Energetic Particle Long Range Frequency Sweeping and Quasi-linear Theory

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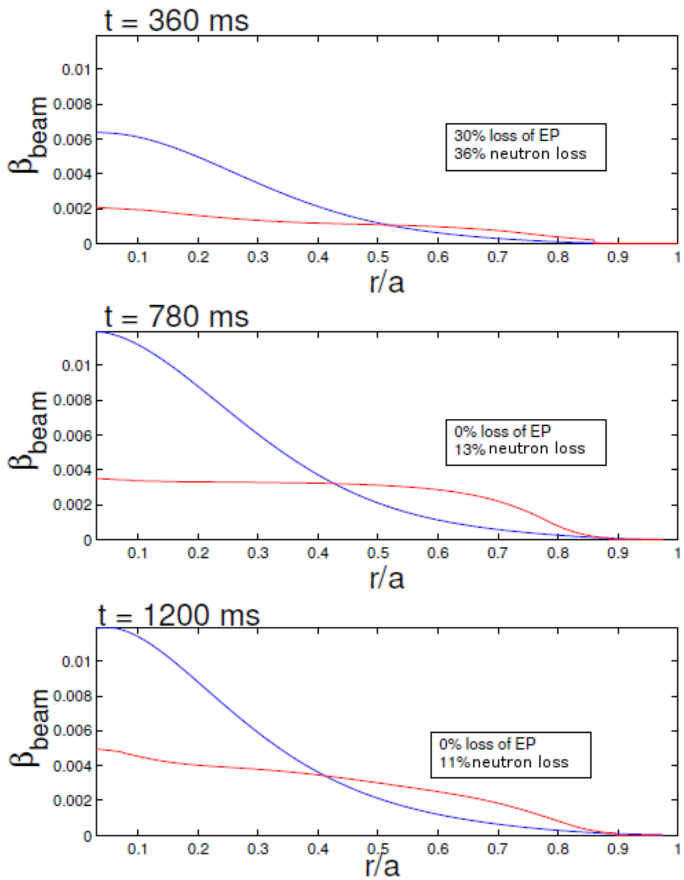
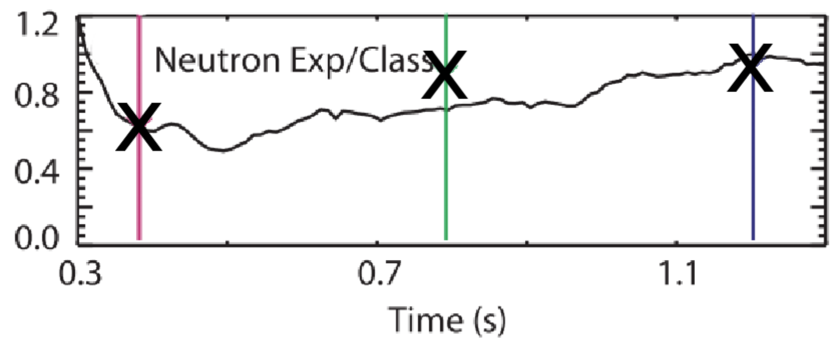
Energetic Particle Transport

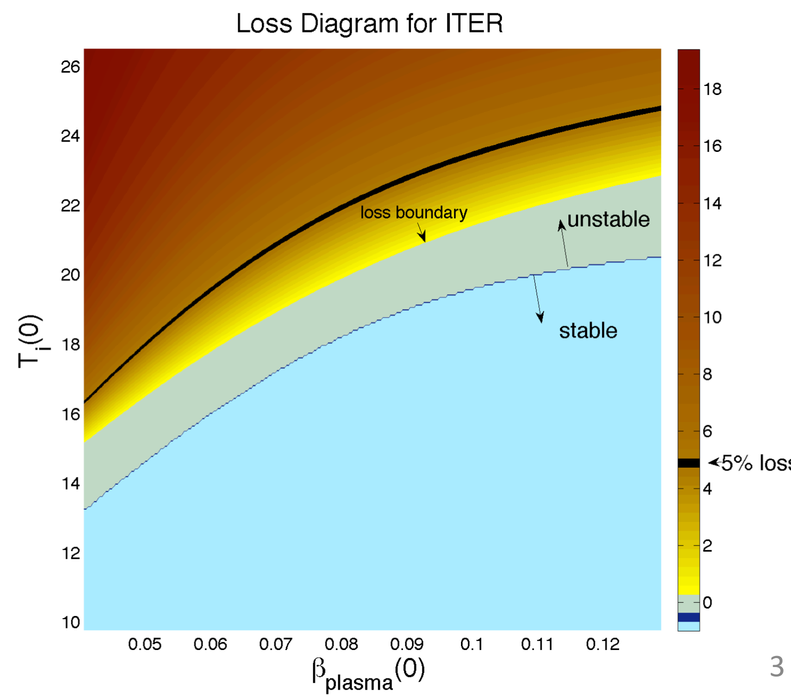
1. Two extreme approaches
   a. Quasi-linear (QL) theory
   b. Spontaneous Frequency Sweeping
2. Simplified QL Model Replicates Experimental Data
3. Transport theory for Long Range Frequency Sweeping in bump-on-tail model
4. Global relaxation caused by single linear unstable mode
5. TAE model simulation for TAE chirp from gap into continuum

Simplified QL (1.5 D) Replicates Experimental

(D-III-D- #112117) Neutron Response (K. Ghantous, et. al. Phys. Plas. (2012)







Quasi-linear Profile Relaxation

Density can change by 33% and ion temperature by 20% with less than loss of alpha

Theory for Long Range Frequency Chirping Nyqvist, et. al. Nucl Fusion (2012)

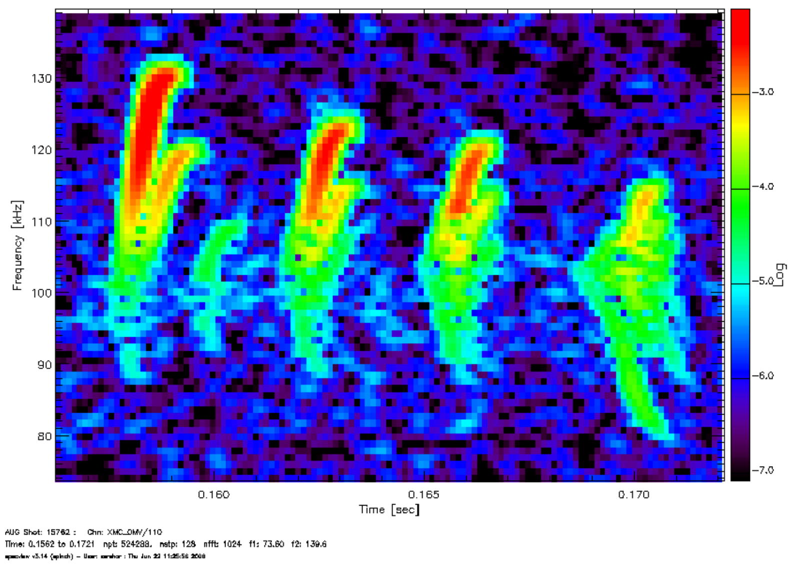
1. Bounced averaged transport equation for distribution f(V,x) during chirping with diffusion-drag-annihilation leads to:

$$J = \frac{m}{\sqrt{2\pi}} \oint dz \sqrt{\varepsilon - U(z,t)}$$

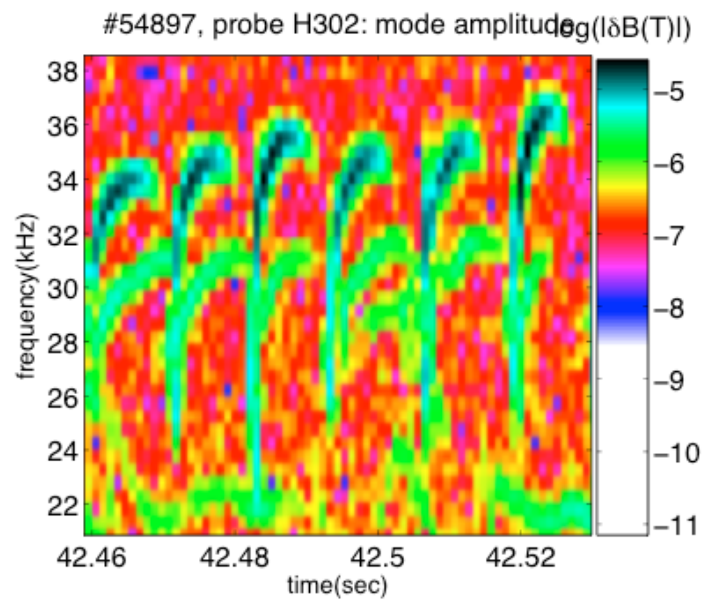
$$\frac{\partial \delta f}{\partial t} + \beta \delta f - \frac{\nu^3}{k^2} \frac{\partial}{\partial J}\left[J \frac{dJ}{d\varepsilon} \frac{\partial \delta f}{\partial J} \right] = \left(\alpha^2 / k + \frac{d^2 s}{dt^2} \right) \frac{dF(\upsilon = \dot{s})}{d\upsilon}$$

2. Consistent solution of Poisson equation with long range chirping deforms sinusoidal response
3. Establishes methodology for chirping in realistic plasma for kinetic-MHD response
4. “Hook” response arising from drag an variation of $\frac{\partial F(\upsilon = \dot{s}(t))}{\partial \upsilon}$

“Hooks” Observed in Experiment

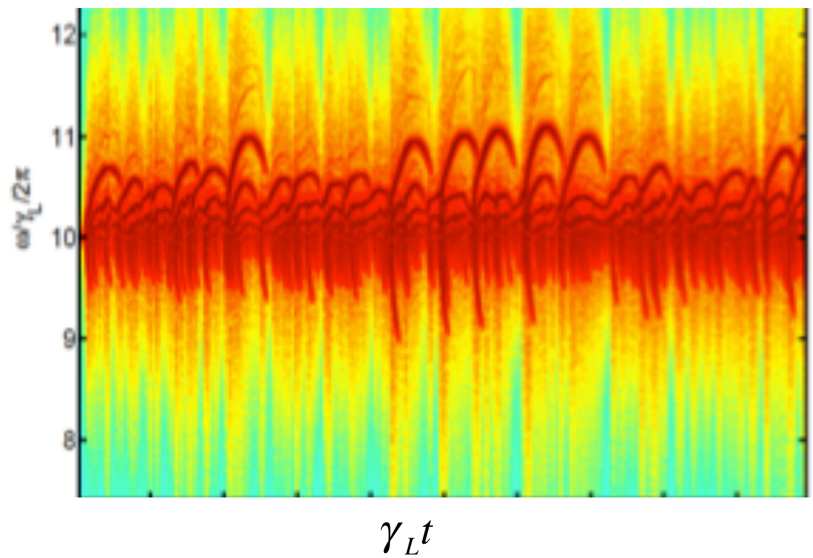


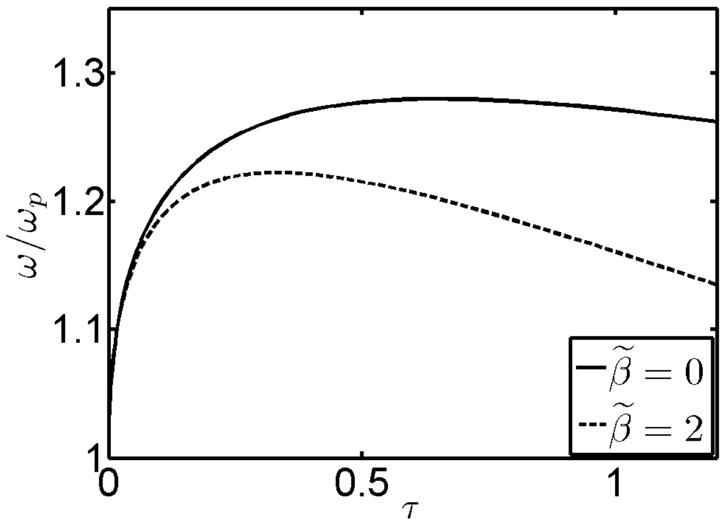
JET (ICRH)



MAST (NB)

“Hook Response”





Simulation: Competition of drag and diffusion allows hole to ‘fall’

Self-consistent transport solution; varying slope induces “hook”. Accentuated by drag

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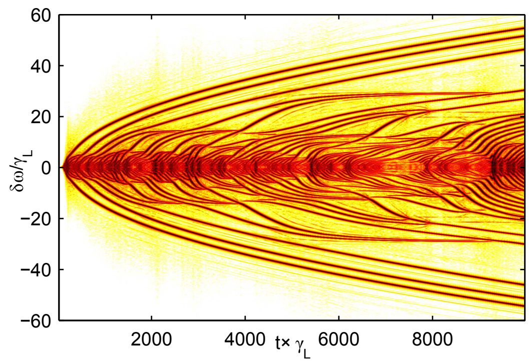
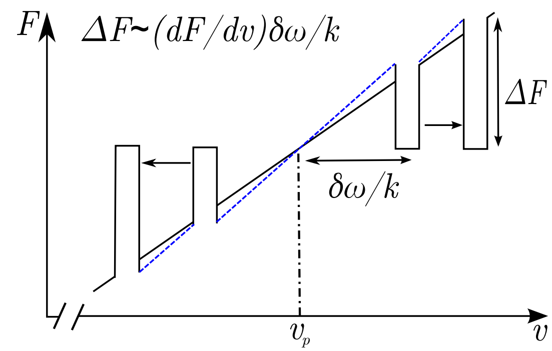
Global Transport from single linear wave

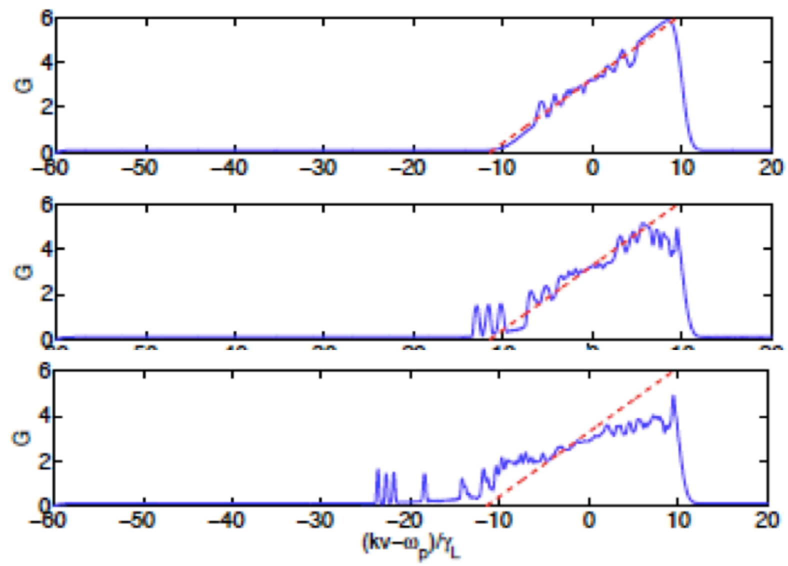
(Lilley, et. al. Phys. Plasmas 2010, NF 2012)

Constant slope ∂F/∂V chirps continually

shocked phase space still "unstable"







Global relaxation to plateau comes from 'erosion' of outer region

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TAE GAP and Continuum

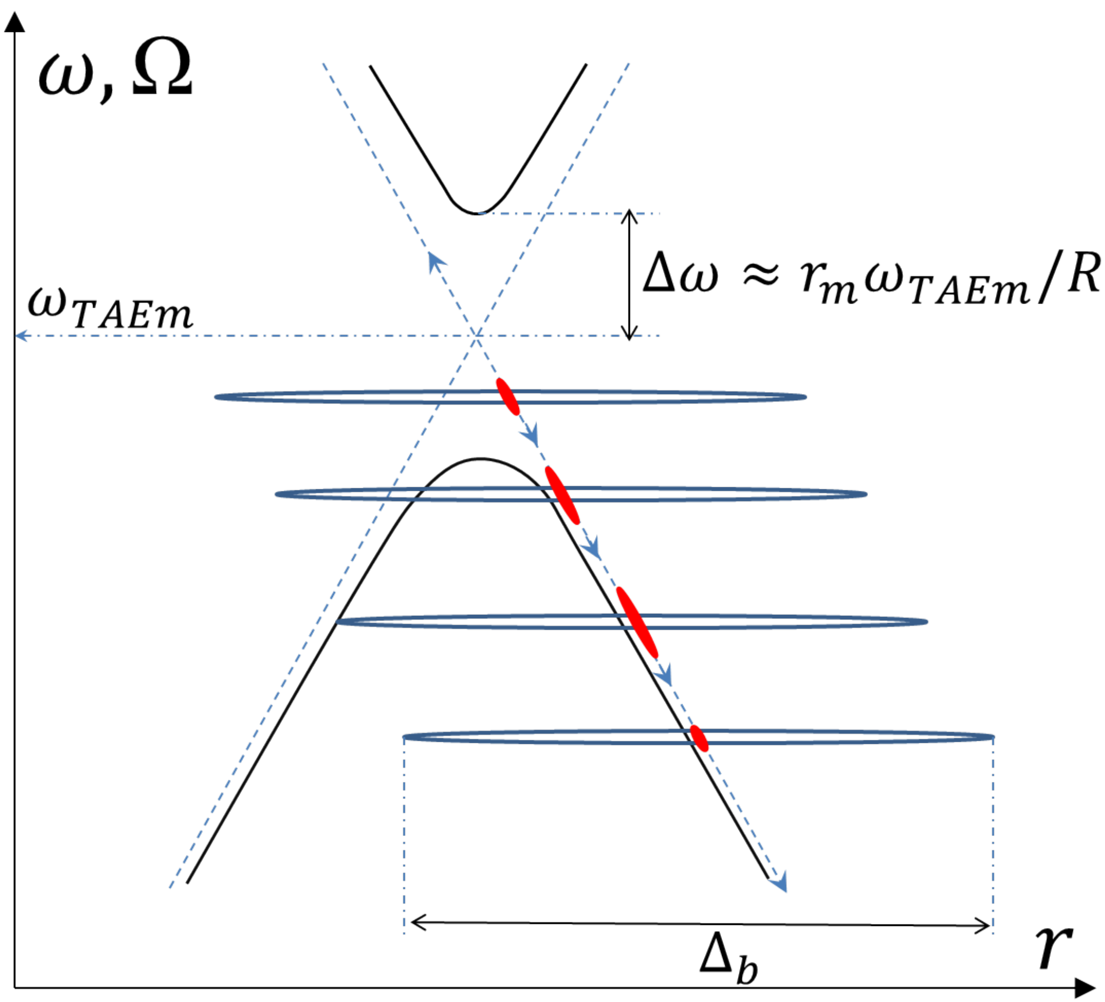
$$\omega_{Am}(r) \equiv V_A\left(n - m/q(r)\right)/R$$

Gap Position:

$$\omega_{Am}(r_m) = -\omega_{Am+1}(r_m)$$

$$\rightarrow q(r_m) = \frac{m+1/2}{n}$$

$$\omega_{TAEm} = \frac{V_A}{2q(r_m)R}$$



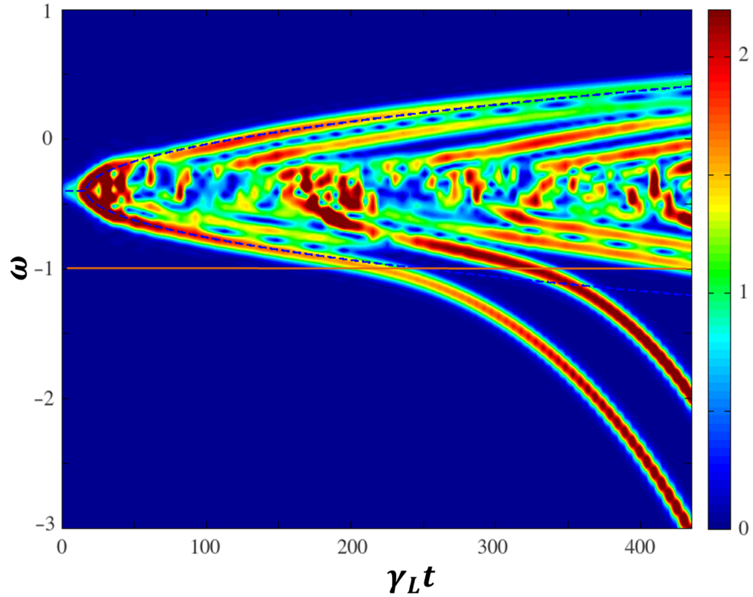
TAE Modeling

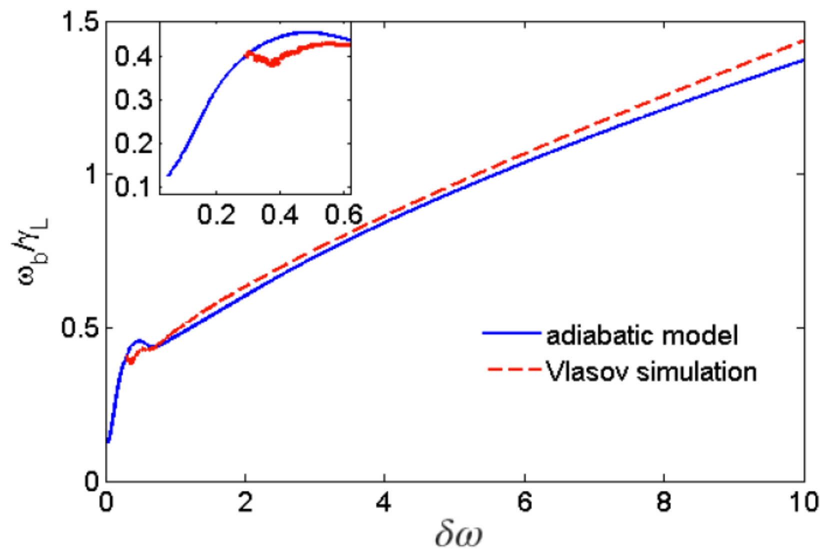
- Model: Large aspect ratio, low beta tokamak
- Wave dynamics uses Rosenbluth, et. al. [PRL, (1991)] theory
- Wave-particle interaction- from BBY map theory
- Relatively simple expressions in frequency domain, takes form of Volterra Integral equation

$$\left(\frac{1+\omega}{1-\omega}\right)^{1/2}\phi(\omega)\rightarrow i\phi(t)+\int_0^t d\tau\left[J_0(\tau)-iJ_1(\tau)\right]\phi(t-\tau)$$

- Wave particle interaction first taken with spatial profile independent of frequency and momentum (same as bump-on-tail); Map model and wave theory force profile to depend on frequency and momentum
- First results of simple model is discussed.

Frequency Sweeping into Lower Continuum





Initial hole clump pair emerge
Upward frequency holes never reach lower continuum
Downward frequency clumps penetrate deep into continuum

Adiabatic theory can explain most of results
Both simulation and adiabatic theory allow clump to pass through lower continuum, while hole does not penetrate upper continuum

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Adiabatic Theory Using Map Model Interaction Term

Interaction Hamiltonian

$$\delta H_{\text{int}} \propto \text{Im}\left[-\eta(\Omega) \pm \sqrt{\eta(\Omega)^2 - 1}\right]\omega_b^2 \cos\xi;$$

depends on momentum variable Ω.

Dynamics of phase function ξ

$$\frac{d\xi}{dt} = n\bar{\omega}_\phi\left(E, P_\phi, \mu\right) \text{-} (m+l)\bar{\omega}_\phi\left(E, P_\phi, \mu\right) - \omega\left(t\right)$$

Momentum Variable Ω

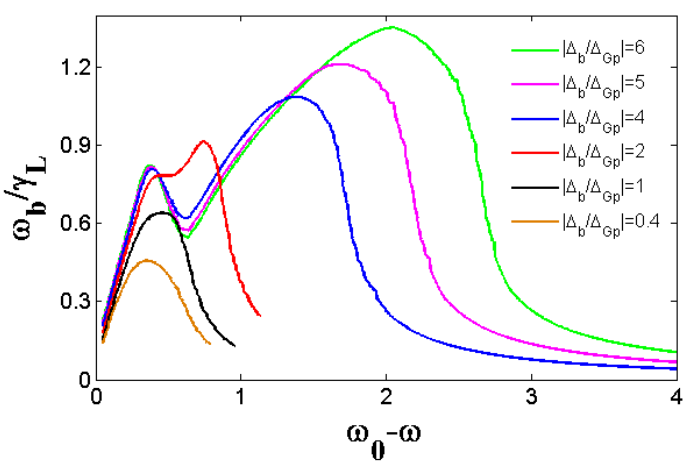
$$\Omega \equiv R\left[n\bar{\omega}_\phi\left(E, P_\phi, \mu\right) \text{-} (m+l)\bar{\omega}_\phi\left(E, P_\phi, \mu\right) - \omega_{TAEm}\right] / r_m$$

$$\eta(\Omega) = \frac{\Delta_{Gp}}{\Delta_b}\left[\Omega + i\left(1 + \frac{\partial^2}{\partial t^2}\right)^{1/2}\right] \xrightarrow{\text{adiabatic}} \eta = \frac{\Delta_{Gp}}{\Delta_b}\left[\Omega + i\sqrt{1 - \omega^2\left(t\right)}\right];$$

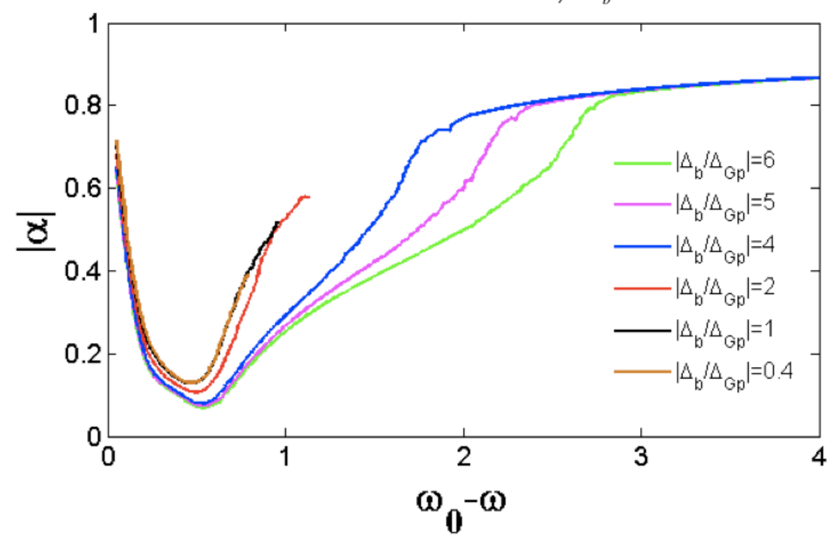
$$\frac{\Delta_{Gp}}{\Delta_b} \equiv \frac{\text{spatial width of gap}}{\text{orbit width of EP}}$$

Adiabatic map model chirp into continuum

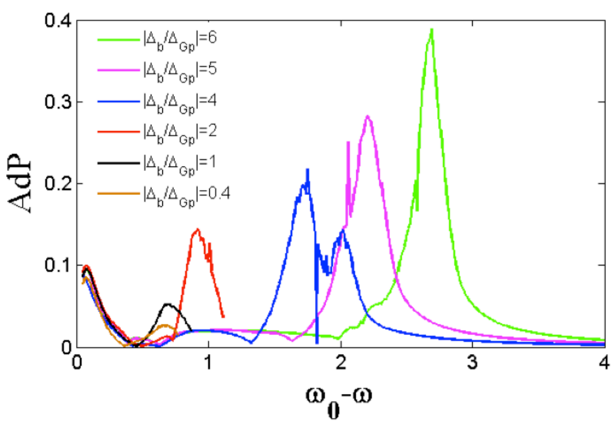
Square root of mode amplitude vs. frequency shift



Chirping rate, $\alpha = {d\omega/dt}\Big/{\omega_b^2}$



Adiabatic parameter (AdP) vs. frequency shift



$$AdP = \left|\frac{d\left(\left(1-\alpha^2\right)^{1/4}\omega_b^2\right)/dt}{\sqrt{1-\alpha^2\omega_b^2}}\right| + \left|\frac{d^2\left(\left(1-\alpha^2\right)^{1/4}\omega_b^2\right)/dt^2}{\left(1-\alpha^2\right)^{3/4}\omega_b^3}\right|$$

$$\frac{d}{dt} = \alpha\omega_b^2\frac{d}{d\omega}$$

1. Chirp of Holes decay in gap, as before
2. Clump's chirp can penetrate continuum if $\Delta_b\Big/\Delta_{Gp} > 1$
3. At max amplitude frequency shift is:

$$|\omega|_{mx} \simeq \Delta_b\Big/2\Delta_{Gp}$$

4. Adiabatic approximation breaks towards end of chirp

Summary

- Fairly simple QL relaxation replicates neutron deficit observed in D-IIID experiment
- Numerical Paradigm, for long range frequency chirping including transport, has been established
- Possibility of global relaxation from single unstable linear mode through chirping
- Model for TAE mode exhibits chirping of clumps into lower continuum, but no chirping of holes into upper continuum
- The rigorous TAE adiabatic model predicts limited downward chirping to the radius in which orbit width just barely intersects both continuum points

Future Work

- Generalize quasi-linear approach to enable planning of plausible alpha particle scenarios in ITER
- Extend long range chirping model to tokamak geometry using adiabatic and transport theory
- Generalize TAE model's numerical algorithm to realistic geometry to enable nonlinear Vlasov simulation using Hamiltonian derived from a map model interaction
- Develop global transport code accounting for QL and chirping routes to relaxation so that a quantitative predictive tool is available for ITER