A New Paradigm for E×B Velocity Shear Suppression of Gyro-kinetic Turbulence and the Momentum Pinch

by
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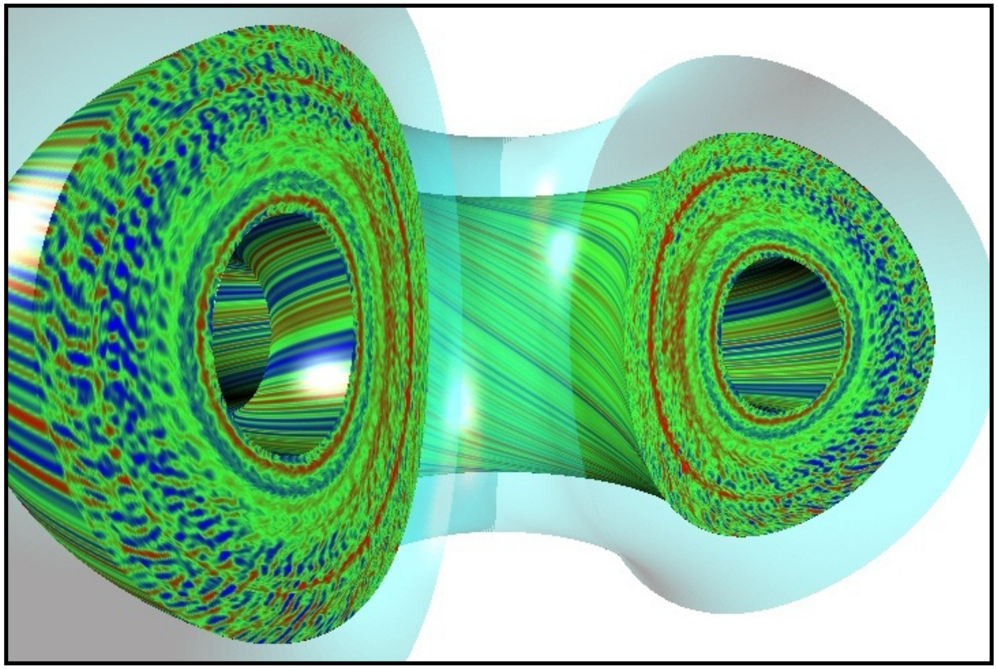

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Overview

- **The shear in the ExB velocity Doppler shift or "Doppler shear" produces a Reynolds Stress with a sign opposite to the parallel velocity shear contribution (i.e. a momentum pinch)**
  - Quasilinear models based on the **decorrelation** (Shaing 1990, Biglari 1990) or **quench rule** (Waltz 1994) paradigms give no Reynolds stress
- **The new "spectral shift" paradigm is based on non-linear gyro-kinetic turbulence simulations with Doppler shear**
  - Doppler shear suppresses turbulence by causing a shift in the radial wavenumber (k_x) spectrum
  - A model that depends only on the k_x-shift robustly fits the non-linear gyro-kinetic spectrum of the RMS electric potential
- **The spectral shift model is implemented in the Trapped Gyro-Landau Fluid (TGLF) quasilinear transport model**
  - The Reynolds stress is produced by the finite k_x-shift of linear modes
- **The first transport predictions with the new model in TGLF are made for a low-torque tokamak discharge where the pinch is important**

Shear in the ExB Velocity Causes a Spectral Shift

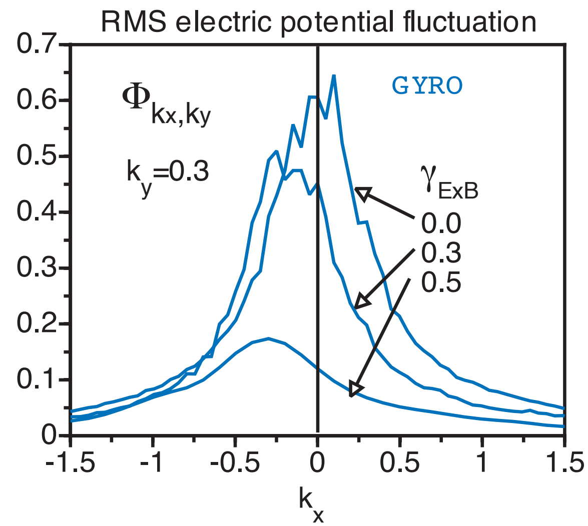
- **The time and flux surface average RMS amplitude of the electric potential fluctuations (Φ_{k_x,k_y}) in non-linear gyro-kinetic simulations reveals a shift in the peak of the radial wavenumber spectrum**

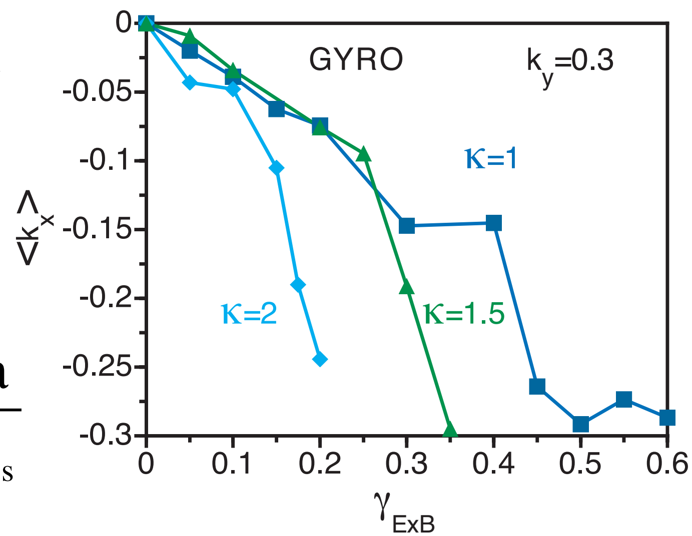
$$k_x = k_r \rho_s \qquad k_y = k_\theta \rho_s \qquad k_x = k_y \hat{s} \theta_0$$

- **The amplitude of the shifted peak is suppressed**
- **The spectral average shift is a non-linear function of the Waltz-Miller shear rate**

$$\langle k_x \rangle = \int_{-\infty}^{\infty} dk_x k_x \Phi^2_{k_x,k_y} \Big/ \bar{\Phi}^2_{k_y}$$

$$\bar{\Phi}^2_{k_y} = \int_{-\infty}^{\infty} dk_x \Phi^2_{k_x,k_y} \qquad \gamma_E = \frac{q}{r}\frac{d}{dr}\left(\frac{c\partial\phi_{-1}}{\partial\psi}\right)\frac{a}{c_s}$$





A Finite Reynolds Stress Requires Parity Breaking

- **The decorrelation model gives a weak intensity suppression (Shaing 1990, Zhang 1992)**

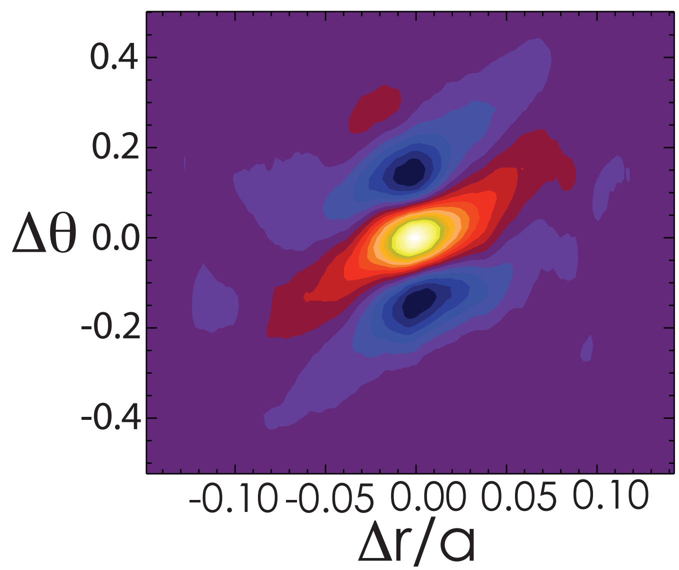
$$\bar{\bar{\Phi}}^2(\gamma_{ExB}) = \bar{\bar{\Phi}}^2(0) \Big/ \left(1+\alpha_c(\gamma_{ExB}\tau_c)^2\right) \qquad \bar{\bar{\Phi}}^2 = \int_{-\infty}^{\infty} dk_y \int_{-\infty}^{\infty} dk_x \Phi^2_{k_x,k_y}$$

- **The quench rule gives a stronger intensity suppression (Waltz 1994).**

$$\bar{\bar{\Phi}}^2(\gamma_{ExB}) = \bar{\bar{\Phi}}^2(0)\,\mathrm{MAX}\left[1-\alpha_E|\gamma_{ExB}|/\gamma_{max},0\right]$$

- **Neither paradigms give a finite Reynolds stress because they do not break the parity of the eigenmodes**

- **The finite spectral average k_x-shift produces breaking of the parity and a tilt of the ballooning eigenmodes and the 2D correlation function shown for** $\gamma_{ExB} = 0.1$



The k_x-Shift is Due to Doppler Shear k_x-Coupling

The RMS amplitude of the electric potential fluctuations from GYRO is well fit by a Lorentzian for $k_y > 0.05$

$$\Phi_{k_x,k_y} = \frac{\gamma_{k_y}^{eff}}{\left(c_y k_y^2 + c_x k_x^2\right)}$$

With a uniform ellipticity $\frac{c_x}{c_y} = 0.56$

The Bernoulli differential equation provides an interpretive model of the **linear growth**, **Doppler shear k_x-coupling** and **non-linear mixing** of the potential

$$\frac{d}{dt}\Phi_{model} = \underline{\gamma_{k_y}^{eff}\Phi_{model}} + \underline{\gamma_{ExB} k_y \frac{\partial \Phi_{model}}{\partial k_x}} - \underline{\left(c_y k_y^2 + c_x k_x^2\right)\Phi_{model}^2} = 0$$

The Doppler shear term is **linearly destabilizing** when $\gamma_{ExB} k_y \frac{\partial \Phi_{model}}{\partial k_x} > 0$
This induces a tilt of the spectrum which is then
re-centered about a shifted peak by the non-linear mixing term

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The New Spectral Shift Paradigm Models the 2D-Spectrum Not Just the Integrated Intensity

The interpretive model has the solution

$$\Phi_{\text{model}} = \frac{\gamma_{k_y}^{\text{eff}}}{\left(c_y k_y^2 + c_x \langle k_x \rangle^2 + c_x \left(k_x - \langle k_x \rangle\right)^2\right)}$$

where $\langle k_x \rangle = -k_y \gamma_{\text{ExB}} / \gamma_{k_y}^{\text{eff}}$

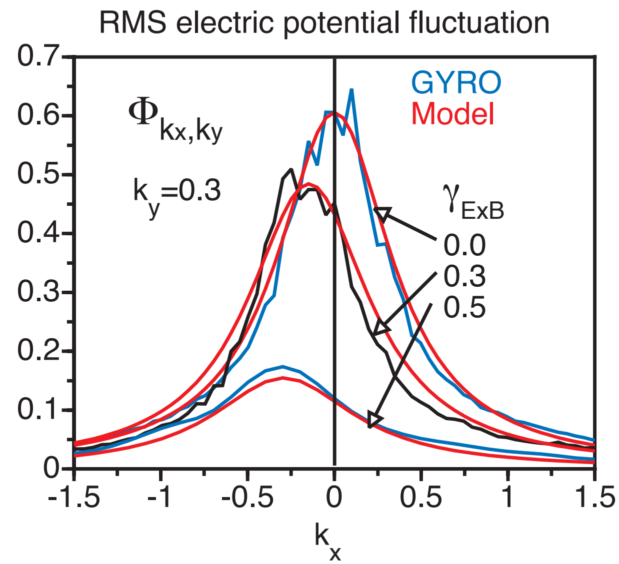
- **The model spectrum shifts in the linearly destabilized direction with a symmetric peak**
- **The amplitude at the shifted peak is reduced but not by enough**

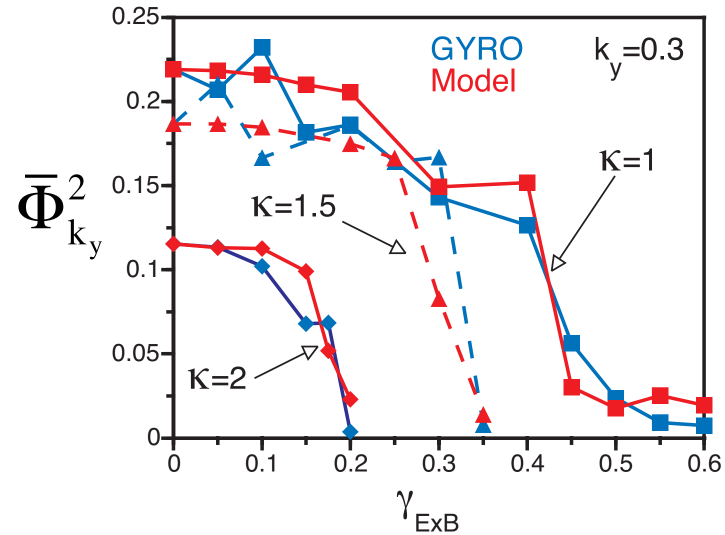
The ansatz

$$\gamma_{k_y}^{\text{eff}}\left(\langle k_x \rangle\right) = \gamma_{k_y}^{\text{eff}}(0) \Big/ \left(1 + \left(\alpha_x \langle k_x \rangle / k_y\right)^4\right)$$

robustly fits the simulations for $\alpha_x = 1.15$ **independent of plasma parameters (q,r/R,s,Ti/Te,shape)**

This completes the "**spectral shift**" model





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The Spectral Shift Model has been Implemented in TGLF

- **The quasi-linear Trapped Gyro-Landau Fluid (TGLF) transport model computes accurate linear drift-wave ballooning eigenmodes and quasi-linear weights for the transport fluxes**
- **A formula for the spectral shift has been fit to GYRO simulations**

$$\langle k_x \rangle_{fit} = k_y 2.0 \mathrm{Tanh}\left[k_{x_e} / k_y 2.0 \right] \quad \text{where}$$

$$k_{x_e} = -k_y \left(\frac{\gamma_{ExB}}{\gamma_0} \right) \left\{ 0.36 + 0.38 \sigma_x \mathrm{Tanh}\left[\left(0.69 \sigma_x \frac{\gamma_{ExB}}{\gamma_0} \right)^6 \right] \right\}$$

$$\sigma_x = \mathrm{MIN}\left[k_y / 0.3, 1.0 \right] \left(1 + 0.42 \left(1 - B_{unit} / B(0) \right)^2 \right)$$

- **The procedure to use the spectral shift model is as follows:**
  - Compute the linear growth-rate spectrum (γ_0) for $k_x = 0$
  - Compute $\langle k_x \rangle$ using the fit formula
  - Re-compute the linear eigenvalues and eigenfunctions to compute the quasi-linear weights at finite $k_x = \langle k_x \rangle$
  - Use the spectral shift model for the change in the peak intensity of the turbulence with $\langle k_x \rangle$ to modify the TGLF intensity model

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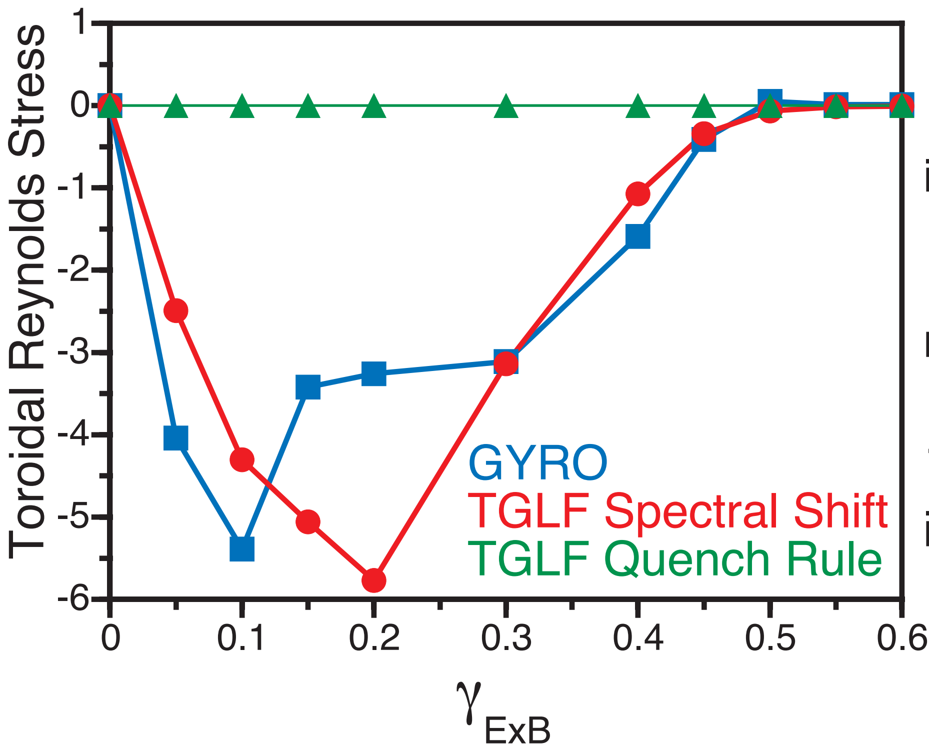
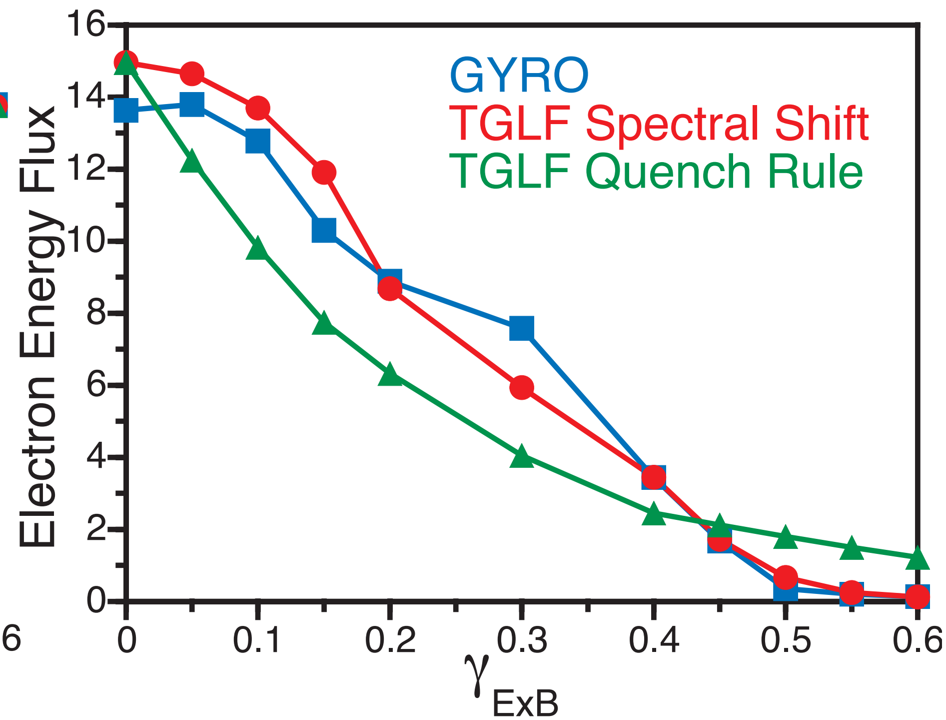
TGLF with the Spectral Shift Model Allows Calculation of the Reynolds Stress Momentum Pinch

- **The Reynolds stress is due to the finite k_x of the linear modes.**
- **The accuracy of TGLF in modeling the GYRO energy transport is improved with the spectral shift compared to the quench rule**





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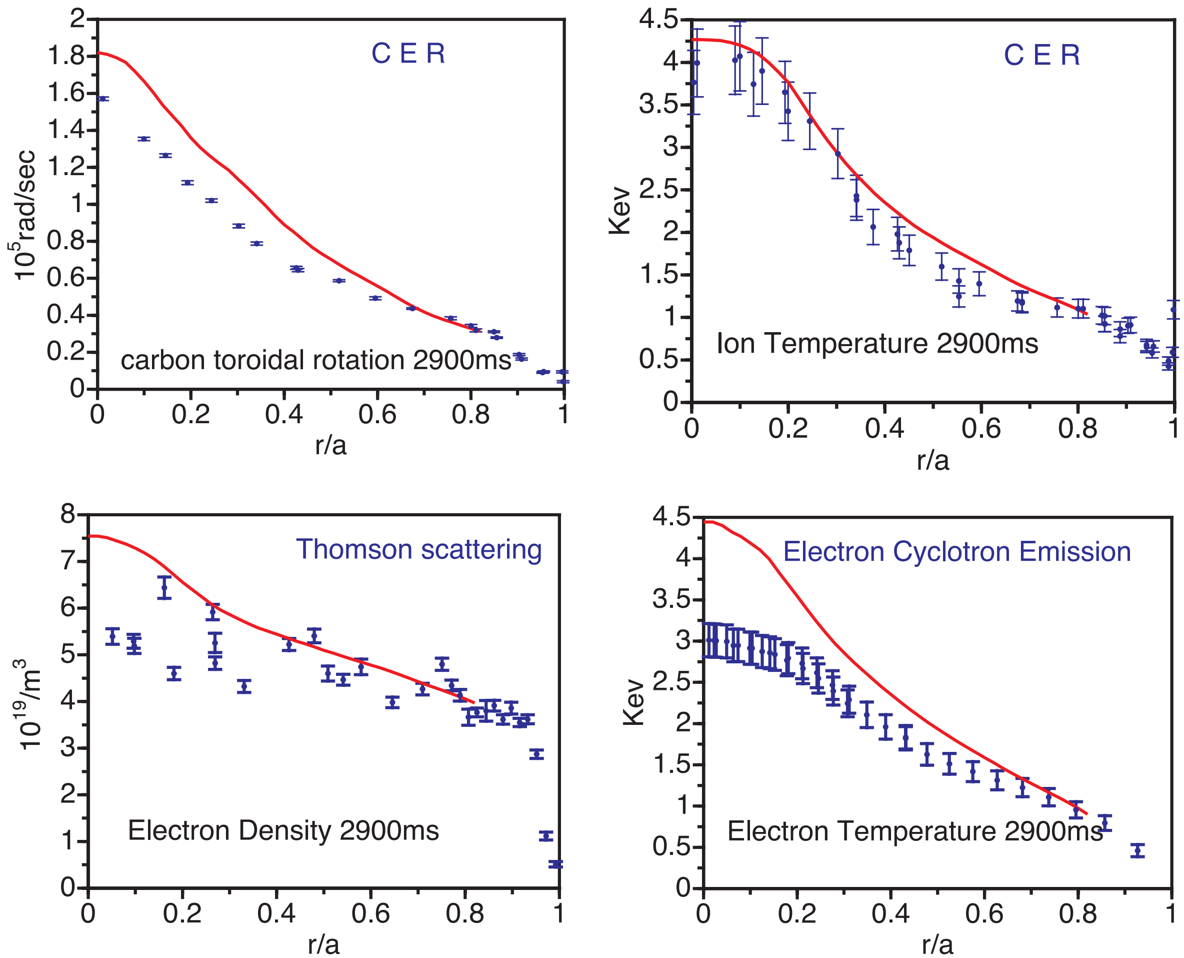
The First Momentum Transport Simulations with the Spectral Shift Model in TGLF

- **The toroidal rotation, electron and ion temperature and electron density are simultaneously predicted using 3 kinetic species: electrons, deuterium and carbon 6**
  - Fast ion dilution is included and the ion/electron densities are fixed
- **DIII-D discharge 125236 is simulated at two times**
  - 2900ms when there is unbalance NBI in the direction of the current
  - 3500ms when there is balanced NBI injection (zero net torque)
  - W.M. Solomon, K.H. Burrell, J.S. deGrassie, et al., Plasma. Phys. Control. Fusion, **49**, B313 (2007)
- **The high accuracy NEO code is used for neoclassical transport.**
  - E. Belli and J. Candy, Plasma Phys. Control. Fusion, **50**, 095010 (2008)
- **TGLF+NEO predictions are done both in the high-ExB velocity ordering, neglecting diamagnetic level flows, and then with these flows included in the parallel flow of each species in TGLF**

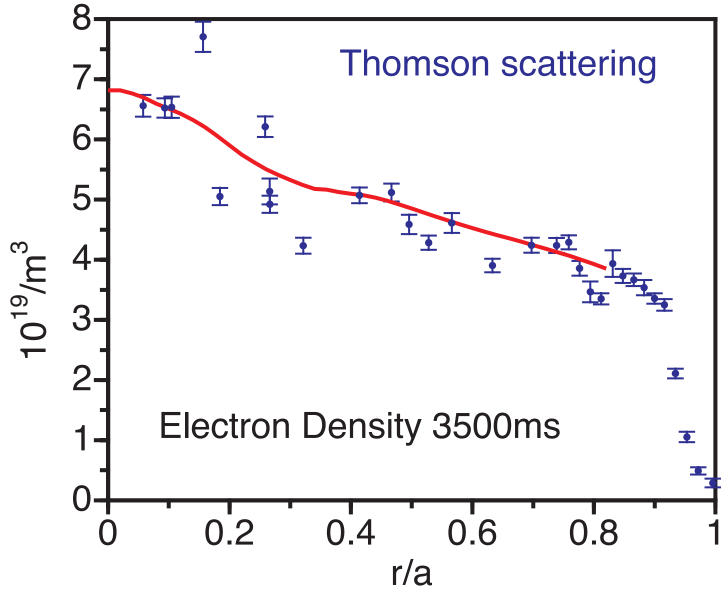
Predictions for Co-injected Beam Phase with only ExB Flows Are Close to the Experimental Level of Momentum Transport

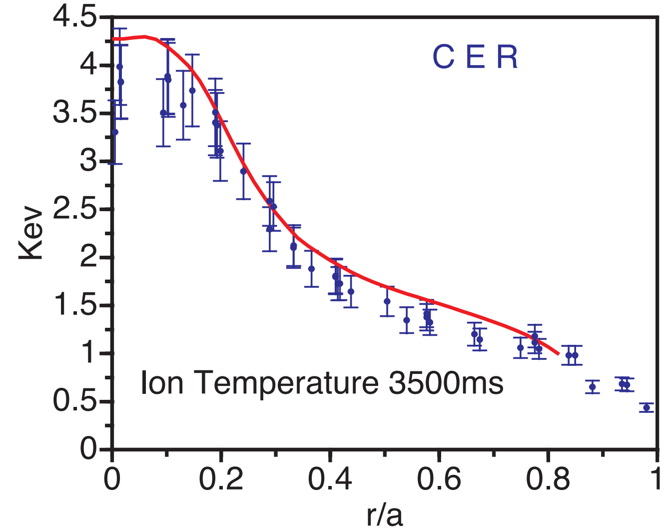
- **The toroidal rotation of carbon is of similar accuracy to the other profile predictions**
- **The ion temperature and density are good but the electron temperature prediction is too high**
  - The stabilzation of ETG modes by kinetic carbon ions contributes
- **Diamagnetic and neoclassical poloidal flows are negligible**

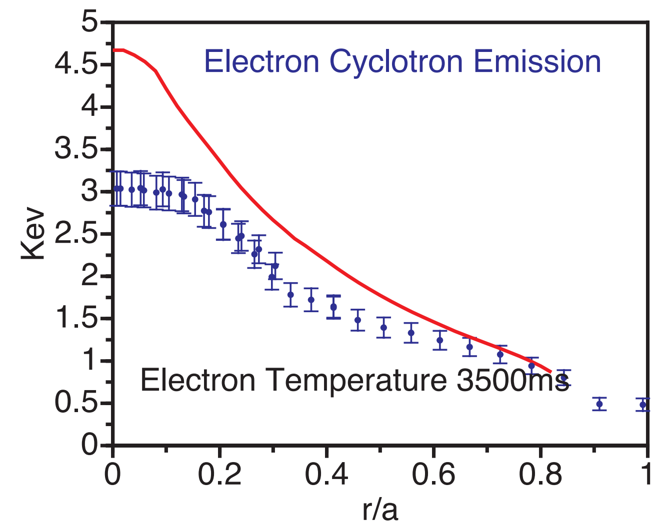


Predictions for the Density and Temperatures of the Balanced Beam Phase are Similar to the Co-phase

- **The predicted ion temperature is within measurement errors**
  - Electron temperature is too high
- **The predicted electron density is in good agreement with the data**
  - Only the beam particle source was included







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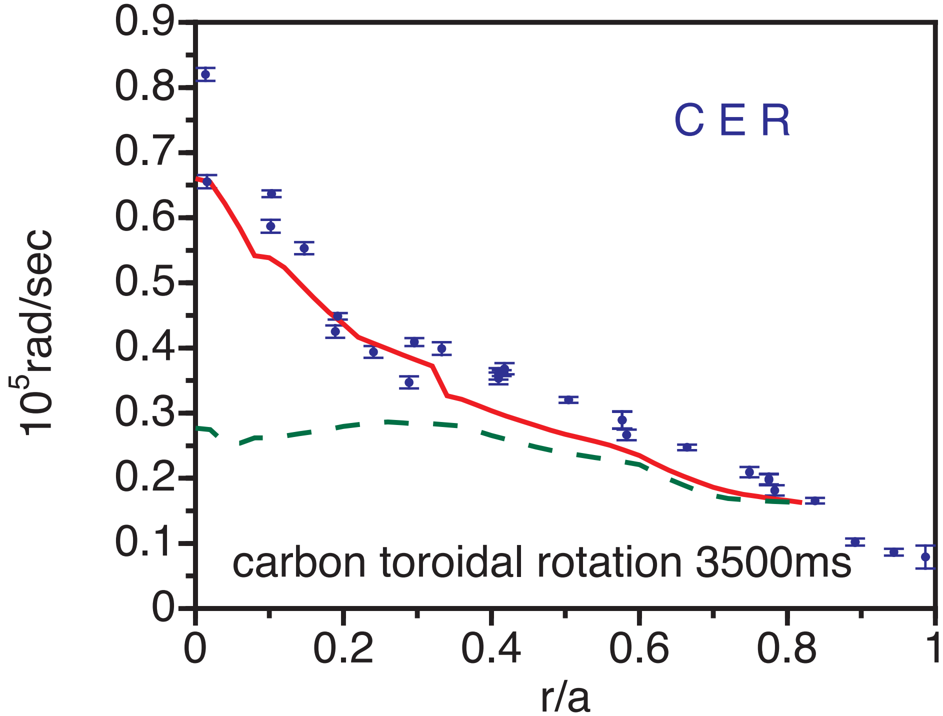
The Toroidal Rotation Prediction for the Balanced Beam Phase is Remarkably Good

- **Only the ExB velocity is included in this simulation. The predicted toroidal rotation is in good agreement with the carbon data for the balanced beam phase**
- **Artificially zeroing out the small beam torque gives the dashed green curve**
  - The peaking in the center is due to beam orbit effects yielding a local net torque
  - toroidal flow shear is negative for r/a<0.3 because the parallel flow "pinch" is outward due to negative magnetic shear



- **The parallel flow and Doppler shear pinches give most of the rotation peaking for r/a>0.3**

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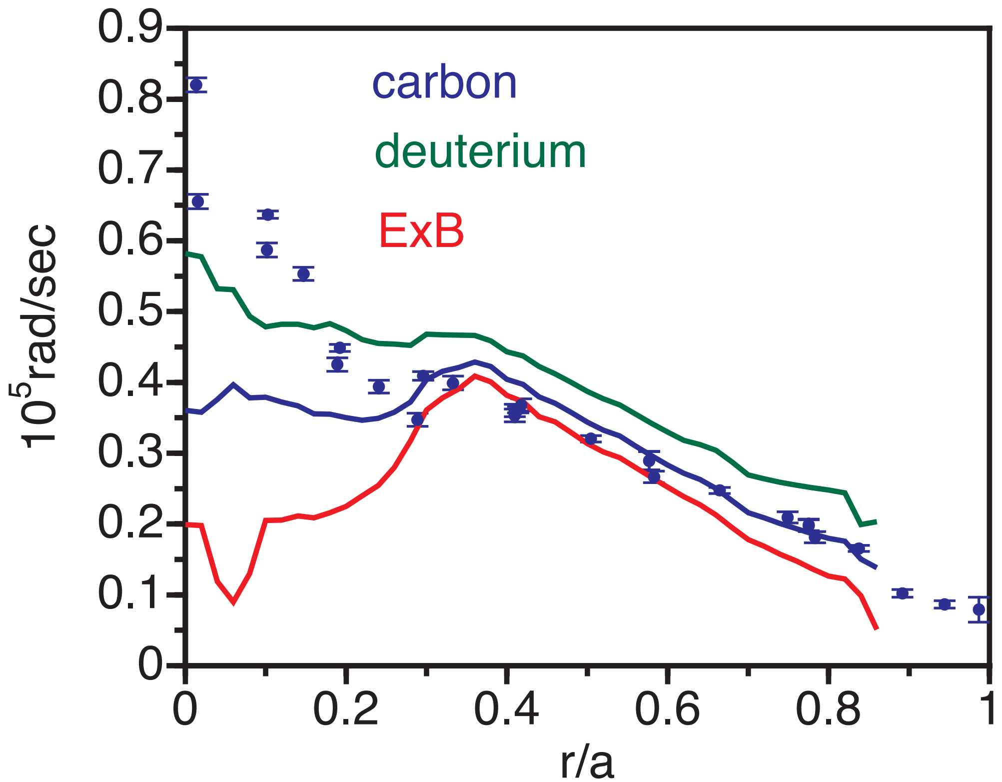
Diamagnetic and Neoclassical Poloidal Flows are Significant in the Balanced Beam Phase

- **Including the diamagnetic and neoclassical poloidal flows in the parallel flows of each species improves the agreement of TGLF with the carbon toroidal rotation for r/a > 0.3**
  - it is not a good match in the deep core where the species differences are larger
- **The Doppler shear pinch allows intrinsic flow reversal when parallel and ExB flow shear are not the same**



$$\Pi_{tor} = \eta\left(\gamma_{\parallel} - \beta\gamma_{ExB}\right) - \mu u_{\parallel}$$

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Summary

- **The new "spectral shift" paradigm for how the turbulence is suppressed by shear in the ExB velocity Doppler shift is based on gyro-kinetic turbulence simulations**
  - The primary stabilization mechanism is identified as a shift in the peak of the radial wavenumber spectrum
  - A model of the 2D-spectrum that depends only on the spectral average radial wavenumber shift has been found to be robust to changes in plasma parameters (q, s, r/R, Ti/Te, shape)
- **The spectral shift model has been implemented in TGLF and shown to be capable of computing the Reynolds stress momentum pinch due to the shear in the ExB velocity Doppler shift**
- **The first simulation of a balanced beam DIII-D discharge using TGLF+NEO shows promise of being able to predict the core intrinsic rotation**
- **Extensive verification of TGLF momentum transport with the spectral shift model has been done with GYRO simulations. (to be published)**
- **The TGLF model has sufficient accuracy to begin large database validation of gyro-kinetic momentum transport with experiment**