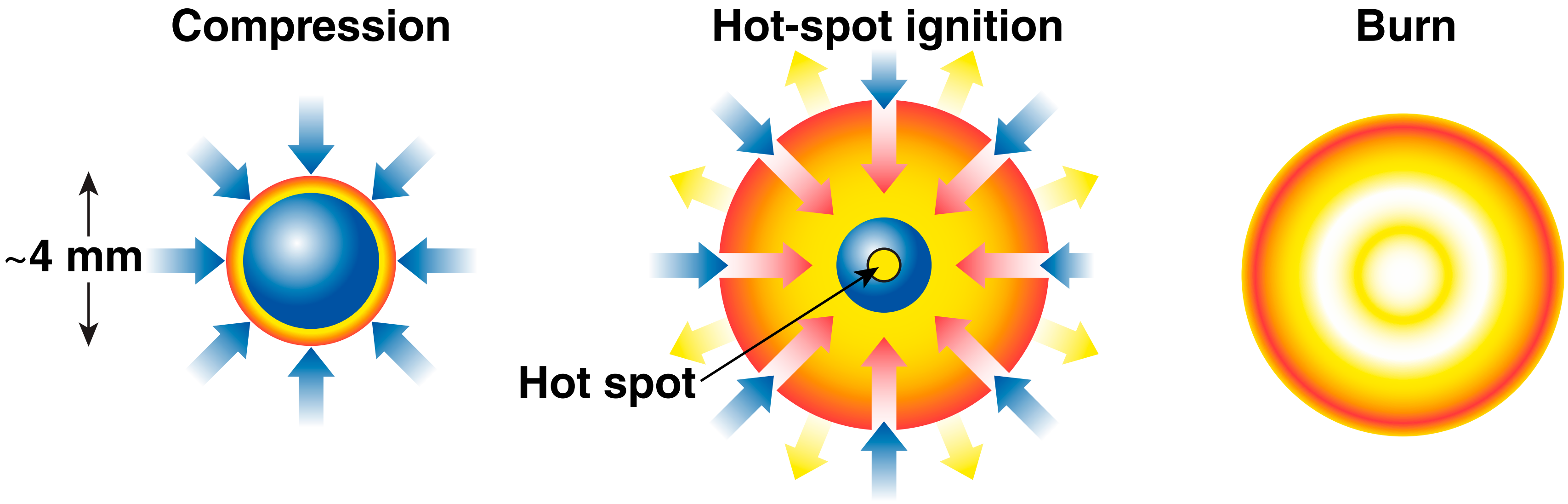
Theory of Ignition and Hydro-Equivalence for Inertial Confinement Fusion



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24th IAEA Fusion Energy
Conference
San Diego, CA
8–13 October 2012

Summary

Inertial confinement fusion (ICF) ignition theory is used to assess National Iginition Facility (NIF) experiments and design ignition-scalable implosions on OMEGA

- ICF ignition theory is used to derive performance parameters that can be measured in experiments
- The theoretical results can be easily related to the Lawson criterion
- Applications to NIF indirect drive implosions show $P\tau$ up to 18 atm s, and pressures up to ~125 Gbar (~350 Gbar is required for ignition)
- Hydrodynamic scaling is used to design direct-drive similarity experiments between OMEGA and the NIF
- OMEGA implosions that scale to ignition at NIF energies require an areal density of ~0.3 g/cm^2 and a neutron yield of ~4 × 10^{13}

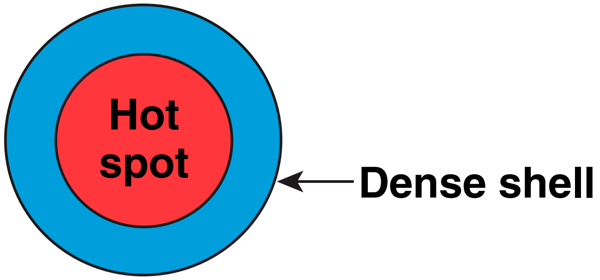
ICF IGNITION THEORY FROM THE LAWSON CRITERION

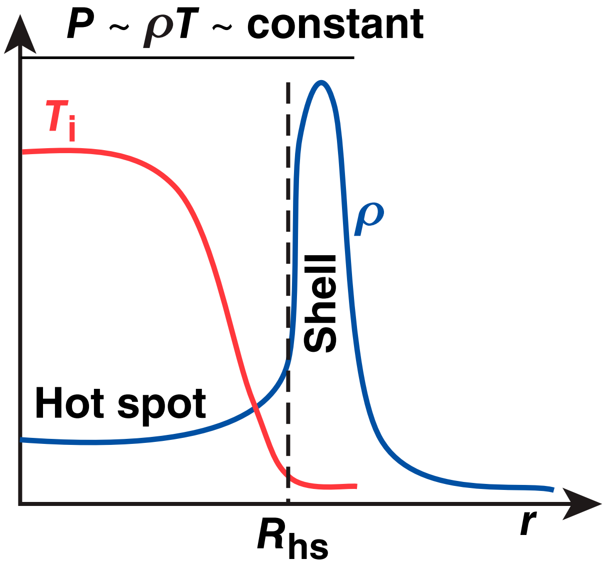
Like in magnetic confinement fusion (MCF), the Lawson criterion determines the ICF ignition condition where ignition occurs in the hot spot

Alpha heating exceeds energy losses → $\frac{n^2}{4}\langle\sigma v\rangle\varepsilon_\alpha > \frac{3}{2}\frac{P}{\tau}$

Ignition parameter → $\chi \equiv \frac{P\tau}{24\langle\sigma v\rangle\varepsilon_\alpha/T^2} = \frac{P\tau}{P\tau(T)_{ig}}$

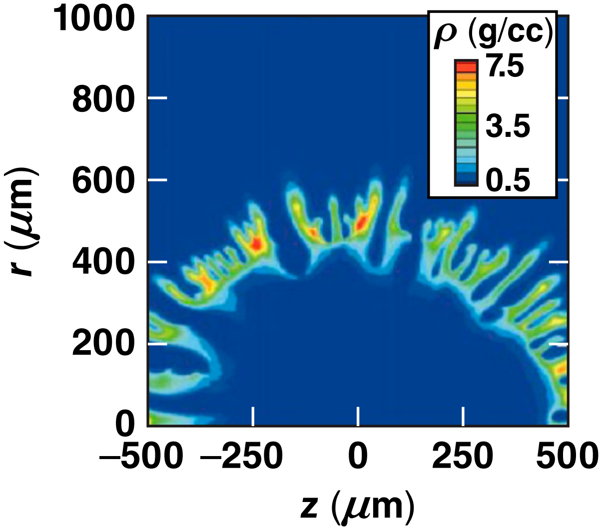
Ignition condition → $\chi > 1$



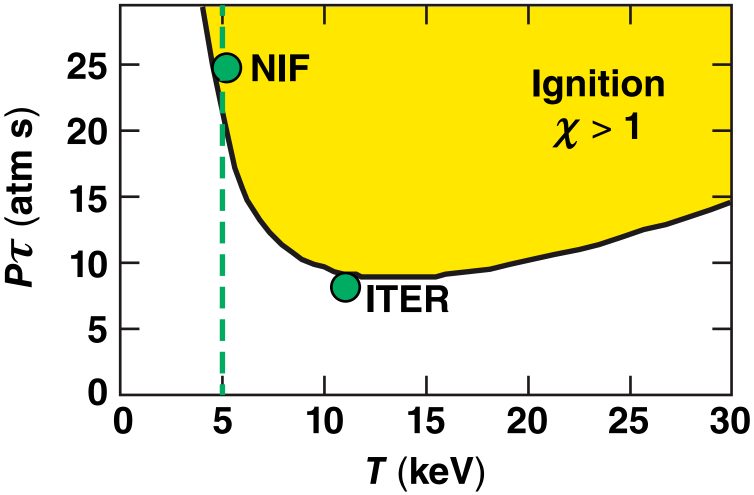


ICF implosions cannot achieve ~10-keV temperatures through compression alone

- High T requires high implosion velocity V_i
- High V_i requires thin shells
- Thin shells break up in flight because of hydrodynamic instabilities →

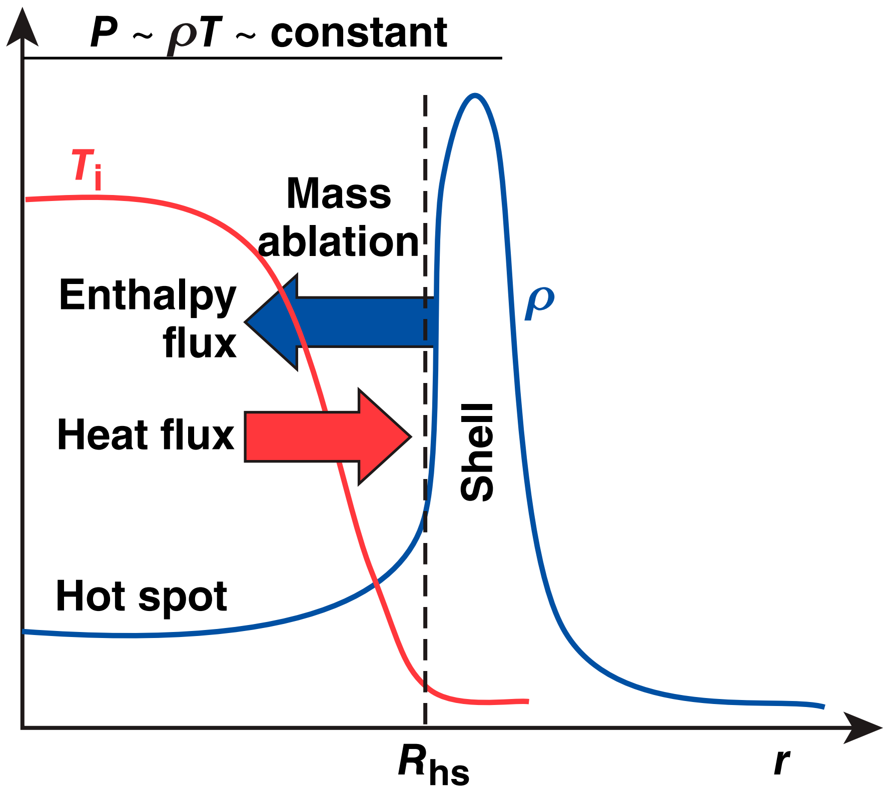


$T \sim V_i$



ICF must ignite at ~5 keV, requiring $V \sim 400$ km/s and $P\tau > 25$ atm s.

Unlike in MCF, heat conduction losses do not reduce the thermal energy (pressure) in the hot spot of ICF capsules



Mass ablation recycles the heat losses back into the hot spot.

$$q_{heat} = -\kappa(T)\nabla T$$

$$\kappa(T) \approx \kappa_0 T^{5/2}$$

Balance heat flux with ablation enthalpy* flux

$$q_{heat} = \frac{5}{2} P V_a$$

Mass ablation rate from shell into the hot spot

$$\boxed{\dot{m}_a = 0.2 \frac{m_i \kappa_0 T_0^{5/2}}{R_{hs}}}$$

TC8536a

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*R. Betti *et al*. Phys. Plasmas 8, 5257 (2001).

The hot spot is confined by a dense shell with the confinement time depending on the shell inertia

- The temperature depends mainly on implosion velocity

$$T \sim \frac{V_i^{1.2}}{\kappa_0^{2/7}}(\rho R)^{0.2}$$

- The confinement time comes from applying Newton's law to the shell

$$M_{\text{sh}}\ddot{R} \sim M_{\text{sh}}\frac{R}{\tau^2} = 4\pi P_{\text{hs}} R^2 \Rightarrow \tau \sim \sqrt{\frac{M_{\text{sh}}}{4\pi P_{\text{hs}} R}}$$

- The 1-D ignition parameter* $\chi \equiv \frac{P\tau}{P\tau(T)_{\text{ig}}}$ is rewritten in terms of measurable quantities

$$\chi \equiv \frac{P\tau}{P\tau(T)_{\text{ig}}} \approx \rho R_{\text{g/cm}^2}^{0.8}\left(\frac{T_{\text{keV}}}{4.7}\right)^{1.6}$$

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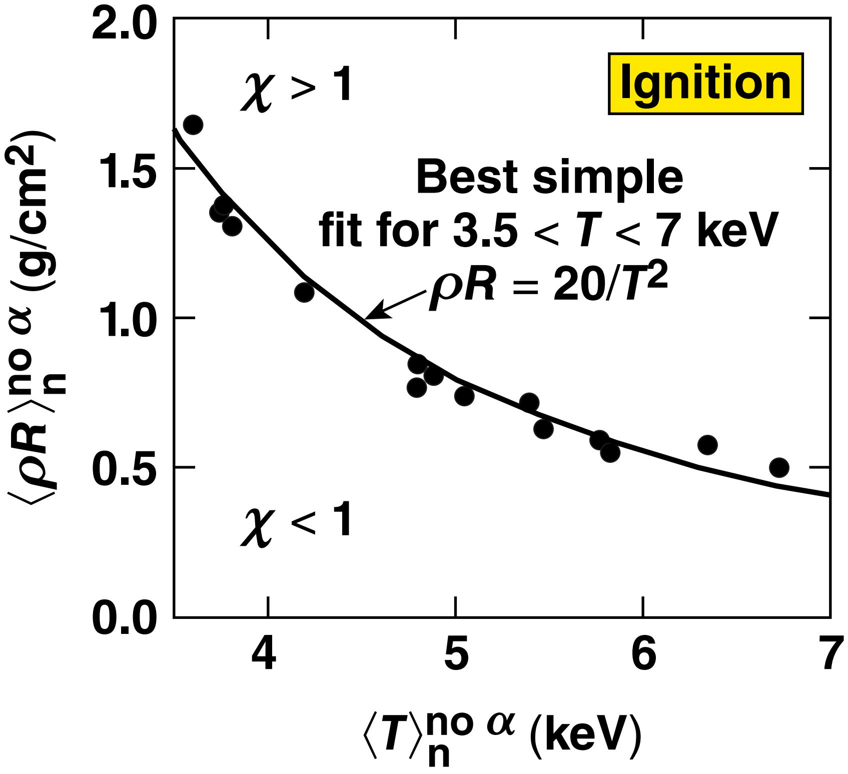
*C. D. Zhou and R. Betti, Phys. Plasmas 15, 102707 (2008).

One-dimensional simulations of gain = 1 targets confirm that the 1-D ignition condition depends on ion temperature and total areal density

- Comparison* of ignition condition with simulation of gain = 1 targets



- Ignition-relevant parameters inferred from measurable quantities

Ignition parameter

$$\chi_{1\text{-D}} \equiv \frac{P\tau}{P\tau(T)_{\text{ig}}} \approx \rho R_{\text{g/cm}^2}^{0.8} \left(\frac{T_{\text{keV}}}{4.7}\right)^{1.6}$$

Lawson's $P\tau$

$$P\tau \approx 8\left(\rho R_{\text{g/cm}^2} T_{\text{keV}}\right)^{0.8} \text{ atm} \cdot s$$

All hydrodynamic quantities are calculated without burn (no alphas).

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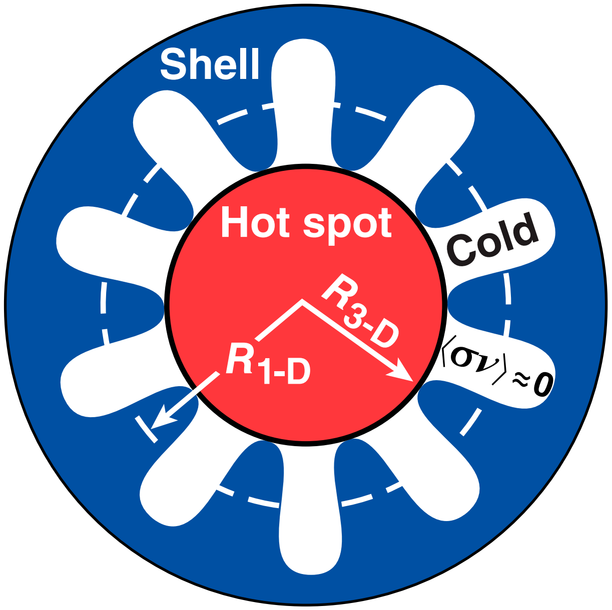
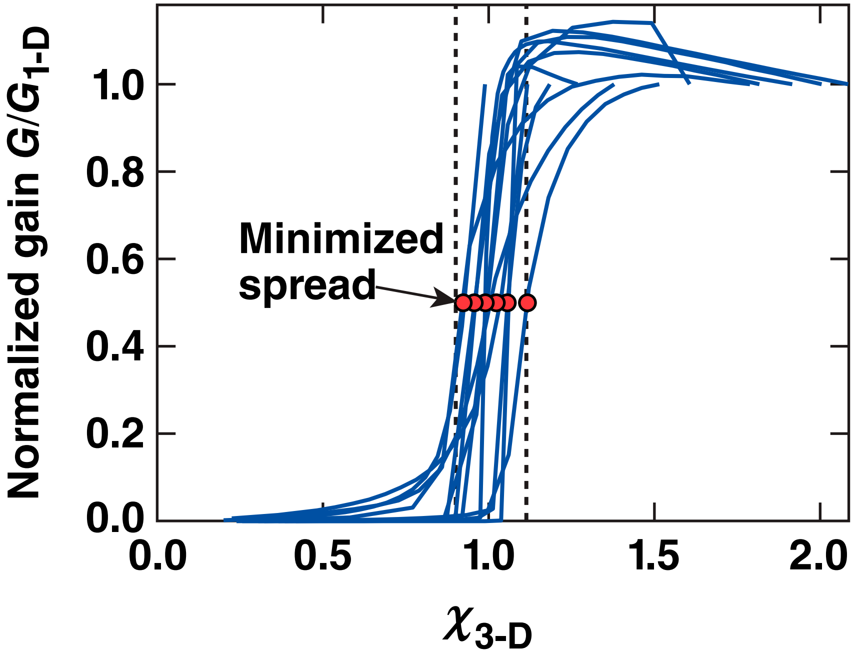
*C. D. Zhou and R. Betti, Phys. Plasmas 15, 102707 (2008).

Three-dimensional effects[1,2] are included through the yield over clean (YOC) and its relation to the hot-spot clean volume

- Fusion reactions occur in the clean volume (red)



$$\text{YOC} \equiv \frac{Y_n^{3\text{-D}}}{Y_n^{1\text{-D}}} \approx \frac{V_{3\text{-D}}}{V_{1\text{-D}}}$$



3-D ignition condition

$$\chi_{3\text{-D}}^{\text{fit}} = \left(\rho R_{\text{g/cm}^2}^{\text{no}\,\alpha}\right)^{0.8} \left(\frac{T_{\text{keV}}^{\text{no}\,\alpha}}{4.7}\right)^{1.6} \text{YOC}_{\text{no}\,\alpha}^{0.4}$$

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[1] P. Y. Chang *et al.*, Phys. Rev. Lett. 104, 135002 (2010).

[2] R. Betti *et al.*, Phys. Plasmas 17, 058102 (2010).

The LLNL ignition threshold factor (ITFx) from fitting of *LASNEX* results is the cubic power of the Lawson parameter

- Rewrite the Lawson criterion by using the 1-D yield Y_N in the YOC[1]

$$\chi_{3\text{-D}} \approx \left(\rho R^{\text{no}\,\alpha}_{\text{g/cm}^2}\right)^{0.6} \left(\frac{0.24\, Y_N^{16}}{M^{\text{mg}}_{\text{fuel}}}\right)^{0.34}$$

- The ignition criterion can be written in terms of energy

$$P\tau \sim P\sqrt{\frac{M_{\text{sh}}}{PR}} \sim P\sqrt{\frac{R^3}{PR}} \sim R \sim E^{1/3} \quad \Longrightarrow \quad \text{ITF} \sim \frac{E}{E^{\text{min}}_{\text{ig}}} \approx \left[\frac{P\tau}{(P\tau)_{\text{ig}}}\right]^3 = \chi^3$$

- Compare LLNL performance parameter[2] ITFx with cubic power of χ for $M_{\text{DT}} = 0.17$ mg

$$\text{ITFx}^{\text{1-D}}_{\text{NIF}} \equiv \frac{Y_N^{16}}{0.32}\left(\frac{\rho R}{1.5}\right)^{2.3} \quad \Longleftrightarrow \quad \chi^3_{3\text{-D}} \equiv \frac{Y_N^{16}}{0.35}\left(\frac{\rho R}{1.5}\right)^{1.8}$$

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[1] R. Betti *et al.*, Phys. Plasmas 17, 058102 (2010).

[2] B. K. Spears *et al.*, Phys. Plasmas 19, 056316 (2012).

The Lawson parameter is used to estimate P and $P\tau$ for NIF indirect drive and OMEGA direct drive

- The hot-spot pressure is inferred from $P\tau$ and the burn duration τ_{burn} ⟶

$$P(\text{Gbar}) \approx 27\,\frac{\chi_{\text{3-D}}}{\tau_{\text{burn}}^{\text{ns}}}\left(\frac{4.7}{T_{\text{i}}^{\text{keV}}}\right)^{0.8}$$

NIF shot	N120321	N120131	N110914	N111215
$P\tau$ atm/s	18	12	15	15
$\chi \equiv P\tau/\lvert P\tau\rvert_{\text{ig}}$	0.48	0.37	0.45	0.44
P Gbar*	124	76	111	94
χ^3/ITFx	0.11/0.12	0.05/0.046	0.09/0.09	0.09/0.09

OMEGA shot	67290	67289	67288	67291
$P\tau$ atm/s	3.6	3.5	3.1	3.3
$\chi \equiv P\tau/\lvert P\tau\rvert_{\text{ig}}$	0.086	0.082	0.080	0.083
P Gbar	34	33	29	31

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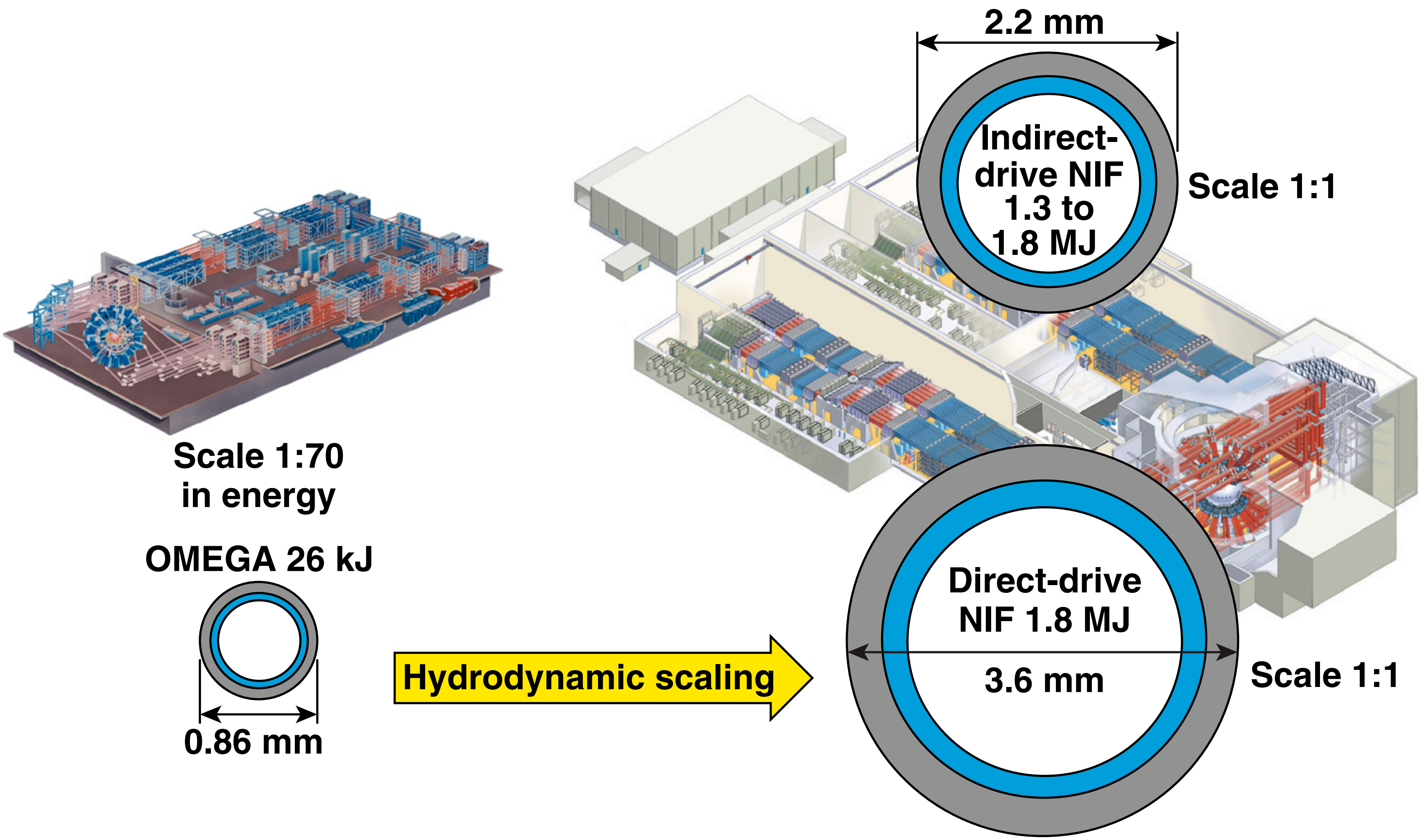
*Similar values are inferred by C. Cerjan (LLNL), private communication.

SCALING FROM OMEGA TO DIRECT-DRIVE NIF
(assume symmetric drive and similar laser smoothing)

OMEGA results can be scaled to NIF energies



Scale 1:70
in energy

OMEGA 26 kJ

0.86 mm

Hydrodynamic scaling

2.2 mm

Indirect-
drive NIF
1.3 to
1.8 MJ

Scale 1:1

Direct-drive
NIF 1.8 MJ

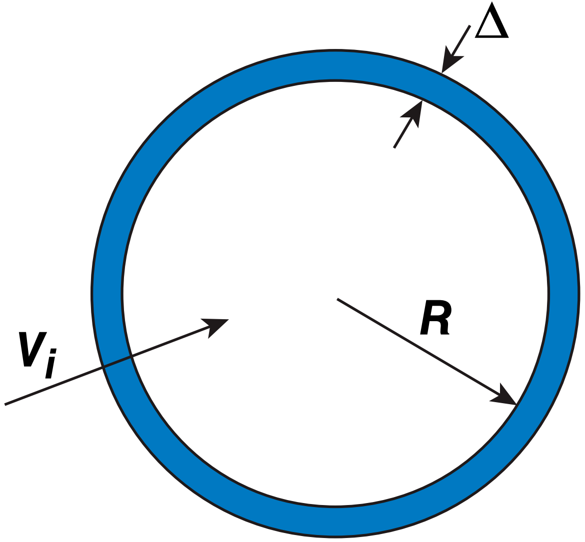
3.6 mm

Scale 1:1

One-dimensional implosion similarity requires equal Mach numbers

- The shell implodes with V_i and expands with C_s
- The Mach number $\frac{V_i}{C_s}$ is the only dimensionless parameter
- Use isentropic implosion condition $P_a \sim \alpha\rho^{5/3}$
- 1-D similarity requires equal Mach numbers

$$\mathrm{Mach}^2 \sim \frac{V_i^2}{\alpha^{3/5} P_a \, (I_L)^{2/5}}$$



Multidimensional implosion similarity imposes additional requirements on entropy and velocity

- The multidimensional behavior is determined primarily by the Rayleigh–Taylor (RT) instability

- Number of e-foldings of RT growth* for wave numbers $k \approx \frac{\ell}{R}$

$$N_e^{RT} = \int_0^{t_i} \gamma_{RT}\,dt = \int_0^{t_i} \left(\sqrt{kg} - 3kV_a\right)dt = \int_0^1 \left(\sqrt{\ell \frac{\ddot{\hat{R}}}{\hat{R}}} - 3\frac{\ell}{\hat{R}}\frac{V_a}{V_i}\right)d\tau$$

- Similar implosions have the same dimensionless R: $\hat{R} = \frac{R}{R_0}$

- 3-D similarity requires same → $\frac{V_a}{V_i} = \frac{\dot{m}_a}{\rho V_i} \sim \frac{\dot{m}_a\left(I_L\right)}{P_a\left(I_L\right)^{3/5}}\,\frac{\alpha^{3/5}}{V_i}$

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*H. Takabe *et al.*, Phys. Plasmas 28, 3676 (1985).

Hydrodynamic similarity leads to geometric and energy scaling of implosion performance

- 1-D hydrodynamic similarity → $\text{Mach}^2 \sim \dfrac{V_i^2}{\alpha^{3/5} P_a (I_L)^{2/5}}$ ①

- 3-D hydrodynamic similarity → $\dfrac{V_a}{V_i} \sim \dfrac{\dot{m}_a (I_L)}{P_a (I_L)^{3/5}} \dfrac{\alpha^{3/5}}{V_i}$ ②

- Laser-energy requirement

$$\frac{4\pi}{3} P_a R_0^3 = \eta E_L \approx \eta 4\pi R_0^2 I_L \frac{R_0}{V_i} \quad \longrightarrow \quad P_a (I_L) V_i \approx 3\eta I_L$$ ③

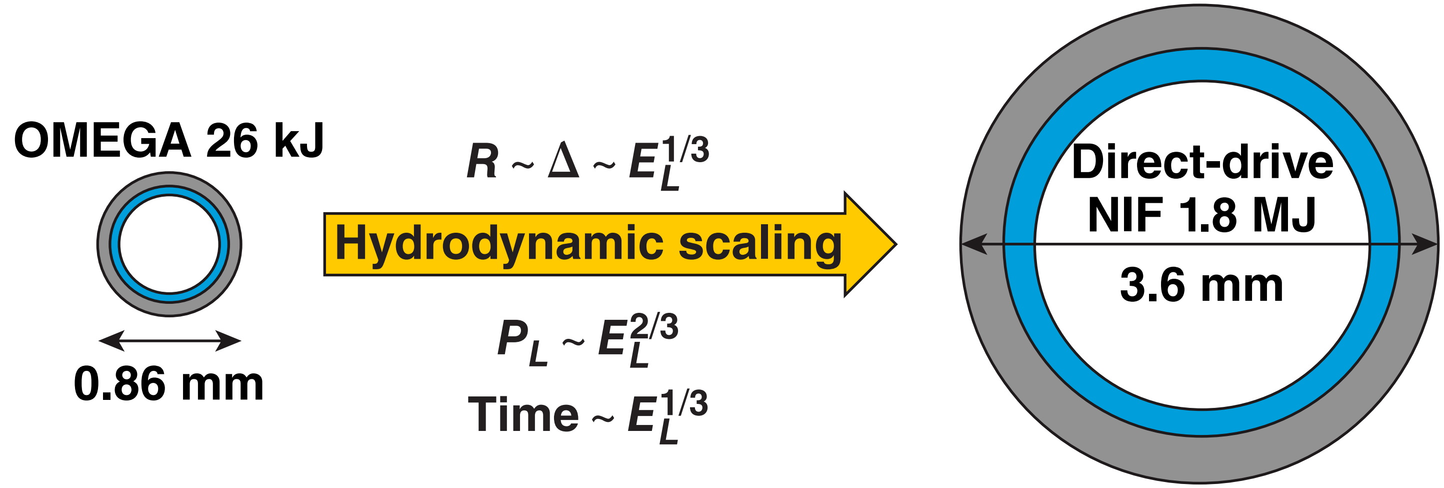
- Three constraints for V_i, α, I_L → hydrodynamic equivalence requires same V_i, α, I_L

Hydrodynamic equivalence:

Fixed V_i, α, I_L, → $E_L \sim R^3$, $P_L \sim R^2$, $M \sim R^3$, $\Delta \sim R$, $t \sim R$

Targets and laser pulses are designed for OMEGA to reproduce direct-drive NIF hydrodynamics

Same V_i, α, I_L and $E_L \sim R^3$, $P_L \sim R^2$, $M \sim R^3$, $\Delta \sim R$, $t \sim R$



OMEGA 26 kJ

0.86 mm

$R \sim \Delta \sim E_L^{1/3}$

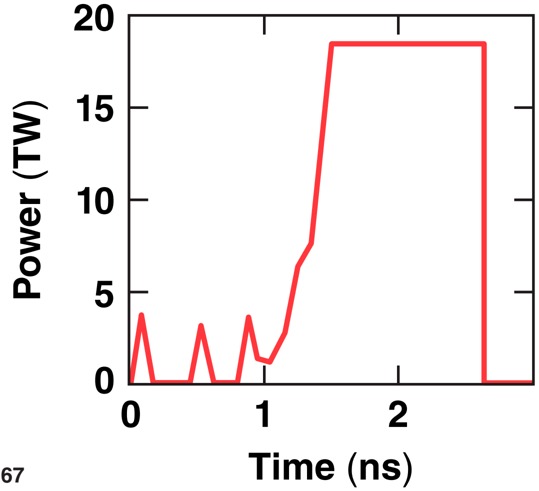
Hydrodynamic scaling

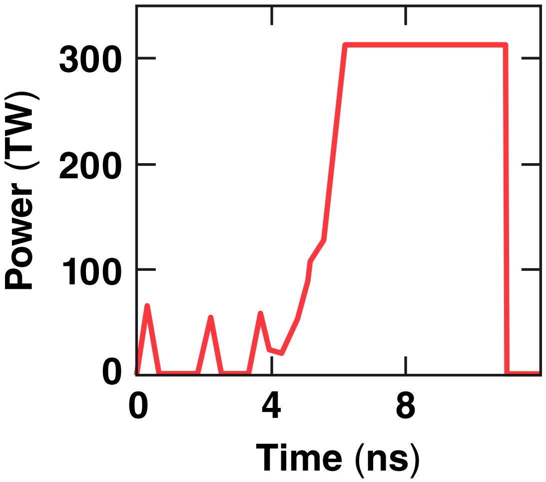
$P_L \sim E_L^{2/3}$

Time $\sim E_L^{1/3}$

Direct-drive NIF 1.8 MJ

3.6 mm





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Ignition theory and hydro-similarity provide the energy scaling of critical parameters

- Energy scaling* of areal density, ion temperature, and fuel mass for hydro-equivalent implosions

$$\rho R \sim E^{0.33} \qquad T \sim E^{0.07} \qquad M_{\text{fuel}} \sim E$$

- Energy scaling for neutron yield without burn (no alphas)

$$Y_n^{\text{no}\,\alpha} \sim E^{1.5}$$

- Energy and YOC scaling for ignition parameter and ITFx

$$\boxed{\chi_{\text{3-D}} \sim E^{0.37}\,\text{YOC}^{0.4}} \qquad \boxed{\text{ITFx} \sim E^{1.3}\,\text{YOC}}$$

$$\left(\frac{E_{\text{NIF}}^{1.8\ \text{MJ}}}{E_{\Omega}^{26\ \text{kJ}}}\right)^{0.37} \approx 5 \qquad \left(\frac{E_{\text{NIF}}^{1.8\ \text{MJ}}}{E_{\Omega}^{26\ \text{kJ}}}\right)^{1.3} \approx 220$$

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*C. D. Zhou and R. Betti, Phys. Plasmas 14, 072703 (2007).

Hydro-equivalent ignition on OMEGA requires $\chi \equiv P\tau/(P\tau)_{\text{ig}} \approx 0.16$

- The energy and YOC scaling is $\chi_{\text{3-D}} \sim E^{0.37}\,\text{YOC}^{0.4}$
- Expect YOC improvement of 2× on NIF versus OMEGA because of larger hot-spot size, more beams, and equal ice roughness (see back-up slides for details)

$$\text{YOC}_{\text{NIF}} \sim 2 \times \text{YOC}_{\Omega}$$

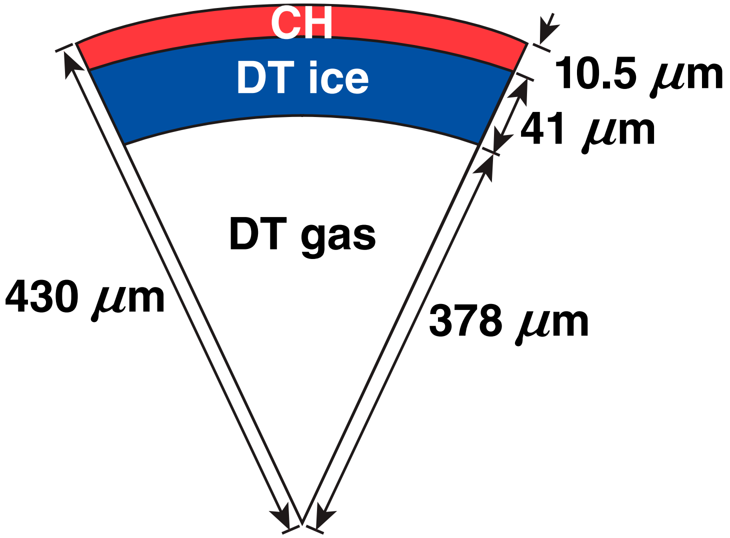
- Apply OMEGA-to-NIF scaling

$$\chi_{\text{DD}}^{1.8\ \text{MJ}} \approx \chi_{\Omega}^{26\ \text{kJ}} \left(\frac{1800\ \text{kJ}}{26\ \text{kJ}}\right)^{0.37} 2^{0.4} \approx 1$$

$$\boxed{\chi_{\Omega}^{\text{eq-ignition}} \approx 0.16}$$

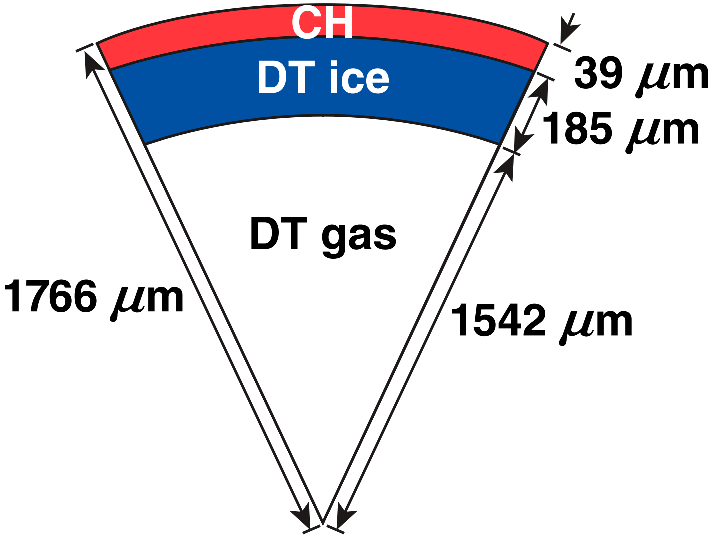
Hydro-equivalent implosions are designed using current OMEGA targets (but with better performance)

OMEGA



E_L kJ	26
V_i km/s	350
α_{min}	1.5
IFAR$_{2/3}$*	22
$\langle \rho R \rangle_n$ g/cm^2	0.3
Y_N (1-D)	1.8×10^{14}

NIF



E_L kJ	1800
V_i km/s	350
α_{min}	1.5
IFAR$_{2/3}$*	22
$\langle \rho R \rangle_n$ g/cm^2	1.3
Y_N (1-D)/gain	$3.4 \times 10^{19}/52$

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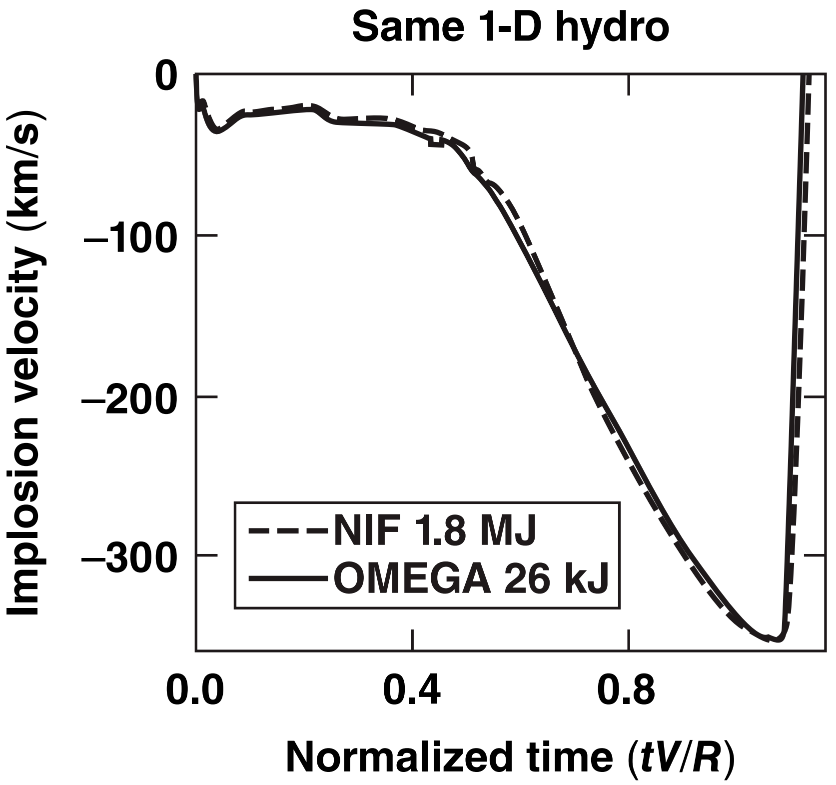
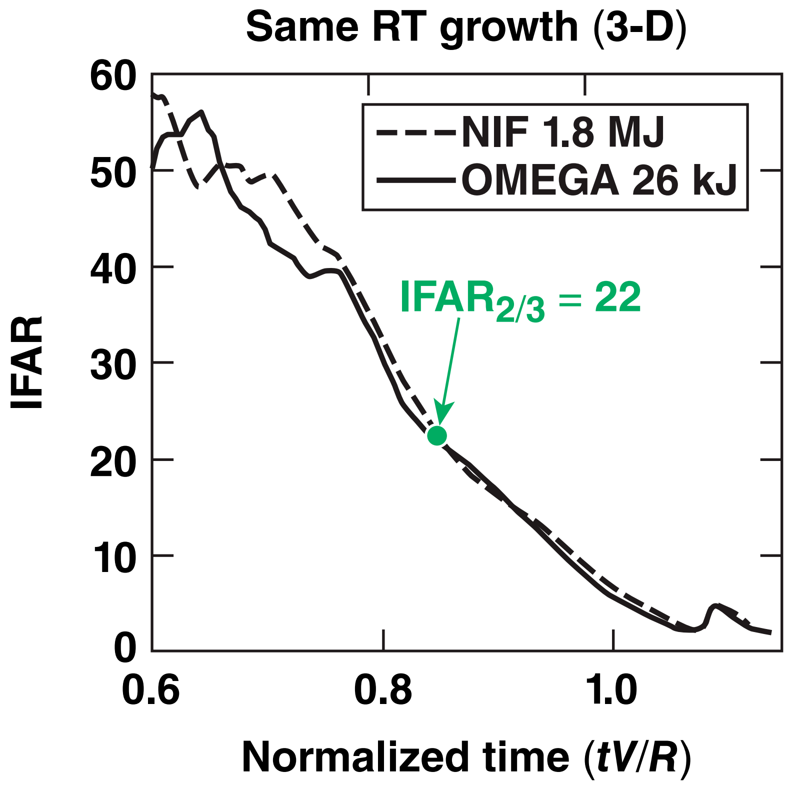
*In-flight aspect ratio

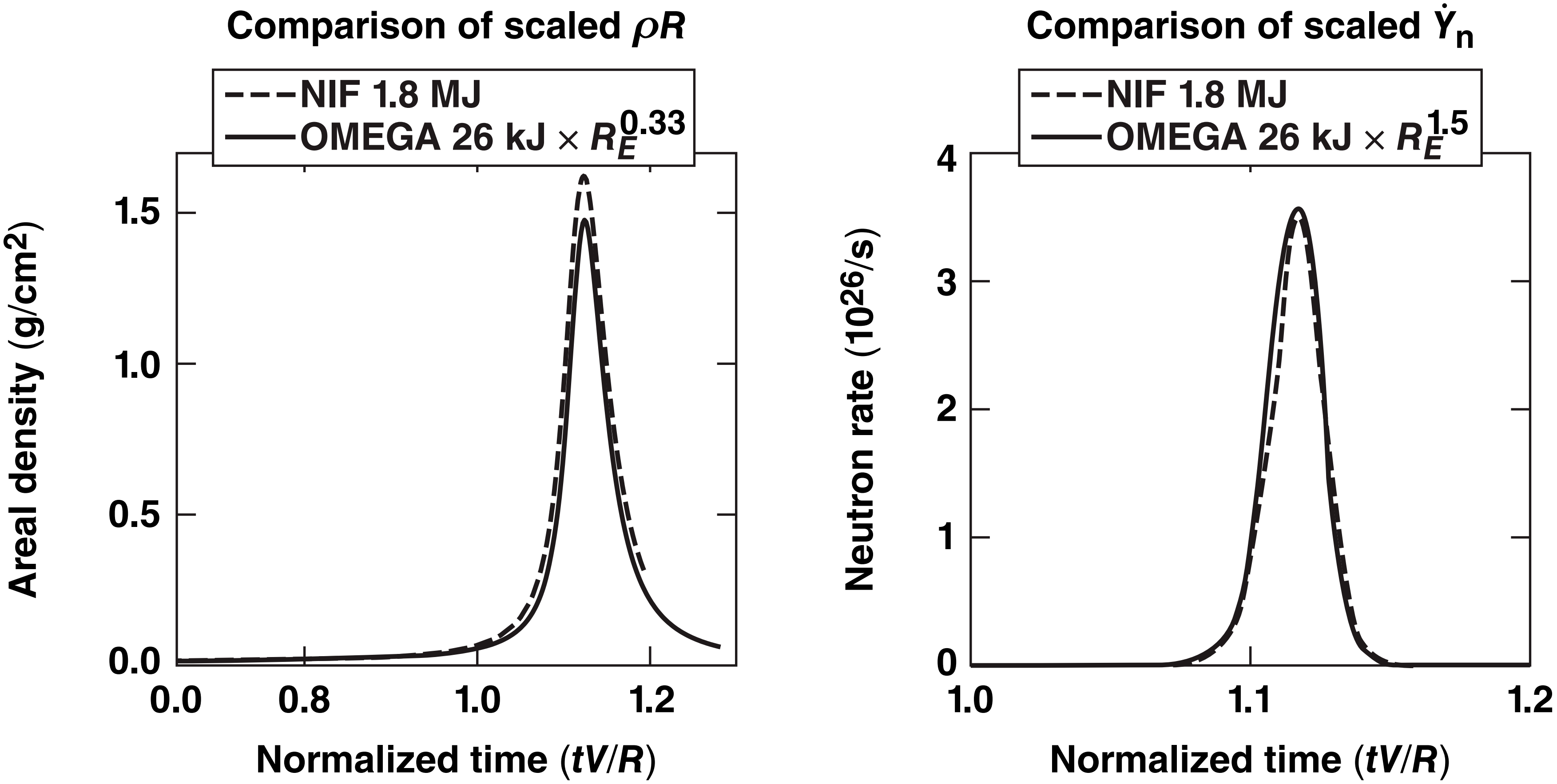
The time evolution of implosion velocity and IFAR are the same for the NIF and OMEGA


Same 1-D hydro


Same RT growth (3-D)

The 1-D areal density and no-burn neutron rate scale as predicted

$$R_E \equiv \frac{E_{\mathrm{NIF}}}{E_{\Omega}}$$



Comparison of scaled ρR

Comparison of scaled $\dot{Y}_n$

$\rho R \sim E^{1/3}$

$\dot{Y}_n \sim E^{3/2}$

The requirements for hydro-equivalent ignition at 26 kJ are confirmed by simulations

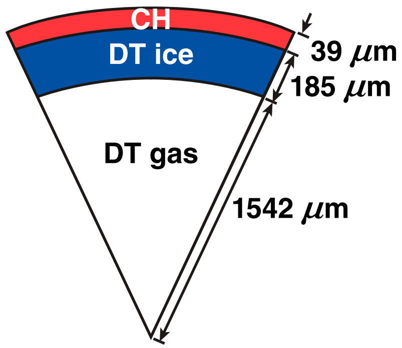
Theory

- $\chi \approx 0.16$ is required for hydro-equivalent ignition at 26 kJ
- Use $\rho R = 0.3$ g/cm^2, $M_{DT} = 0.021$ mg in $\chi \approx \left(\rho R^{\text{no}\,\alpha}_{\text{g/cm}^2}\right)^{0.61} \left(\frac{0.24\, Y_N^{16}}{M^{\text{mg}}_{\text{fuel}}}\right)^{0.34}$
- Required neutron yield at 26 kJ is 3.7×10^{13}

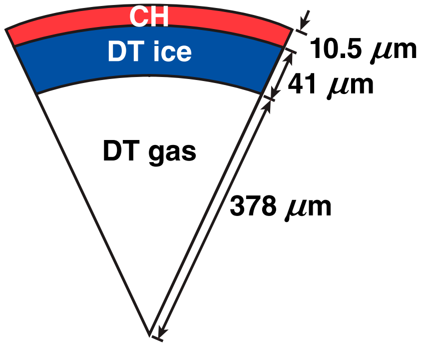
Simulations

- 1.8-MJ targets ignite (gain = 1) with 44% of 1-D no-burn neutron yield (use YOC model*) ⟶
- Required yield at 26 kJ is ~22% of the 1-D yield ($Y_{1\text{-D}} = 1.8 \times 10^{14}$); i.e., required yield is 4×10^{13} ⟶

1.8 MJ



26 kJ



Hydro-equivalent ignition parameters for OMEGA: $\rho R \approx 0.3$ g/cm^2 and $Y_N \approx 4 \times 10^{13}$

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*P. Y. Chang *et al.*, Phys. Rev. Lett. 104, 135002 (2010).

Summary/Conclusions

Inertial confinement fusion (ICF) ignition theory is used to assess National Iginition Facility (NIF) experiments and design ignition-scalable implosions on OMEGA

- ICF ignition theory is used to derive performance parameters that can be measured in experiments
- The theoretical results can be easily related to the Lawson criterion
- Applications to NIF indirect drive implosions show $P\tau$ up to 18 atm s, and pressures up to ~125 Gbar (~350 Gbar is required for ignition)
- Hydrodynamic scaling is used to design direct-drive similarity experiments between OMEGA and the NIF
- OMEGA implosions that scale to ignition at NIF energies require an areal density of ~0.3 g/cm^2 and a neutron yield of ~4 × 10^{13} (a 1.6× improvement in areal density and 3× improvement in yield with respect to best implosion to date)

Hydrodynamic scaling indicates lower 3-D degradation in performance on NIF versus OMEGA

- **Yield-over-clean (YOC): the required YOC on OMEGA is difficult to estimate. Use simple clean volume analysis:**

$$R_{3\text{-D}} = R_{1\text{-D}} - \underset{\substack{\uparrow\\ \boxed{\text{RT spike amplitude}}}}{\Delta R_{\mathrm{RT}}}$$

$$\Delta R_{\mathrm{RT}} \sim \underset{\substack{\nearrow\\ \boxed{\text{Initial seed}}}}{\sigma_0}\, \underset{\substack{\uparrow\\ \boxed{\text{Growth factor}}}}{G_{\mathrm{RT}}}$$

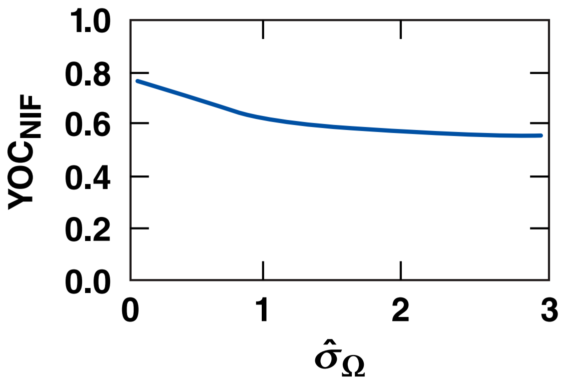
$$\underset{\boxed{\text{Hydro-equivalency}}}{G_{\mathrm{RT}}^{\mathrm{NIF}} \approx G_{\mathrm{RT}}^{\Omega}}$$

$$\mathrm{YOC}_{\mathrm{NIF}} \approx \left[1 - \frac{\sigma_0^{\mathrm{NIF}}}{\sigma_0^{\Omega}}\left(\frac{E_L^{\Omega}}{E_L^{\mathrm{NIF}}}\right)^{1/3}\left(1 - \mathrm{YOC}_{\Omega}^{1/3}\right)\right]^3$$

A YOC of 30% on OMEGA extrapolates (approximately) to a 60% YOC on NIF

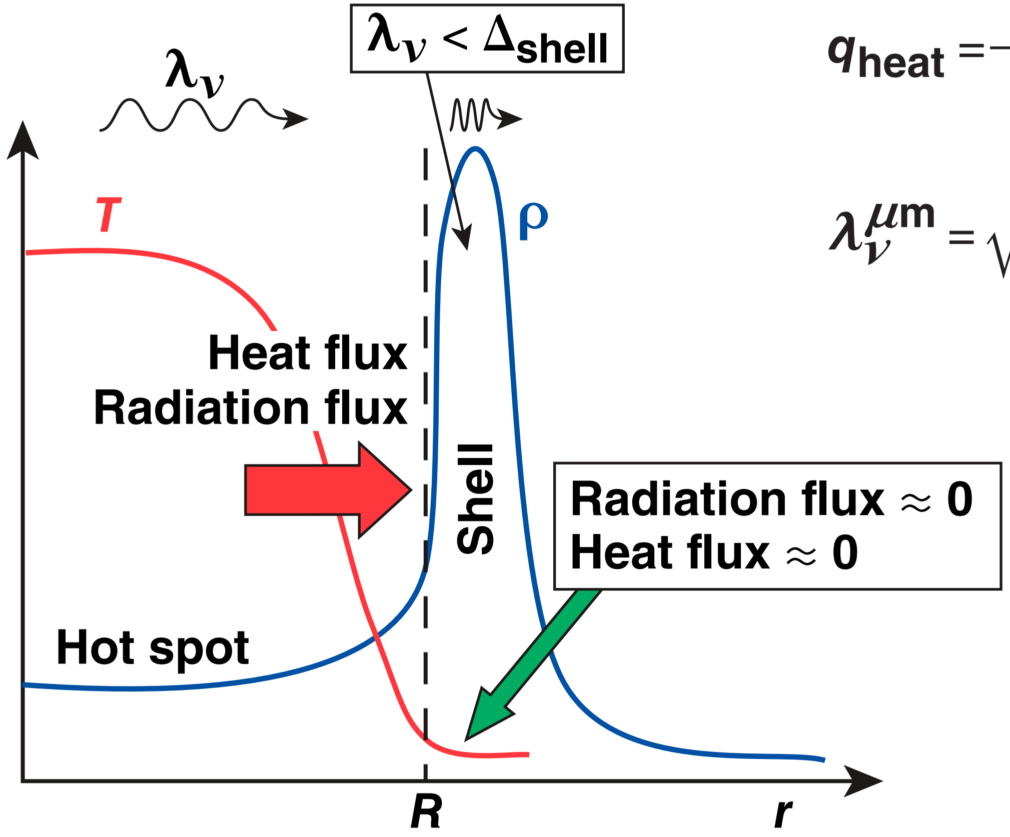
- Seeds for the RT come from the ice roughness and the laser nonuniformities: $\sigma_0 \approx \sqrt{\sigma_{\text{ice}}^2 + \sigma_{\text{laser}}^2}$
- Beta layering makes NIF targets as smooth as OMEGA's: $\sigma_{\text{ice}}^{\Omega} \approx \sigma_{\text{ice}}^{\text{NIF}}$
- Laser nonuniformities grow with size $\left(E_L^{1/3}\right)$ and are reduced by a larger number of overlapping beams (N_b) $\sigma_{\text{laser}} \sim E_L^{1/3} N_b^{-1/2}$

$$\frac{\sigma_0^{\Omega}}{\sigma_0^{\text{NIF}}} \approx \sqrt{\frac{1+\hat{\sigma}_{\Omega}^2}{1+\left(\frac{E_L^{\text{NIF}}}{E_L^{\Omega}}\right)^{2/3}\left(\frac{N_b^{\Omega}}{N_b^{\text{NIF}}}\right)\hat{\sigma}_{\Omega}^2}} \qquad \hat{\sigma}_{\Omega} \equiv \frac{\sigma_L^{\Omega}}{\sigma_{\text{ice}}^{\Omega}}$$



NIF YOC extrapolated from measured 30% YOC on OMEGA

The heat and radiation energy lost by the hot spot does not propagate through the dense cold shell; no heat or radiation flux at the hot-spot boundary



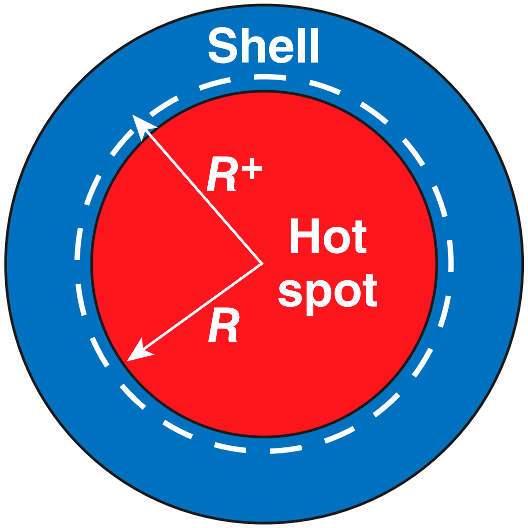
$\lambda_\nu < \Delta_{shell}$

Radiation flux $\approx$ 0
Heat flux $\approx$ 0

$$q_{heat} = -\kappa(T)\nabla T \quad \kappa_{Sp}(T) \approx \kappa_0 T^{5/2}$$

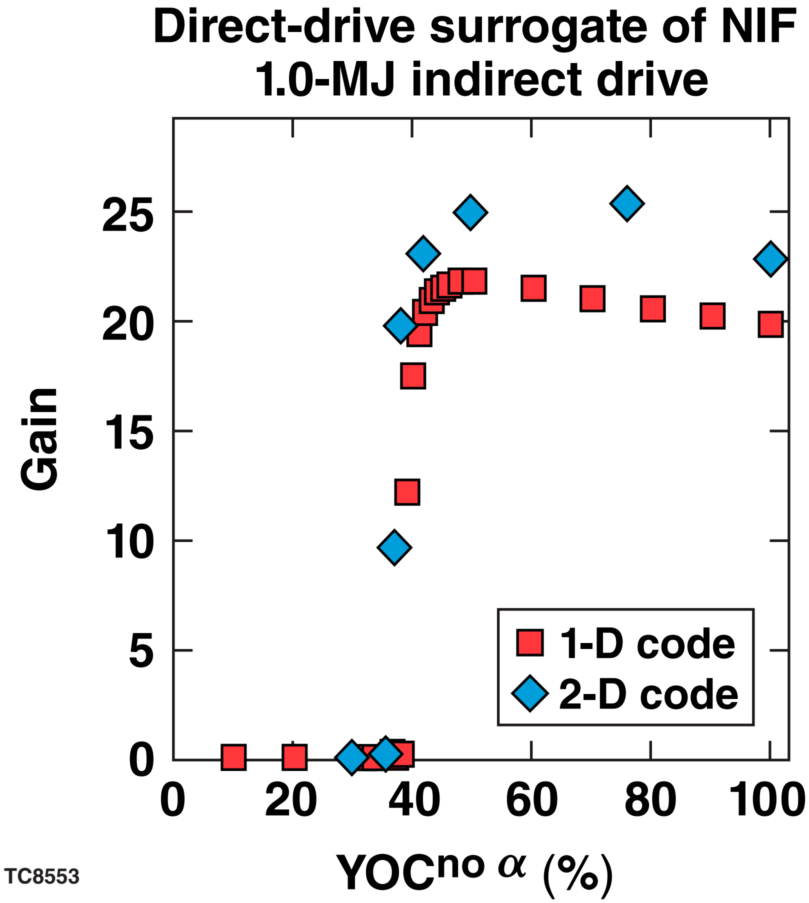
$$\lambda_\nu^{\mu m} = \sqrt{\frac{T_{eV}^{shell}}{200}}\left(\frac{h\nu_{keV}}{5}\right)^3\left(\frac{1000}{\rho_{g/cc}}\right)^2$$

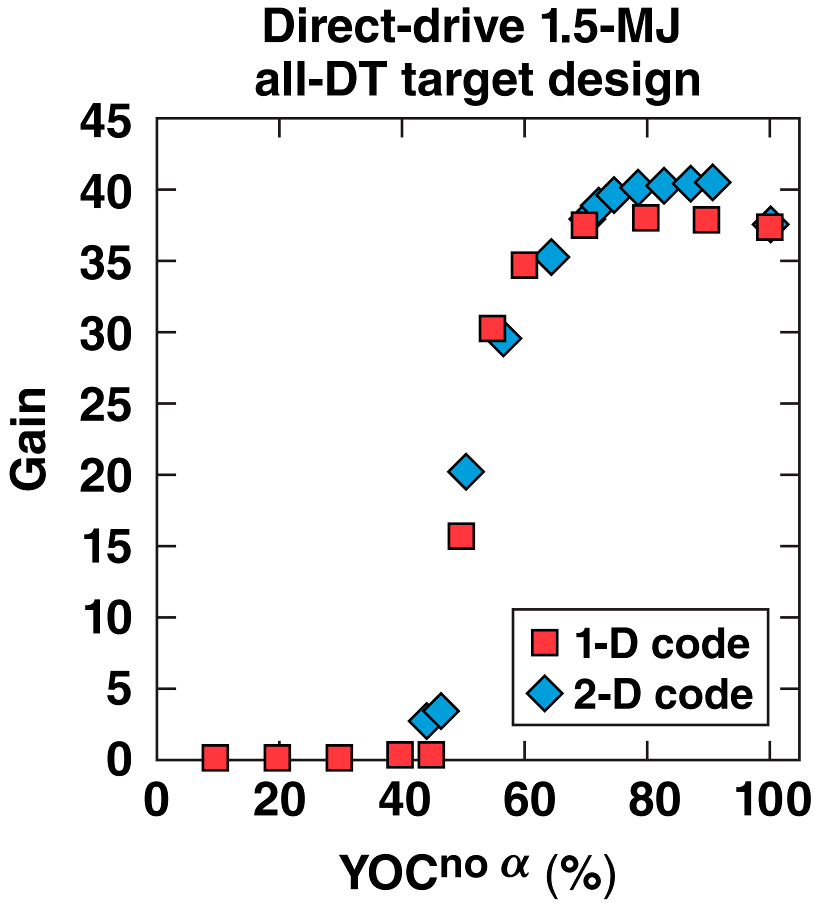
$$q_{heat}\left(r = R^+\right) \approx 0 \quad q_{rad}\left(r = R^+\right) \approx 0$$



The clean volume analysis is validated by comparing 2-D simulations with inner-surface roughness and 1-D simulations having reduced $\langle \sigma v \rangle \rightarrow \langle \sigma v \rangle V_{\text{clean}}/V_{\text{1-D}} \approx \langle \sigma v \rangle \text{YOC}^{\text{no } \alpha}$

- In the 1-D simulations, $\langle \sigma v \rangle$ is reduced by the YOC (or clean volume fraction) until the hot-spot temperature reaches 10 keV.

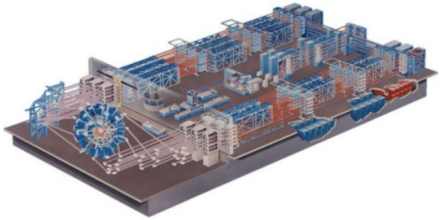




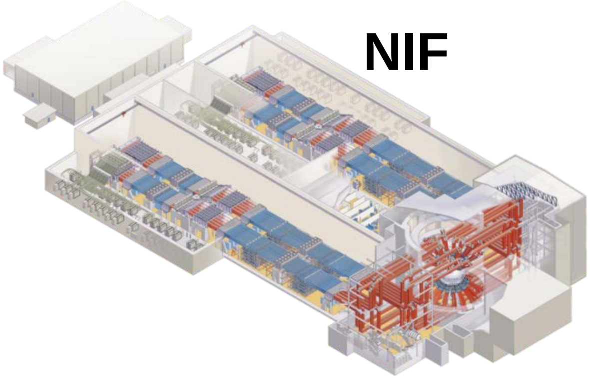
TC8553

Physics other than hydrodynamics (based on purely theoretical extrapolations)

OMEGA



NIF



• Laser-energy collisional absorption	Less	better →	More
• Laser plasma instabilities and hot electron preheat	Less	worse →	More
• Cross-beam energy transfer	Less	worse →	More
• Radiation preheat	More	better →	Less