Drift-kinetic Simulation Studies on Neoclassical Toroidal Viscosity in Tokamaks with Small Magnetic Perturbations

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Outlines

Non-axisymmetric magnetic field perturbations in tokamak (RMP coils, toroidal ripples, MHD modes, …)

$$\delta B/B_0 = O(10^{-3} \sim 10^{-4})$$

- Even small perturbation can damp the toroidal rotation drastically by neoclassical toroidal viscosity (**NTV**) (in JET, DIII-D, NSTX, …).

- RMP can mitigates ELMs, but toroidal rotation is related to other stabilities (RWM, locked modes, …).
- Perturbation field penetration is shielded by plasma rotation.

Accurate calculation method for NTV is required to predict / control the effect of magnetic perturbations on NTV and rotation damping.

- ◆ FORTEC-3D δf drift-kinetic code is applied to NTV calculations.
- ◆ Verification of the numerical method has been done with Park's analytic formula for $E \times B \rightarrow 0$ case.→ good agreement. (Satake, PRL 2011)

- ◆ Here, new benchmark results for the finite-$E \times B$ cases are reported.
- ◆ **It is found that the magnetic shear and the resonant drift motion of passing particles makes a double-peak profile of NTV when $\boldsymbol{E} \times \boldsymbol{B}$ rotation is fast enough.**

Basic relations for NTV calculations

● Momentum balance equation

$$\frac{\partial}{\partial t}(mn\mathbf{u}) = -\nabla\cdot\overleftrightarrow{\mathbf{P}} + en\mathbf{u}\times\mathbf{B} + en\mathbf{E} + \mathbf{F} + \mathbf{S}_m \quad \left\{ \begin{array}{lll} \mathbf{F} & = \int d^3v\, m\mathbf{v}C(\delta f) & \text{: Friction force} \\ \mathbf{S}_m & = \int d^3v\, m\mathbf{v}S & \text{: Momentum input} \end{array} \right.$$

● Radial particle flux [$\langle \nabla\psi\cdot\left(\frac{\mathbf{b}}{B}\right)\times \text{above eq.}\rangle$]

$$\Gamma_\psi = \left\langle \frac{\mathbf{F}_\perp\cdot\mathbf{B}\times\nabla\psi}{eB^2}\right\rangle + \langle n\mathbf{v}_{\mathbf{E}\times\mathbf{B}}\cdot\nabla\psi\rangle + \left\langle \frac{\mathbf{S}_{m\perp}\cdot\mathbf{B}\times\nabla\psi}{eB^2}\right\rangle - \left\langle \frac{\mathbf{B}\times\nabla\psi\cdot\nabla\cdot\overleftrightarrow{\mathbf{P}}}{eB^2}\right\rangle + \left\langle \frac{\partial}{\partial t}\left(\frac{n}{\Omega}\mathbf{b}\times\mathbf{u}_\perp\right)\cdot\nabla\psi\right\rangle$$

- Γ_{cl}: Classical flux
- $\Gamma_{E\wedge}$: Radial $\mathbf{E}\times\mathbf{B}$ flux

(neglected in DKE)

- $\Gamma_{s\perp}$: Momentum input-driven flux
- Γ_{NC}: Neoclassical flux
- Γ_{pl}: Polarization flux

● Neoclassical flux

$$\Gamma_{NC} = -\left\langle \frac{\mathbf{B}\times\nabla\psi\cdot\nabla\cdot\overleftrightarrow{\mathbf{P}}}{eB^2}\right\rangle = -\frac{G}{e\iota}\left\langle \frac{\mathbf{B}\cdot\nabla\cdot\overleftrightarrow{\mathbf{P}}}{B^2}\right\rangle + \frac{1}{e\iota}\left\langle \mathbf{e}_\zeta\cdot\nabla\cdot\overleftrightarrow{\mathbf{P}}\right\rangle = \Gamma_{p_\parallel} + \Gamma_{TV}.$$

Flux driven by toroidal viscosity (non-ambipolar)

$\Gamma_{p_\parallel}$: (Banana-plateau & Pfirsch-Schluter fluxes) → **ambipolar**

● Time evolution of toroidal angular momentum

$$\sum_a \left\langle \frac{\partial}{\partial t}\mathcal{L}^a_\zeta\right\rangle = \sum_a \left(-\left\langle \mathbf{e}_\zeta\cdot\nabla\cdot\overleftrightarrow{\mathbf{P}}\right\rangle + T^a_\zeta\right) + \langle \mathbf{J}\times\mathbf{B}\cdot\mathbf{e}_\zeta\rangle\,.$$

$\langle\mathcal{L}_\zeta\rangle \equiv \langle mn\mathbf{u}\cdot\mathbf{e}_\zeta\rangle$

a: particle species

Toroidal rotation is accelerated by the toroidal viscosity $\left\langle \mathbf{e}_\zeta\cdot\nabla\cdot\overleftrightarrow{\mathbf{P}}\right\rangle = e\iota\Gamma_{TV}$, external torque input T_ζ , and the torque of $\mathbf{J}\times\mathbf{B}$ force.

The δf Monte-Carlo method in FORTEC-3D code

Drift-kinetic equation for $\delta f(\mathbf{x}, \mathcal{K} = \frac{mv^2}{2}, \mu = \frac{mv_\perp^2}{2B}) = f - f_M$:

$$\frac{D}{Dt}\delta f \equiv \left[\frac{\partial}{\partial t} + (\mathbf{v}_\| + \mathbf{v}_d)\cdot\nabla + \dot{\mathcal{K}}\frac{d}{d\mathcal{K}}\right]\delta f - C_{TP}(\delta f) = -\left[\mathbf{v}_d\cdot\nabla + \dot{\mathcal{K}}\frac{d}{d\mathcal{K}} - \mathcal{P}\right] f_M$$

Linearized collision term (for ion-ion collisions)

$C_{TP}(\delta f)$: expressed by random-walk of markers in the $(v_\|, v_\perp)$-space. (test-particle operator)

$\mathcal{P} f_M$: defined so that it satisfies the following properties. (field-particle operator)

$\int d^3v \mathcal{M}(C_{TP}(\delta f) + \mathcal{P} f_M) = 0$ for $\mathcal{M} = 1, v_\|, v^2.$ (conservation low)

$C_{TP}(\delta f) + \mathcal{P} f_M = 0$ for $\delta f = (c_0 + c_1 v_\| + c_2 v^2) f_M.$ (null-space)

The two-weight scheme

For each marker (distribution function: *g*), two types of weights are assigned that satisfy :

$$wg = \delta f \quad , \quad pg = f_M.$$

Each marker moves according to the LHS of the linearized DKE, $Dg/Dt = 0.$

Then, the change of these weights are given as follows :

$$\dot{w} = -\frac{p}{f_M}\left(\mathbf{v}_d\cdot\nabla + \dot{\mathcal{K}}\frac{\partial}{\partial\mathcal{K}} - \mathcal{P}\right) f_M - \eta(w - \bar{w}),$$

$$\dot{p} = \frac{p}{f_M}\left(\mathbf{v}_d\cdot\nabla + \dot{\mathcal{K}}\frac{\partial}{\partial\mathcal{K}}\right) f_M - \eta(p - \bar{p}).$$

Noise-reduction terms
($\bar{w}, \bar{p}$ $\rightarrow$ ensemble average)

☆ In the present study, the radial electric field profile is given as an input parameter.

☆ Only ion drift-kinetic equation is solved, and the ion-electron collision is neglected.

Evaluation of pressure tensor and NTV in the δf simulation

The guiding-center distribution function : $f = f_M(\psi, v) + \delta f(\psi, \theta, \zeta, v_{||}, v_\perp)$

$$\Rightarrow \quad \overleftrightarrow{\mathbf{P}} = p_0(\psi)\overleftrightarrow{\mathbf{I}} + \delta P_{||}\mathbf{bb} + \delta P_\perp(\overleftrightarrow{\mathbf{I}} - \mathbf{bb}).$$

Solution of δf from FORTEC-3D

Taking an flux surface average $\Rightarrow \left\langle \mathbf{e}_\zeta \cdot \nabla \cdot \overleftrightarrow{\mathbf{P}} \right\rangle = \frac{1}{2}\left\langle \frac{\partial}{\partial \zeta}\delta P \right\rangle$,

where $\delta P = \delta P_{||} + \delta P_\perp = m\int d^3v(v_{||}^2/2 + v_\perp^2)\delta f.$

Instead of evaluating $\partial \delta P/\partial \zeta$ directly, we make use of the fact that the magnetic field in FORTEC-3D is given in Fourier series in Boozer coordinates.

$$B(\psi, \theta, \zeta) = B_0\left[1 - \sum_{m\geq 1}\epsilon_m(\psi)\cos(m\theta) + \sum_{m\geq 0, n\neq 0}\delta_{m,n}(\psi)\cos(m\theta - n\zeta)\right]$$

$$\frac{1}{2}\left\langle \frac{\partial}{\partial \zeta}\delta P \right\rangle = \frac{G+\iota I}{2V'}\oint d\theta d\zeta \frac{1}{B^2}\frac{\partial \delta P}{\partial \zeta} = \frac{G+\iota I}{V'}B_0\oint d\theta d\zeta \frac{\delta P}{B^3}\sum_{m,n} n\delta_{m,n}\sin(m\theta - n\zeta).$$

Then, the toroidal viscosity is evaluated by decomposing into the following form:

$$\left\langle \mathbf{e}_\zeta \cdot \nabla \cdot \overleftrightarrow{\mathbf{P}} \right\rangle = \sum_{m,n}\left\langle \mathbf{e}_\zeta \cdot \nabla \cdot \overleftrightarrow{\mathbf{P}} \right\rangle_{m,n} = B_0\sum_{m,n} n\delta_{m,n}Q_{m,n} \ .$$

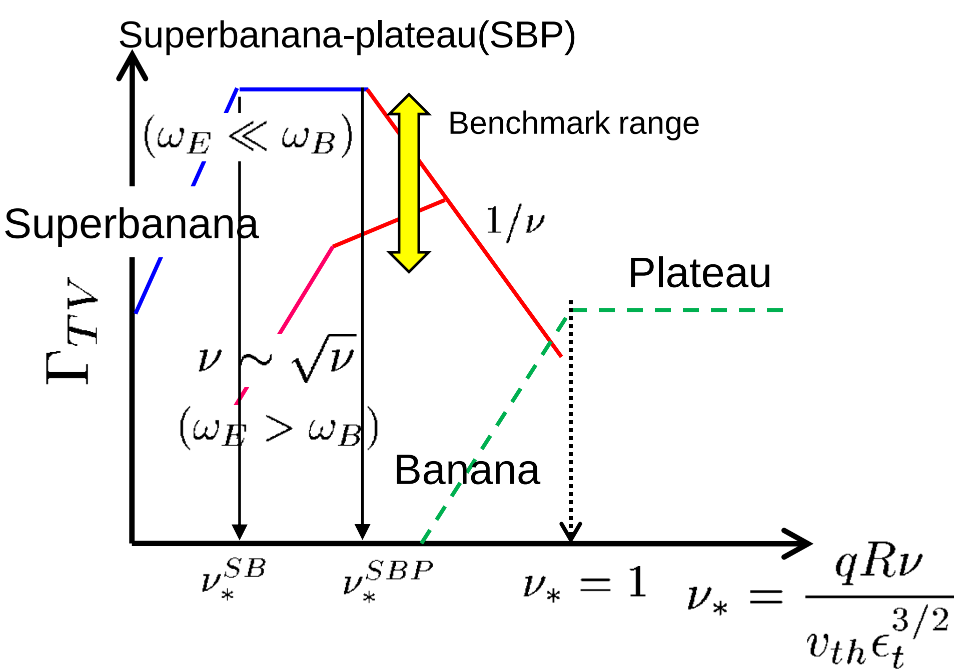
$$Q_{m,n} \equiv \left\langle \frac{\delta P}{B}\sin(m\theta - n\zeta) \right\rangle.$$

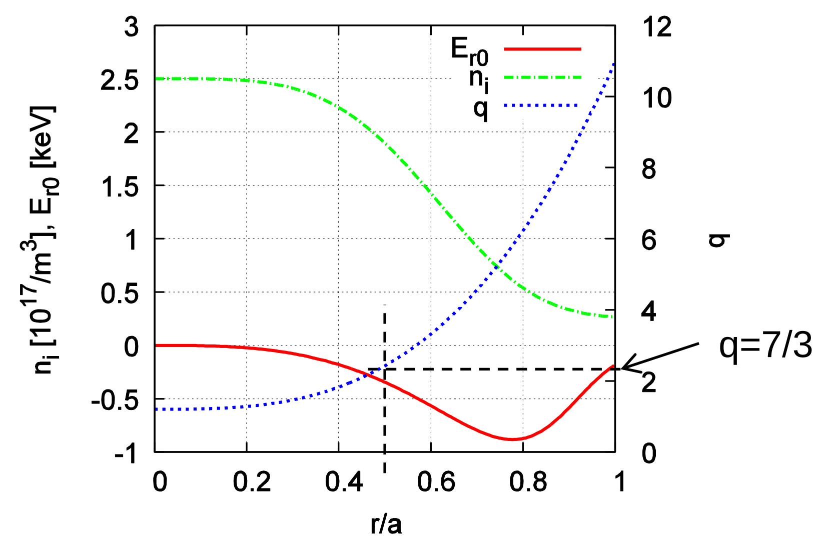
In this expression, one needs to evaluate **only the $Q_{m,n}$ components which has corresponding non-zero $\delta_{m,n}$ $(n \neq 0)$ perturbations applied.**

Settings of the numerical simulation model

- Large-aspect-ratio, circular cross-section tokamak $(R_0 = 10m, a = 2.5m, B_0 = 10T)$.
- For simplicity, $T_i = 0.4$keV$= const$. Banana width is small, $\Delta_b/a \sim 10^{-3}$.
  - → Finite-orbit-width effect in FORTEC-3D is negligible when comparing with local theory.
- Single-helicity perturbation, $\delta_{7,3} = 0.01x^2 \cos(7\theta - 3\zeta)$ where $x = r/a$, is superimposed on the equilibrium tokamak field.
- Plasma collisionality is in the $1/\nu$-regime, $\nu_* \approx 0.06$.
- q-profile is $q(x) = 1.2 + 9.8x^3$. <u>Resonant flux surface exists at r≒0.49a, where q=7/3.</u>

- Dependence of NTV on $E \times B$ rotation speed is investigated by varying the E_r profile.





Combined analytic NTV theory

- Including the missing components in conventional bounce-average theories:
  - Resonance between bounce motions and electric precession
  - Resonance between magnetic and electric precession (SBP and SB)
- Do not use assumptions that limits the range of collisionality (such as $1/\nu$, superbanana, etc.)
- Combined formula for NTV torque has been derived with effective Krook collision operator

$$\langle\hat{\phi}\cdot\vec{\nabla}\cdot\overleftrightarrow{\Pi}_a\rangle_\ell = \frac{\epsilon^{-1/2}p_a}{\sqrt{2}\pi^{3/2}R_0}\int_0^1 d\kappa^2\delta_{w,\ell}^2\int_0^\infty dx\mathcal{R}_{a1\ell}\left[u^\varphi + 2.0\sigma\left|\frac{1}{e}\frac{dT_a}{d\chi}\right|\right] \quad (11)$$

Torque (Transport) $\propto \sum_{m,m'}\delta_{m,n}\delta_{m',n}$ Resonance Rotation with offset

$$\mathcal{R}_{ay\ell} = \frac{1}{2}\frac{n^2(1+(\frac{\ell}{2})^2)\frac{\nu_a}{2\epsilon}x(x-\frac{5}{2})^y e^{-x}}{\left[\ell\frac{\pi\sqrt{\epsilon}}{4\sqrt{2}}\omega_{ta}\sqrt{x} - n\omega_E - n\sigma\frac{q^3}{4\epsilon}(\omega_{ta}^2/\omega_{ga})x\right]^2 + \left[(1+(\frac{\ell}{2})^2)\frac{\nu_a}{2\epsilon}\right]^2 x^{-3}}. \quad (12)$$

Bounce Frequency ω_b | Electric Precession ω_E | Magnetic Precession ω_B | Collisionality ν_K

[Park et al, PRL102, 065002 (2009)]

Benchmark of NTV : Finite-E_r case

How are simulations with $E \times B$ rotation compared with theory?

Park's formula ($\chi =$ poloidal flux)

$$\langle \mathbf{e}_\zeta \cdot \nabla \cdot \mathbf{\Pi} \rangle \simeq -\frac{\sqrt{\epsilon} p u_l^\zeta}{\sqrt{2} e \pi^{3/2} R_0} \sum_l \sum_{nmm'} \int_0^1 d\kappa^2 \delta_{l;mm'n}^2(\kappa) \int_0^\infty dx \mathcal{R}_{1l}(x),$$

$$\mathcal{R}_l(x) \sim \frac{\omega_b \nu_K}{[l\omega_b - n(\omega_E + \omega_B)]^2 + \nu_K^2}, \quad A_1 = \frac{d \ln p}{d\chi} + \frac{e}{T}\frac{d\Phi}{d\chi}, \quad A_2 = \frac{d \ln T}{d\chi}$$

$$u_l^\zeta \equiv u^\zeta - \frac{T}{e}\left[k + \frac{\int dx \mathcal{R}_{2l}(x)}{\int dx \mathcal{R}_{1l}(x)}\right] A_2, \quad R_{2l}(x) \equiv (x - 5/2) R_{1l}(x) \ ,$$

$$u^\zeta = -T(A_1 - kA_2)/e$$

← **From the radial force balance relation**

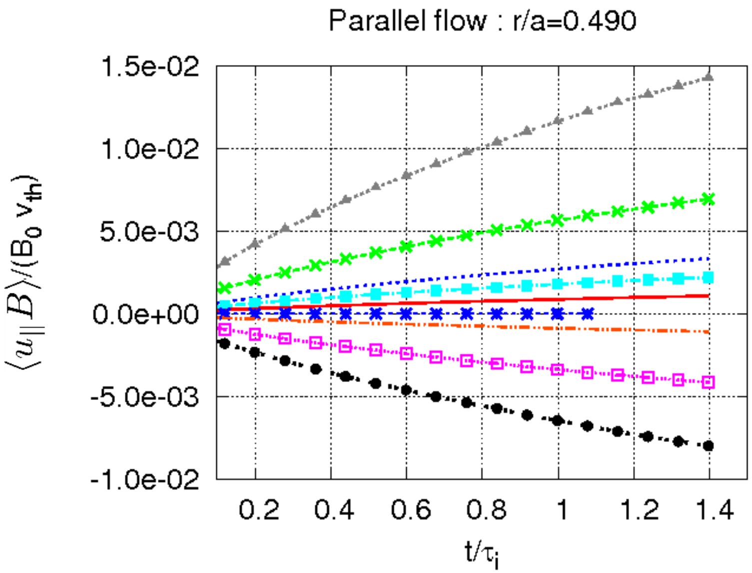
- In order to avoid ambiguity coming from approximating $k(\epsilon_t, \nu_*) \approx 1$ used in the definition of u_l^ζ ($k \simeq 1.17$ in the $\epsilon_t, \nu_* \to 0$ limit), benchmarks are carried out by setting $A_2 \propto dT/dr = 0$.

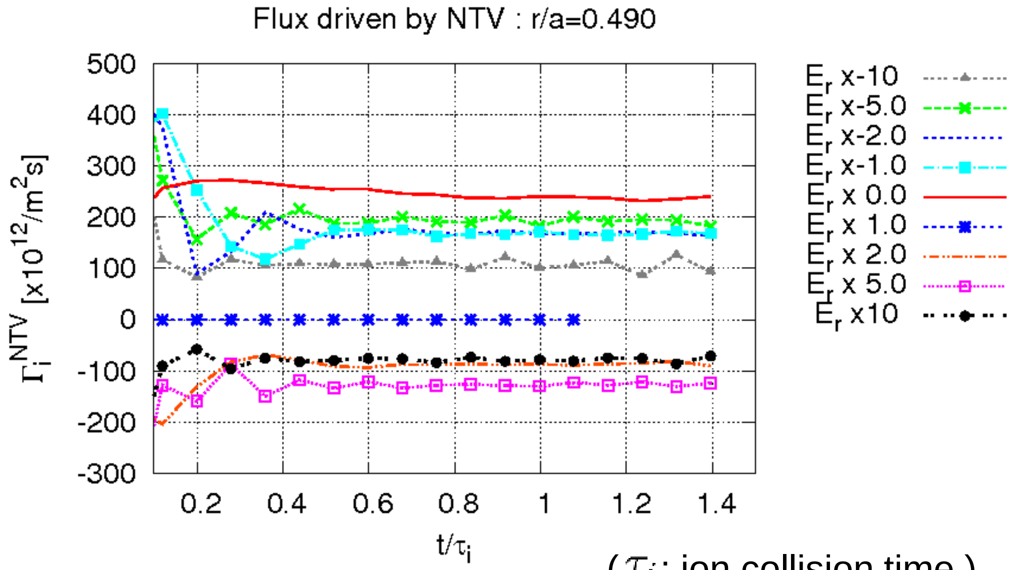
- In FORTEC-3D, an E_r profile is given from the force balance relation with assuming $u_\zeta \simeq 0$, e.g., $E_r = \frac{T_i}{e}\frac{d}{d\chi}\ln n_i$. **This E_r profile is called "the E_{r0} case".**

- **To see the NTV dependence on E_r, the E_{r0} case profile is multiplied by ± 1, ± 2, etc.**

Benchmark of NTV : Finite-E_r case

Convergence check of the Monte Carlo simulation



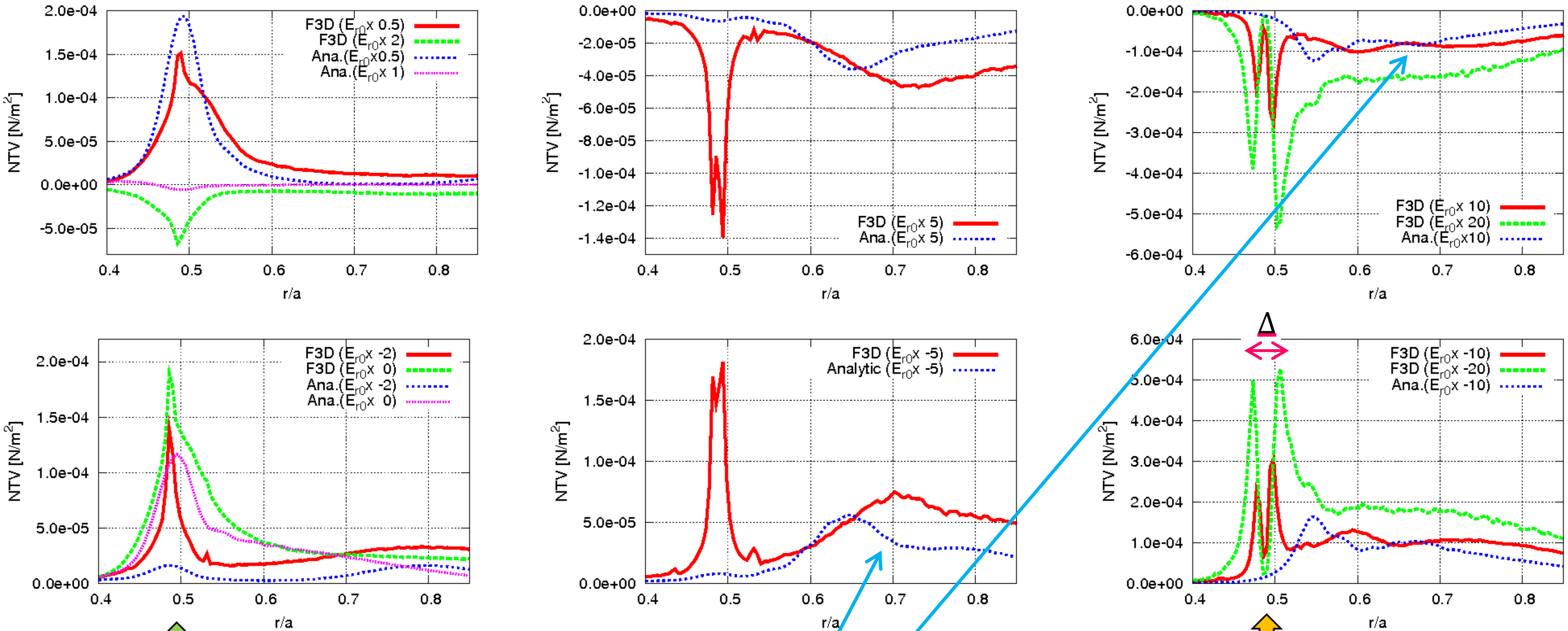


(τ_i: ion collision time)

- Though $u_{\|}$(and u_ζ) evolves slowly in a simulation run, it remains small , $u_{\|}/v_{th} \ll 1$, in the present benchmark calculations.
- The evolution of $u_{\|}$ does not affect the evaluation of NTV in the simulations even in the large-$|E_r|$ cases.
- NTV is evaluated by taking time average in the quasi-steady phase after $\tau_i > 1$.
- Total simulation marker number is 3.2×10^7 and the radial mesh points are 240.
- Each run takes about 20 hours on Helios supercomputer.

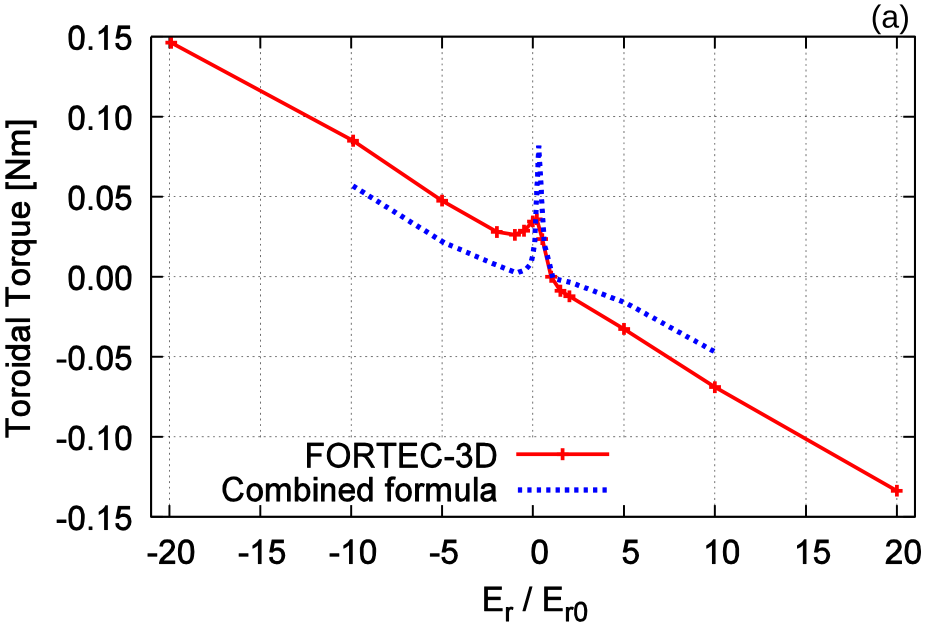
Comparison of the NTV radial profile in Finite-E_r cases

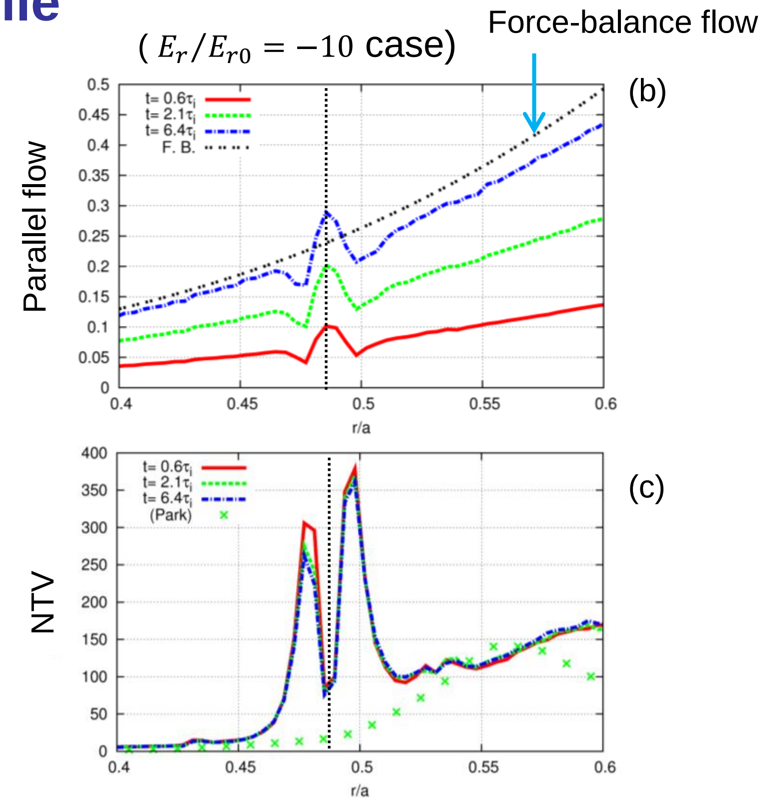
$|E_r|$ small ⟷ $|E_r|$ large



- Agreement b/w FORTEC-3D and the combined analytic formula at the resonant surface when $|E_r/E_{r0}| < 1$.
- Even $|E_r/E_{r0}| \gg 1$, in the off-resonant region (r/a>0.6) two calculations agree within factor $\mathcal{O}(1)$.
- Double peaks appear only in FORTEC-3D. Peak at the q=7/3 resonant disappears in the analytic formula when $|E_r/E_{r0}|$ is large.
- Distance of the peaks is much larger than the banana width, and $\Delta \propto |E_r|/B$.

NTV Dependence on the E_r amplitude : Total toroidal torque and parallel flow profile





- **The total toroidal torque (NTV integrated on the whole volume) is also compared.** (fig. (a))

- The off-resonant region, where FORTEC-3D agrees well with the analytic formula, contributes mainly to the total torque.
- Both calculation methods shows the same linear dependence of total toroidal viscosity on E_r/E_{r0} .

- The peak of NTV at $E_r \sim 0$ is because of the resonant of the trapped particles, $\omega_E + \omega_B = 0$.

- In FORTEC-3D simulations, parallel (toroidal) flow evolves so that it satisfies the force balance relation, $u^\zeta = -T(A_1 - kA_2)/e$.
- In large-$|E_r|$ cases, $V_\parallel$-shear is observed around the resonant flux surface (fig.(b)).
- However, the evolution of the shear flow does not affect the double-peak profile of NTV (fig. (c)).

The origin of the double-peak:

From the bounce-average analytic formula ($T_i = const.$ case):

$$\langle \mathbf{e}_\zeta \cdot \nabla \cdot \mathbf{P} \rangle \sim \sum_{l \geq 0} \sum_{mn} u^\zeta \delta_{mn}^2 \int d\kappa^2 F_{lmn}(\kappa) \int dx \mathcal{R}_l(x),$$

$$\mathcal{R}_l(x) \sim \frac{\omega_b \nu_K}{[l\omega_b - n(\omega_E + \omega_B)]^2 + \nu_K^2}, \qquad u^\zeta = -\frac{1}{e}\left(\frac{d\ln n_i}{d\chi} + e\frac{d\Phi}{d\chi}\right)$$

$\omega_E = d\Phi/d\chi$: $E \times B$ rotation freq.
ω_b : Bounce freq.
ω_B : Magnetic drift freq.
ν_K : Collision freq.

Strong peak of NTV will occur when $l\omega_b - n(\omega_E + \omega_B) \approx 0$ is satisfied. (resonance)

- $l = 0$ harmonics makes the single peak when $|E_r/E_{r0}| < 1$.
- $l \geq 1$ harmonics contributions are dominant when $|E_r|$ is large.
- $\omega_b \approx n\omega_E$ can be satisfied by $v \sim v_{th}/2$ particles at $r/a \sim 0.5$.

However, the analytic formula does not predict the double-peak profile of NTV.

Resonance between ω_E and the transit frequency $\omega_{tr} = v_\parallel/qR$ of **passing particles**, which is neglected in the bounce-average theory, is possible?

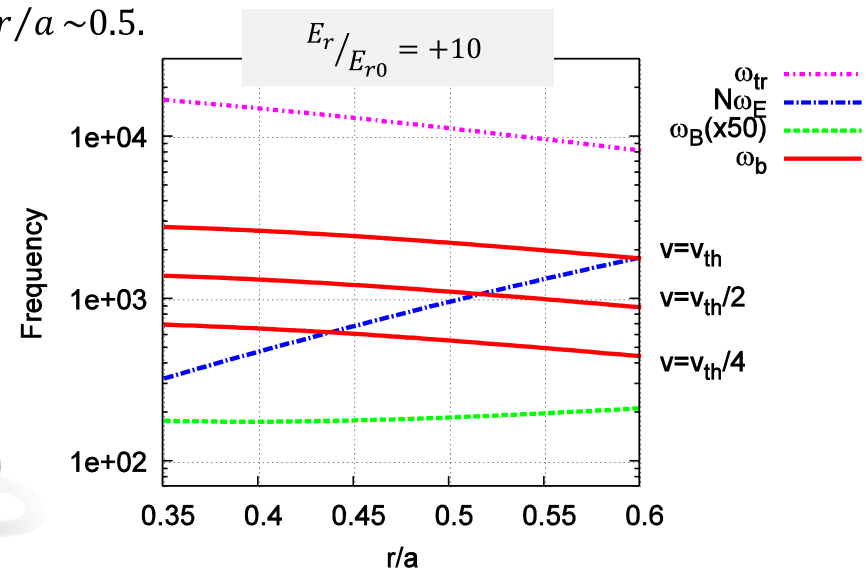


Fig: Comparison of the drift frequencies and the bounce frequency.

Note: ω_{tr} in the right fig. is evaluated by $v_\parallel = v_{th}$.

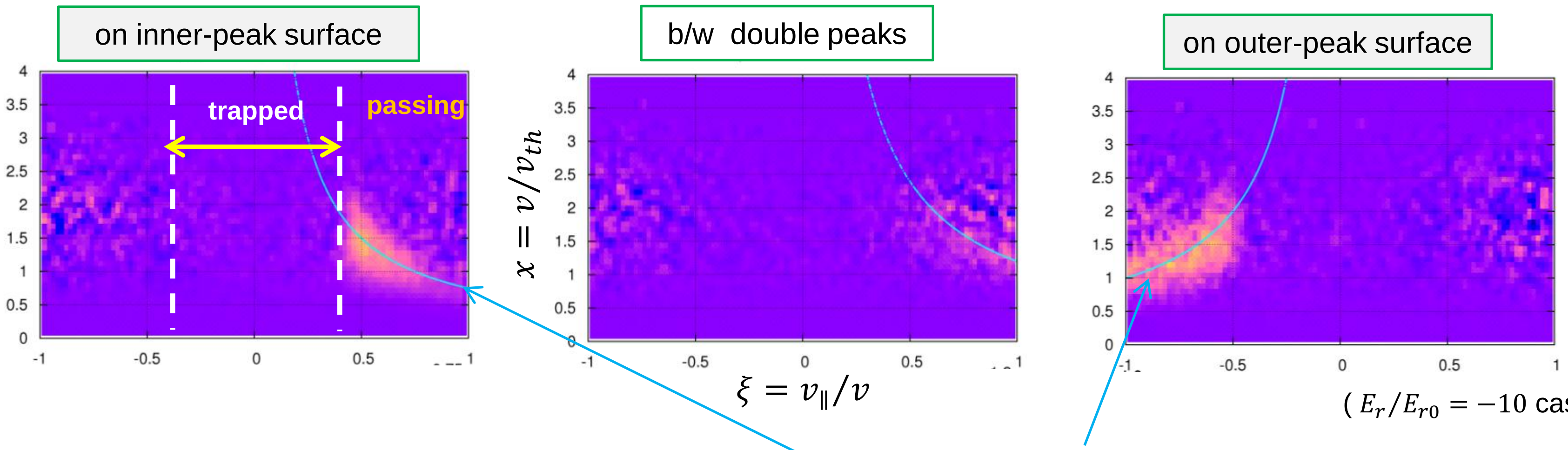
Passing particle resonance makes the double-peak of NTV (1)

In FORTEC-3D, NTV is evaluated as follows:

$$\langle \mathbf{e}_\zeta \cdot \nabla \cdot \mathbf{P} \rangle = \sum_{m,n\neq 0} \langle \mathbf{e}_\zeta \cdot \nabla \cdot \mathbf{P} \rangle_{m,n} \equiv B_0 \sum_{m,n\neq 0} n\delta_{m,n} Q_{m,n},$$

$$Q_{m,n} \equiv \left\langle \frac{\delta P}{B} \sin(m\theta - n\zeta) \right\rangle = \frac{1}{V'} \oint d\theta \underbrace{\oint d\zeta \frac{J_B}{B} \sin(m\theta - n\zeta) M_i \int d^3 v \delta f \left(v_\parallel^2 + \frac{v_\perp^2}{2} \right)}_{dQ_{m,n}}.$$

To see what kind of particles makes the double-peak NTV, the integrand $dQ_{m,n}$ in the velocity space $(x, \xi) = (v/v_{th},\ v_\parallel/v)$ at $\theta \cong 0$ is observed.

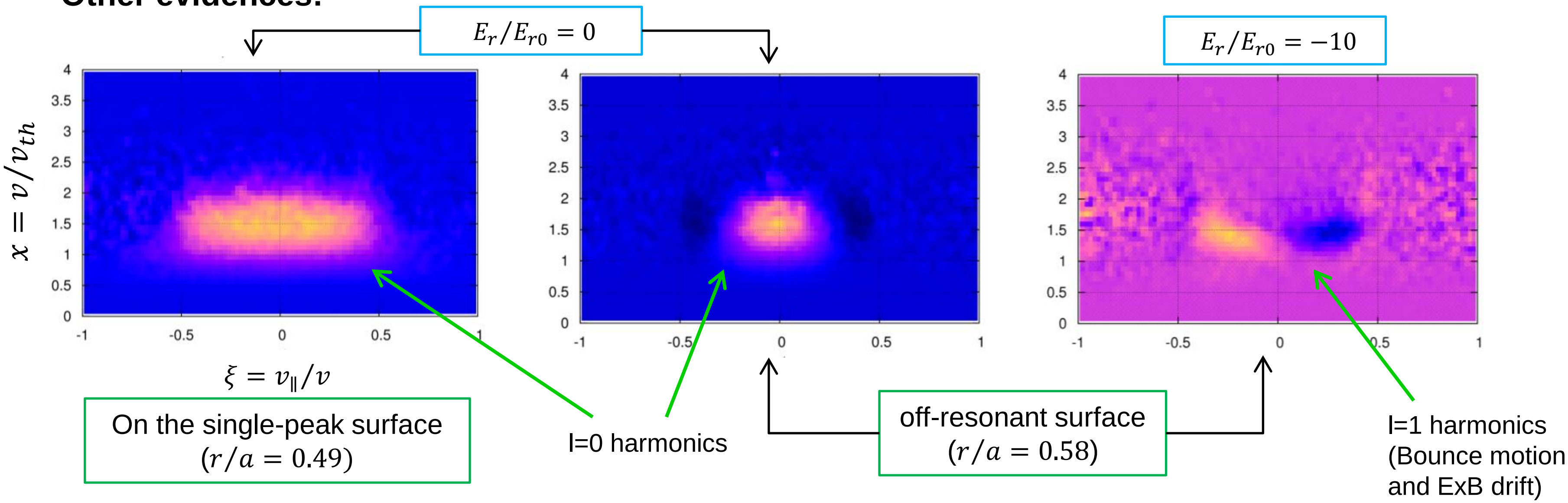


($E_r/E_{r0} = -10$ case)

- In passing region, strong resonant appears on $v_\parallel = \mathrm{const.}$ line.
- The sign of resonant $v_\parallel$ is opposite b/w the inner and outer peaks.
- The sign also changes according to the sign of E_r.
- When the resonance of $dQ_{m,n}$ appears at $x \approx 1$, strong peak of NTV is created on that flux surface.

Passing particle resonance makes the double-peak of NTV (2)

Other evidences:



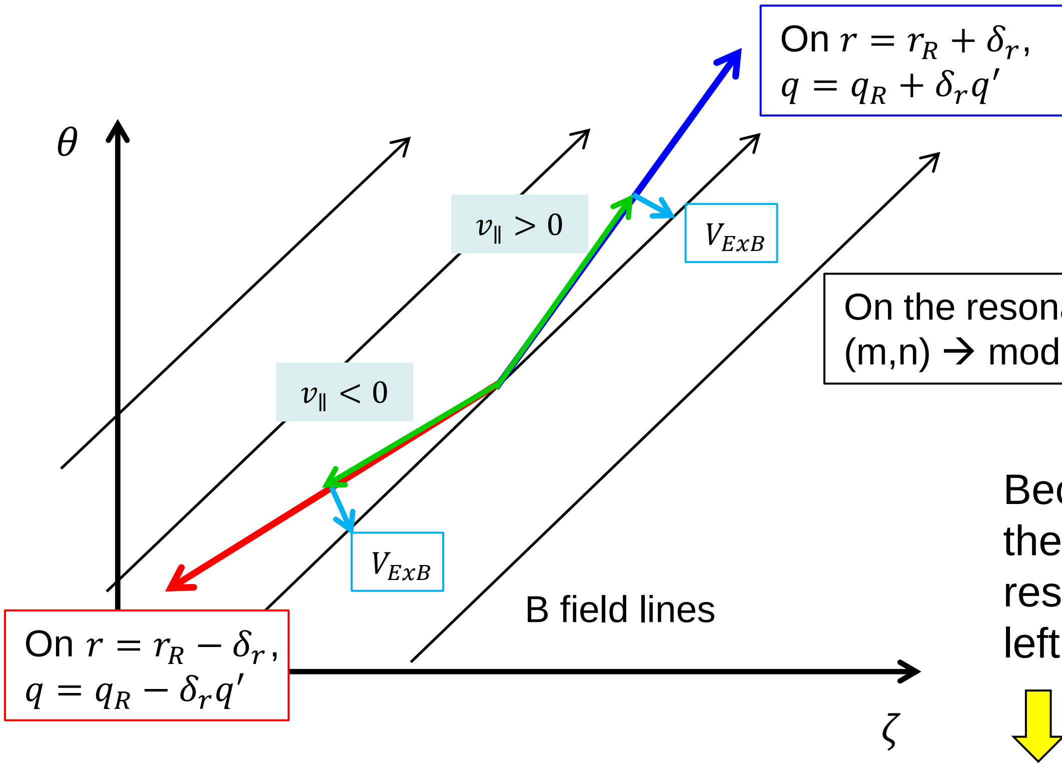
- When $E_r \approx 0$, the single peak of NTV on the resonant surface ($q = m/n$) is created mainly from the trapped particles. Passing contribution is non-zero, but small.

- In the off-resonant region (away from the resonant surface), NTV is almost determined by the trapped particles.

- The $l = 0$ ($E_r \approx 0$) and $l = 1$ ($|E_r/E_{r0}| \gg 1$) type harmonics in the velocity space, which is predicted in the analytic theory, can clearly be distinguished in FORTEC-3D simulation, as shown in the right two figures.

The difference between FORTEC-3D and the combined analytic formula comes from the passing particles' resonance when $\omega_{tr} \sim \omega_E$.

The passing particle resonance condition



On the resonant flux surface $(r = r_R)$: $q = q_R = m/n$.
(m,n) → modes of perturbation field

Because of the magnetic shear q', the field lines on the both sides of the resonant surface $r = r_R$ tilt as in the left figure.

- If the $E \times B$ drift is large enough and the drift velocity $v_\parallel \mathbf{b} + \mathbf{v}_{E\times B}$ for $v \approx v_{th}$ passing particles on $r = r_R \pm \delta_r$ surfaces is aligned with the $q = m/n$ field lines on the resonant surface, such passing particles continuously feel the constant phase of perturbed magnetic field, $\delta_{m,n} \propto \cos(m\theta - n\zeta)$.
- If such condition is satisfied, the resonant passing particles' orbits are affected by the perturbed magnetic field, resulting in non-ambipolar radial flux and therefore NTV.

- Large $V_{E\times B}$ → Large δ_r is required to satisfy the condition for $v \approx v_{th}$ particles. This explains why the distance b/w the double peaks increases with $|E_r|$.

Summary

- **FORTEC-3D code was applied to calculate neoclassical toroidal viscosity in tokamak plasmas with asymmetric magnetic field.**
- **Benchmark tests were carried out with the combined analytic formula for finite-E_r cases .**

✓ NTV profile shows significant difference b/w two calculations when $|E_r/E_{r0}| \gg 1$. In FORTEC-3D simulations, a **double-peak profile of NTV** appears around the resonant rational flux surface as $|E_r/E_{r0}|$ increases.

✓ **Passing particle resonance**, which occurs if magnetic shear dq/dr and large E_r co-exist near the resonant flux surface, makes a significant contribution to NTV to make the double-peak profile.

✓ The localized double-peak of NTV will also affect the toroidal rotation damping there.

Future Tasks

- Investigate the collisionality dependence of NTV when $E_r \neq 0$.
  - Is NTV really scales as $\nu \sim \sqrt{\nu}$ as asymptotic formula predicts?
- Include the passing-particle resonance effect in the combined analytic formula.
- What if the perturbed field makes explicit island structure (the present model is vacuum approximation).
  - Use $\delta \mathbf{B} = \sum_{m,n} \nabla \times (\alpha_{m,n} \mathbf{B})$ -type expression, for example.
- **Application to helical devices** : EX/P3-27 "*Transition to Improved Confinement Mode by Electrode Biasing in the Large Helical Device*" by Kitajima et al.