# An Investigation of Coupling of the Internal Kink Mode to Error Field Correction Coils in Tokamaks

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# An Investigation of Coupling of the Internal Kink Mode to Error Field Correction Coils in Tokamaks.

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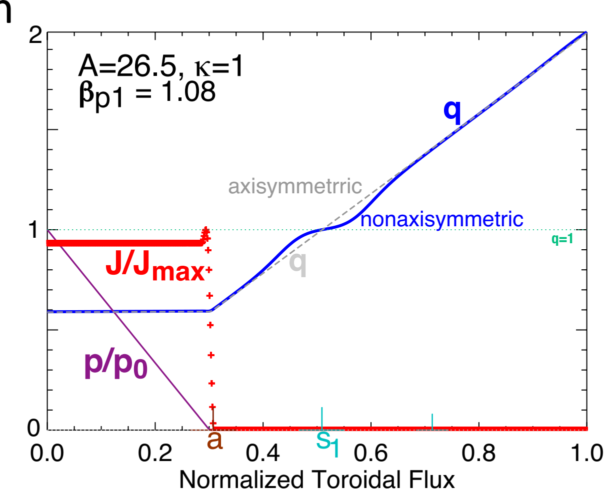
- The saturated internal kink that was studied analytically in the cylindrical tokamak, (Rosenbluth, *et.al.*, 1973) is examined in 3d geometry using the VMEC equilibrium code. While the mode is stabilized by toridicity, the boundary *m/n=1/1* perturbation needed to restore the saturated kink is surprisingly small; with the perturbation amplitude, normalized to major radius, ($\delta R/R_0$) in the range $10^{-4} – 10^{-3}$.
- Characterization of solutions with the pressure and $\delta R/R_0$ is presented for a large aspect ratio circle and a small aspect ratio ellipse.
- It is proposed that such field errors are the explanation for the sawtooth results in devices where $q_0$ remains below unity throughout the sawtooth cycle.

The saturated kink exhibits a flattening of the q-profile around the q=1 region and a helical core.




**These results may lead to efficient control of the sawtooth amplitude in large devices that have error-field correction coils. The possibility of sustained operation at $q_0$<<1 in a high-$\ell_i$ scenario is also discussed.**

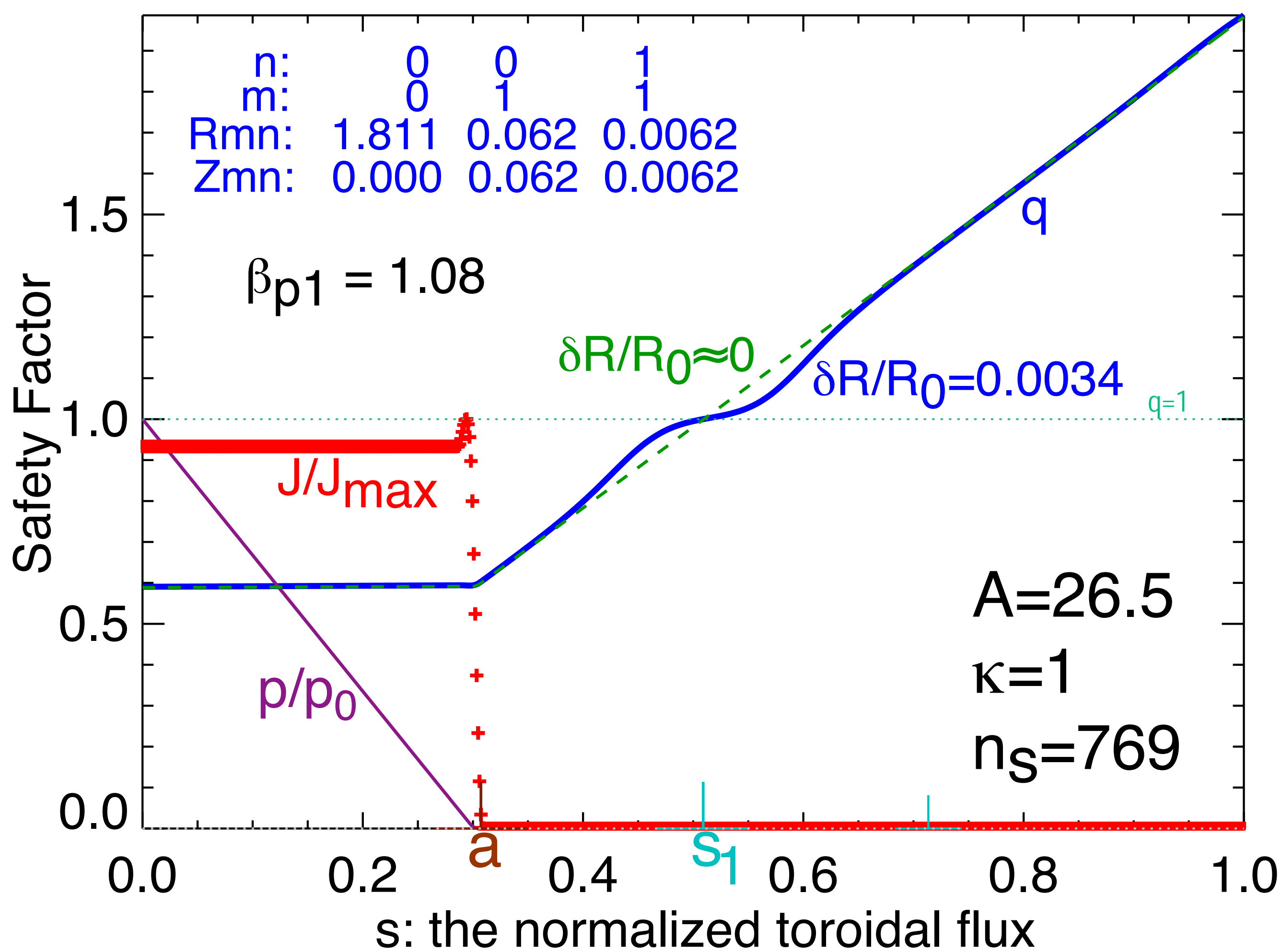
# In this work I examine the saturated internal kink in 3D geometry, starting from the work of Rosenbluth, Dagezian & Rutherford [Phys.Fluids **16**(1973)1894.].

- That work showed the existence of a saturated internal kink in the limit of a cylindrical tokamak. It was an analytical calculation.
  - The profiles, with $a < \rho_1 < b$ ,where $\rho_1$ is the $q\!=\!1$ surface
  - and $b$ is the boundary, are $p(\rho) = p_0 \cdot (1 - (\rho^2/a^2)$ and
  - $J_\phi(\rho) = J_0$ for $\rho \leq a$. Both $p$ & $J$ are zero for $\rho > a$.
  - The VMEC radial coordinate is $s$, the normalized toroidal flux.
  - Here we take $\rho \equiv \sqrt{s}$.
- In this work with those same profiles equilibria are computed numerically in 3D toroidal geometry.

While the internal kink is stabilized by toridicity, **a surprisingly small boundary *m/n=1/1* perturbation needed to restore the saturated kink**; with the perturbation amplitude, normalized to major radius, $(\delta R/R_0)$ in the range $10^{-4}$ — $10^{-3}$. Elongation and reduced aspect ratio appear to reduce the external drive requirements ($\delta R/R_0$ & $\beta_{p1}$ ).

# 3D solutions are characterized by a flattening of the q-profile in a region centered on the q=1 radius.

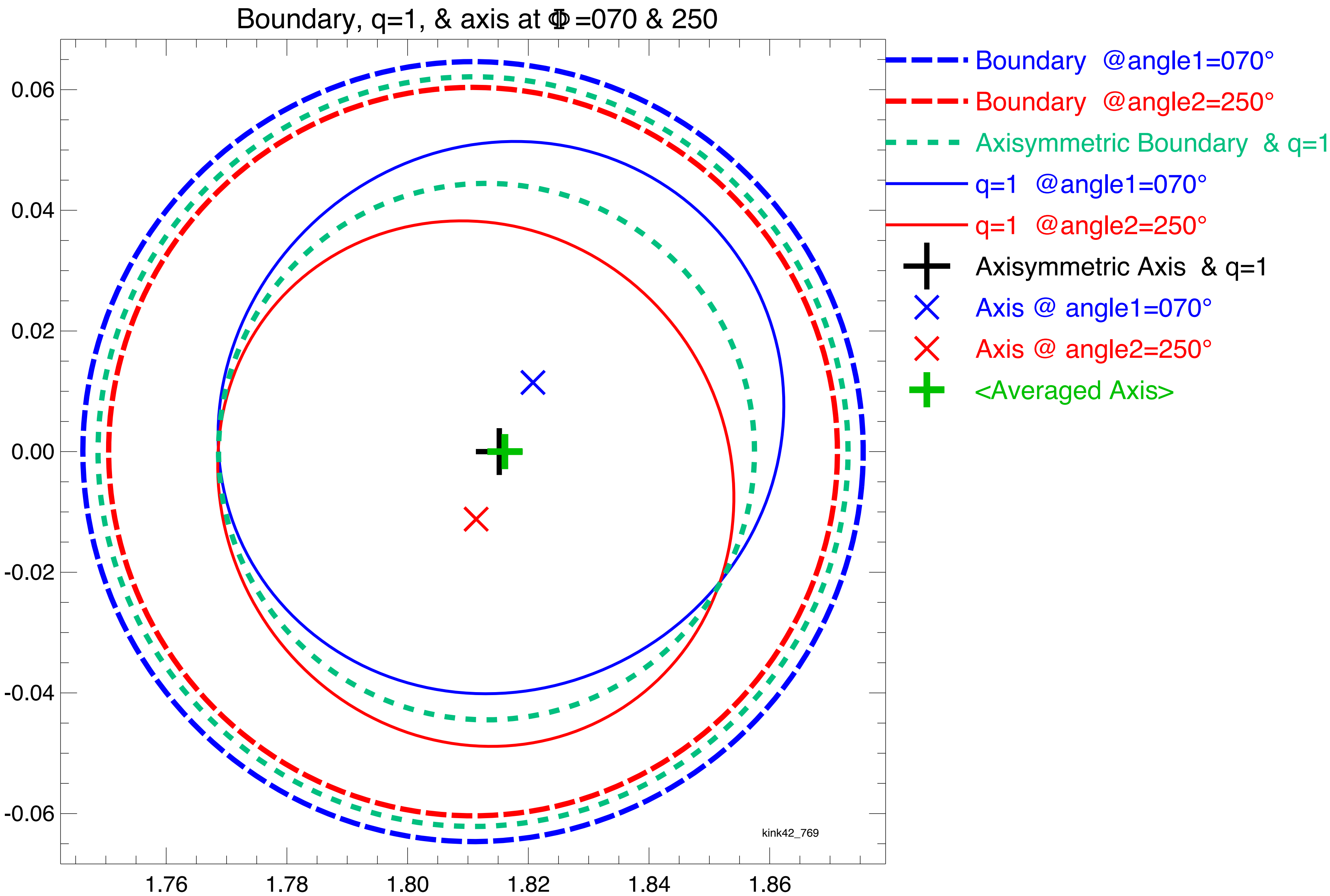
The "typical result" is shown here.



# The equilibrium has a helical core.

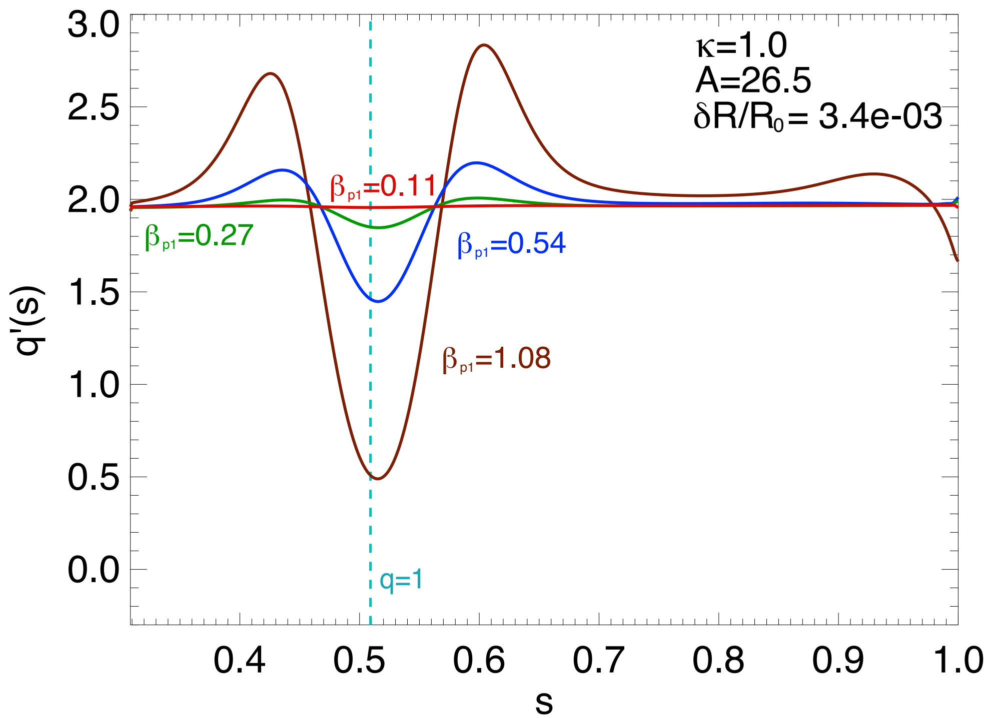
- In the figure above, shown are the solutions for q, the safety factor along with the normalized driving functions $p/p_0$ (purple line) and $\langle J^\Phi/J^\phi_{max}\rangle$ (red + signs). The q-profile from an infinitesimal boundary perturbation is also shown (green dashed line).
- In no cases have we found evidence for a saturated internal mode when $\delta R/R_0 = 0$, *i.e.,* we do not obtain convergence at a $\beta$ that exceeds some critical value.
- In the figure below, surfaces in toroidal planes are shown at two angles separated by 180°. It is seen that the perturbation of the q=1 surface (and the axis) are large compared to the boundary perturbation. The toroidally averaged axis , also shown, is near that for the axisymmetric case.
- These results differ from the work of Cooper, *et.al.,* [PRL **105** (2010) 035003.] in two significant ways:
  - Here the q=1 surface is inside the plasma and $q_0 << 1$, whereas Cooper has $q_{min} \approx 1$ and $1 < q_0 < 3/2$. *i.e.,* this is an internal kink and Cooper's result is a quasi-interchange.
  - Here, the helical core vanishes when $\delta R/R_0 = 0$.

# The edge perturbation is amplified in the core.

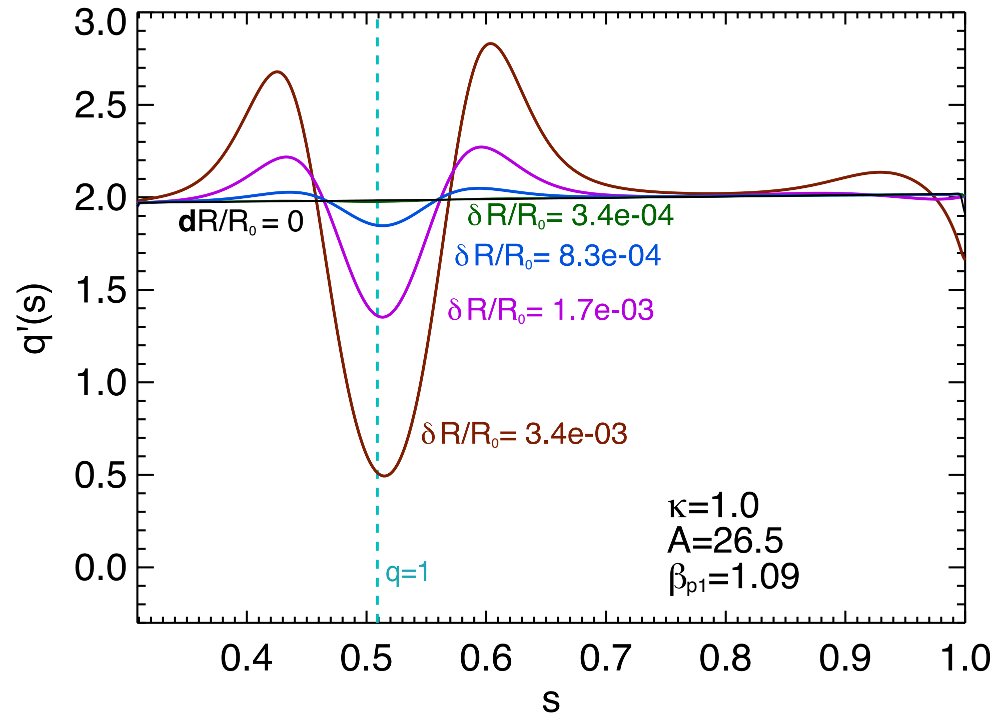


# The derivative $q\prime(s)$, is taken as a proxy for mode strength. Here are results for the large aspect ratio circles (LAC).

Variation of $q\prime(s)$ with $\beta_{p1}$ at $s > a$.




Variation of $q\prime(s)$ with $\delta R/R_0$ at $s > a$.

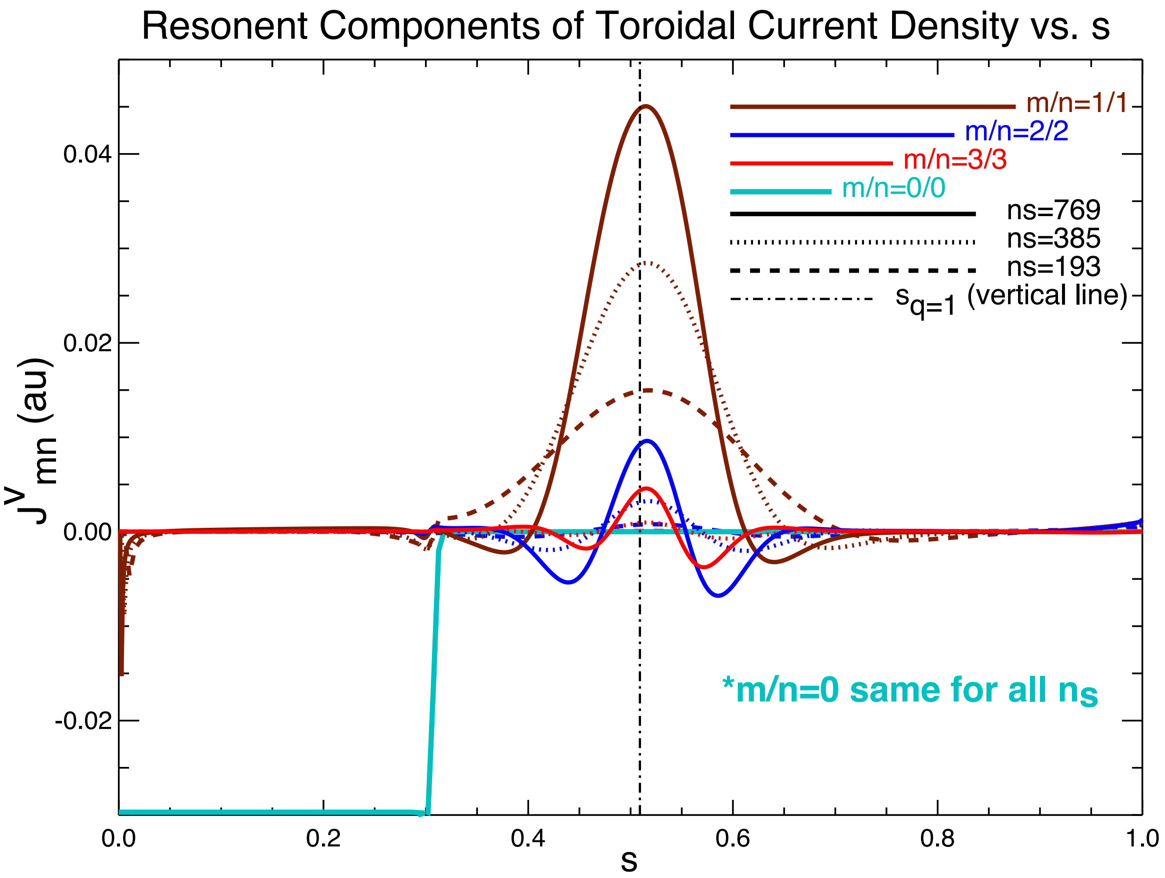



The internal structure is not visible for $\beta_{p1} \lesssim 0.1$ in the LAC.

The internal structure vanishes for $\delta R/R_0 = 0$.

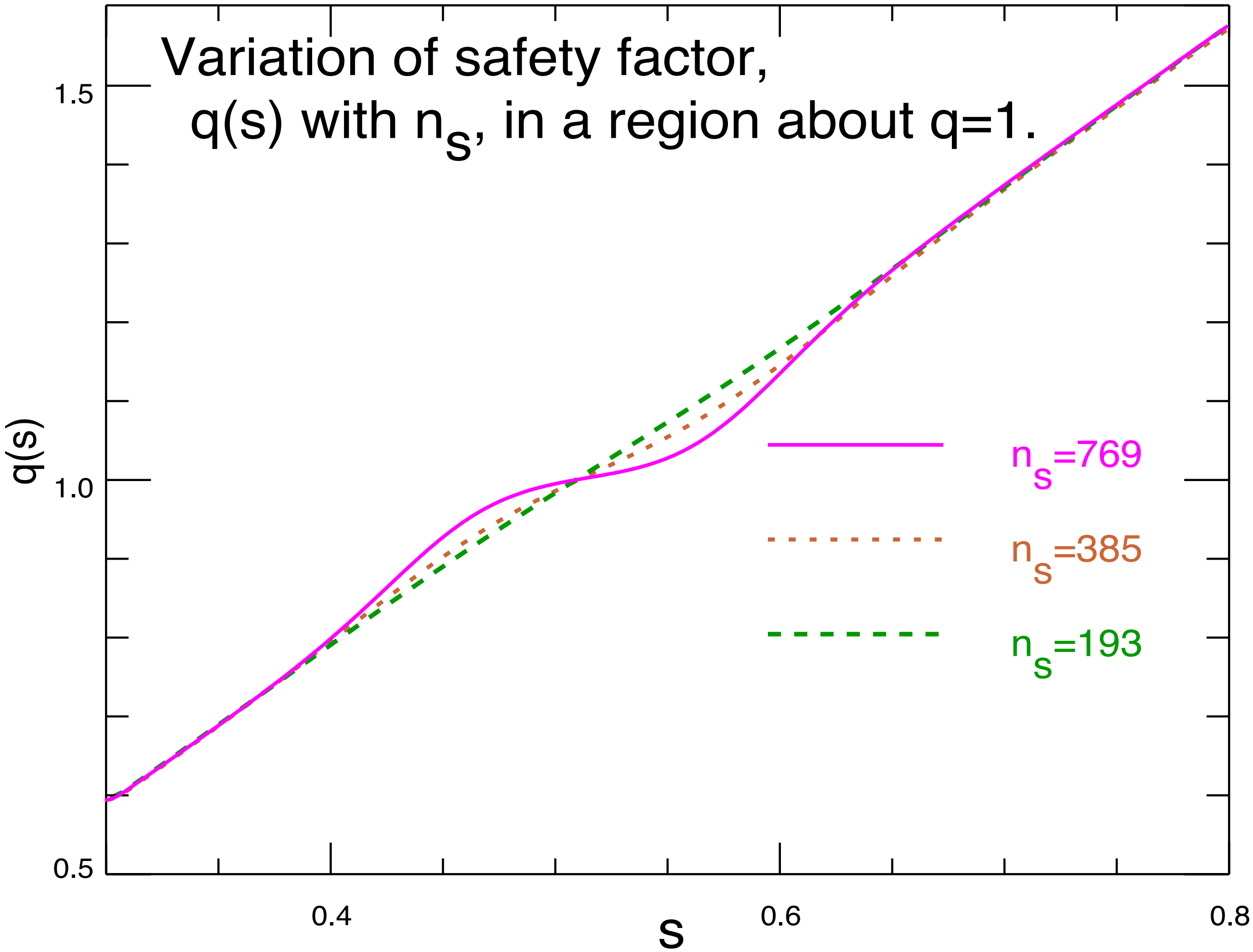
# The behavior of VMEC varies with the number of radial grid points, $n_s$ .

The results on the previous slide were obtained with $n_s$=769. Unfortunately, the result is strongly dependent on the radial resolution. A sequence, $n_s$ =[193, 385, 769] is shown here.




- The resonant structure narrows and peaks as $n_s$ is increased.
- The next step, $n_s$ =1537, does not converge.
- This structure of the resonant terms affects the q-profile.

# The calculated q-profile for the internal mode requires a fine radial grid.

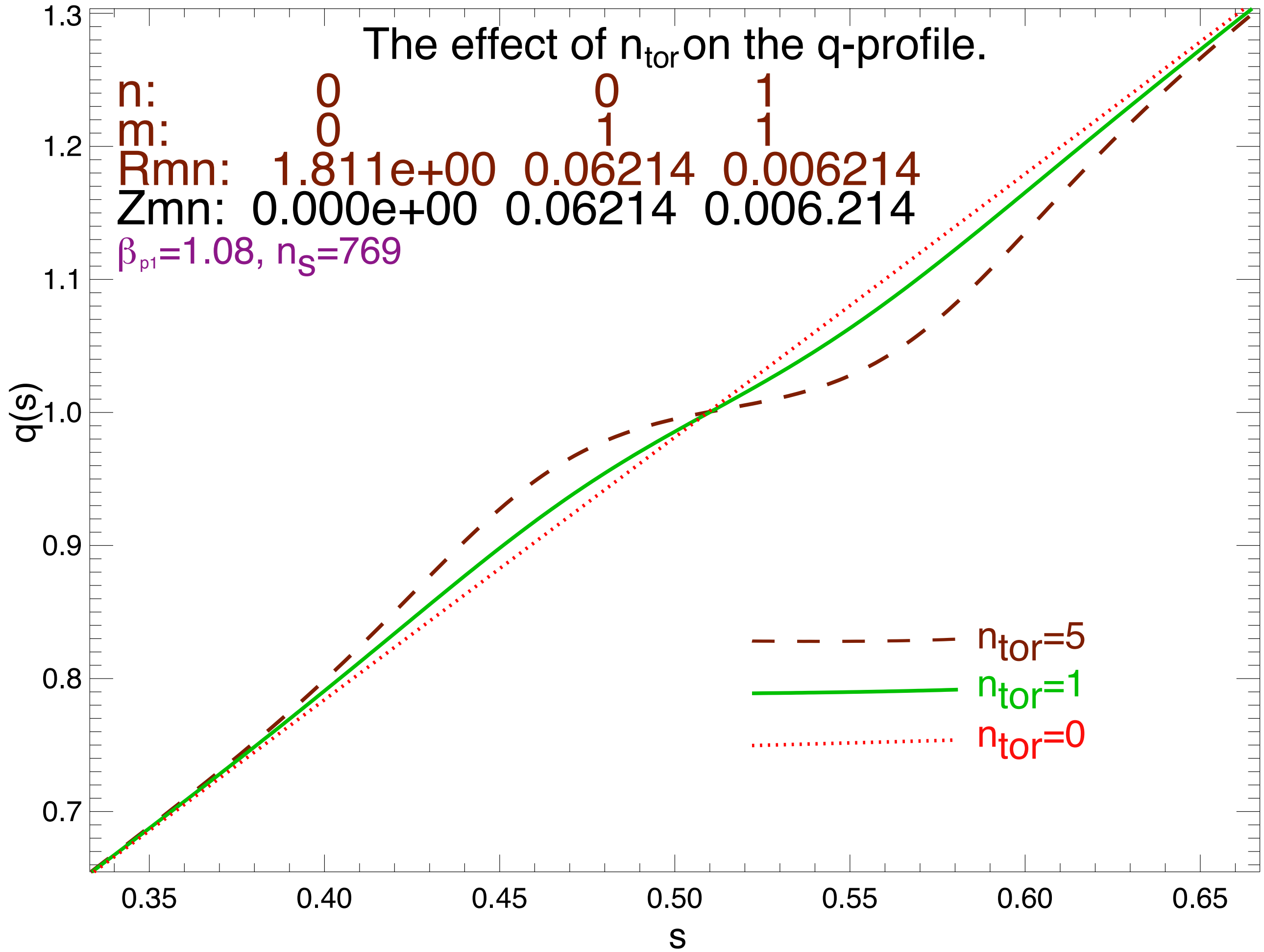



Clearly, the VMEC behavior is less-than-desired.
On the otherhand, each case is a converged equilibrium; *i.e.*, $\nabla B = 0$ & $\nabla p = J \times B$

# VMEC convergence and plasma shape.

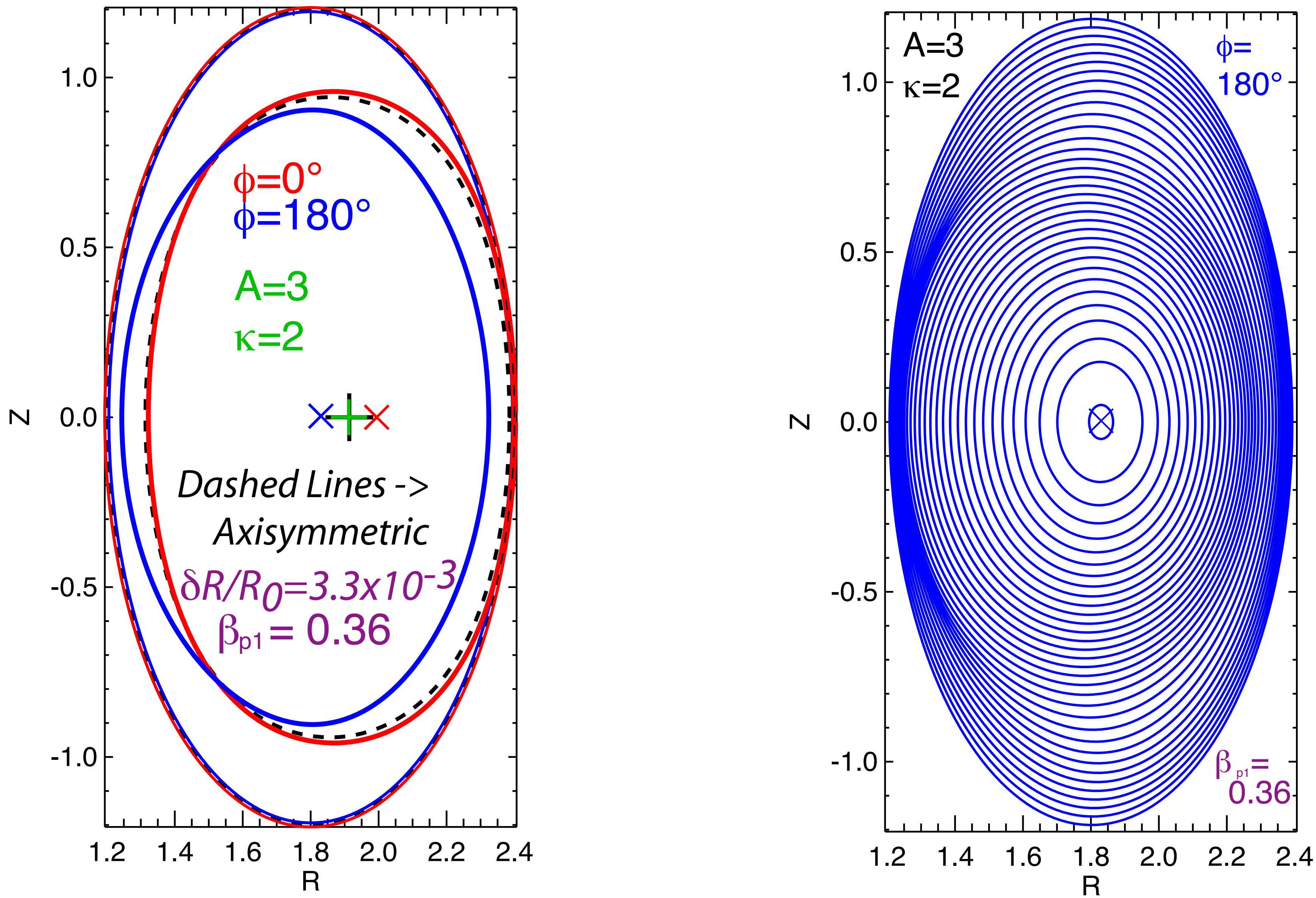
- The cases presented above are well-converged: $ftol < 1 \times 10^{-12}$ @ $n_{tor} = 5$ & $m_{pol} = 12$
- Another, similar run with $ftol < 1 \times 10^{-17}$ produced virtually identical results.
- In attempting to lower the aspect ratio or increase elongation, I cannot obtain convergence. Replacing the step functon $\langle J_\phi \rangle$ with a sigmoid did not improve convergence. The only way I found to achieve convergence at $A \approx 3$ or $\kappa > 1$ was to truncate the Fourier series at $n_{tor} = 1$.
- This is quite severe, resulting in substantial reduction in the mode structure ( see figure below).
- I pursue this because the saturated kink appears stiill more robust at $A \approx 3$ , $\kappa \approx 2$

# Restriction to $n_{tor} = 1$ in VMEC is a severe truncation. The solutions to be presented are well-converged.

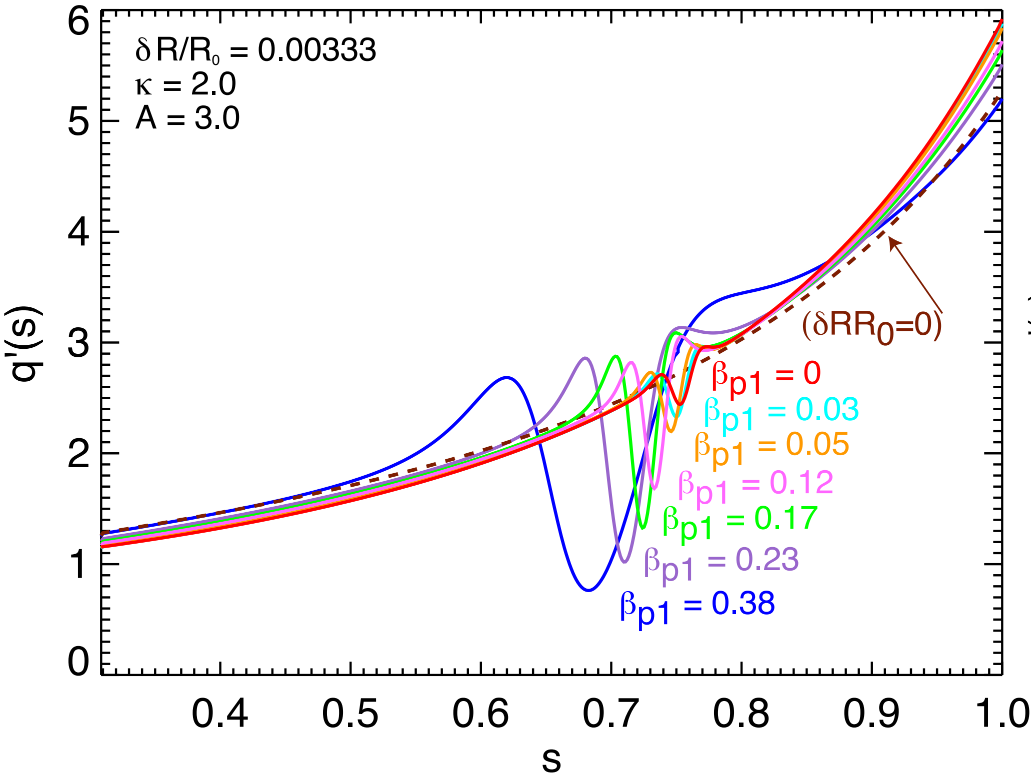


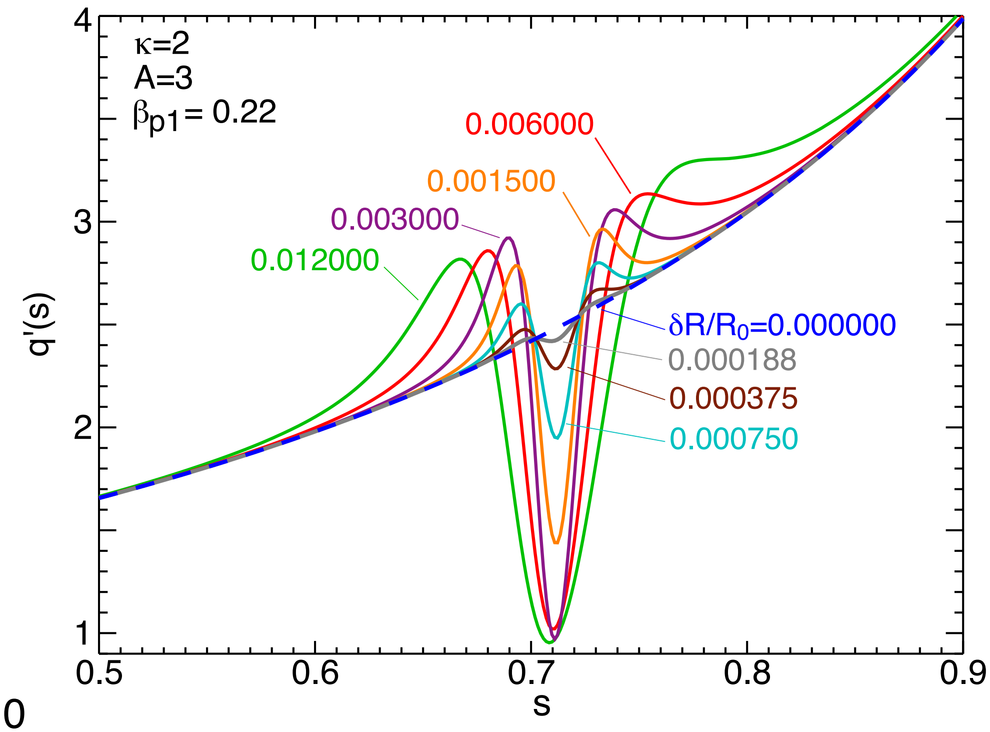

First, reexamine an LAC case where $ntor > 1$ converges. **I interpret this to mean that the calculation with $n_{tor} = 1$ will significantly *under-estimate* the mode strength.**

# The mode structure for $A = 3$, $\kappa = 2$.



# The mode scaling for $A = 3$, $\kappa = 2$ with $\delta R/R_0$ & $\beta_{p1}$.




Notice that the mode still exists at $\beta_{p1}$ =0; solutions were not obtained with $\beta_{p1} > 0.36$ although, in the large aspect ratio circle, $\beta_{p1} > 1$ was possible. Also, the saturated kink is still visible at smaller $\delta R/R_0$ ($\lesssim 2 \times 10^{-4}$) than seen in the LAC.

# Comparison to Experiment (1).

- The VMEC solutions presented here are believed to be accurate; *i.e.*, $\nabla B = 0$ & $\nabla p = J \times B$.
- Nevertheless, they have a character that is not aesthetically pleasing with dependences on $n_s$ and $n_{tor}$.
- A reason they were pursued is that they may help explain experimental results.
- In particular, we examine results from Textor and DIII–D .
  - Textor reports $q_0 \gtrsim \frac{2}{3}$ and $q_0$ only varies by about 0.06 during a sawtooth cycle, *never* reaching unity.
  - DIII-D reports that $q_0$ *always* returns to unity when the sawtooth crashes.

# Comparison to Experiment (2) – TEXTOR.

Below is shown part of Fig. 4 from [Soltwisch, *et.al.* , 11th IAEA FEC (1986, Kyoto) "Sawtooth modulation of the poloidal field in TEXTOR under ohmic heating conditions."

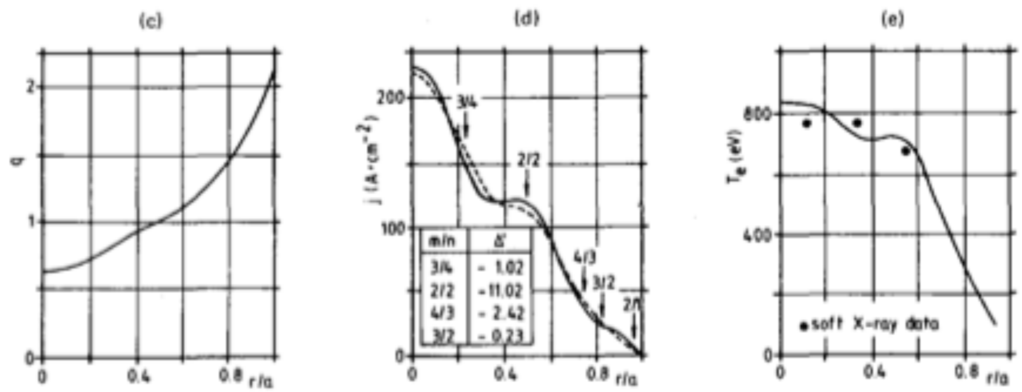
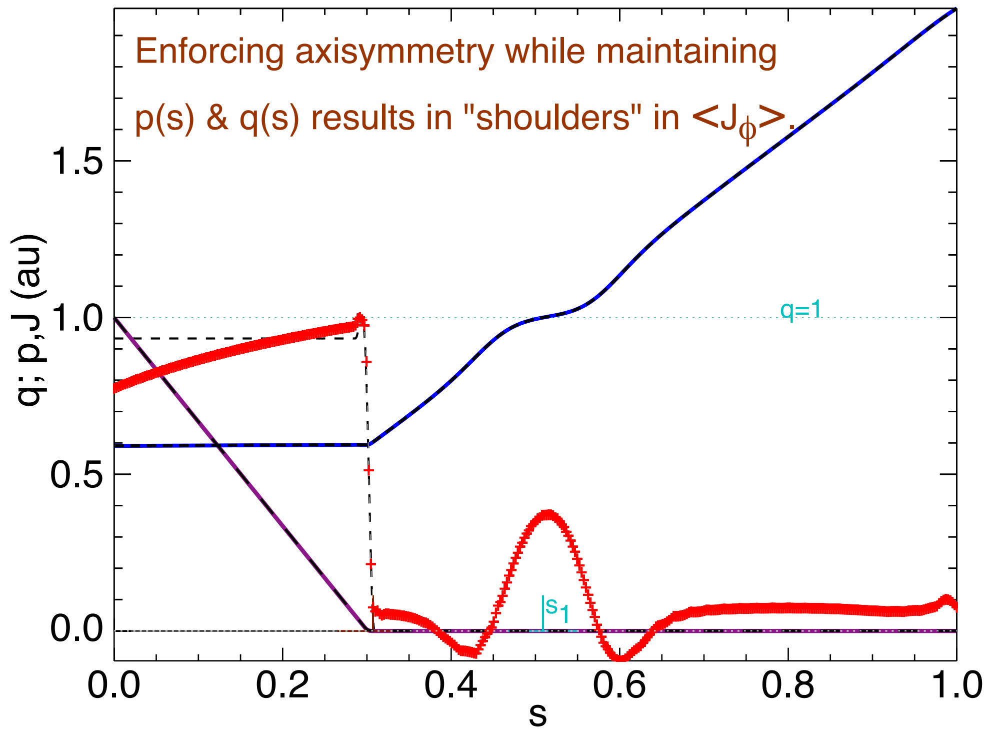



*FIG. 4. Low-q discharge (I = 485 kA, $B_T$ = 1.57 T, q(a) = 2.1) suddenly developing a large m = 1 mode after about 500 ms of stationary behaviour (frame (b) showing the fluctuations of line-integrated electron densities at the chord positions indicated in frame (a)). Profiles about 25 ms before this event: (a) electron density, (c) safety factor, (d) current density, showing a pronounced shoulder at the q = 1 radius, and (e) electron temperature calculated from the j-profile using neoclassical resistivity with $U_\ell$ = 1.50 V and $Z_{eff}$ = 2 (dots represent average temperatures in the time interval 400–800 ms measured by soft X-ray pulse height analysis).*

The important point, in the context of this work, is that an axisymmetric equilibrium reconstruction requires *shoulders* on $J_\phi$ that are centered at q=1. It is unclear how such a current profile would be maintained in an Ohmc plasma.

# Comparison to Experiment (3) – shoulders.

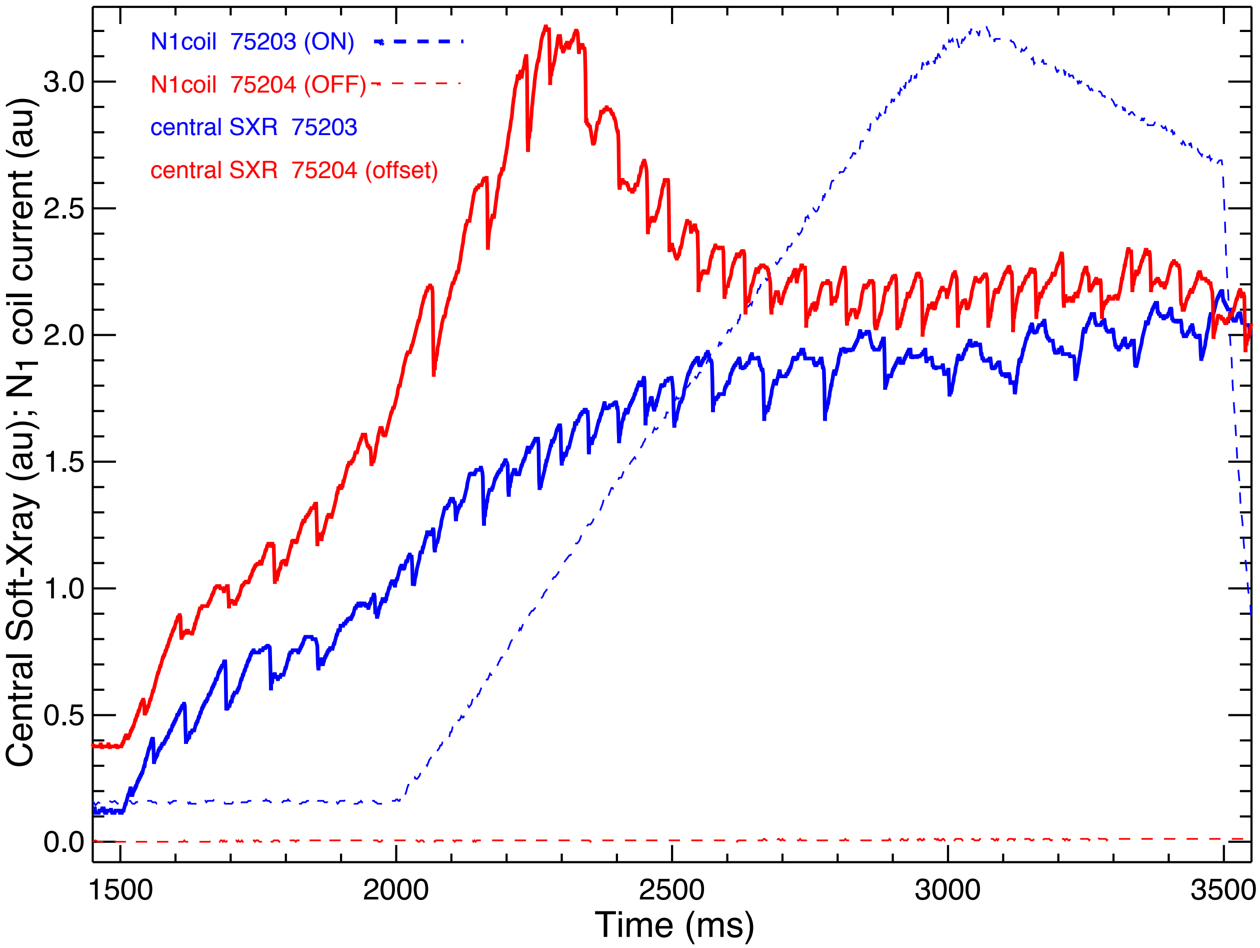
Were we to take our solution from the large aspect ratio circle and, instead of using $p(s)$ and $J_\phi(s)$ as driving functions with 3D geometry, we insist on axisymmetry while using $p(s)$ and $q(s)$ , we would obtain "shoulders", as shown in the figure below. The existence of an n=1 error field that results in a saturated kink seems to me more plausible than structure on the toroidal current profile.



# Comparison to Experiment (4)

- We note that numerous other experiments (Tokapole, JET, TFTR, PBX-M) have also reported that $q_0$ remains below unity throughout the sawtooth cycle.
- Linear stability for the internal kink was explored for the case of the TEXTOR "shoulders" in [Lutjens, *et.al.,* Nucl. Fusion **32**(1992)1625.] (as well as the destabilizing effect of elongation). They found that placing the current peak shoulders at the q=1radius was quite destabilizing, as compared to a current shoulder (reduced shear) well inside of $r_{q=1}$. That is, the profiles depicted in the previous figure have a very low critical $\beta_{p1}$ .
- DIII-D reported [Snider, *et.al.,* Nucl. Fusion**34**(1994)483.] changes in sawtooth behavior when the "N1-coil" was activated. (The N1-coil was a crude predecessor to the C-coils.) DIII-D was developing error field correction techniques well before it had the capability for q-profile measurement.
- It is likely that DIII-D, sees q=1 returning to unity because of insufficient *m/n=1/1* boundary perturbation.

# The application of a nonaxisymmetric field alters sawtooth behavior on DIII-D.




SXR data from the 1992 DIII-D experiment.
SXR with/without N1-coil displaced for clarity.

- The N1 perturbing field alters the sawtooth waveform.
- With the N1-coil on, the distinct sawtooth character disappears.

# Some Conclusions

- **3D equilibrium calculations with VMEC suggest the existence of an equilibrium state with $q_0 << 1$ and a q=1 surface well-inside the plasma.**
  - Convergence of the equilibrium calculations was difficult, with results depending on VMEC run parameters
  - Even for the large aspect ratio circle, a convergence study was not possible; solution does not converge at sufficiently dense radial grid.
  - Results at $A \sim 3$ or $\kappa > 1$ or $\delta \neq 0$ are still less reliable ($n_{tor} = 1$).
- The result offers a plausible explanation for the observed phenomenology, namely, that **most tokamaks with measurement capability show that $q_0$ does not return to unity at the sawtooth crash is likely due to small n=1 error fields**.
- It seems a more likely explanation of TEXTOR results than "shoulders" on the current profile.
- Additionally, there is evidence from DIII–D that altering the error field compensation alters sawtooth behavior.
- Given the fragility of the equilibrium calculations, it is likely that further insight will come from experiment; not additional VMEC calculations. (SIESTA was also tried by S.P. Hirshman and found to be less stable than VMEC for this class of equilibria.)

# Sawtooth Control?

- The desirability of control of the sawtooth amplitude in ITER is well known: avoiding NTM's, avoiding large power transients on the divertor plates, etc.
- Error-field correction coils (designed into ITER) are used to avoid locked-modes. In normal operation the error-field correction can be turned off after the $I_p$ ramp phase. These coils are big window-panes, far from the plasma. They are excellently positioned for producing an $m/n=1/1$ field.
- Learning how to make the transient of the sawtooth crash less severe with an error-field will require experimental effort – preferably on more than a single tokamak.
- This seems to me to be more likely to provide robust sawtooth control than the various localized wave heating/current-drive schemes that have been proposed.
- This is unlikely to be expensive. It is merely changing the spectrum of the n=1 field being applied by existing coils.

# Other Possible Applications – high $\ell_i$

The possility of sustained operation at $q_0 < 1$ opens the possibility of new tokamak operating scenarios.

- The results provide a plausible basis for operation with $q_0 < 1 \sim \frac{2}{3}$, as is seen on TEXTOR.
- High $\ell_i$ would seem more feasible with $q_0 \gtrsim \frac{2}{3}$.
- Ellipicity is strongly destablilizing to the internal kink [Troyon & Turnbull, Nucl Fusion **29**(1989)1887.
- In previous work [Lazarus *et.al.*, Phys.Plasmas **14**(2007)055701.] we found that an elliptical shape ($\delta = 0$), as compared to a bean shape, had large $T_i$ sawteeth and small $T_e$ sawteeth. In further investigation with central ECH it appeared that the nature of electron transport inside q=1 was perhaps parallel, in that a $\nabla T_e$ could not be produced, while $T_i$ transport was about the neoclassical level.

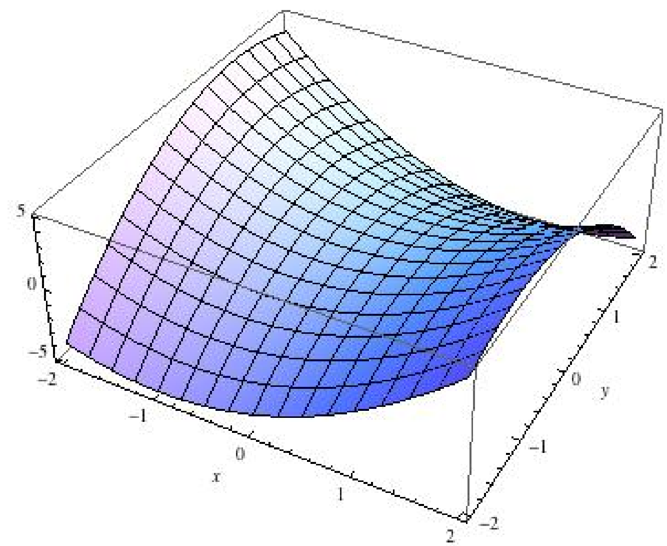
These all might combine in a fortuitous manner to produce a hot-ion mode, even with $\alpha$-heating, with great ion confinement and exceptionally poor electron confinement. As $\ell_i$ is high, $\beta_n$ can be high and the beta wasted on electrons in the core is minimized.

# Is the G-S Equation Adequate in Tokamaks?

- In some notable work [1,2] equilibria have been found that have **axisymmetric boundaries** but the **minimum energy state is <u>not</u> axisymmetric**.
- The work presented here is in a similar vein; the internal 3D distortion is large compared to the boundary distortion from 2D.

[1] Garabedian, PNAS **103** (2006) 19232.
[2] Cooper, *et.al.*, PRL **105** (2010) 035003.

**Saddle Point?**




If our equilibrium is limited to a trajectory (Grad-Shafranov equation.) that is a saddle point, when a nearby nonaxisymmetric minimum exists, stability calculations will merely demonstrate that our equilibrium was not the minimum energy state.