TH/P2-18

Calculation of transport profiles in modulation experiments with source without transport codes

D.F. Escande[1,2], F. Sattin[2], Y. Camenen[1], A.T. Salmi[3], T. Tala[3]

[1] UMR 7345, CNRS/Aix-Marseille Université, Marseille, France
[2] Consorzio RFX, Associazione EURATOM-ENEA, Padova, Italy
[3] Association Euratom-Tekes, VTT, P.O. Box 1000, FI-02044 VTT, Finland

dominique.escande@univ-amu.fr

CONSORZIO RFX — Ricerca Formazione Innovazione — Euratom-ENEA Association

IAEA FEC 2012, October 8-13, 2012, San Diego, California, USA

Association Euratom–Tekes

Abstract

Estimating the coefficients of the Convection-Diffusion Equation (CDE) from experimental measurements is critical in order to compare experiments against first-principle theories, but is a tough problem, both due to the complexities related to performing the experiment, and because of the intrinsic ill-conditioned nature of the mathematical inverse problem. Here we concentrate on a particular class that can be reduced to a linear algebraic problem, with explicit solution. Ill-conditioning of the problem corresponds to the vanishing of one eigenvalue of the matrix to be inverted. The comparison with algorithms based upon matching experimental data against numerical integration of the CDE sheds light on the accuracy of the parameter estimation procedures, and suggests a path for a more precise assessment of the profiles and of the related uncertainty. Several instances of the implementation of the algorithm to real data are presented.

The algorithm

Step 1: Convection-Diffusion Equation written explicitly in cylindrical geometry and all quantities Fourier transformed: from PDE to ODE

$$\frac{\partial \xi}{\partial t} = -\nabla \cdot \Gamma(\xi) + S_\xi, \qquad \Gamma = -D\nabla\xi + V\xi$$

$$-i\omega\xi = -\frac{1}{r}\frac{\partial}{\partial r}\left[r\left(-D\frac{\partial \xi}{\partial r} + V\xi\right)\right] + S_\xi$$

Step 2: A first integration over radius is made together with null flux boundary condition at r = 0. By rearranging the equation one gets a linear algebraic equation in (D,V), once ξ, S, are known

$$-i\omega\xi = -\frac{1}{r}\frac{\partial}{\partial r}\left[r\left(-D\frac{\partial \xi}{\partial r} + V\xi\right)\right] + S_\xi$$

$$-D\frac{\partial \xi}{\partial r} + V\xi = -\frac{1}{r}\int_0^r dz\, z\left[S_\xi + i\omega\xi\right]$$

Step 3: as usual in modulated experiments, perturbed quantities are written in complex form in terms of an amplitude and a phase $\xi = Ae^{i\phi}$

By taking separately the real and imaginary part, we write the previous equation as 2 real equations and we set it in a matrix-vector formulation

$$-D\frac{\partial \xi}{\partial r} + V\xi = -\frac{1}{r}\int_0^r dz\, z\left[S_\xi + i\omega\xi\right]$$

$$\mathbf{M}\cdot\mathbf{Y} = \mathbf{\Gamma} \qquad \mathbf{Y} = \begin{pmatrix} D \\ V \end{pmatrix}, \quad \mathbf{M} = \begin{bmatrix} -A'\cos\phi + A\phi'\sin\phi & A\cos\phi \\ -A'\sin\phi - A\phi'\cos\phi & A\sin\phi \end{bmatrix},$$

$$\mathbf{\Gamma} = \begin{bmatrix} r^{-1}\int z[\mathrm{Re}(S_\xi(z)) - \omega A(z)\sin\phi(z)]dz \\ r^{-1}\int z[\mathrm{Im}(S_\xi(z)) + \omega A(z)\cos\phi(z)]dz \end{bmatrix} \qquad f' \equiv \partial_r f$$

[Krieger, *et al*, Nucl. Fusion **30**, 2392 (1990), Takenaga, *et al*, PPCF **40**, 183 (1998)]

(D,V) are obtained by inverting this linear system at each radius r

Issue of invertibility (ill-conditioning)

Inverse problems are often ill-conditioned: small variations in the data can lead to large variations in the reconstructed coefficients.

Simple example: under steady state conditions, (D,V) cannot be assessed unambiguosly.

All couples (D+D₀, V+V₀) such that $\nabla\cdot(-D_0\nabla\xi_{meas} + V_0\xi_{meas}) = 0$ ***are equivalent***

Because of this, time-dependent experiments are designed.

The algebraic system is **not** invertible when det(**M**) = 0

det(**M**) = A² φ' $\rightarrow r_s : \varphi'(r_s) = 0$

Is this condition likely to be met anywhere ?

Inspection of the literature, supported by analytical considerations, shows the answer is 'yes': singular points are actually encountered in experiments, close to the source locations.

What does happen close to singular points?

Let us introduce the eigenvalues $\lambda_{0,1}$ and eigenvectors $\mathbf{E}_{0,1}$ of the matrix **M**

$$\mathbf{M}\cdot\mathbf{E}_i = \lambda_i\mathbf{E}_i \qquad \mathbf{Y} = y_0\mathbf{E}_0 + y_1\mathbf{E}_1 \qquad \mathbf{\Gamma} = g_0\mathbf{E}_0 + g_1\mathbf{E}_1$$

Eigenvectors and eigenvalues depend on the data (A, φ, A', φ'). At a singular point one eigenvalue must vanish ($\lambda_0 = 0$)

Experimental errors upon (A, φ) propagate upon (**E**, λ):

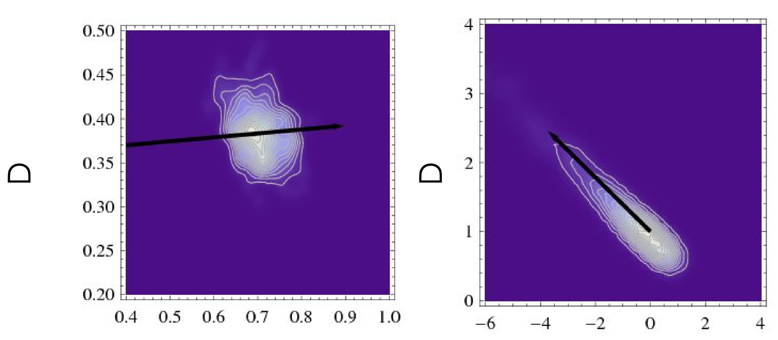
$$\mathbf{Y} = \frac{g_0}{\lambda_0}\mathbf{E}_0 + \frac{g_1}{\lambda_1}\mathbf{E}_1 \rightarrow \delta\mathbf{Y} = \left(\frac{\delta g_0}{\lambda_0} - \frac{g_0}{\lambda_0^2}\delta\lambda_0\right)\mathbf{E}_0 + \left(\frac{\delta g_1}{\lambda_1} - \frac{g_1}{\lambda_1^2}\delta\lambda_1\right)\mathbf{E}_1 + \frac{g_0}{\lambda_0}\delta\mathbf{E}_0 + \frac{g_1}{\lambda_1}\delta\mathbf{E}_1$$

$$\delta\mathbf{Y} \approx -\frac{g_0\delta\lambda_0}{\lambda_0^2}\mathbf{E}_0, \quad \lambda_0 \rightarrow 0$$

At the singular points, all solutions aligned along one axis in the (D,V) plane are allowed. Therefore: even explicit time-dependence in experiments does not automatically warrant a reliable assessment of (D,V)

Example

The clouds show the density distribution of **Y** in response to different measurement errors in a sinthetic case. The black lines are the local direction of the eigenvector $\mathbf{E}_0$



Regular point $\lambda_0 \neq 0$ — **Singular point** $\lambda_0 \approx 0$

How the algorithm works in actual cases: an analysis of fusion plasmas data

Recipe:

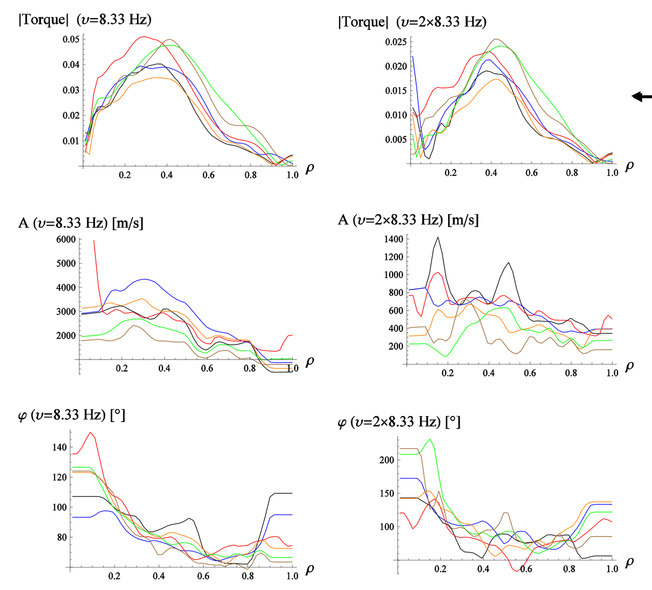
- (A, φ,S) are known from the experiments on a discrete set of radial points, interpolate them using smooth curves (e.g., splines).
- Feed **M**, **Γ** with these curves, and solve for **Y**
- Estimate error bars via Monte Carlo analysis: perturb (A, φ,S) by random noise and re-evaluate **Y** on the new data. Repeat the calculation over a large number of independent runs and take the statistical moments (mean, variance) for **Y** and its estimated error

Training ground: JET experimental data

Toroidal rotation transport studies
Each color refers to a different shot

JET pulses 73701-2,73704,73707-9

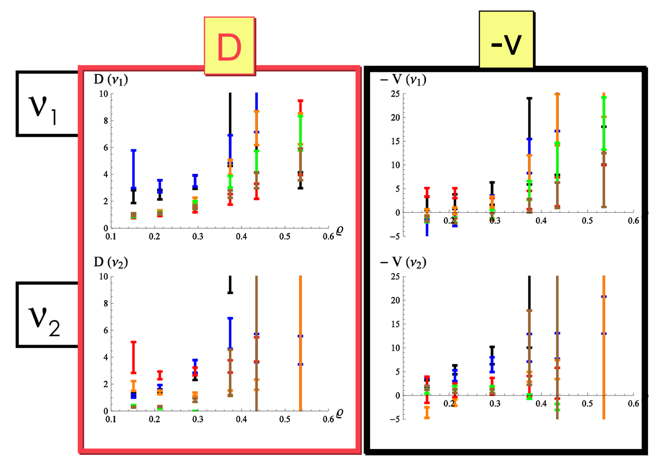
[T. Tala, *et al*, Nucl. Fusion **51**, 123002 (2011)]



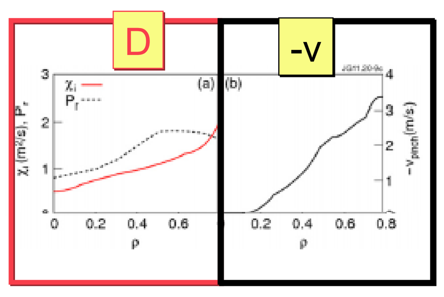
Torque obtained by modulating NBI, at fundamental frequency ν_1 = 8.33 Hz and its first overtone $\nu_2 = 2\times\nu_1$
Rotation measured via CXRS

Results from present method

(first and second harmonics ν_1 ν_2)



…and from [Tala NF 2011]



Direct method yields by construction the only solution fully compatible with the data: global minimum

Results obtained by minimizing the spread between profiles computed by transport codes (JETTO) and experimental profiles.

Iterative methods minimizing the global spread between simulation and data: "local" or "global" minimum ?

Accounting for further effects

Let us generalize the convection-diffusion equation including further effects:

$$\Gamma = -D\nabla\xi + V\xi \rightarrow -D\nabla\xi + V\xi + R$$

R includes the fraction of flux that does not depend explicitly on the quantity ξ.

It may appear when there is some degree of coupling with other quantities: nonzero off-diagonal terms of the stress tensor.
E.g., during experiments of momentum transport several additional torques may appear, due to breaking of axisymmetry (Neoclassical Toroidal Viscosity Torque, …), and are modelled via *R*.

R may also account for partial ignorance about the source term.

It is straightforward to generalize the linear system to: $\mathbf{M}\cdot\mathbf{Y} + \mathbf{R} = \mathbf{\Gamma}$

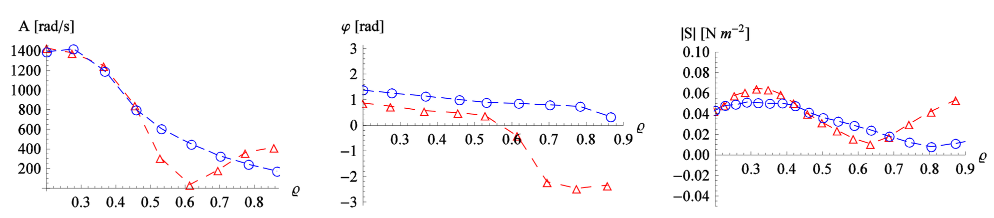
R is a 2-dimensional array containing the real and imaginary part of the additional flux

This linear system has two equations and four unknowns, if **R** is considered as a variable. It cannot be solved by one experiment alone: needs at least two experiments that share the same transport

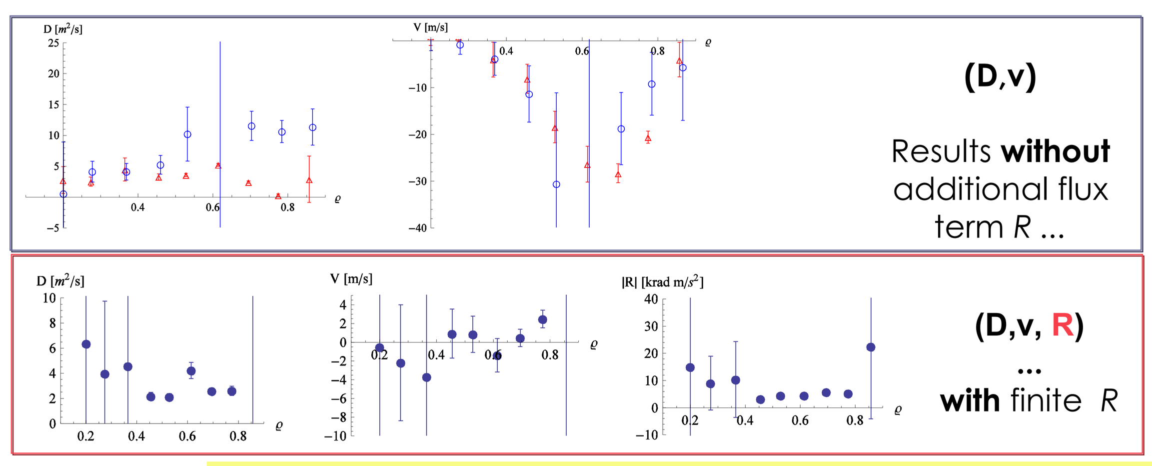
$$\begin{cases} \mathbf{M}_1\cdot\mathbf{Y} + \mathbf{R} = \mathbf{\Gamma}_1 \\ \mathbf{M}_2\cdot\mathbf{Y} + \mathbf{R} = \mathbf{\Gamma}_2 \end{cases}$$

NBI modulation with magnetic ripple: breaking of axisymmetry

Measurements of toroidal rotation with breaking of axisymmetry. Appear promising candidates for investigating a possible role of R



Ref: A.T. Salmi, *et al*, Plasma Phys. Control. Fusion **53**, 085005 (2011) JET Pulses 77090 & 77091



(D,v)

Results **without** additional flux term *R* …

(D,v, R) … **with** finite *R*

No definitive conclusions but some considerations about the merits of including/excluding R:
-w/o R
- Different transport (red-blue curves): is it justified by differences in the discharges?
- D close to suspiciously null or negative values
- On the average, D, |V| larger than expected (see, e.g., previous exercise)

- with R
- D,|V| look closer to independent standard estimates
- The flux driven by R is quite close to flux driven by diffusion
- Of course, the presence of R must be supported on physical grounds

Concluding remarks

- We have reviewed a fast and simple algorithm for extracting (D,V)--and possibly other parameters--from perturbative periodically modulated experiments.
- The algorithm does not adopt regularization of transport profiles, but just a smoothing of experimental data; hence, there are not issues of converging to spurious solutions caused by constraints placed on (D,V) curves, the solution is exact by construction
- Reliable error bars, exclusively due to measurement errors.