\psi)_z}{\gamma_z^2+\Delta_T^2},$$

$$\delta\psi_z = -2\left(\frac{c}{B_0}k_\theta k_z|A_0|\right)^2\left(\frac{b_0}{b_+}\right)\frac{\epsilon_0\omega_0^2/\omega_A^2}{\Lambda_T(\omega_0)}\frac{2\omega_0/\omega_A^2}{\partial D/\partial\omega_0}\frac{\Delta_T/\omega_0}{\gamma_z^2+\Delta_T^2}(\delta\phi-\delta\psi)_z\equiv -\alpha_{\psi T}\frac{(\delta\phi-\delta\psi)_z}{\gamma_z^2+\Delta_T^2}.$$

- The zonal mode dispersion relation becomes:

$$\gamma_z^2 = \alpha_{\psi T}-\alpha_{\phi T}-\Delta_T^2 \quad ;$$

- Kinetic behaviors and equilibrium geometry enter via χ_{iz} (polarizability), Λ_T (gap structure), Δ_T (sideband frequency shift), $\omega_0\partial D/\partial\omega_0$ (wave energy density).

Modulational instability onset condition

- From the zonal mode dispersion relation, modulational instability will set in when (main result)

zonal current zonal flow

$$\left(\frac{c}{B_0\omega_0}k_\theta k_z|A_0|\right)^2\left(\frac{b_0}{b_+}\right)\frac{\epsilon_0}{\Lambda_T(\omega_0)}\frac{4\omega_0/\omega_A^2}{\partial D/\partial\omega_0}\left[\frac{\Delta_T}{\omega_0}\frac{\omega_0^2}{\omega_A^2}+\frac{b_z}{\chi_{iz}}\left(\frac{\omega_0^2}{\omega_A^2}-\frac{1}{4}\right)\right]>\left(\frac{\Delta_T}{\omega_0}\right)^2 .$$

- Typical ordering gives $|\Delta_T/\omega_0| \sim O(\epsilon_0)$ and $|b_z(1-\omega_A^2/4\omega_0^2)/\chi_{iz}| \sim O(\epsilon_0^{3/2}/q^2) \Rightarrow$ zonal current effect is dominant over the usual zonal flow.

- Furthermore, generally $\omega_0\partial D/\partial\omega_0 > 0$ in the ideal MHD first stability region. Thus, onset condition becomes approximately $\Delta_T/\omega_0 > 0$ and

$$\left(\frac{c}{B_0\omega_0}k_\theta k_z|A_0|\right)^2\left(\frac{b_0}{b_+}\right)\frac{\epsilon_0\omega_0^2/\omega_A^2}{\Lambda_T(\omega_0)}\frac{4\omega_0/\omega_A^2}{\partial D/\partial\omega_0}>\left(\frac{\Delta_T}{\omega_0}\right) .$$

- The sign of Δ_T/ω_0 depends on specific equilibria and parameters and must be calculated for individual cases. For $\Delta_T/\omega_0 < 0$, modulational instability can still be excited for $\omega_0^2 > \omega_A^2/4$ and small $|\Delta_T/\omega_0|$; however, with $\delta\phi_z$ dominating over $\delta\psi_z$.

- Quantitative estimates for the onset condition of the modulational instability assume (from TAE linear theory)

$$\frac{\epsilon_0\omega_0^2/\omega_A^2}{\Lambda_T(\omega_0)}\frac{2\omega_0/\omega_A^2}{\partial D/\partial\omega_0} \sim 1 \; .$$

- Furthermore, assume $b_z \lesssim k_\theta^2\rho_i^2 \sim \epsilon_0 b_0$: zonal mode modulates the TAE envelope on the distance between rational surfaces (radial width of poloidal Fourier harmonics). Meanwhile, $k_\| \simeq 1/2qR_0$.

- Threshold condition becomes (for the maximum $b_0 \sim \epsilon_0$)

$$\left(\frac{c}{B_0\omega_0}k_\theta k_z|A_0|\right)^2 \sim \left|\frac{\Delta_T}{\omega_0}\right| \sim \epsilon_0\frac{b_z}{k_\theta^2\rho_i^2} \sim \frac{b_z}{\epsilon_0} \; .$$

- In terms of $\delta B_r/B_0$,

$$\left|\frac{\delta B_r}{B_0}\right|^2_{th} \sim \frac{\rho_i^2}{4\epsilon_0(qR_0)^2} \quad .$$

- For some typical tokamak parameters, $|\delta B_r/B_0|^2_{th} \sim O(10^{-8}) \Rightarrow$ spontaneous excitation of zonal structures may be a process effectively competing with other nonlinear dynamics in determining the saturation level of TAE modes.

- Above threshold, one can estimate

$$\gamma_z \simeq \epsilon_0^{-1/2} b_z^{1/2} k_z v_A \left|\delta B_r/B_0\right| \quad .$$

- For the most unstable growing zonal structures, $b_z \sim \epsilon_0^2$ with $\gamma_z \simeq \epsilon_0^{1/2} k_z v_A |\delta B_r/B_0|$.

Importance of the Alfvénic state

- This work assumes $|k_\perp \rho_i|^2 \sim |k_z \rho_i|^2 < \epsilon = r_0/R_0 < 1$: reasonable and usually applies for TAEs excited by energetic ions in burning plasmas.
- For shorter wavelengths, or equivalently $\epsilon \to 0$, both $\delta\phi_z$ and $\delta\psi_z$ become increasingly smaller, since $\omega_0^2/\omega_A^2 - 1/4 \to 0$ and $\Delta_T/\omega_0 \to 0$.
- This is due to the cancellation of the Reynolds and Maxwell stresses, yielding the well known properties of the Alfvénic state, which is broken in the present case by the toroidal geometry of the considered plasma equilibrium.
- This result shows the importance of equilibrium geometry in determining both linear and nonlinear plasma dynamic behaviors.
- At sufficiently short wavelengths or in simpler plasma equilibria, present analysis must be suitably modified to account for the breaking of the Alfvénic state, e.g., by finite ion Larmor radius effects.

Discussions and Conclusions

- Spontaneously excited zonal structures are dominated by the zonal current instead of the usual zonal flow because of magnetically trapped-ion enhanced polarizability, $\chi_{iz} \simeq 1.6q^2\epsilon^{-1/2}k_z^2\rho_i^2$ [Rosenbluth and Hinton 98]

$$|\delta\phi_z|/|\delta\psi_z| \approx |k_z\rho_i|^2/|\chi_{iz}| \approx O(\epsilon^{1/2}/q^2) < 1.$$

- The MHD model without trapped ions yields $\chi_{iz} \simeq k_z^2\rho_i^2$, and, correspondingly, $\delta\phi_z \approx \delta\psi_z$. Spontaneous excitation of zonal structures is still possible for $\Delta_T/\omega_0 > 0$ and $\omega_0^2/\omega_A^2 < 1/4$, but with larger threshold condition.

- For $\Delta_T/\omega_0 < 0$, spontaneous excitation of zonal structures is found only in the upper half of the TAE frequency gap, $\omega_0^2/\omega_A^2 > 1/4$; contrary to the case including the proper trapped ion responses.

- The importance of including proper trapped-ion dynamics for the spontaneous excitation of zonal flows by electrostatic drift-type turbulence in toroidal plasmas was pointed out [Chen etal 00, Guzdar etal 01].

- In the drift-type turbulence case, dropping trapped ion dynamics yields a quantitative difference. In the present TAE case, results change qualitatively and quantitatively.

- Including kinetic thermal ion treatment and proper equilibrium geometry in the nonlinear simulations of Alfvénic modes [Holod etal 09, Bass and Waltz 10, Wang etal 11] is, thus, an important ingredient for realistic comparisons with experimental measurements.

- The quantities Δ_T/ω_0 and b_z/χ_{iz} regulate the branching ratio (relative strength) of zonal flows and currents and the onset condition for the modulational instability. It, therefore, will be interesting, by a suitable extension of these terms, to generalize the present theoretical framework to other toroidal configurations.